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# Markov Decision Processes and ARIMA models to analyze and predict Ice Hockey player's performance

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**Abstract**

In this thesis, player's performance on ice hockey is modelled to create new metrics by match and season for players which aim is to take management about them. AD-trees have been used to summarize ice hockey matches using state variables, which combines context and action variables to estimate the impact of each action under that specific state using Markov Decision Processes. With that, an impact measure has been described and four player metrics have been derived by match for regular seasons 2007-2008 and 2008-2009. General analysis have been performed for these metrics and ARIMA models have been used to analyze and predict players performance. The best prediction achieved in the modelling is the mean of the previous matches. The combination of several metrics including the ones created in this paper could be combined into quantile analysis on player's performance using salary ranges to indicate whether a player is worth hiring/maintaining/firing.

# Acknowledgments

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Additionally, I would like to express the deepest appreciation to my opponent **CHAGE-FORNAME** for reviewing my thesis in depth and giving valuable feedback.

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# 1 Introduction

## 1.1 Motivation

Sport analytics has been traditionally used to understand the relevant features of a sport for a player to succeed as well as to summarize the most important information on matches and players using statistical Key Performance Indicators (KPI). Evaluating performance and characteristics of elite athletes [6] or assessing important metrics by sport categories (e.g. net and wall games, invasion games, and striking and fielding games) [9] have tried to give better understanding on sport dynamics.

The usage of video-recording games, the increase in computer power and the rise of machine learning techniques have enabled to evaluate sports in a way impossible before. The evaluation of traditional and new metrics can provide insights to further understanding the game dynamics of a particular sport (e.g. core strategies, effects of particular events on a sequence of events).

In a sport as ice hockey, it is crucial for teams to understand well game dynamics, players' performance and successful scoring strategies to develop and improve their playing performance. This may result, with good management, to more audience in matches, better sponsors, more money to fulfill and increase the objectives of the team.

Markov Decision Processes (MDP), in the view of Reinforcement Learning, provides a general framework to deal with decision making under particular events. With that, one can not only learn which are the best actions to take under a particular state, but also being able to see through historical data which are the most useful/successful strategies to win a match. Also, the creation of more complex valuation metrics for teams and players' valuation (e.g. understanding which players play best/worst together) can be of useful insight for hiring/firing purposes, even for gamblers to bet with meaningful knowledge.

## 1.2 Aim

The underlying purpose of this project is to analyze the usage of MPDs on players and events' evaluation using the NHL data used in Routley and Schulte's article [20].

## 1.3 Research questions

The research questions that this paper addresses are:

1. Can a MDP/RL be used to evaluate actions under certain time-series events ?
2. How can I store time series data for the usage of a MDP?
3. Can I predict player's performance based on their performances in the previous matches ?
4. Is there a way to use MDPs to create a metric that evaluates players for hiring/maintaining/firing purposes?

## 1.4 Limitations

The scope of this project is assessing the usefulness of the Markov Decision Process model to provide new meaningful insights for ice hockey for the NHL dataset. The project does not try to improve or optimize the model nor the solving algorithms but rather study the usage of the MDP in ice hockey as well as Time Series Modelling for the impacts derived from MDPs using ARIMA models.

## 1.5 Literature Review

Markov Decision Processes (MDP) is a framework classically used for taking sequential decisions under uncertainty in a perfectly observable state. It was developed during the 1950s by Bellman [3], and it is currently used in several fields such as robotics, economics, sports, medicine or manufacturing.

In order to learn which actions are the best ones (i.e. they give higher rewards), the sequence of events with their associated actions need to be explored. Littman [13] explained the theory of a MDP in the view of the Q-learning algorithm from the reinforcement learning theory to find transition probabilities to states and best strategies on the framework of two agents with opposed goals while being in a common environment [8].

There is a lot of research made on player performance for different sports. Some related papers on other sports are fittting a MDP on rugby matches to evaluate which sequences of events are more prone to scores to find the most effective rugby strategies [2] or evaluating the EPV (Expected Point Value) per possession on basketball taking into account players configuration in the field and the ball position [4].

In ice hockey, one of the main metric used across years is the +/- metric: the difference on scoring between your team and the opponent relative to the current players on the ice. In other words, if you are a player on the ice rink and your team scores, you will be awarded with a +1. On the other hand, if you are on the ice ring and the opponent team scores, you as a player will get -1 point. This non-marginal metric has been tried to be improved in several papers [17, 16] [7] by using regressions (eg. logistic, ridge regression) trying to assess which player has had more impact marginalizing other players' valuation.

Schuckers [21] gave a value to all events performed as a measure to how nearer the possibility of a goal was in the following 20 seconds. Routley and Schulte [20] used the NHL dataset without a window event taking into account the following context: the Zone an event was happening, the Manpower Differential (MD) across teams, the Period in which the match was happening and the Goal Difference (GD) between teams. With this, they created a MDP process that gives players a valuation based on the improved probability of an event to scoring vs receiving a score, extending it lately to a better dataset taking into account more actions and better locations on the field [22], [23]. Also, Liu and Schulte [14] used LSTM (Deep Neural Networks) to calculate player's performance on the NHL database using more context variables such as Time Remain of the match or whether and action was a success.

MDPs have also been used to calculate the probability of winning a match given that an action is performed by a player under certain context variables (Goal and Manpower differential) [10]. Moreover, the impact of players goals under context has been calculated to know which players are more valuable [18]. Also, the best pair of players based on goal performance for each team has been done taking into account positions of players and the time these players has played together each match [15].

# 2 Theory

This thesis deals with discrete variables and states. Therefore, the theory described in this project is focused on discrete data, even if a large part of it is common with continuous data. The notation as well as the explanation used in the article follows the notation and the structure of the MIT book *Decision Making Under Uncertainty: Theory and Application (2015)* [11].

## 2.1 Markov Decision Processes (MDP)

Markov Decision Processes (MDPs) provide a framework for sequential decision making in situations where the outcome of a given action is non-deterministic. More specifically, an agent chooses action $a_t$ at time t observing a state $s_t$ based on an associated Reward $R(s_t, a_t)$. This state evolves according to a defined state transition function $T(s'|s_t, a_t)$ which depends on the action $a_t$ taken at that particular state. The system modeled is assumed to follow the Markov property, meaning that its state at time $t$ depends only on its state at time $t-1$.

Illustration 2.1 represents graphically a Markov Decision Process with States (S), Actions (A), Rewards (R) for 3 general sequential processes.

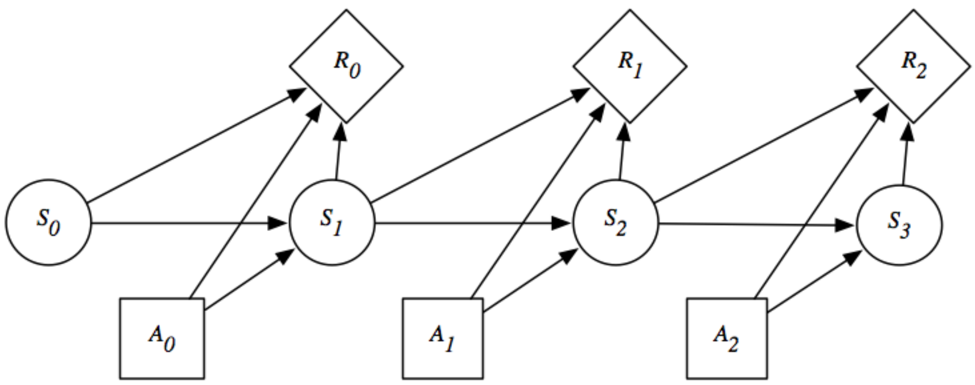



Figure 2.1: Graphical illustration of a Markov Decision Process with States (S), Actions (A), Rewards (R) for 3 general sequential processes(from *Artificial Intelligence: Foundations of Computational Agents* [19], figure 9.12).

In other words, a MPD can be explained by the tuple (S,A,T,R):

- S, the set of states
- A, the set of actions the agent can engage in
- T: S $\mapsto \prod(S)$, the transition function
- R: S x A $\mapsto \mathbb{R}$, the reward function dependent on the action taken at each state

**MDP example**

A clear example of a MDP is the modelling of an ice hockey player's decision in control of the puck in the offensive zone of the field: The 4-tuple presented before <(S,A,R,T,)> would be used to describe what each variable would represent:

- States, representing the set of conditions a specific player might be in under which she/he performs an action (e.g. player shots the puck in the offensive zone (Zone), winning 1-0 (score of the match), same number of players per team (manpower differential ) and first period (period the match is in)).
- Actions the player can perform, which might be passing, shot, dribble.
- Rewards: The player receives a reward for engaging into an action that improves statistically the probability of scoring against the probability of performing an action that reduces the probability of score or increases the probability of receiving a goal. As an example, the reward might be set to 1 for scoring a goal and -1 for receiving a goal.
- Transition function, representing the set of probabilities of the following state given the action performed in a particular current state (e.g. related to the example set in States: it describes, among others, the probability of scoring given a shot in the light of the context the player is in, as well as the possibility of scoring given that you pass the puck to a player that is alone in the offensive zone)

**The Utility and Reward function**

The choice of an action at each point in time has an associated reward r. In a finite horizon problem (finite number of states, i.e. n states), the total utility is defined by the sum of associated rewards to the action performed:

$$\sum_{t=0}^{n-1} r_t \tag{2.1}$$

The problem of this simple approach is that for an infinite time horizon (i.e. $t = \infty$) the reward function could reach the value $\infty$, so that no specific action would be preferred under this case. The simplest way to solve it is to add a discount factor $\gamma^t$ (which can take values between 0 and 1) that is affected by time t. With that, a nearer reward would be preferable in case of equal rewards.

$$\sum_{t=0}^{n-1} \gamma^t r_t \tag{2.2}$$

That is one way to deal with the infinite horizon problem, but there are other ways such as calculating the average reward:

$$\lim_{x \to \infty} \frac{1}{n} \sum_{t=0}^{n-1} r_t \tag{2.3}$$

### Defining policies and utilities

A policy that determines the choice of an action at a point in time t is expressed as $\pi_t(s)$. When transitions and rewards are stationary (i.e. they are equal across time), a policy $\pi$ is only dependent on the current state s at that point in time (i.e. recall the Markov assumption from the beginning of the section: the current state s defines all previous states). The optimal policy $\pi^*$ at state s is defined as the policy that maximizes the expected utility:

$$\pi^*(s) = \arg\max_{\pi} U^{\pi}(s) \tag{2.4}$$

The optimal policy of a MDP is defined by the sequence of actions that maximize the Expected Utility of the whole MDP. That is the sum of the expected reward at each time step, or equivalently, the sum of expected rewards associated to n actions and states. The calculation of the total policy for a certain number of actions is called policy evaluation.

$$U_t^{\pi}(s) = R(s, \pi(s)) + \gamma \sum_{s'} T(s'|s, \pi(s)) U_{t-1}^{\pi}(s') \tag{2.5}$$

For an infinite time horizon with discounted rewards (i.e. with discount factor $\gamma$), equation 2.5 shows that the expected Utility of a sequence of policies is explained by the reward got at the current state given the policy $\pi$ chosen at that state plus the sum of all future states defined by the Transition function (which defines with probability the exact future state given the current policy and state) and its associated Utility function at that future state multiplied by the discount factor $\gamma$.

### Optimization of MDP

If the transition probabilities and the Rewards for each action are known, one can calculate the optimal policy $\pi^*$ as follows:

- **Policy evaluation:** For any given policy $\pi_l$, the expected utility of the policy $U^{\pi_k}$ is calculated
- **Policy Improvement**: Calculating a new policy using the pseudocode shown below in *Algorithm 1.*

The pseudo-code for the whole algorithm, called policy iteration, can be found below: Each iteration on the policy iteration algorithm is computationally expensive. Thus, another similar algorithm that is used is the value iteration algorithm. For an associated time horizon n, the optimal utility (only for time horizon equal to n) is carried out following equation 2.6

$$U_n(s) = \max_a (R(s, a) + \sum_{s'} T(s'|s, a) U_{t-1}(s')) \tag{2.6}$$

In the case of an infinite time horizon, the optimal utility is calculated using equation 2.7

$$U*(s) = \max_a (R(s, a) + \gamma \sum_{s'} T(s'|s, a) U^*(s')) \tag{2.7}$$

Then, the optimal value function $\pi(s)$ can be extracted from the optimal policy $U_n^*(s)$ by just taking the actions that optimize the utility at each state. That is :

$$\pi(s) = \arg\max_a (R(s, a) + \gamma \sum_{s'} T(s'|s, a) U^*(s')) \tag{2.8}$$

Then value iteration algorithm uses equation 2.7 to estimate and update $U^*$ (being v(s) in the pseudocode), proceeding as follows:

**Algorithm 1** Policy iteration (using iterative policy evaluation from chapter 4.3 in Sutton and Barto's book [24]

1.Initialization
$v(s) \in$ and $\pi(s) \in A(s)$ arbitrarily for all s ∈ S
2.Policy Evaluation
**repeat**
    $\Delta \leftarrow 0$
    **for** each s ∈ S **do**
        temp $\leftarrow v(s)$
        $v(s) \leftarrow \sum_{s',r} p(s',r|s,\pi(s))[r+\gamma v(s')]$ which is $U_t^\pi$ from 2.5 with infinite time horizon
        $\Delta \leftarrow$ max($\Delta$, | temp-v(s) | )
    **end for**
**until** $\Delta < \theta$ small positive value
3.Policy Improvement $policy_s table \leftarrow$ True
**for** each s ∈ S **do**
    old-action $\leftarrow \pi(s)$
    $\pi(s) \leftarrow \arg\max_a \sum_{s',r} p(s',r|s,a)[r+\gamma v(s')]$
    **if** $temp \neq \pi(s)$ **then**
        $policy - stable \leftarrow false$
    **end if**
**end for**
**if** $policy - stable$ **then**
    return stop and return v and $\pi$
**else**
    go to Policy Evaluation (to 2)
**end if**

**Algorithm 2** Value iteration (from chapter 4.4 from Sutton and Barto's book [24]

Initialize array v arbitrarily (e.g. $v(s) = 0$ for all s ∈ $S^+$)
**repeat**
    $\Delta \leftarrow 0$
    **for** each s ∈ S **do**
        temp $\leftarrow v(s)$
        $v(s) \leftarrow \max_a \sum_{s',r} p(s',r|s,a) * [r+\gamma v(s')]$ which is 2.7
        $\Delta \leftarrow$ arg max($\Delta$, | temp-v(s) | )
    **end for**
**until** $\Delta < \theta$ small positive value
Output deterministic policy $\pi$ such that
$\pi(s) \leftarrow \arg\max_a \sum_{s',r} p(s',r|s,a)[r+\gamma v(s')]$

In order to speed up convergence on algorithm 2, convergence is set to be reached when $|(U_k - U_{k-1})| < \delta$ being $\delta$ a parameter of the programmer's choice known as the Bellman residual.

Both algorithms will return the global optimal value function. On a rule of thumb though, policy iteration will converge in fewer iterations but is more computationally complex than value iteration. The difference between $S^+$ and S in the pseudocode relates to the sequential processes of a MDP. S denotes the set of all non-terminal states in a MDP, which are all but the last one. $S^+$ denotes all non-terminal states plus the terminal state. Value iteration is able to combine in each iteration policy evaluation and policy improvement, using the max operator as the main difference for it.

### Learning a MDP

MDP provides a framework to deal with sequences of states: it enables to learn the transition probabilities between states given actions taken at each point in time. With that, one can decide which events happening are meant to have a reward, and thus evaluating which events are more significant for a specific event to happen.

The transition probabilities are explained following Littman's notation [13]:

- Occ($s$) is the number of times state $s$ occurs in the dataset under specific context variables
- Occ($s$, $s'$) is the number of times state $s$ is followed by state $s'$.
- The transition probability $T(s'|s, a)$ then is learned via

$$\frac{Occ(s, s')}{Occ(s)}$$

### Parallelism with Reinforcement Learning

This section is merely informative to understand the MDP and the value iteration algorithm from another perspective.

The main objective of reinforcement learning is to make an agent engage in optimal actions in a space. This is learned through performing actions and seeing its results. Those actions which results are understood as being the optimal ones are given positive rewards whereas those with bad results are given a penalty. As an example, we might say that the main objective in hockey is scoring a goal. Thus, whatever action that goes towards the direction of scoring is given a reward, otherwise, it is given a penalty.

However, at the beginning, we do not know which actions in the space give positive/negative rewards. Thus, the Q-table is used to store and see which actions are indeed positive/negative. In other terms, the Q-table is the expected maximum future reward for each action. In order to do so, the Q-learning algorithm is used, presented as a flowchart below:

Explained in a little more detail, the functioning is as follows:

1. Initiate Q-values from Q(s,a) arbitrarily for all pairs of (s,a).
2. While the number iterations is smaller than the maximum number of iterations stated or while there is no convergence (normally a threshold number is set which accounts for convergence when change is smaller than that number):
   a) Choose an action a in the current state s based on current Q-value estimates (s,·)
   b) Take the action chosen and see the next state s′ and the reward r
   c) Update $Q(s, a) = Q(s, a) + \epsilon[r + \gamma \max_{a'}(Q(s', a') - Q(s, a))]$

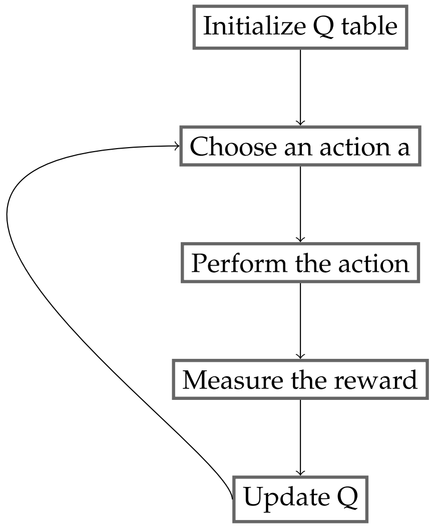



Figure 2.2: Process of the Q learning algorithm

The Q-learning process is iterative (see figure 2.2). On the first time, initial values will be set (normally zeros). Then, for as many iterations as the user specifies or until convergence, the following procedure will be carried out: an action will be chosen and performed, a reward will be given to that action based on the Q current estimates and then the Q table is updated. However, at the beginning, all actions are seen equally interesting (a matrix of zeros). Therefore, a value for $\epsilon$ must be specified. This value is set at the beginning to 1 because we do not know anything about the Q-value of an action. Progressively, as we know more about the space, this epsilon will decrease, and by drawing a random number between zero and 1, if that random number > $\epsilon$, then the best action (which we have already calculated) will be taken, else, a random action will be chosen. This allows a balance between exploitation (choosing best action with the information we have) and exploration (choosing a random action). On the 'c' part of the explanation, the updated Q(s,a) is the current Q value for that state and action (Q(s,a)), $\epsilon$ is the learning rate , R(s,a) is the reward for a specific action in that state, $\gamma$ is the discounted rate and $\max_a'(Q(s',a'))$ is the maximum new expected future reward given the new state s' and new all possible actions at that new state.

## 2.2 AD-tree

### Motivation for AD-trees on MDP

Finding the best policy at each time step (i.e. performing the best action every time with the current information), is unfeasible for MDPs with a large amount of state variables given its time constraint and its computer complexity.

Therefore, showing data in a compact way such that the transition probabilities of states and the rewards associated to each state must be easily and rapidly used for the usage of the MDP to calculate the evaluation of players.

### Introduction to the AD-tree

An AD-tree is a machine learning method developed by B. Anderson and A. Moore [1] to summarize relations of events that happen sequentially in a database. Their main purpose was establishing associative rules to extract some general rules based on support and confidence.

Following their article's notation, a literal is an attribute-value pair such as "event = shot". Then, L is all possible combinations of literals in the database. An association rule implies that $S1 \rightarrow S2$, where $S1, S2 \subset L$ , and $S1 \cap S2 = \emptyset$. S1 will refer to the first event, being the antecedent, and S2 to the conditional event, called the consequent. Then, we can understand a time series dataset as a compound of rules, each literal associated by time with the following literal. As an example, $S1 \rightarrow S2$, could be on its simplest form as S1 being event = shot and S2 being event = goal.

The support measure is a metric defined as the number of records in the database that match all the attribute-value pairs in S. In other words, it is counting how many times an association of events S1, S2 occur on the dataset, also expressed as $supp(S1 \cup S2)$. The more times an association occurs, the higher its support and the higher its significance.

The confidence measure is another metric defined as the percentage of records that matches $S1 \cup S2$ out of all records that match S1. That is, the fraction between support of the association rule $S1 \cup S2$ and the support of the antecedent S1. It is expressed as $conf(S1 \cup S2) = \frac{supp(S1 \cup S2)}{supp(S1)}$. The higher the confidence measure, the stronger and the more probable an association is, giving more importance to that specific association rule.

### The AD-tree Data Structure

An AD-tree is a summary of a database with support measures for each state. This means that all possible queries have been calculated in advanced to provide basic statistics (support measure) for each query. In other words, the support of a literal can be found next to each literal.

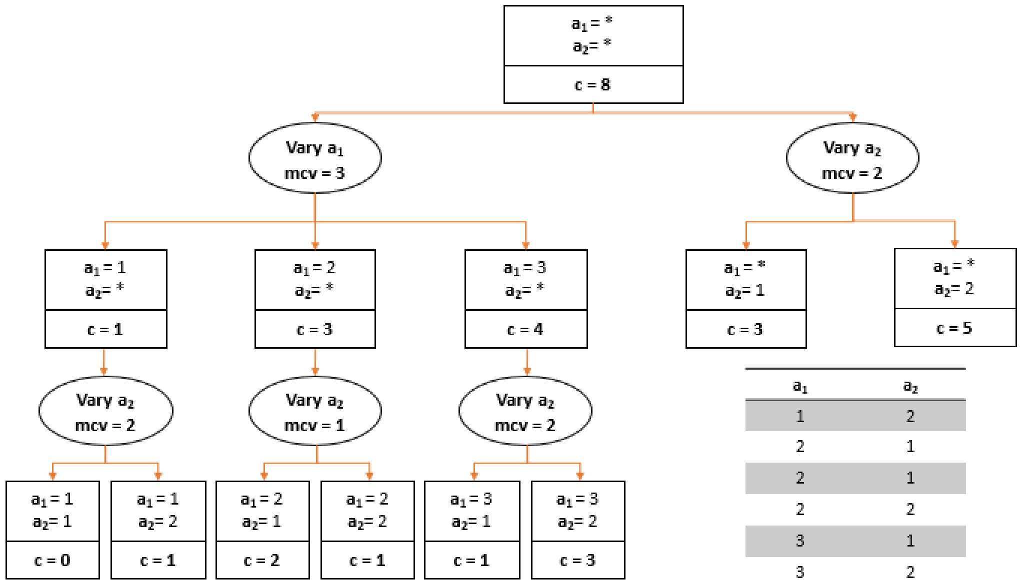
To do that, a basic structure to record all the queries is needed (a tree structure with its support). There are two main components in the AD-tree structure. An ADnode represents a query and stores the number of times that query happens. Each ADnode, (shown as rectangles in figure 2.3) has child-nodes called "vary nodes" (shown as ovals). These "vary nodes" do not store counts but group ADnodes with one only feature. It can also contain the most common value (mcv). The "vary nodes" child of an ADnode has one child for each value $v_j$ for feature $a_i$. These grandchildren ADnodes specialize the grandparent's query by storing the counts of the specific queries.

## 2.3 Time series

The time series explained in this thesis only serve the purpose of understanding the thesis work. Therefore, theory and models will not be explained in depth. To better understand the models that are going to be presented in this paper, *Time Series Analysis With Applications in R*, [5] is a book that provides a lot of practical examples using R's software. Also, the notation used in this part of the thesis follows the book's notation.

### Motivation for Time Series modelling

The main objective behind time series modelling is being able to predict certain behaviours of variables through time. Given that the objective of the thesis is trying to predict player's performance valuation based on previous performances, time series modelling can be quite handy due to the ability of modelling different types of trends from previous observations and then forecast the following performances. It is often alleged that forecasting player's performance for match 26 may not depend on matches 1 and 2, but on the most recent previous matches (e.g. the last 5 matches). This could be taken as an evaluation of how good that player is in that specific point in time, therefore used to predict their next match performance.




| $a_1$ | $a_2$ |
|---|---|
| 1 | 2 |
| 2 | 1 |
| 2 | 1 |
| 2 | 2 |
| 3 | 1 |
| 3 | 2 |
| 3 | 2 |
| 3 | 2 |

Figure 2.3: Graphical representation of an ADtree based on the dataset shown in the same figure. ADnodes are shown as rectanges. Vary nodes are shown as ovals.

### The concept of Stationarity

In order to evaluate the structure of a stochastic process, it is assumed that the laws in which all stochastic processes occur do not change over time. This assumption is called stationarity. The 3 main laws that define whether a process is stationary (referring in this thesis for weak stationarity) are:

- The mean function is constant over time: $\mathbf{E}(Y_t) = \mathbf{E}(Y_{t-k})$
- The covariance between $Y_t$ and $Y_s$ being s<t depends only on time difference ∣t-s∣ but not on actual times of s and t. Therefore: $Cov(Y_t, Y_s) = Cov(Y_{0,|t-s|})$
- The variance is : $Var(Y_t) < \infty$

The most typical stationary process is the white noise process, which is a sequence of a independent and identically distributed (iid) random variables $e_t$.

### ARMA Models

Inside Stationarity, there are two main processes. An autoregressive process (AR) is a regression process where the current observation is a combination of the p previous observations plus some difference explained as $e_t$. This model was proposed by Yule in 1926:

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + ... + \phi_p Y_{t-p} + e_t$$

A moving average process (MA) is a another regression process based on q previous white-noises with some specific weights. This model was firstly introduced by Slutsky in 1927:

$$Y_t = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - ... - \theta_q e_{t-q}$$

### ARIMA Models

In real life, though, time series data is not often stationary. Nevertheless, there exist techniques which might help us dealing with it.

Differencing is one of the most used methods for making data stationary, which is calculating the difference in Y for lags. As an example, a first order difference would be $\nabla Y_t = Y_t - Y_{t-1}$. Another method that is used together with differencing is taking logarithms of the data to avoid modelling exponential growth in the data set.

An Integrated autoregressive moving average models (ARIMA), introduced by Box and Jenkins in 1976, is a regression process that takes into account AR, differencing and MA processes to model time series. Considering $W_t$ as $Y_t - Y_{t-1}$, an ARIMA process can be defined as

$$W_t = \phi_1 W_{t-1} + \phi_2 W_{t-2} + ... + \phi_p W_{t-p} + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - ... - \theta_q e_{t-q} \tag{2.9}$$

which can be expressed as :

$$\begin{aligned} Y_t - Y_{t-1} = \phi_1(Y_{t-1} - Y_{t-2}) + \phi_2(Y_{t-2} - Y_{t-3}) + ... + \phi_p(Y_{t-p} - Y_{t-p-1}) \\ + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - ... - \theta_q e_{t-q} \end{aligned} \tag{2.10}$$

and therefore

$$\begin{aligned} Y_t = (1+\phi_1)Y_{t-1} + (\phi_2 - \phi_1)Y_{t-2} + ... + (\phi_p - \phi_{p-1})Y_{t-p} + \theta_p Y_{t-p-1} \\ + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - ... - \theta_q e_{t-q} \end{aligned} \tag{2.11}$$

which looks like an ARMA process. By using ARIMA processes, AR and MA processes are taken into account on one single process making it really interesting for its application.

# 3 Domain: Hockey Rules and Hockey Data

## 3.1 Hockey Rules

Ice hockey, documented for the first time in 1773 in the book *Sports and Pastimes, to Which Are Prefixed, Memoirs of the Author: Including a New Mode of Infant Education*, by Richard Johnson, is a two-team team sport played on an ice rink with 5 players plus a goalkeeper for each team. The objective is to score more goals than the opponent in 3 periods of 20 minutes. On regular season, if the 3rd period ends up with a draw, there is an extra period of 5 minutes where the first to score immediately wins. In case there is no score, on a regular season, there will be shootouts for both teams. In playoffs, new periods of 20 minutes will continue until one team scores.

After a penalty occurs, the player who occasioned it will sit in the penalty box for a certain amount of minutes depending on the severity of the fault. The penalized team will be short-handed, creating a Manpower Differential (MD) between the teams. A team has *powerplay* when their team has more players on the field than their opponent, resulting into manpower advantage. A *powerplay* goal means that a team scored with positive *Manpower differential*. The opposite is called a *shorthanded goal*.

## 3.2 Hockey Data

The datasets this project will use are the NHL dataset from Routley and Schulte's paper in 2015 [20], scrapped from *www.nhl.com*.

### NHL Hockey Dataset

The NHL dataset is a relational database with 2,827,467 play-by-play events recorded by the NHL, containing complete 2007-2014 seasons (i.e. regular season and playoff games) and the first 512 games of the 2014-2015 regular season. General information about the dataset is displayed in Table 3.1a.

As shown in table 3.1b, each event can either be an Action Event or Start/End event. An Action Event represents those events performed by players, whereas Start/End events are events that stop time not performed by players. Each time event is continuous of another time event. That means we have a time series dataset. Additionally, each event performed by

| | |
|---|---|
| **Nº of events** | 2,827,467 |
| **Nº sequences of events** | 590,924 |
| **Nº of games** | 9,220 |
| **Nº of players** | 1,951 |
| **Nº of teams** | 32 |

(a) General Information

| **Action Events** | **Start/End Events** |
|---|---|
| Faceoff | Period Starts |
| Missed Shot | Period Ends |
| Shot | Early Intermission Starts |
| Hit | Early Intermission Ends |
| Blocked Shot | Stoppage |
| Giveaway | Penalty |
| Goal | Game End |
| Takeaway | Shootout completed |
| | Game off |

(b) Types of events

Figure 3.1: The NHL Dataset

a player (an action) has an associated Zone in which the event occurs (Home, Neutral, Away) and which team performs that event (Home or Away team).

# 4 Method

## 4.1 The AD-tree creation

### Definition of Context variables

In sports, not only is it important to understand the actions players take, but also under which circumstances that player is deciding and performing that specific action. In ice hockey, the probability between scoring a goal given that one team has more players in the field might be higher than when the same team has less. Probably, a reckless pass with fewer players in the rink might imply a higher risk of receiving a takeaway and therefore receiving a score. Also, it might not be equal the probability of scoring an extra goal in period 3 when you are losing by 1 goal than when you are winning by 3.

Thus, taking into consideration the context in which events are performed might be crucial to understand a game itself and evaluate which are the players that play better. With that, the choice of context variables that are important for the game are *Goal Differential*, *Manpower Differential* and *Period* [20]. *Goal Differential* (GD) is defined by how many goals the Home team is winning by. $GD = (HomeGoals - AwayGoals)$. A GD of 1 means that the Home team wins by 1 goal whereas a GD of -1 stands for the Away team winning by 1. *Manpower Differential* is defined as the difference of players in the rink between both teams following the exact same logic as GD. $MD = HomePlayersinRink - AwayPlayersinRink$. Those context variables are the following ones displayed in the table 4.1

The table is showing the range in which those context variable might happen. As a summary of them, that means our events might happen under 17x7x7 combinations of context variables. That results in a total of 833 context states where each action could happen.

| **Notation** | **Variable Name** | **Range** |
|---|---|---|
| GD | Goal Differential | [-8,8] |
| MD | Manpower Differential | [-3,3] |
| P | Period | [1,7] |

Table 4.1: Context Variables associated to each event

### Definition of a play sequence

As explained in the dataset information section 3.2, there are 8 main actions a player can perform (see table 3.1b). Those actions have associated a Zone Z in which the action is performed (Offensive, Neutral or Defensive) subjected to the teams T performing the action (Home, Away). A playing sequence is a series of action events. Each play sequence begins with a start marker. If the play sequence is complete, it will end with an end marker. However, it may be that there are sequences that are not complete. Start/End markers are those events in the right column of table 3.1b, adding faceoffs and shots as starting markers and goals as ending markers. To display it clearer here, action events will be explained in the notation form of a(T,Z) [12], where T is Team and Z is Zone. Goal(Home, Offensive) is an example of the a(T,Z) structure.

An illustrative example is displayed below in figure 4.1:

| GameID | Period | Sequence Number | Event Number | Event |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | PERIOD START |
| 1 | 1 | 1 | 2 | Faceoff(Away, Neutral) |
| 1 | 1 | 1 | 3 | Hit(Home, Defensive) |
| 1 | 1 | 1 | 4 | Shot(Away, Offensive) |
| 1 | 1 | 1 | 5 | Goal(Away, Offensive) |
| 1 | 1 | 2 | 6 | Faceoff(Home, Neutral) |
| 1 | 1 | 2 | 7 | Hit(Away, Offensive) |
| 1 | 1 | 2 | 8 | Hit(Away, Offensive) |
| ... | | | | |

Figure 4.1: Illustrative example of the variables got in the time series dataset for a hockey match with format a(T,Z) of the events

### Definition a state

Following Routley and Schulte's notation, a state is denoted as s $< x, h >$ where x englobes the combination of context variables (being GD, MD and P) and h the action history (the variables in table 3.1b with Zone, and the Team Performing that action). If the sequence h is empty, the state is purely a context node. An example of this can be found in figure 4.2.

## 4.2 The AD-tree structure

In order to deal with a high dimensional space, an AD-tree has been created using a SQL Database with four tables. Each table tries to store different basic useful information for the posterior calculus of the evaluation of players using MDP.

### The Nodes table

The Nodes table is the AD-tree table with all the variables related to s$< x, h >$. Additionally, a variable NodeId is added to each Node working as a unique key for each node and an Occurrence variable, standing for how many times each state event happens in the whole dataset, or in other words, storing the support of each query (see figure 4.2).

### The Edges table

The Edges table stores the links between nodes in the dataset and its occurrence. The Variables FromId and ToId refers to the link created between two nodes (see figure 4.3a). These are the only paths considered later on as possible transitions from one node to another node

| NodeId | NodeType | NodeName | GD | MD | Period | Team | Zone | Occurrence |
|---|---|---|---|---|---|---|---|---|
| 1 | root | root | 0 | 0 | 0 | | | 590578 |
| 2 | state | state | 0 | 0 | 1 | | | 78126 |
| 3 | event | period start | 0 | 0 | 1 | | | 9081 |
| 4 | event | faceoff | 0 | 0 | 1 | away | neutral | 4417 |
| 5 | event | missed shot | 0 | 0 | 1 | away | offensive | 175 |
| 6 | event | shot | 0 | 0 | 1 | home | offensive | 17 |
| 7 | event | hit | 0 | 0 | 1 | home | offensive | 1 |
| 8 | event | blocked shot | 0 | 0 | 1 | home | defensive | 1 |
| 9 | event | stoppage | 0 | 0 | 1 | | | 1 |
| 10 | event | faceoff | 0 | 0 | 1 | away | offensive | 10595 |
| 11 | event | blocked shot | 0 | 0 | 1 | home | defensive | 1755 |
| 12 | event | giveaway | 0 | 0 | 1 | home | defensive | 48 |
| 13 | event | hit | 0 | 0 | 1 | home | defensive | 3 |
| 14 | event | hit | 0 | 0 | 1 | away | offensive | 1 |
| 15 | event | hit | 0 | 0 | 1 | home | defensive | 1 |
| 16 | event | stoppage | 0 | 0 | 1 | | | 1 |
| 17 | event | faceoff | 0 | 0 | 1 | home | neutral | 9807 |
| 18 | event | shot | 0 | 0 | 1 | home | offensive | 1085 |
| 19 | event | stoppage | 0 | 0 | 1 | | | 345 |
| 20 | event | faceoff | 0 | 0 | 1 | home | offensive | 12117 |
| 21 | event | shot | 0 | 0 | 1 | home | offensive | 2792 |
| 22 | event | stoppage | 0 | 0 | 1 | | | 894 |
| 23 | event | faceoff | 0 | 0 | 1 | away | defensive | 11674 |
| 24 | event | hit | 0 | 0 | 1 | home | defensive | 539 |
| 25 | event | shot | 0 | 0 | 1 | away | offensive | 81 |

Figure 4.2: Example of the Nodes table

| FromId | ToId | Occurrence |
|---|---|---|
| 1 | 2 | 78126 |
| 2 | 3 | 9081 |
| 3 | 4 | 4417 |
| 4 | 5 | 175 |
| 5 | 6 | 17 |
| 6 | 7 | 1 |
| 7 | 8 | 1 |
| 8 | 9 | 1 |
| 9 | 2 | 1 |
| 2 | 10 | 10595 |
| 10 | 11 | 1755 |
| 11 | 12 | 48 |
| 12 | 13 | 3 |
| 13 | 14 | 1 |
| 14 | 15 | 1 |
| 15 | 16 | 1 |
| 16 | 2 | 1 |
| 2 | 17 | 9807 |
| 17 | 18 | 1085 |
| 18 | 19 | 345 |
| 19 | 2 | 340 |
| 2 | 20 | 12117 |
| 20 | 21 | 2792 |
| 21 | 22 | 894 |
| 22 | 2 | 885 |

(a) The Edges table

| GameId | EventNumber | StartingId | EndingId |
|---|---|---|---|
| 2013020001 | 1 | 2 | 3 |
| 2013020001 | 2 | 3 | 4 |
| 2013020001 | 3 | 4 | 5 |
| 2013020001 | 4 | 5 | 6 |
| 2013020001 | 5 | 6 | 7 |
| 2013020001 | 6 | 7 | 8 |
| 2013020001 | 7 | 8 | 9 |
| 2013020001 | 8 | 2 | 10 |
| 2013020001 | 9 | 10 | 11 |
| 2013020001 | 10 | 11 | 12 |
| 2013020001 | 11 | 12 | 13 |
| 2013020001 | 12 | 13 | 14 |
| 2013020001 | 13 | 14 | 15 |
| 2013020001 | 14 | 15 | 16 |
| 2013020001 | 15 | 2 | 17 |
| 2013020001 | 16 | 17 | 18 |
| 2013020001 | 17 | 18 | 19 |
| 2013020001 | 18 | 2 | 20 |
| 2013020001 | 19 | 20 | 21 |
| 2013020001 | 20 | 21 | 22 |
| 2013020001 | 21 | 2 | 23 |
| 2013020001 | 22 | 23 | 24 |
| 2013020001 | 23 | 24 | 25 |
| 2013020001 | 24 | 25 | 26 |
| 2013020001 | 25 | 2 | 10 |

(b) The Node information table

| NodeId | RewardGoal |
|---|---|
| 1 | 0 |
| 2 | 0 |
| 3 | 0 |
| 4 | 0 |
| 5 | 0 |
| 6 | 0 |
| 7 | 0 |
| 8 | 0 |
| 9 | 0 |
| 10 | 0 |
| 11 | 0 |
| 12 | 0 |
| 13 | 0 |
| 14 | 0 |
| 15 | 0 |
| 16 | 0 |
| 17 | 0 |
| 18 | 0 |
| 19 | 0 |
| 20 | 0 |
| 21 | 0 |
| 22 | 0 |
| 23 | 0 |
| 24 | 0 |
| 25 | 0 |

(c) The Reward table

Figure 4.3: Examples of the AD-tree tables

in the MDP. Non-existent pair of sequences are taken as impossible to occur and therefore with transition probability equal to 0.

#### The Node Information table

The Node information table stores the original pair of unique keys from the original table (GameId, EventNumber) with the matching NodeId called EndingId (see figure 4.3b). Each NodeId is unique, but different (GameId, EventNumber) tuples can have the same NodeId.

#### The Reward table

The Reward table stores the reward values associated to each node id. The values the RewardGoal variable takes are 0, 1 or -1 (see figure 4.3c). Since the main objective of Ice Hockey is scoring goals to win a match, the Reward associated for scoring a Goal is determined to be +1. The reward for receiving a goal then is -1, and 0 otherwise. In other papers such as in Routley and Shulte, the rewards given are 0 and 1 for Goals. By giving -1 to receiving a goal, certain actions under which there are a lot of possible events which can finish in other teams scoring will have a lower q-value, which could be seen as well as a measure of risk under certain situations.

## 4.3 The Markov Decision Process

### The reward's value per state

The reward given here differs from Routleys paper and it is defined as follows: For any state with a play sequence starting with Home/Away and finishing with that same team scoring Goal Home/away, they will get a 1 $R_{Away}(GoalAway) = 1$ and $R_{Home}(GoalHome) = 1$. However the state of receiving a goal by the other team $R_{Home}(GoalAway) = -1$ and $R_{Away}(GoalHome) = -1$. Alternatively, the reward is set to be 0.

In order to set that in the rewards table, everything is set for the Home Team so Home Goals are +1 and Away Goals -1. Only the sign is necessary to be changed to calculate the q-values for Away team scoring.

Once the AD-tree has been created, a Markov Decision Process has been trained to compute the q-values related to likeness of the next home/away team scoring, being two MDP with 100,000 iterations and a convergence of 0.0001 (using Bellman's convergence). I provide below the pseudocode for the Value Iteration algorithm [24] using the notation used in section 2.1 .

**Algorithm 3** Dynamic Programming for Value Iteration

**Require:** Markov Game Model, convergence criterion c, maximum number of iterations M

lastValue = 0
currentValue = 0
converged = False
**for** $i = 1; i \leqslant M, i \leftarrow i + 1$ **do**
  **for** all states s in the Markov Game Model **do**
    **if** $converged == False$ **then**
      $Q_{i+1} = R(s) + \frac{1}{Occ(s)} \sum_{(s,s') \in E} Occ(s,s') x Q_i(s'))$
      $currentValue = currentValue + |Q_{i+1}(s)|$
    **end if**
  **end for**
  **if** $converged == False$ **then**
    **if** $\frac{(currentValue - lastValue)}{currentValue} < c$ **then**
      converged = True
    **end if**
  **end if**
  lastValue = currentValue
  currentValue = 0
**end for**

Once it converges, the two variables (probability for Home and Away team to score) are stored in a table with the node id reference.

## 4.4 Creation of metrics

Once each pair of (GameId, EventNumber) with an event a(T,Z) (as in figure 4.1) has been associated with its q-value, several metrics has been created for posterior evaluation.

In order to understand the metrics, a couple of measures are introduced:

- The impact of a player's action stands for how much that specific player has contributed to score with the action performed compared to the previous state (e.g. If we are in GameId, eventNumber with NodeId = n and q-value = 0.3 and an action performed by a player, then previous state is related to the Node's information table where relations between that ids are also stored according to (GameId, EventNumber). If the previous n-1 NodeId value is 0.4, then the impact or contribution of the player with his action is $0.3 - 0.4 = -0.1$).
$$impact(s,a) = q_T(s,a) - q_T(s)$$
where T is the Team Performing the current action, s and a refer to Node state, and a to action.

- The time each player has played in each match has also been calculated from the data set.

All metrics are calculated for the regular season, which are 82 matches in the NHL. These metrics are:

- Direct Impact: it measures the individual performance of a player for each match. It will be represented in tables and plots as *Direct*.

- Collective Impact: it measures the collective valuation of the actions the team has performed while a specific player is on the rink, represented as that particular specific player's valuation. It will be represented in tables and plots as *Collective*.

- Direct Impact / time (in hours): it is the same metric as in Direct Impact but taking into account time spent by each player on the rink per match. It will be represented in tables and plots as *Directh*.

- Collective Impact Valuation / time (in hours): it is the same metric as in Collective Impact but taking into account time spent by each player on the rink per match. It will be represented in tables and plots as *Collectiveh*.

Moreover, the latter metrics will be divided, according to each player, by the amount of games they have played in that specific season. In this case, the metric will have the same name with *GP* in front of the metric (e.g.*GPDirect*).

For Regular Seasons 2007-2008 and 2008-2009, these metrics are plotted and evaluated in general terms to understand their behavior in the *results* section. Total Regular season metrics such as Position (Position), Salaries (Salary), Goals (G), Goals Assisted (GA), Points (P) or PlusMinus (PlusMinus) have been scraped from *http://www.dropyourgloves.com*.

Also, for each metric, a data set has been created with player's performance for each regular match. That means there are 82 matches x n players per year with valuations per match. For those cases where there are players that have played in 2 or 3 different teams during the same season, performance has been joined in a sole vector using the *GameId* variable as an order variable. In the case where a player plays for 2 teams in a specific number match (eg. match 32), the match with higher performance is the one taken into account.

## 4.5 General analysis of the metrics

In this section, a general analysis has been done on player's valuation per match. Also, the top 10 players for the four metrics are extracted. Using the data scrapped from *http://www.dropyourgloves.com*, scatterplots are displayed to evaluate whether it exists some relation between Salary and player performance for the Regular Season and by match, taking into consideration the position of the player (whether it is a Central (C), Defender (D), Right

Wing (RW), Left Wing (LW), Goalie (G) or Forward (F). Note that C, RW, LW and F are all Forward positions, and the analysis is performed replacing these positions for the Forward one, since they indeed are.

## 4.6 Time series forecasting of metrics

In this section, it will be briefly explained how are the ARIMA models chosen for prediction purposes and how the player's performance forecast has been done.

### The choice of the best ARIMA models

For each player taken into consideration in each metric, an ARIMA model has been fitted to evaluate which are the most typical ARIMA models taking into account the number of Games Played by each player (later classified in ranges of 10 for the seasons 2007-2008 and 2008-2009 (e.g. the 10 range accounts for players that has played between 5-14 matches, and so on)) and the Position the player plays in the rink, as described in section 4.5.

### Forecast of player's performance using ARIMA models

The selection of ARIMA models performed in the previous subsection establishes the best general models to forecast over data. Therefore, one best ARIMA model is established for each player each Season using Maximum Likelihood Estimation (MLE). The MLE is a typical method to estimate parameters based on the data one has. In this case, it is used to know how many parameters are to be used, evaluating how the data is better fitted given different values to parameters and therefore deciding on the number of parameters. All ARIMA models which occurred to be the best one over a threshold of 2% of the times have been selected for this part. In total, only five ARIMA models surpass this threshold. Additionally, one extra ARIMA model which will be called *best* is added, accounting for the ARIMA model that better fits the previous m matches at each moment. The choice of how many observations to take has been settled to 5 and predict on the sixth one. The motivation behind this choice is that we are interested in trying to predict player's performance through season based on their last previous matches, intuitively understanding that as their current playing status. Additionally, a short number of observations has also been chosen given the general fit of the ARIMA models over the threshold, which is max(p,d,q) equal to one, displayed later in the results section. For doing forecast, all possible 6 consecutive combinations of matches have been selected for each player and Season, eliminating those combinations where player's performance of matches for the fit was zero, since the prediction would be too obvious. The distribution of the errors is also shown for each case and metric.

# 5 Results

In this section, it is firstly presented a general analysis of the four different player performance metrics for Seasons 2007-2008 and 2008-2009 (section 5.1), showing as well the top 10 players for each metric. Additionally, players valuation is regressed to Salary for each metric to put into context player's performance position in the rink.

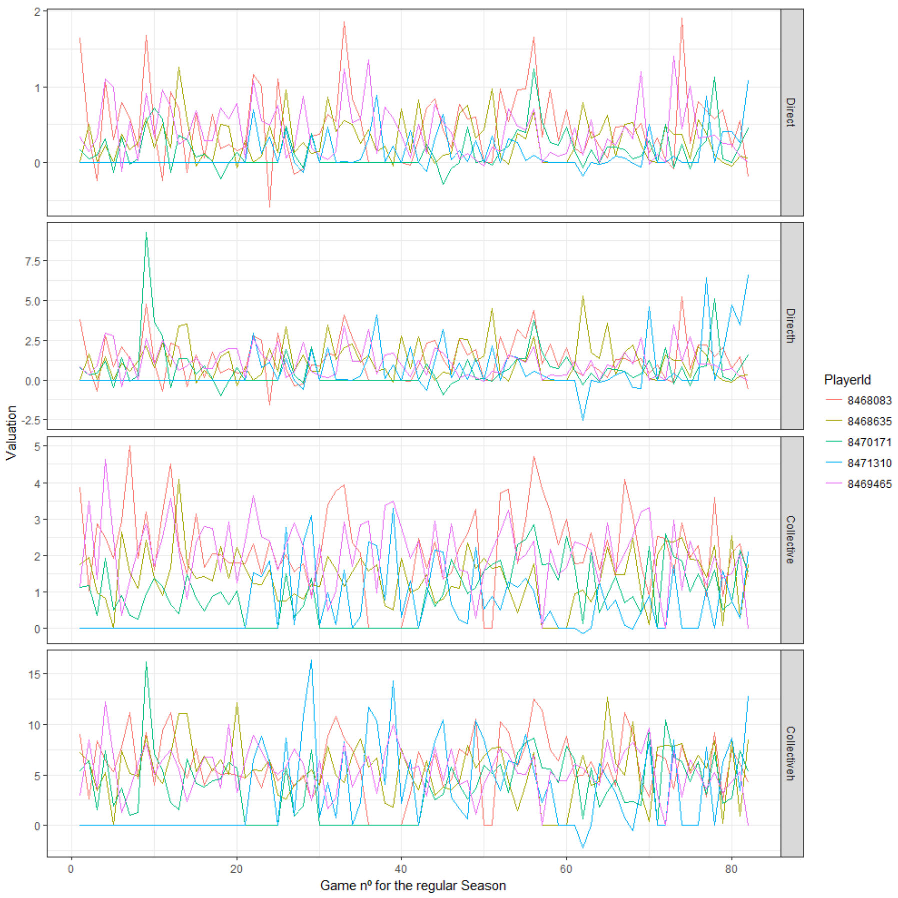
After that, ARIMA analysis is presented (section 5.2) for each metric taking into consideration position and number of games played per player, discretized in ranges of 10. After that, forecasting has been done for Seasons 2007-2008 and 2008-2009 with the 5 most frequent ARIMA models and the best ARIMA model selected each time (called *best*) fitting the previous five games and then predicting the next game player's performance. It is presented then some accuracy measures for the predicted data such as ME, RMSE or MAE to identify the best models.

## 5.1 General evaluation of the metrics

In figure 5.1, the valuation of five players chosen at random through the regular season are exemplified. It seems that player's valuation are quite random across the league, and in some cases, it even looks like some noise across the mean. In figure 5.2, it can be seen the distribution of the 99.8% player's valuation in matches during Regular Seasons 2007-2008 and 2008-2009 is displayed (excluding 0.01% tails) for each metric. The players valuation distribution on matches for both the histogram and the boxplot are really similar across both years. It can also be seen in all metrics that the valuation distribution is skewed to the right.

In figure 5.3, a quantile regression is shown through the 82 matches for seasons 2007-2008 and 2008-2009, as well as a table showing the general quantiles year performance with its mean, standard deviation and change compared to the previous year. The quantile plot shows that players range of performance through season is quite stable. All metrics except the Collectiveh (Collective/time (hours)) have a clear separation between the mean and the median (the 50% quantile).

In figure 5.3, a regression is shown between player's valuation metrics and Salary (Valuation $\sim$ Salary) for each position, scrapping the data from *www.dropyourgloves.com*. The first plot takes into account the sum of players valuation at the end of the regular season, whereas the second takes into account the sum of players valuation at the end of the regular season divided by the number of Games Played each player has performed. Additionally, other met-



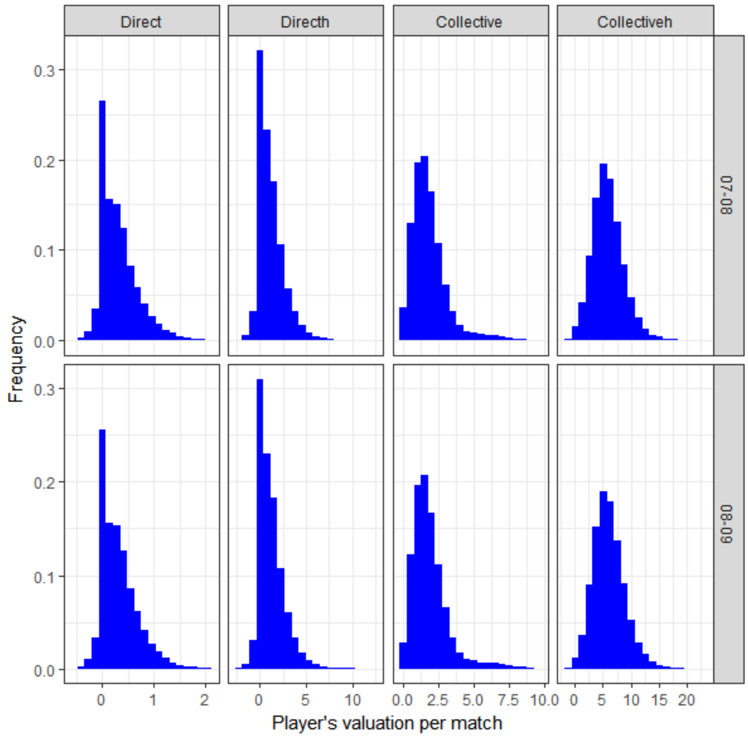

(a)

Figure 5.1: Example of five players valuation through the regular season

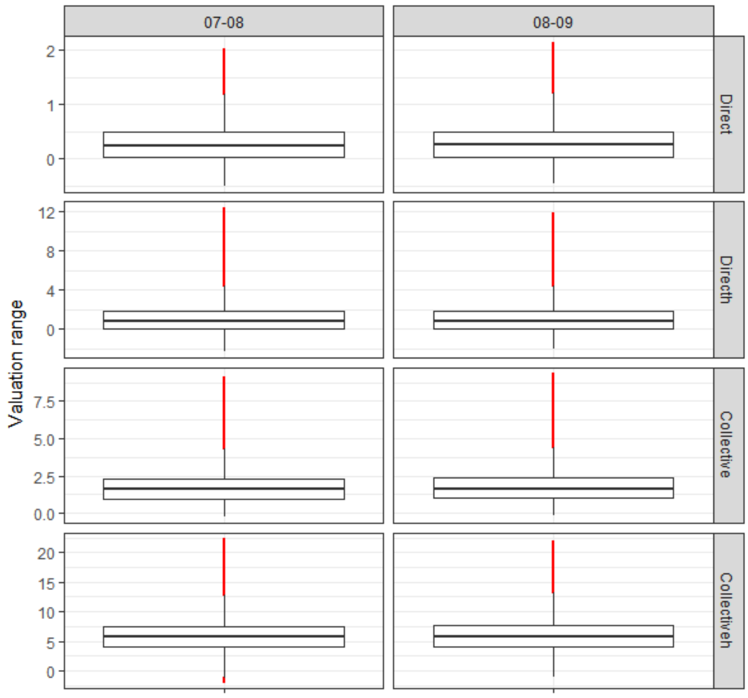
rics such as Points (Goals +Assists) and the $+/-$ (PlusMinus) measure are also regressed to Salary. Positions are Defender (D), Forward (F) and Goalie (G).

| PlayerName | Position | Age | Salary | GP | G | GA | PlusMin | Points | **Direct** | Directh | Collective | Collectiveh |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **2007** | | | | | | | | | | | | |
| Alex Ovechkin | F | 22 | 3.83 | 82 | 65 | 47 | 28 | 112 | **71.96** | 182.65 | 232.56 | 588.85 |
| Dion Phaneuf | D | 22 | 0.94 | 82 | 17 | 43 | 12 | 60 | **59.22** | 134.05 | 246.12 | 559.67 |
| Rick Nash | F | 23 | 5.50 | 80 | 38 | 31 | 3 | 69 | **59.01** | 181.80 | 158.82 | 485.99 |
| Jarome Iginla | F | 30 | 7.00 | 82 | 50 | 48 | 27 | 98 | **58.94** | 161.92 | 204.12 | 560.88 |
| Dustin Brown | F | 23 | 1.18 | 78 | 33 | 27 | -13 | 60 | **53.78** | 156.41 | 171.40 | 501.48 |
| Brenden Morrow | F | 28 | 4.10 | 82 | 32 | 42 | 23 | 74 | **51.15** | 146.62 | 171.59 | 504.57 |
| Zdeno Chara | D | 30 | 7.50 | 77 | 17 | 34 | 14 | 51 | **50.74** | 117.69 | 203.78 | 468.89 |
| Trent Hunter | F | 27 | 1.55 | 82 | 12 | 29 | -17 | 41 | **50.31** | 167.65 | 153.36 | 508.27 |
| Mike Green | D | 22 | 0.85 | 82 | 18 | 38 | 6 | 56 | **48.26** | 122.63 | 219.72 | 545.08 |
| Pavel Datsyuk | F | 29 | 6.70 | 82 | 31 | 66 | 41 | 97 | **48.22** | 134.68 | 198.44 | 559.41 |
| **2008** | | | | | | | | | | | | |
| Alex Ovechkin | F | 23 | 9.00 | 79 | 56 | 54 | 8 | 110 | **75.93** | 194.34 | 239.89 | 612.23 |
| Dustin Brown | F | 24 | 2.60 | 80 | 24 | 29 | -15 | 53 | **59.76** | 177.60 | 178.34 | 540.84 |
| Shea Weber | D | 23 | 4.50 | 81 | 23 | 30 | 1 | 53 | **53.14** | 136.10 | 201.19 | 511.36 |
| Evgeni Malkin | F | 22 | 3.83 | 82 | 35 | 78 | 17 | 113 | **50.76** | 134.92 | 220.41 | 591.75 |
| Dion Phaneuf | D | 23 | 7.00 | 79 | 11 | 36 | -11 | 47 | **50.34** | 122.64 | 240.57 | 532.49 |
| Vincent Lecavalier | F | 28 | 7.17 | 77 | 29 | 38 | -9 | 67 | **49.46** | 143.99 | 188.17 | 549.37 |
| Sheldon Souray | D | 32 | 6.25 | 81 | 23 | 30 | 1 | 53 | **49.38** | 125.86 | 203.08 | 514.73 |
| Jeff Carter | F | 24 | 4.50 | 82 | 46 | 38 | 23 | 84 | **48.88** | 141.78 | 189.35 | 548.30 |
| Rick Nash | F | 24 | 6.50 | 78 | 40 | 39 | 11 | 79 | **48.88** | 145.11 | 171.59 | 498.26 |
| Martin St. Louis | F | 33 | 5.00 | 82 | 30 | 50 | 4 | 80 | **47.82** | 135.55 | 204.19 | 569.06 |

Table 5.1: Top 10 Players performance for 2007-2008 and 2008-2009 for the Direct Metric.

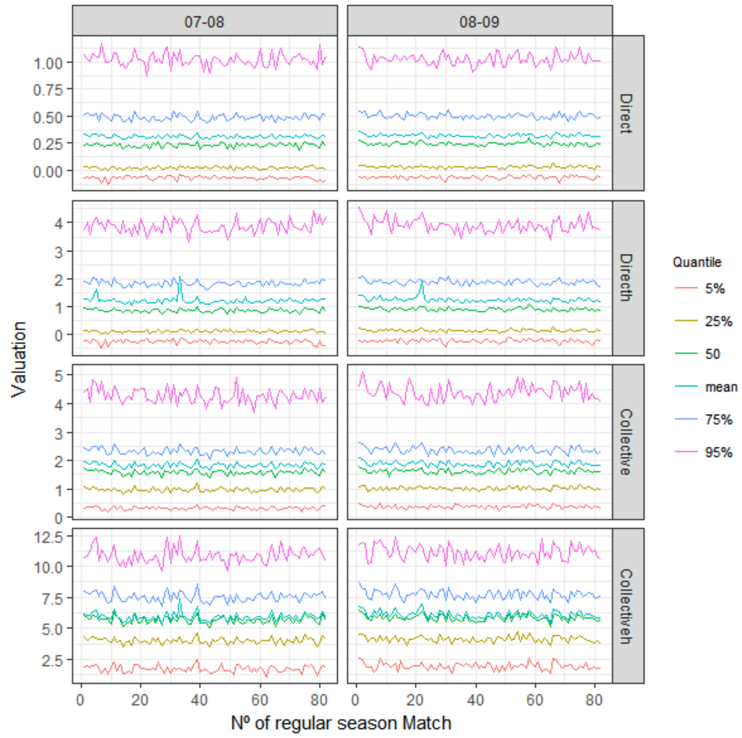



(a) Histogram distribution of player's valuation in matches




(b) Box plot distribution of player's valuation in matches. The box encloses values from $Q1 - 1.5(Q3 - Q1)$ to $Q3 + 1.5(Q3 - Q1)$, where Q1 and Q3 are the 25% and 75% quantiles respectively

Figure 5.2: Distribution of the 99.8% player's valuation in matches during Regular Seasons 2007-2008 and 2008-2009 per metric (excluding 0.01% tails)

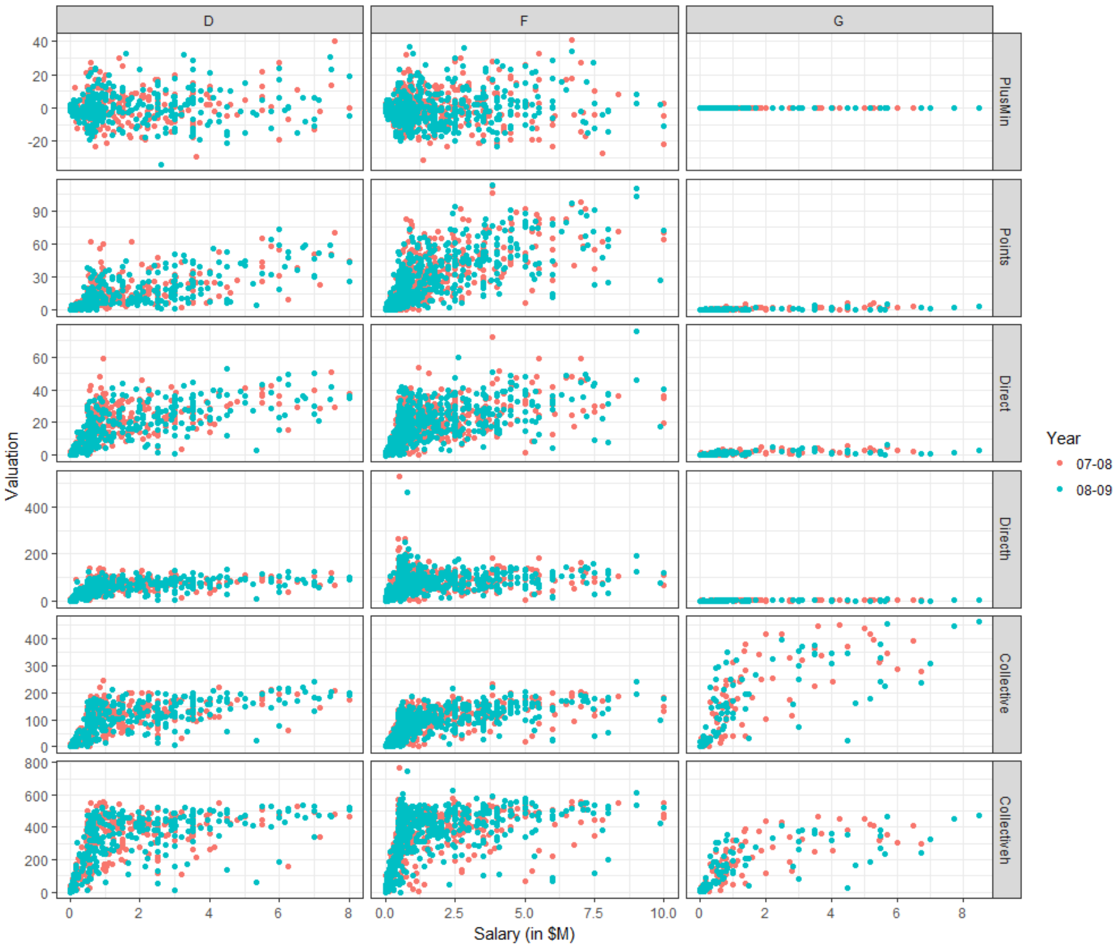



(a) Quantile plot of players valuation per match, Season and Metric

| | Direct | | | Direct/h | | | Collective | | | Collective/h | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **07-08** | **08-09** | **% change** | **07-08** | **08-09** | **% change** | **07-08** | **08-09** | **% change** | **07-08** | **08-09** | **% change** |
| 5% | -0.069 | -0.063 | **-7.730** | -0.284 | -0.259 | **-8.730** | 0.318 | 0.372 | **17.078** | 1.685 | 1.901 | **12.817** |
| 25% | 0.024 | 0.031 | **30.208** | 0.086 | 0.111 | **30.081** | 0.971 | 1.021 | **5.210** | 3.998 | 4.125 | **3.183** |
| 50% | 0.234 | 0.246 | **5.409** | 0.845 | 0.887 | **5.023** | 1.583 | 1.620 | **2.336** | 5.653 | 5.823 | **3.002** |
| 75% | 0.489 | 0.501 | **2.463** | 1.820 | 1.865 | **2.476** | 2.323 | 2.372 | **2.120** | 7.528 | 7.742 | **2.842** |
| 95% | 1.023 | 1.029 | **0.576** | 3.873 | 3.939 | **1.715** | 4.309 | 4.415 | **2.465** | 10.890 | 11.170 | **2.575** |
| mean | 0.312 | 0.321 | **2.839** | 1.194 | 1.230 | **3.008** | 1.833 | 1.887 | **2.921** | 5.926 | 6.111 | **3.125** |
| sd | 0.362 | 0.364 | **0.766** | 2.688 | 2.201 | **-18.103** | 1.329 | 1.351 | **1.668** | 3.593 | 3.362 | **-6.445** |

(b) Quantile Table of players valuation Season and Metric taking into account the standard deviation of the metrics

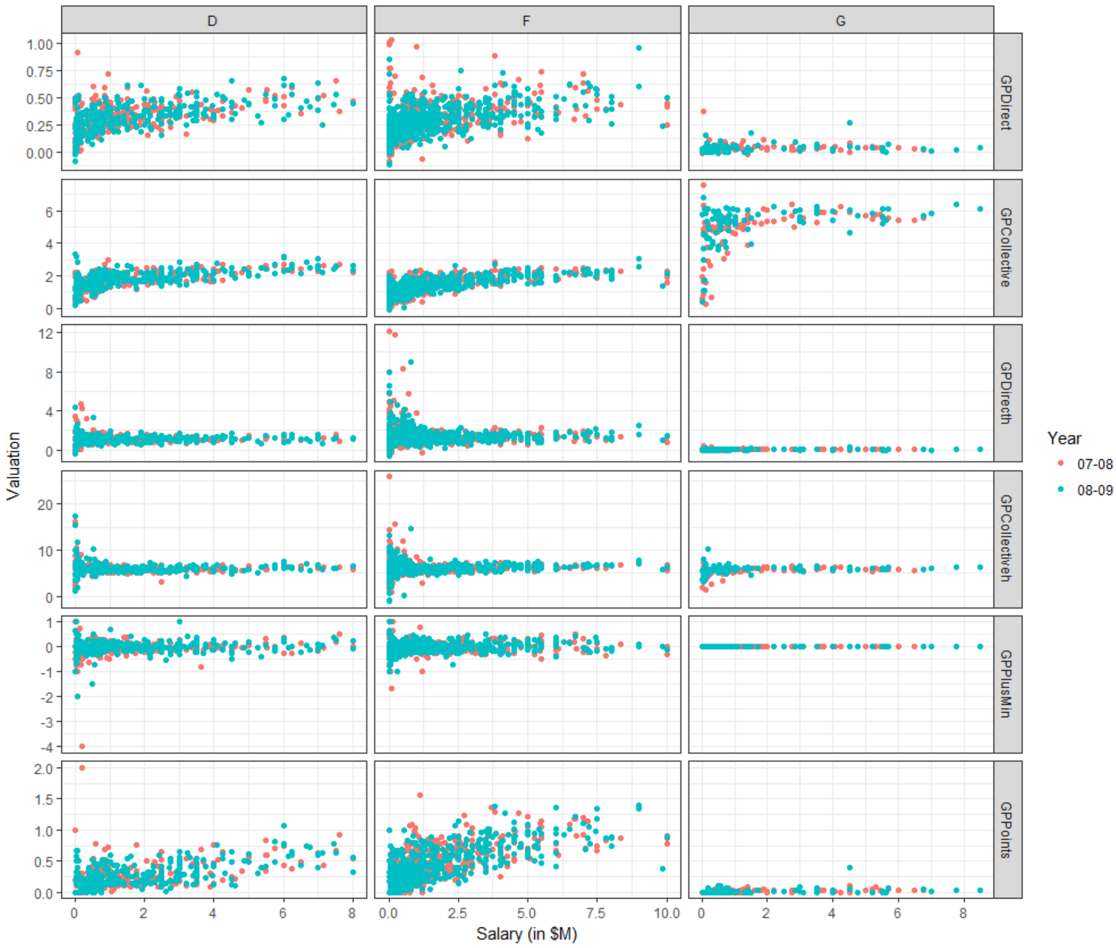
Figure 5.3: Quantile analysis of player's valuation in matches during Regular Seasons 2007-2008 and 2008-2009




(a) Regression of player's Valuation Metrics on Salary at the end of the season

| PlayerName | Position | Age | Salary | GP | G | GA | PlusMin | Points | Direct | **Directh** | Collective | Collectiveh |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **2007** | | | | | | | | | | | | |
| Mike Rupp | F | 27 | 0.5 | 64 | 3 | 6 | -8 | 9 | 15.74 | **529.76** | 49.09 | 769.03 |
| Riley Cote | F | 25 | 0.48 | 70 | 1 | 3 | 2 | 4 | 14.55 | **263.64** | 38.90 | 570.09 |
| Jeff Cowan | F | 31 | 0.72 | 46 | 0 | 1 | -5 | 1 | 10.62 | **262.03** | 39.03 | 316.68 |
| DJ King | F | 23 | 0.5 | 61 | 3 | 3 | -4 | 6 | 12.06 | **227.25** | 36.18 | 485.79 |
| Eric Godard | F | 27 | 0.47 | 74 | 1 | 1 | -8 | 2 | 10.80 | **214.19** | 35.18 | 505.55 |
| Jared Boll | F | 21 | 0.74 | 75 | 5 | 5 | -4 | 10 | 27.41 | **201.09** | 66.26 | 472.20 |
| Alex Ovechkin | F | 22 | 3.83 | 82 | 65 | 47 | 28 | 112 | 71.96 | **182.65** | 232.56 | 588.85 |
| Rick Nash | F | 23 | 5.5 | 80 | 38 | 31 | 3 | 69 | 59.01 | **181.80** | 158.82 | 485.99 |
| Raitis Ivanans | F | 28 | 0.48 | 73 | 6 | 2 | -10 | 8 | 20.88 | **176.36** | 57.73 | 445.11 |
| David Clarkson | F | 23 | 0.8 | 81 | 9 | 13 | 1 | 22 | 33.91 | **168.22** | 97.97 | 476.47 |
| **2008** | | | | | | | | | | | | |
| Derek Boogaard | F | 26 | 0.8 | 51 | 0 | 3 | 3 | 3 | 10.83 | **460.55** | 33.48 | 747.36 |
| Eric Godard | F | 28 | 0.72 | 71 | 2 | 2 | -3 | 4 | 14.15 | **249.95** | 35.40 | 539.28 |
| Dan Carcillo | F | 23 | 0.83 | 73 | 3 | 11 | -15 | 14 | 33.14 | **219.86** | 94.20 | 501.42 |
| Jody Shelley | F | 32 | 0.72 | 70 | 2 | 2 | -6 | 4 | 20.11 | **196.58** | 52.38 | 505.46 |
| Alex Ovechkin | F | 23 | 9 | 79 | 56 | 54 | 8 | 110 | 75.93 | **194.34** | 239.89 | 612.23 |
| Evgeni Artyukhin | F | 25 | 0.89 | 73 | 6 | 10 | 1 | 16 | 35.72 | **189.73** | 94.61 | 504.91 |
| Mike Rupp | F | 28 | 0.5 | 72 | 3 | 6 | -2 | 9 | 24.52 | **188.26** | 73.94 | 517.70 |
| Cam Janssen | F | 24 | 0.55 | 56 | 1 | 3 | -5 | 4 | 13.11 | **187.93** | 36.51 | 478.01 |
| Matt Cooke | F | 30 | 1.17 | 76 | 13 | 18 | 0 | 31 | 36.32 | **185.57** | 117.48 | 500.05 |
| Riley Cote | F | 26 | 0.52 | 63 | 0 | 3 | -7 | 3 | 12.10 | **184.08** | 36.36 | 543.95 |

Table 5.2: Top 10 Players performance for 2007-2008 and 2008-2009 for the Directh Metric (Direct/time(hours))




(b) Regression of player′s Valuation Metrics on Salary at the end of the season divided by Games Played for each player

Figure 5.3: Regression of player′s Valuation Metrics on Salary (Valuation∼ Salary) based on players general Position

| PlayerName | Position | Age | Salary | GP | G | GA | PlusMin | Points | Direct | Directh | **Collective** | Collectiveh |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **2007** | | | | | | | | | | | | |
| Henrik Lundqvist | G | 25 | 4.25 | 72 | 0 | 0 | 0 | 0 | 1.31 | 1.29 | **449.62** | 453.19 |
| Miikka Kiprusoff | G | 31 | 3.60 | 76 | 0 | 2 | 0 | 2 | 4.13 | 4.07 | **447.26** | 464.24 |
| Evgeni Nabokov | G | 32 | 5.00 | 77 | 0 | 2 | 0 | 2 | 3.00 | 2.98 | **440.79** | 453.59 |
| Martin Brodeur | G | 35 | 5.20 | 77 | 0 | 4 | 0 | 4 | 4.58 | 4.55 | **419.29** | 418.00 |
| Ryan Miller | G | 27 | 2.50 | 76 | 0 | 1 | 0 | 1 | 4.11 | 4.55 | **418.40** | 430.52 |
| Cam Ward | G | 23 | 2.00 | 69 | 0 | 1 | 0 | 1 | 1.67 | 1.70 | **417.03** | 440.68 |
| Tomas Vokoun | G | 31 | 5.30 | 69 | 0 | 6 | 0 | 6 | 2.87 | 2.87 | **398.65** | 413.01 |
| Roberto Luongo | G | 28 | 6.50 | 73 | 0 | 3 | 0 | 3 | 2.09 | 1.78 | **393.72** | 407.58 |
| Vesa Toskala | G | 30 | 1.38 | 66 | 0 | 5 | 0 | 5 | 1.73 | 1.74 | **380.84** | 391.68 |
| Rick DiPietro | G | 26 | 4.50 | 63 | 0 | 6 | 0 | 6 | 5.75 | 5.67 | **372.26** | 382.34 |
| **2008** | | | | | | | | | | | | |
| Miikka Kiprusoff | G | 32 | 8.50 | 76 | 0 | 3 | 0 | 3 | 3.03 | 3.31 | **462.40** | 476.85 |
| Marty Turco | G | 33 | 5.70 | 74 | 0 | 5 | 0 | 5 | 6.24 | 6.48 | **454.18** | 467.39 |
| Henrik Lundqvist | G | 26 | 7.75 | 70 | 0 | 2 | 0 | 2 | 1.66 | 1.62 | **447.99** | 450.00 |
| Cam Ward | G | 24 | 2.50 | 67 | 0 | 1 | 0 | 1 | 2.24 | 3.09 | **396.95** | 410.44 |
| Evgeni Nabokov | G | 33 | 5.50 | 62 | 0 | 1 | 0 | 1 | 2.59 | 2.59 | **380.95** | 381.38 |
| Marc-Andre Fleury | G | 24 | 3.50 | 62 | 0 | 1 | 0 | 1 | 2.87 | 2.87 | **375.41** | 385.18 |
| Niklas Backstrom | G | 30 | 3.10 | 71 | 0 | 0 | 0 | 0 | 4.61 | 5.22 | **373.02** | 384.31 |
| Dwayne Roloson | G | 39 | 3.00 | 63 | 0 | 1 | 0 | 1 | 2.62 | 3.25 | **353.05** | 374.33 |
| Steve Mason | G | 20 | 0.85 | 61 | 0 | 0 | 0 | 0 | 1.63 | 1.65 | **352.33** | 357.26 |
| Ilya Bryzgalov | G | 28 | 4.00 | 64 | 0 | 2 | 0 | 2 | 0.69 | 0.70 | **346.39** | 364.53 |

Table 5.3: Top 10 Players performance for 2007-2008 and 2008-2009 for the Collective Metric

The collective measure for a player stands for the collective impact of their mate players and himself while on the rink. On a general purpose, players tend to go and lose the puck after a shot or nearer the offensive zone while retaking it nearby their zone, usually trying to make movements to improve the chances on scoring. Then, it is normal that the collective measure is positive since actions performed by the other team are not taken into account on the opponents. Therefore, those who play during a longer time on the rink are those with highest valuation, which is accounting for the valuation of their team. That is why goalkeepers are in the 10 first positions of this measure, despite of the fact they do not score goals. Also, the more matches they play, the more time they play as well, making their collective performance bigger.

| PlayerName | Position | Age | Salary | GP | G | GA | PlusMin | Points | Direct | Directh | **Collective** | Collectiveh |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **2007** | | | | | | | | | | | | |
| Dion Phaneuf | D | 22 | 0.94 | 82 | 17 | 43 | 12 | 60 | 59.22 | 134.05 | **246.12** | 559.67 |
| Alex Ovechkin | F | 22 | 3.83 | 82 | 65 | 47 | 28 | 112 | 71.96 | 182.65 | **232.56** | 588.85 |
| Tomas Kaberle | D | 29 | 4.25 | 82 | 8 | 45 | -8 | 53 | 38.32 | 93.36 | **221.93** | 551.72 |
| Mike Green | D | 22 | 0.85 | 82 | 18 | 38 | 6 | 56 | 48.26 | 122.63 | **219.72** | 545.08 |
| Andrei Markov | D | 29 | 5.75 | 82 | 16 | 42 | 1 | 58 | 42.37 | 105.18 | **213.81** | 530.37 |
| Nicklas Lidstrom | D | 37 | 7.60 | 76 | 10 | 60 | 40 | 70 | 29.04 | 66.41 | **205.68** | 480.18 |
| Jarome Iginla | F | 30 | 7.00 | 82 | 50 | 48 | 27 | 98 | 58.94 | 161.92 | **204.12** | 560.88 |
| Zdeno Chara | D | 30 | 7.50 | 77 | 17 | 34 | 14 | 51 | 50.74 | 117.69 | **203.78** | 468.89 |
| Lubomir Visnovsky | D | 31 | 2.05 | 82 | 8 | 33 | -18 | 41 | 32.64 | 83.52 | **201.34** | 523.00 |
| Roman Hamrlik | D | 33 | 5.50 | 77 | 5 | 21 | 7 | 26 | 37.79 | 93.89 | **201.29** | 509.39 |
| **2008** | | | | | | | | | | | | |
| Dion Phaneuf | D | 23 | 7.00 | 79 | 11 | 36 | -11 | 47 | 50.34 | 122.64 | **240.57** | 532.49 |
| Alex Ovechkin | F | 23 | 9.00 | 79 | 56 | 54 | 8 | 110 | 75.93 | 194.34 | **239.89** | 612.23 |
| Evgeni Malkin | F | 22 | 3.83 | 82 | 35 | 78 | 17 | 113 | 50.76 | 134.92 | **220.41** | 591.75 |
| Dan Boyle | D | 32 | 6.67 | 77 | 16 | 41 | 6 | 57 | 36.11 | 88.65 | **219.94** | 539.81 |
| Chris Pronger | D | 34 | 6.25 | 82 | 11 | 37 | 0 | 48 | 43.40 | 99.89 | **217.92** | 503.72 |
| Mike Green | D | 23 | 6.00 | 68 | 31 | 42 | 24 | 73 | 46.41 | 106.62 | **214.33** | 493.09 |
| Nicklas Backstrom | F | 21 | 2.40 | 82 | 22 | 66 | 16 | 88 | 37.12 | 111.83 | **214.19** | 630.43 |
| Braydon Coburn | D | 23 | 1.20 | 80 | 7 | 21 | 7 | 28 | 40.78 | 100.10 | **211.64** | 516.12 |
| Andrei Markov | D | 30 | 5.75 | 78 | 12 | 52 | -2 | 64 | 38.03 | 96.17 | **209.18** | 527.62 |
| Mark Streit | D | 31 | 4.10 | 74 | 16 | 40 | 6 | 56 | 39.38 | 97.60 | **206.59** | 504.31 |

Table 5.4: Top 10 Players performance for 2007-2008 and 2008-2009 for the Collective Metric without Goalie Positions

| PlayerName | Position | Age | Salary | GP | G | GA | PlusMin | Points | Direct | Directh | Collective | **Collectiveh** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **2007** | | | | | | | | | | | | |
| Mike Rupp | F | 27 | 0.50 | 64 | 3 | 6 | -8 | 9 | 15.74 | 529.76 | 49.09 | **769.03** |
| Alex Ovechkin | F | 22 | 3.83 | 82 | 65 | 47 | 28 | 112 | 71.96 | 182.65 | 232.56 | **588.85** |
| Nicklas Backstrom | F | 20 | 2.40 | 82 | 14 | 55 | 13 | 69 | 28.80 | 86.12 | 187.96 | **577.10** |
| Riley Cote | F | 25 | 0.48 | 70 | 1 | 3 | 2 | 4 | 14.55 | 263.64 | 38.90 | **570.09** |
| Jarome Iginla | F | 30 | 7.00 | 82 | 50 | 48 | 27 | 98 | 58.94 | 161.92 | 204.12 | **560.88** |
| Viktor Kozlov | F | 32 | 2.50 | 81 | 16 | 38 | 28 | 54 | 32.25 | 108.51 | 168.31 | **560.82** |
| Dion Phaneuf | D | 22 | 0.94 | 82 | 17 | 43 | 12 | 60 | 59.22 | 134.05 | 246.12 | **559.67** |
| Pavel Datsyuk | F | 29 | 6.70 | 82 | 31 | 66 | 41 | 97 | 48.22 | 134.68 | 198.44 | **559.41** |
| Eric Staal | F | 23 | 4.50 | 82 | 38 | 44 | -2 | 82 | 34.85 | 96.67 | 199.04 | **552.43** |
| Tomas Kaberle | D | 29 | 4.25 | 82 | 8 | 45 | -8 | 53 | 38.32 | 93.36 | 221.93 | **551.72** |
| **2008** | | | | | | | | | | | | |
| Derek Boogaard | F | 26 | 0.80 | 51 | 0 | 3 | 3 | 3 | 10.83 | 460.55 | 33.48 | **747.36** |
| Nicklas Backstrom | F | 21 | 2.40 | 82 | 22 | 66 | 16 | 88 | 37.12 | 111.83 | 214.19 | **630.43** |
| Alex Ovechkin | F | 23 | 9.00 | 79 | 56 | 54 | 8 | 110 | 75.93 | 194.34 | 239.89 | **612.23** |
| Arron Asham | F | 30 | 0.64 | 78 | 8 | 12 | 0 | 20 | 18.24 | 143.65 | 83.85 | **605.67** |
| Evgeni Malkin | F | 22 | 3.83 | 82 | 35 | 78 | 17 | 113 | 50.76 | 134.92 | 220.41 | **591.75** |
| Eric Staal | F | 24 | 5.00 | 81 | 40 | 35 | 15 | 75 | 39.45 | 110.57 | 203.62 | **581.78** |
| Brandon Dubinsky | F | 22 | 0.64 | 82 | 13 | 28 | -6 | 41 | 38.83 | 137.89 | 165.14 | **580.76** |
| Nikolai Zherdev | F | 24 | 3.25 | 82 | 23 | 35 | 6 | 58 | 37.57 | 133.72 | 165.89 | **579.38** |
| Chris Thorburn | F | 25 | 0.54 | 81 | 7 | 8 | -10 | 15 | 23.15 | 143.05 | 93.92 | **570.34** |
| Martin St. Louis | F | 33 | 5.00 | 82 | 30 | 50 | 4 | 80 | 47.82 | 135.55 | 204.19 | **569.06** |

Table 5.5: Top 10 Players performance for 2007-2008 and 2008-2009 for the Collectiveh Metric (Collective/time(hours))

In this latter part, the top 10 players for each year and metrics have been included. The Top 10 players on the Direct metric seems to perform relatively well in terms of Points (Goals and Assists). This makes sense since both metrics offer quite a similar regression line with Salary. As previously stated, the top 10 Players in the Collective measure are Goalkeepers, since they are the ones gathering all the impact valuation of their teams 5.3. Therefore, those metrics should be taken into account under Position circumstances, and not only under General Total Valuation.

Metrics accounting for time (Directh and Collectiveh) outputs players with low Goals and Assists on a general basis who might have played little in a lot of matches but who have had a great impact while they were in the rink.

## 5.2 Time Series Analysis

### The choice of an Arima Model

| ArimaModel | Position | Dataset | Frequency | Prob |
|---|---|---|---|---|
| 0,0,0 | F | Direct | 367 | 0.7758985 |
| 0,0,0 | F | Directh | 378 | 0.8025478 |
| 0,0,0 | F | Collective | 286 | 0.6244541 |
| 0,0,0 | F | Collectiveh | 304 | 0.6652079 |
| 0,0,0 | D | Direct | 188 | 0.7736626 |
| 0,0,0 | D | Directh | 194 | 0.7667984 |
| 0,0,0 | D | Collective | 140 | 0.5982906 |
| 0,0,0 | D | Collectiveh | 141 | 0.5899582 |
| 0,0,0 | G | Direct | 75 | 0.9615385 |
| 0,0,0 | G | Directh | 74 | 0.9367089 |
| 0,0,0 | G | Collective | 46 | 0.6865672 |
| 0,0,0 | G | Collectiveh | 51 | 0.6800000 |

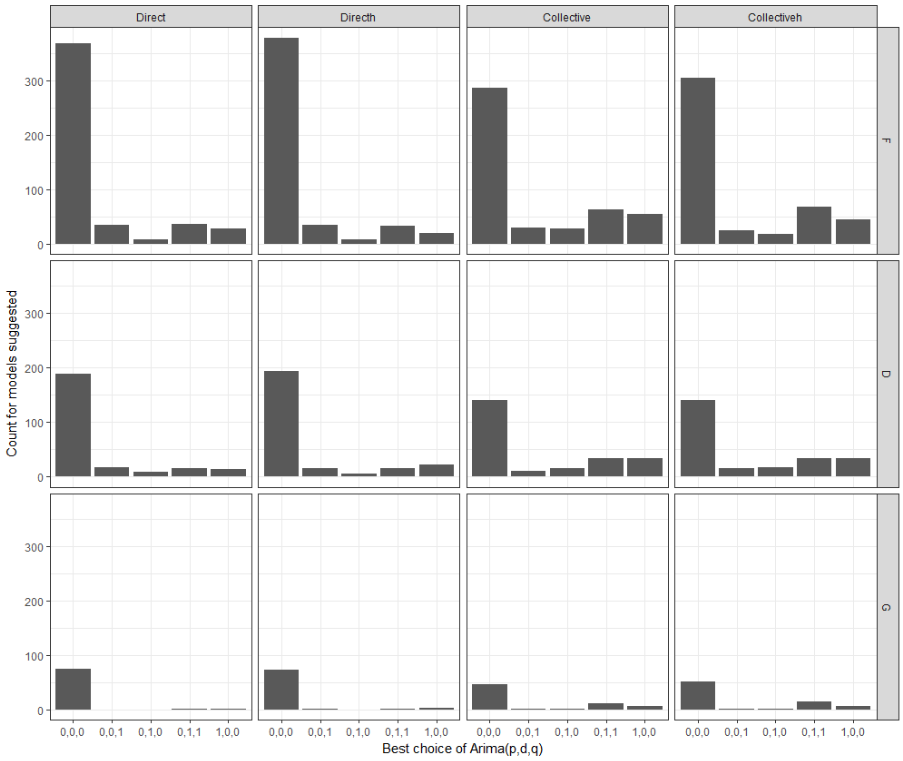
Table 5.6: Top Arima Models Per Dataset (Metric) and position for years 2007-2008 and 2008-2009 together. Only models with the highest frequency per position are taken into account and shown in the table)

In table 5.6, the most and only typical model is displayed for each metric and Position. It can be seen that in all cases the most preferred model is the ARIMA(0,0,0), which stands for the mean of the previous observations.

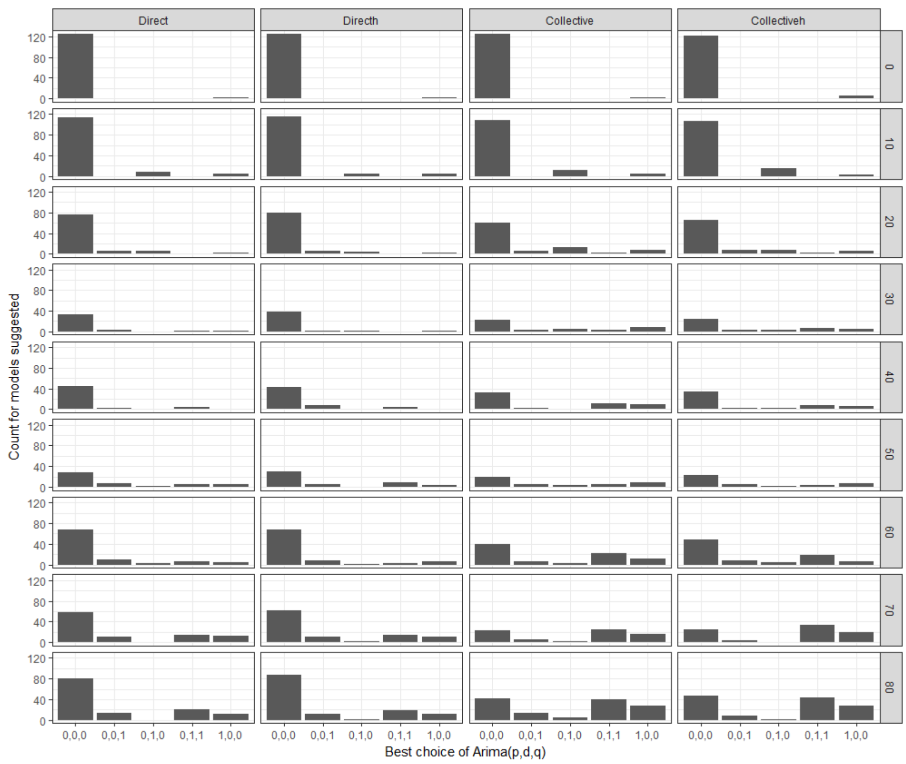
In plots 5.3 it is shown the distribution of the 98% most frequent Arima Models by Position(a), Range(b) and Metric. Both plots show quite an outstanding dominance of the ARIMA(0,0,0) model. The ranges in the second histogram have been discretized rounding up the ranges. For instance, range 0 will englobe those players who have played between 0 and 4 matches. Then, range 10 will englobe those players who have played between 5 and 14 matches, and so on.

### Forecast of Player's performance using ARIMA models

In this section, the distribution of the residuals based on the forecast of each model have been plotted for each metric, showing as well a summary table to understand the forecast prediction of each metric.




(a) Histogram distribution of the 98% most frequent Arima Models by Position and Dataset




(b) Histogram distribution of the 98% most frequent Arima Models by Range and Dataset

Figure 5.3: Histogram distribution of the 98% most frequent Arima Models by Range, Position and Metric

| Model | ME | RMSE | MAE | Dataframe |
|---|---|---|---|---|
| 0,1,0 | 0.006 | 0.46 | 0.33 | Direct |
| 0,0,1 | 0.015 | 0.41 | 0.31 | Direct |
| 1,0,0 | 0.011 | 0.41 | 0.30 | Direct |
| 0,1,1 | 0.034 | 0.44 | 0.31 | Direct |
| 0,0,0 | 0.007 | 0.37 | 0.27 | Direct |
| best | -0.204 | 0.44 | 0.29 | Direct |
| 0,1,0 | 0.037 | 3.72 | 1.34 | Directh |
| 0,0,1 | 0.069 | 3.31 | 1.30 | Directh |
| 1,0,0 | 0.059 | 3.00 | 1.25 | Directh |
| 0,1,1 | 0.180 | 3.77 | 1.31 | Directh |
| 0,0,0 | 0.040 | 2.83 | 1.13 | Directh |
| best | -0.817 | 2.77 | 1.14 | Directh |
| 0,1,0 | 0.024 | 1.08 | 0.80 | Collective |
| 0,0,1 | 0.027 | 1.03 | 0.80 | Collective |
| 1,0,0 | 0.033 | 1.01 | 0.78 | Collective |
| 0,1,1 | 0.063 | 1.03 | 0.77 | Collective |
| 0,0,0 | 0.027 | 0.91 | 0.71 | Collective |
| best | -0.241 | 1.07 | 0.79 | Collective |
| 0,1,0 | 0.122 | 5.22 | 3.12 | Collectiveh |
| 0,0,1 | 0.148 | 4.91 | 3.14 | Collectiveh |
| 1,0,0 | 0.173 | 4.63 | 3.04 | Collectiveh |
| 0,1,1 | 0.328 | 5.15 | 3.03 | Collectiveh |
| 0,0,0 | 0.131 | 4.24 | 2.77 | Collectiveh |
| best | -1.054 | 4.68 | 3.00 | Collectiveh |

Table 5.7: Analysis of the residuals for the predicted player performance on the most frequent ARIMA models

In table 5.7, the Mean Error, the Root Mean Squared Error and the Mean Absolute Error are the measures used to evaluate the forecast of the player's performance. It can be seen that the best model is again the ARIMA (0,0,0) for all cases since it is the one being smaller for most metrics and measures with being smaller in almost all residuals, together with the (0,1,0), which only takes into account differences on results.

# 6 Discussion

In this section, the General and time-series forecast and analysis are discussed for each metric. Then, methods used in this thesis are also discussed.

## 6.1 Results

### General evaluation of the metrics

Player's valuation varies a lot from match to match. Beforehand, it seems very challenging to find a model that would fit well player's performance. As already mentioned in the results section, players valuation distribution on matches for the histogram, boxplot and the quantile plots in 5.2 and 5.3 is really similar across both years and stable. That means that the metrics are therefore usable for other years prediction, meaning that there are no peaks or big changes on valuation for some specific weeks or years. It has also been commented the fact that the valuation distribution is skewed to the right, possibly meaning that players often perform between some ranges, and that there are few players who consistently may have some outstanding performances that are larger than the average of the players performance.

In 5.3, all metrics except the Collectiveh (Collective/time (hours)) have a clear separation between the mean and the median (the 50% quantile). This also makes sense since the distribution of matches seen in 5.2 is skewed to the right in all cases but in the Collectiveh metric where the distribution, although it is still a little bit skewed, looks much more normal than the other ones.

In figure 5.3, the PlusMinus measure does not seem to be a good metric to predict players Salary at all. The Direct and the Points measures seem to be fairly good to predict players Salary for Foward and Defender players. Also, the Collective measure performs better for the prediction in general players performance. This might be misleading since the Collective measure takes into account the sum of all impact actions of players in the rink while that specific player is in the rink. Of course, if a player has been playing good through the season, that player will continue playing even more time since he is proven useful to the team. Also, this measure can be misleading and must be interpreted only between players playing in similar position and similar number of matches. Since Goalies are the players who play most of the time, they are getting all impact actions of their teams and that is why their performances are the highest ones. Both Directh and Collectiveh seem to have some kind of logarithmic modelling, which could be interesting to predict players salary based on performances.

The second figure in 5.3, which is related to the same metrics but divided by Games Played in each case, shows that GPPoints, GPDirect and GPCollective are be able to capture linearly the Salary Players based on their performance for Foward and Defender players. Nevertheless, metrics divided by time do not seem to have any impact on players performance. That makes sense, since the Games Played (GP) measure takes into account to some extent the time a player plays (e.g.the more matches a player plays, the more is playing).

### Time series forecasting of metrics

The best time series ARIMA model for forecasting players performance has proven to be an ARIMA (0,0,0) which is indeed the mean of all previous matches or the or an ARIMA (0,1,0), which is differencing between values with lag 1. In other words: based on my original dataset and on my impact measures, I cannot predict better than the mean. This means that players performance may not depend on the n previous matches as sometimes believed when people say that a player is *on fire* for some consecutive matches but it just happens that player has normal ups and downs, non-related to n previous matches performances, but just on their average level performance through the whole season. This goes in line with the hot hands fallacy, which tries to proof that a person who experiences a successful outcome in a random event (e.g. a match) has not a greater chance of success in successive trials.

## 6.2 Method

### The definition of a state

In this thesis, I have assumed to have as state nodes the combination of 3 context variables (Manpower Differential, Goal Differential, Period) and actions associated with the Team (T) having the puck or winning the action and a categorical Zone (Z) in which the action is happening based on the team's performing the action (i.e a(T,Z)). It would be interesting to include new state variables such as a time remaining in the quarter, meaning that maybe when almost no time is left, the actions taken may have a lower impact on the scoring due to time pressure or duration of the possession until the moment of the action, meaning that the probability of scoring given that you have had the puck might differ when the length of the possession is significantly different.

### The choice of the reward function

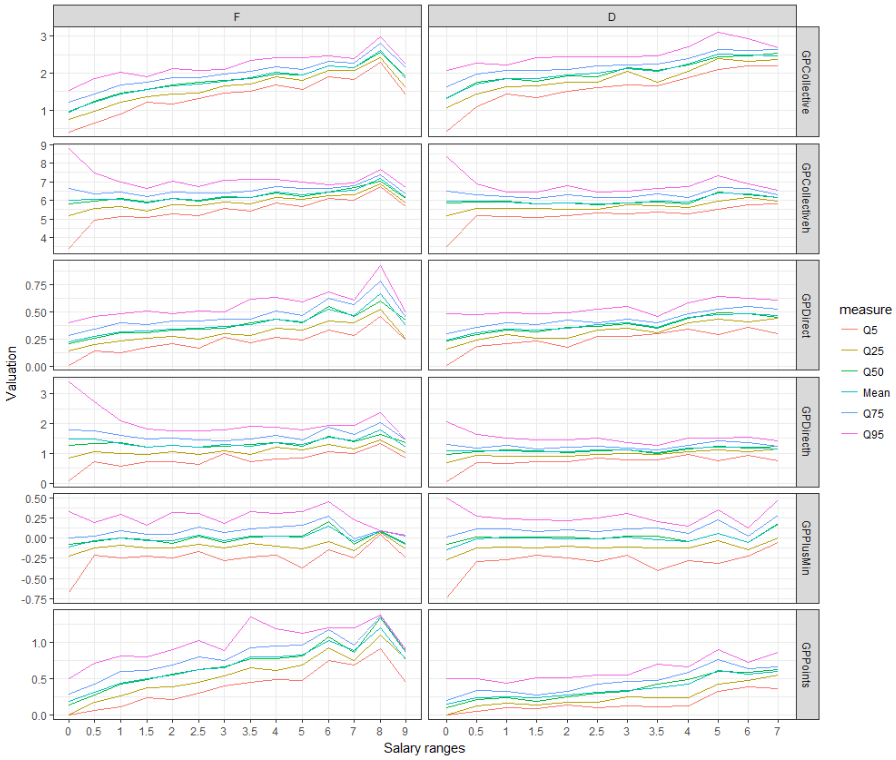
The choice of the reward value for actions in the markov decision process has not been discussed in this article nor on the ones I have read. Nevertheless, it would be interesting to do an study of the impact of different sets of rewards for more than one action at a time, and seeing how that affects the impact of a specific actions and their change.

### Time series forecasting

In terms of time series modelling and prediction, only ARIMA models have been evaluated. Other models such as Neural Networks or Gaussian Processes could be tried to evaluate whether there could be a general model that could fit players evaluation better than the mean.

### Suggestion for further research

In the light of the results got and the shape of players performance, I sincerely do not think that any model would fit significantly well the data. Instead, I would try to get a richer data set in terms of performing actions. For instance, having passes or pucks in the action variable




(a)

Figure 6.1: Quantile distribution of the regression on players performance by game to Salary Range (Valuation Salary). The range of Salary for a specific number is meant to be from that specific number to the next, (eg. 0 is from 0-0.5M). This is done for each Combination of Positions and Metric for Season 2007-2008 and 2008-2009, excluding Goalkeepers and Forward

would enable to have a better precision of the action's impact and therefore of the player performance as in [22], [23] or [14]. Also, the new dataset could have non-discretized or less discretized the Zone variable to quantify better the impact actions as in the latter papers above.

As a matter of fact, I have not given any clear rule to decide whether to hire/fire/maintain a player. Some new research could involve quantile analysis to decide whether a player would be fired/hired/maintained. In figure 6.1, I have discretized the salary range of players such that a specific number goes from that number to the next one on the x-axis (e.g. if 0, from 0-0.5M). All players performance have been allocated in Salary ranges and then, for each discrete salary range, quantile performances are shown by position and range for several metrics. With that, a combination of metrics could be settled such that if a player appears to be over the 75% quantile for their specific range, then that player could be a potential hiring, and viceversa.

### Ethical issues

The data used in this thesis is public and therefore there are no sensitivity nor private potential issues. Nevertheless, the automatization of the hiring/firing process in sports and other jobs in the future could be a potential issue to discuss, as well as the implications this might arise.

## 6.3 Limitations

The limitations of this thesis are explained in section 1.4. Besides time constraints, the purpose of the thesis is to learn about MDPs in Ice Hockey using the NHL Dataset from [20]. It also studies the usage of the MDP in Ice Hockey as well as Time Series Modelling for the impacts derived from MDPs using ARIMA models to forecast player's performance and taking decisions based on these predictions and the current performances. The thesis purpose does not try to improve or optimize the model nor the solving algorithms.

# 7 Conclusion

In this thesis, I have used AD-trees to summarize Ice Hockey matches using state variables, which englobes context and action variables, and then using Markov Decision Processes to estimate the impact of each action under that specific state. With that, an impact measure has been described and four player metrics have been derived by match for regular Seasons 2007-2008 and 2008-2009. From that, general data analysis have been performed to understand the metrics, model player's performance using ARIMA and forecasting the following match through season.

1. **How can a MDP/RL be used to evaluate actions under certain time-series events ?** By using AD-trees to summarize actions events under context, one can possibly reward the actions one is interested in happening (e.g. as in my thesis GOAL) to evaluate how other actions have an impact in reaching that action.

2. **How can I store time series data for the usage of a MDP?** An AD-tree is a good solution to summarize actions under context. It is important to understand well data and also the context under which specific variables are happening to provide a meaningful insight. In this case, context variables such as manpower differential (MD), goal differential (GD) and Period (P) has been chosen as context variables and the player actions taking into account the Team performing the action and the Zone in which the action is occurring.

3. **Can I predict player's performance based on their performances in the previous matches ?** One can try to do it using ARIMA models. Nevertheless, the result got is that no better result is achieved rather than the mean of the previous $m$ matches.

4. **Is there a way to use MDPs to create a metric that evaluates players for hiring/maintaining/firing purposes?**

   Several metrics has been created to achieve that purpose (Direct, Direct/h, Collective and Collective/h). Neverthess, none of them seems good enough by itself to decide using one specific metric whether a player should be hired/fired/maintained. Therefore, not an specific answer has been given to that particular question. Nevertheless, in the further research section 6.2, a combination of relevant metrics per position of a player could be done using quantile analysis and salary range of a player to decide whether to hire/fire/maintain a specific player.

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