Neural Networks and Learning Systems
TBMI 26, 2018

# Lecture 4
# Ensemble Learning

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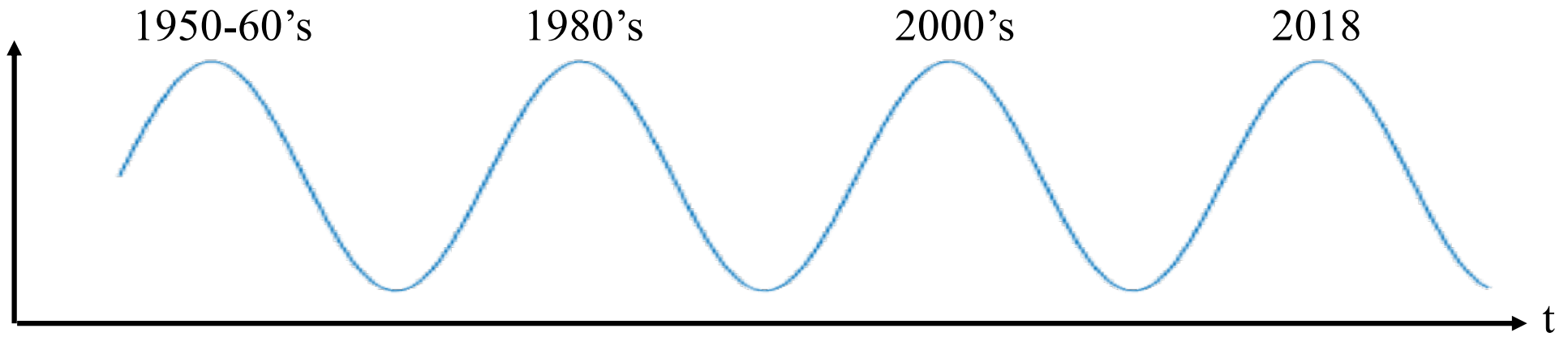
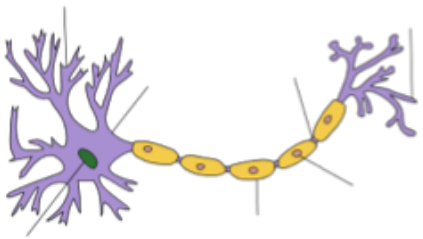
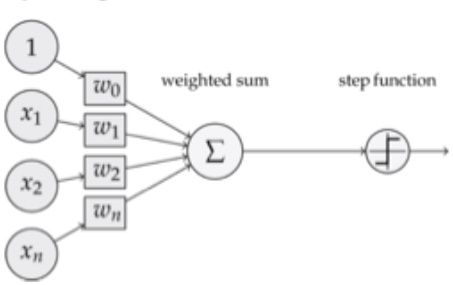
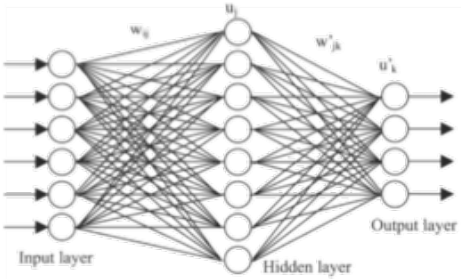
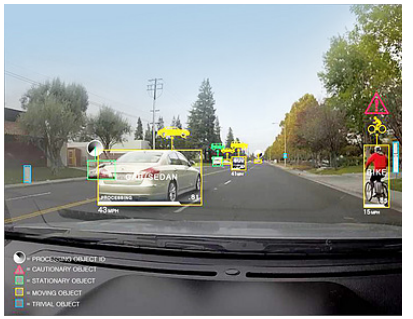
# Machine Learning History

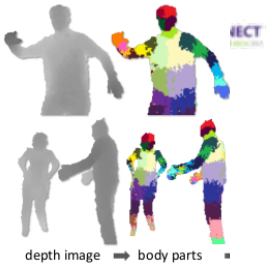


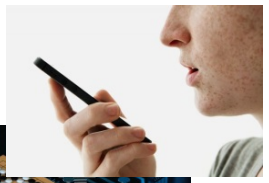



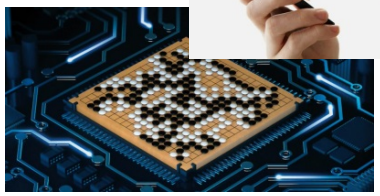






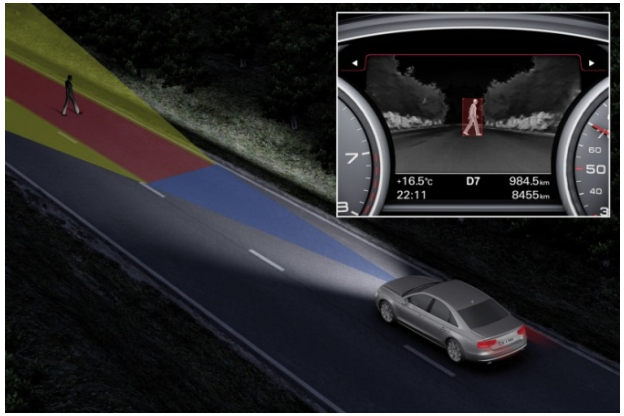
# History

(roughly)

- 1960's (and before): Linear methods, perceptron, LDA
- 1980's: Nonlinear breakthroughs, neural networks
- 1990's-2010:
  - Kernel methods, Support Vector Machines
  - Ensemble learning, Boosting, Random Forest
- 2010 -
  - Deep neural networks

# Applications

Face detection



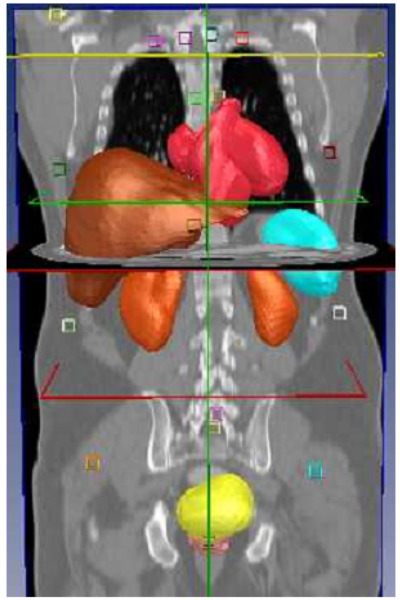
Pedestrian detection


J. Shotton et al., *"Real-Time Human Pose Recognition in Parts from Single Depth Images"*

Pose estimation

Organ detection

Barbu et al. *"Marginal Space Learning for Fast Object Detection in Medical Imaging"*

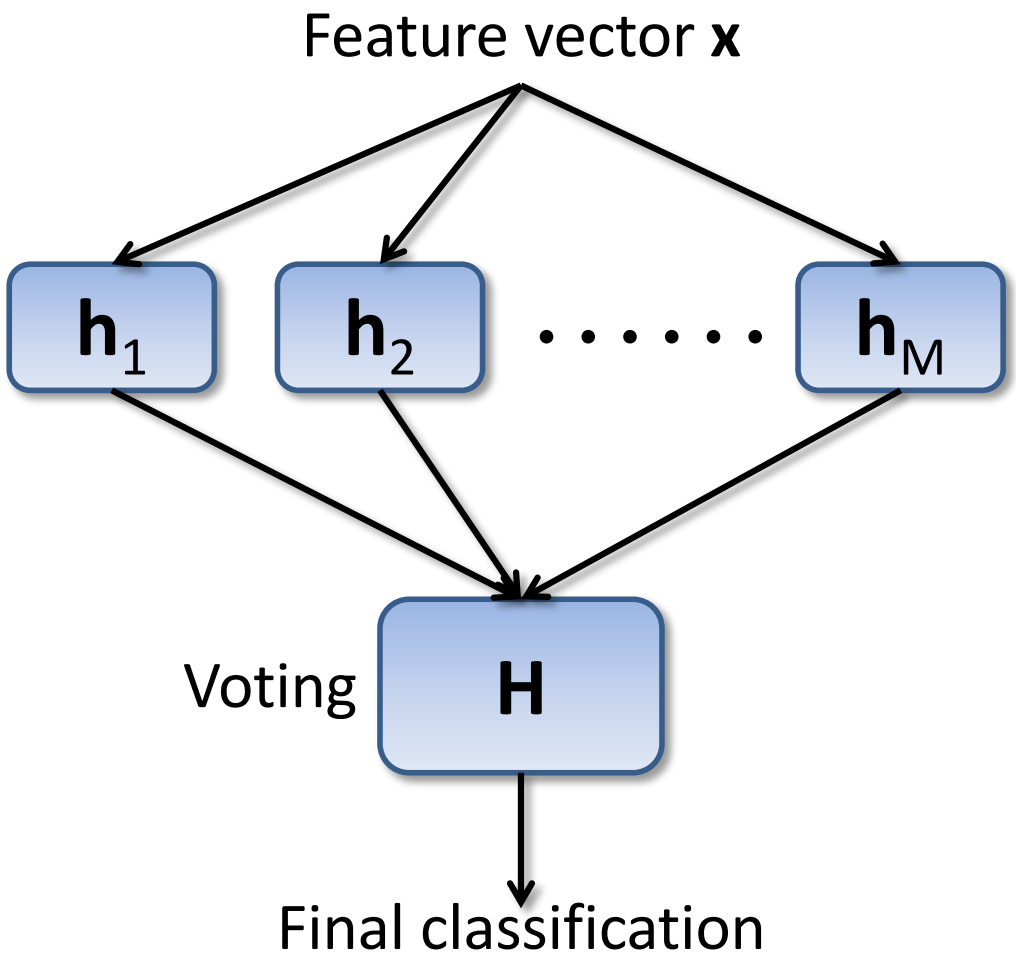


# One big vs. Many small

HE FOUND THAT HIS ARMS AND LEGS WERE TIGHTLY FASTENED TO THE GROUND.

# Classifier ensemble

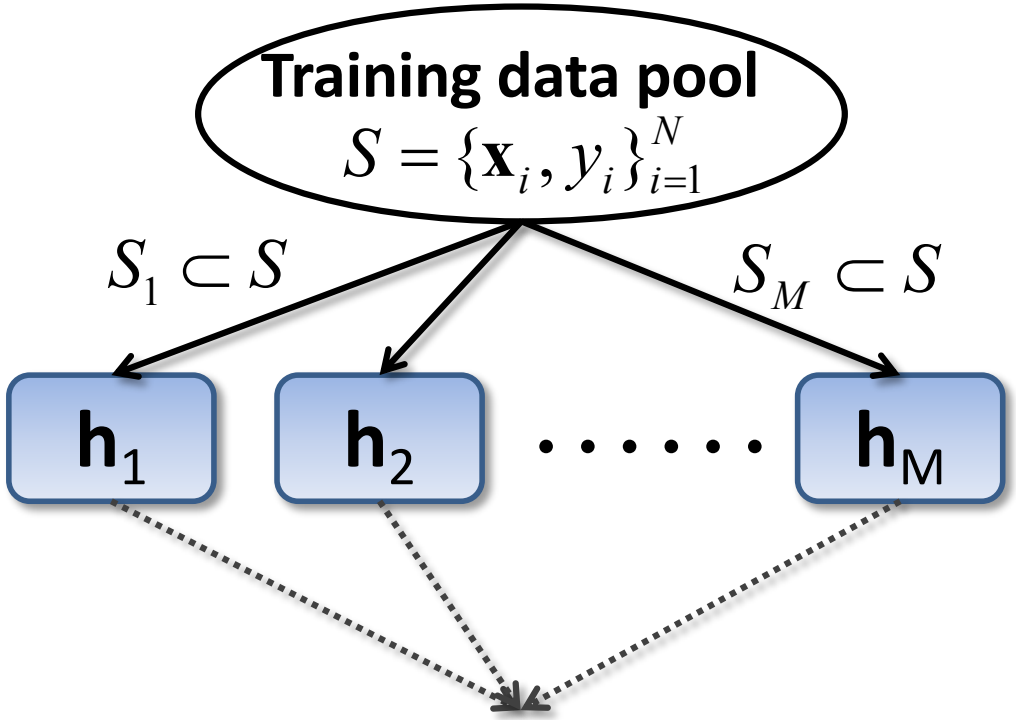
Ensemble/committee of different *base-classifiers*



# **Bootstrap Aggregating** (Bagging)

Breiman,1994

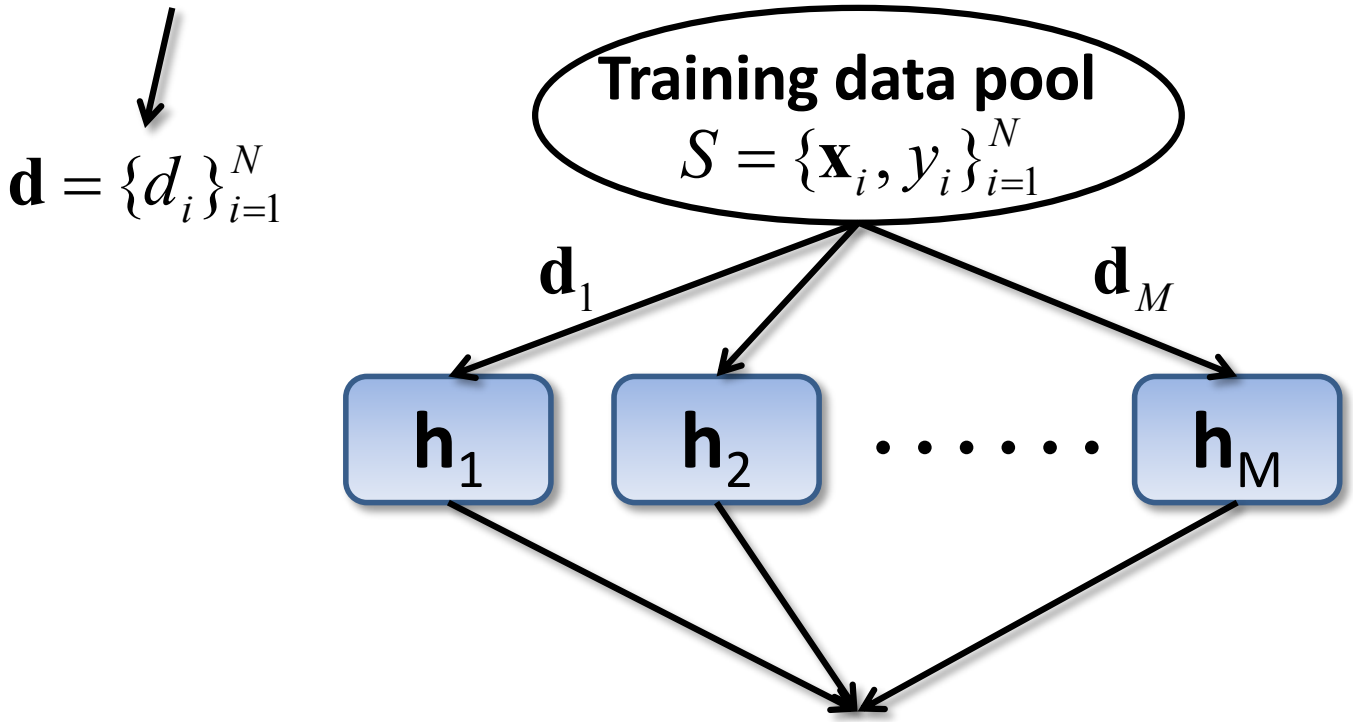
Train each base-classifier using a subset of the training data



# Boosting

Schapire and others, (1989-1990)

Train each base-classifier using all training data but with weights indicating how important each training sample is.



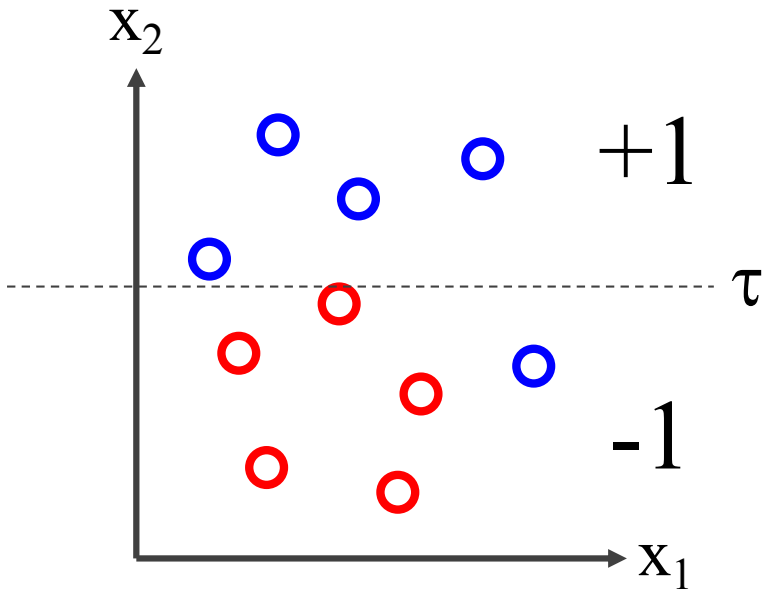
# Simple/Weak classifiers

cf. ”a rule of thumb”

$$\mathbf{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_N \end{pmatrix}$$

Example: Threshold ***one*** feature

$$h(x_2) = \begin{cases} +1 & x_2 \geq \tau \\ -1 & x_2 < \tau \end{cases}$$

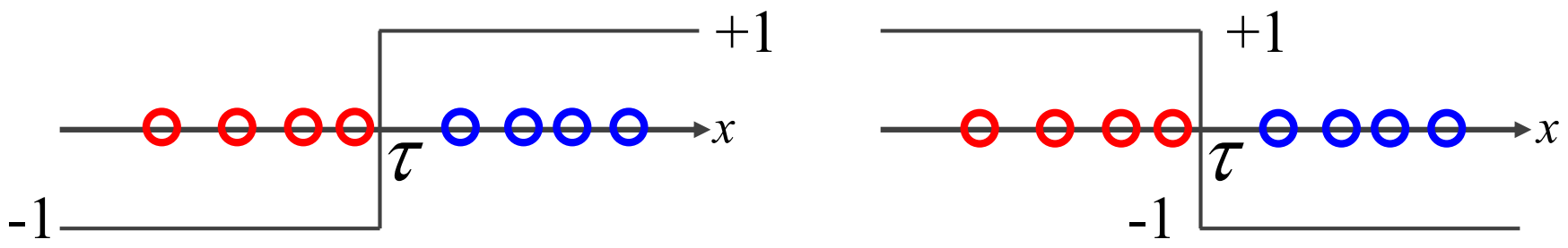


# Decision stump

Polarity $p = \{-1, 1\}$

$p = 1$ $\qquad$ $p = -1$

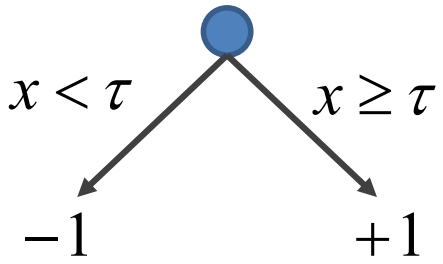
$$h(x) = \begin{cases} +1 & x \geq \tau \\ -1 & x < \tau \end{cases} \qquad h(x) = \begin{cases} +1 & x \leq \tau \\ -1 & x > \tau \end{cases}$$



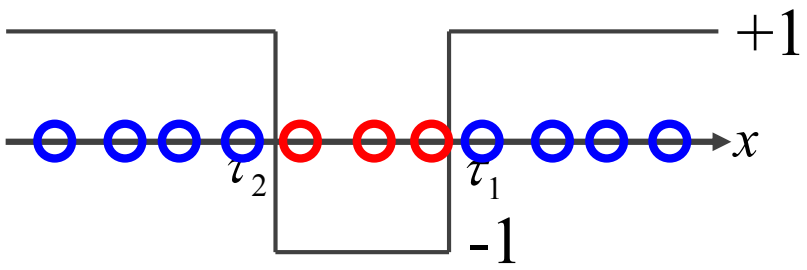
# Classification and Regression Trees (CART)

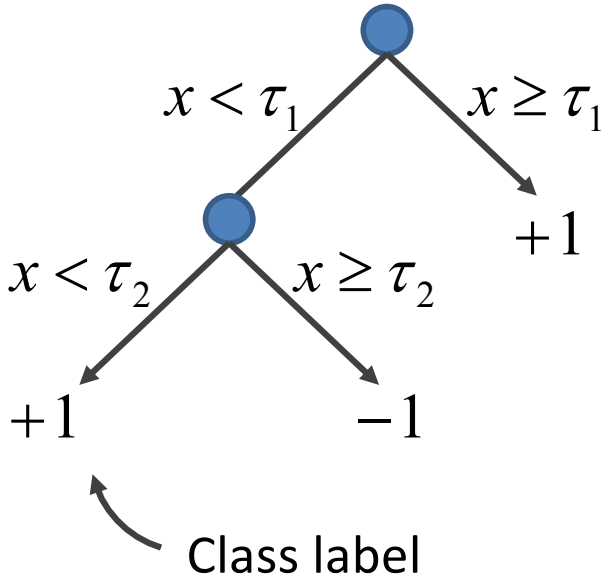
$$h(x) = \begin{cases} +1 & x \geq \tau \\ -1 & x < \tau \end{cases}$$

Decision stump

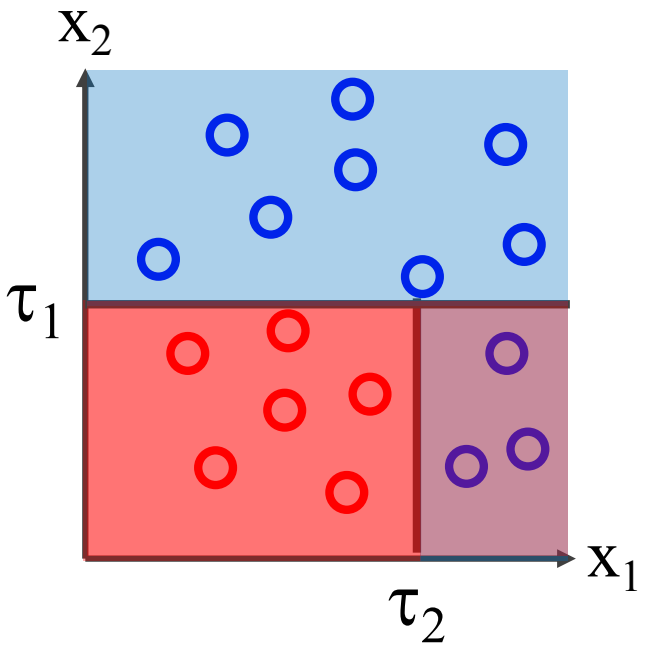


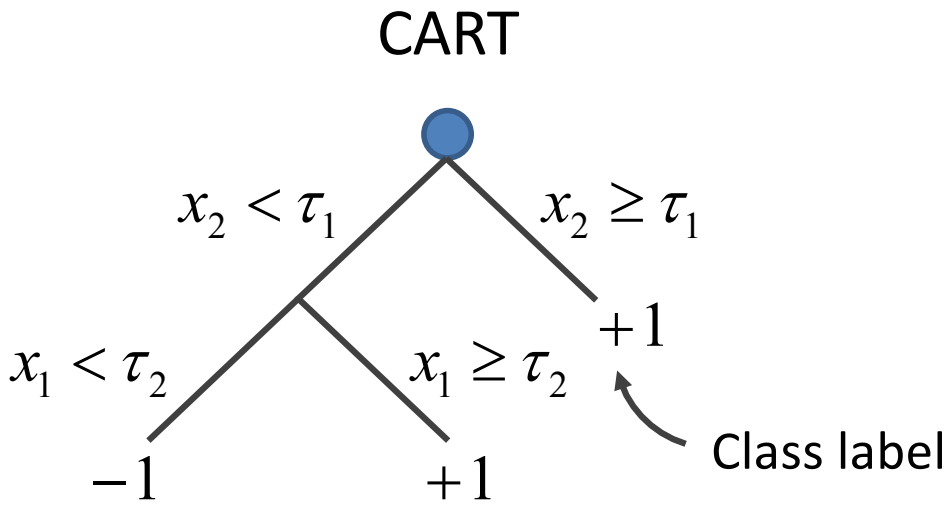

$$h(x) = \begin{cases} +1 & x \geq \tau_1 \\ -1 & x < \tau_1 \text{ and } x \geq \tau_2 \\ +1 & x < \tau_2 \end{cases}$$



# CART – 2D example




Piecewise flat classification function

$$f(\mathbf{x}; \underbrace{w_1, \ldots, w_k}) \rightarrow \Omega$$

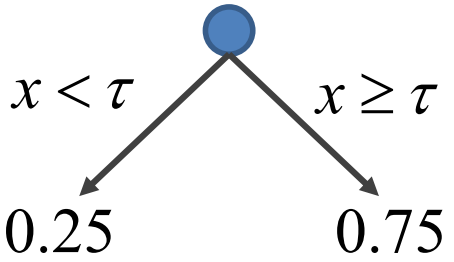
Feature index and thresholds

# Regression Tree

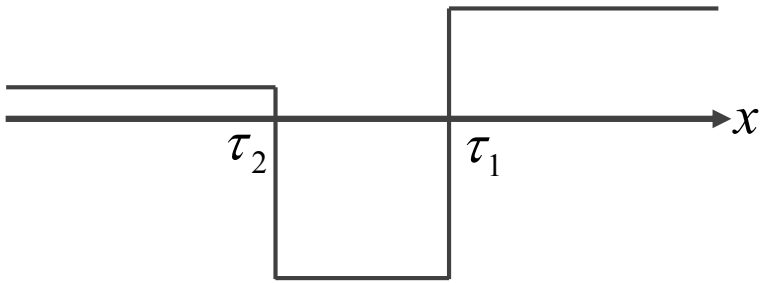
Same, but with real-valued output!

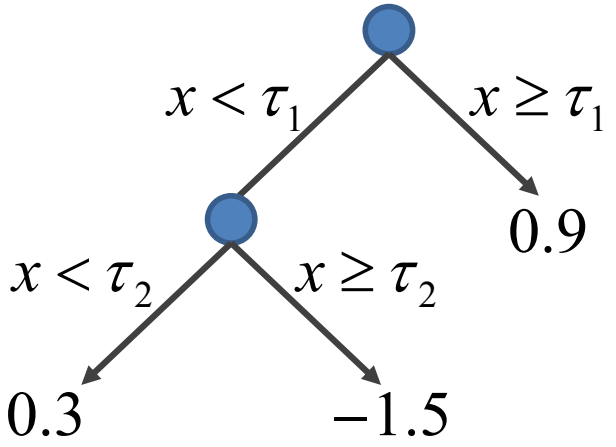
$$h(x) = \begin{cases} 0.75 & x \geq \tau \\ 0.25 & x < \tau \end{cases}$$

Decision stump

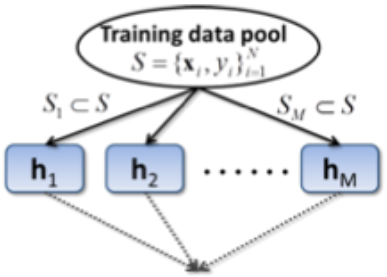



$$h(x) = \begin{cases} 0.9 & x \geq \tau_1 \\ -1.5 & x < \tau_1 \text{ and } x \geq \tau_2 \\ 0.3 & x < \tau_2 \end{cases}$$

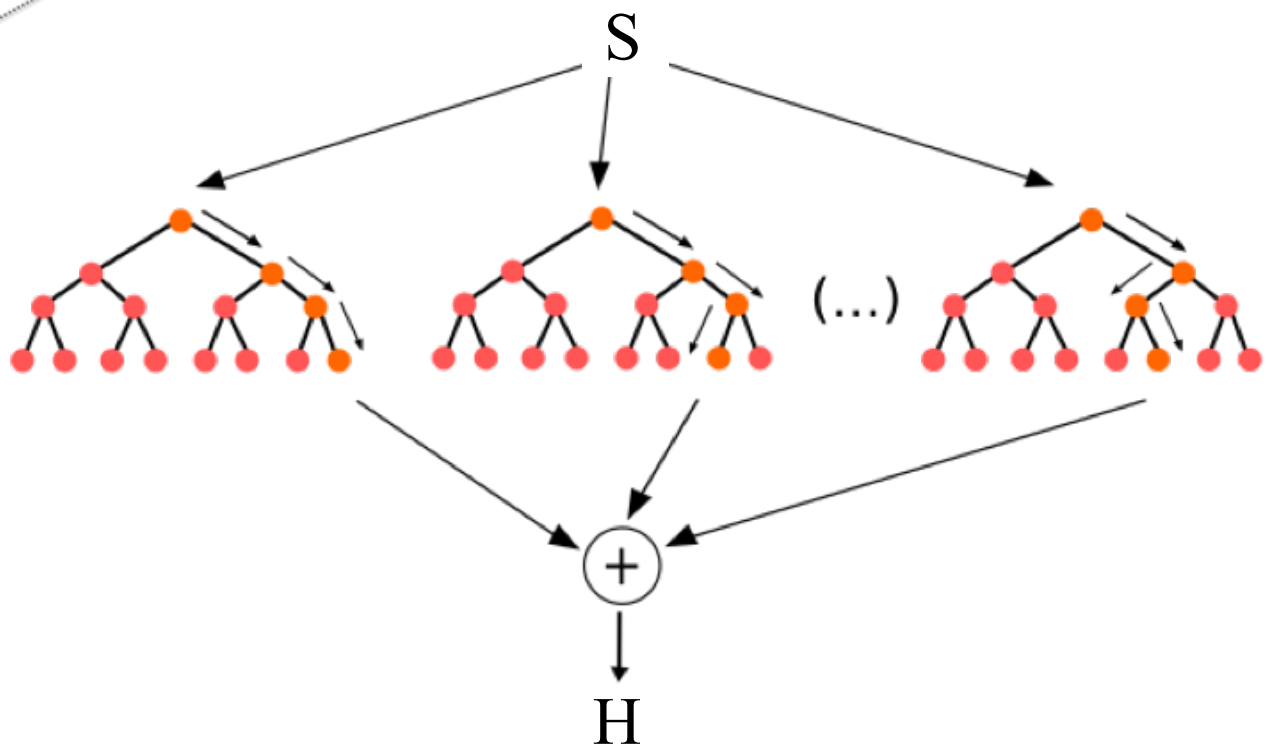


# Random Forest




Bagging + Decision Trees




1. For each tree, select a random subset of the training samples
2. For each split, select a random subset of the features

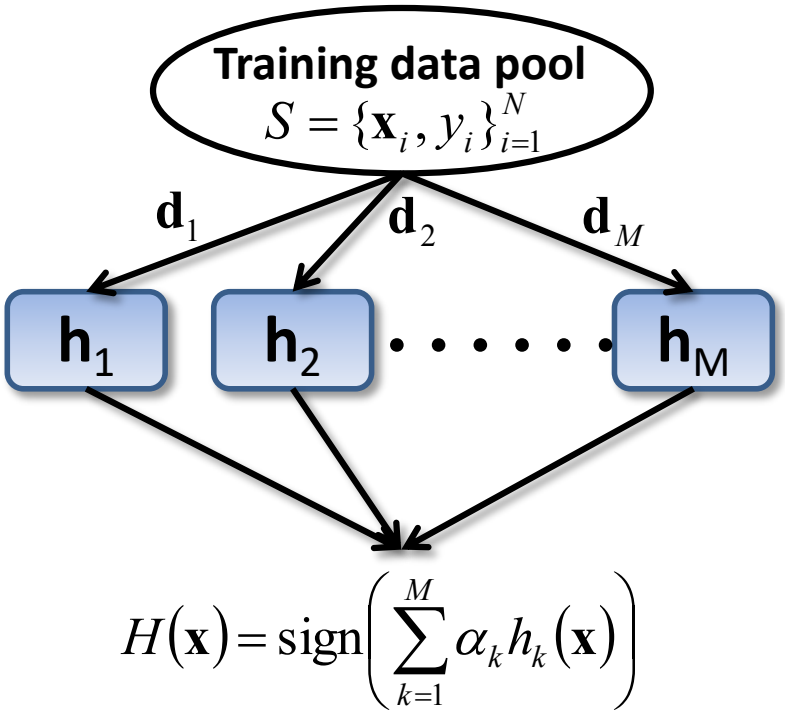
} Make the trees "independent"



Image from https://towardsdatascience.com/random-forest-learning-essential-understanding-1ca856a963cb

# General boosting algorithm

Train weak classifiers sequentially!




1. Set weights $d_1$=1/N
2. Train weak classifier $h_1(\mathbf{x})$ using weights $\mathbf{d}_1$
3. Increase and decrease weight for wrongly and correctly classified training examples respectively -> $\mathbf{d}_2$
4. Train weak classifier $h_2(\mathbf{x})$ using weights $\mathbf{d}_2$
5. Repeat until $h_M(\mathbf{x})$

# Training a decision stump

Find best split threshold $\tau$!

Class label {-1,+1}

Training input: $\{x_i, y_i, d_i\}_{i=1}^{M}$

Consider only one feature

Weight

Normalized weights: $\sum_{i=1}^{M} d_i = 1$

Threshold function: $h(x;\tau,p) = \begin{cases} +1 & p\,x \geq p\,\tau \\ -1 & p\,x < p\,\tau \end{cases}$

Polarity {-1,1}

0-1 cost function: $\min_{\tau,p} \varepsilon(\tau,p) = \sum_{i=1}^{M} d_i \, I\big(y_i \neq h(x_i;\tau,p)\big)$

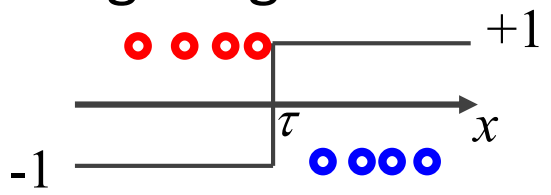
1 for false classifications

# Training a decision stump, cont.

$$\min_{\tau,p} \varepsilon(\tau,p) = \sum_{i=1}^{M} d_i \, I\big(y_i \neq h(x_i;\tau,p)\big) \text{ is always} \leq 0.5!$$

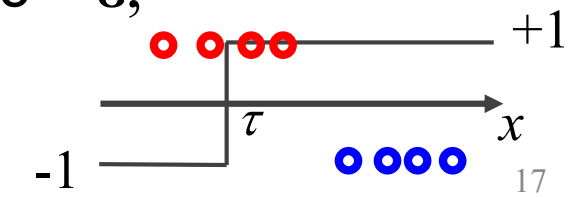
Why? If we classify <u>all</u> training samples wrong we get:

$$\varepsilon(\tau,p) = \sum_{i=1}^{M} d_i = 1$$




But we can then just change polarity/sign and get all samples correct, i.e., ε = 0!

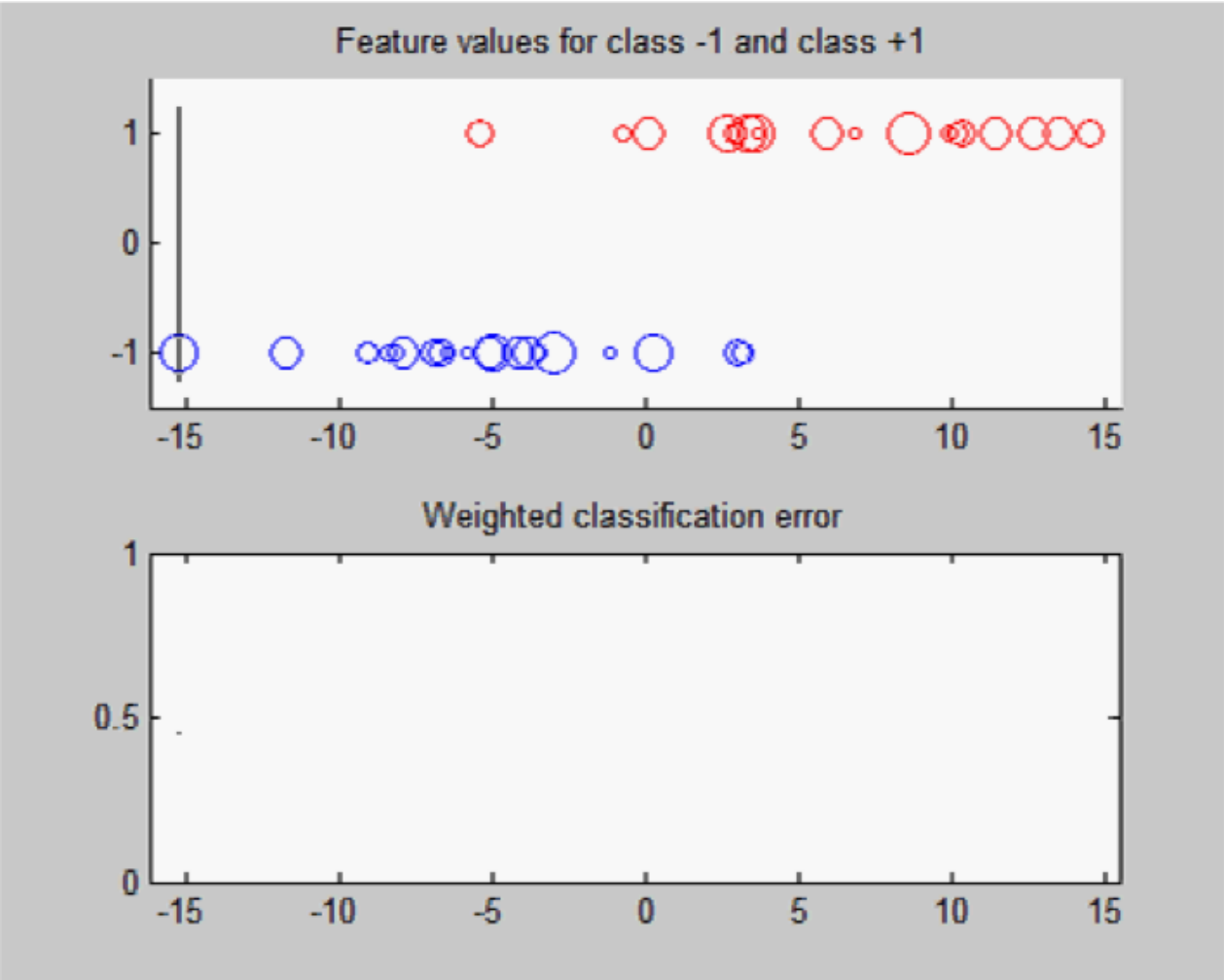
In general, if we obtain an error ε between 0.5 and 1.0, we can switch polarity and get the error 1.0 – ε, which is smaller than 0.5.



# Brute force optimization

$$\min_{\tau,p} \varepsilon(\tau,p) = \sum_{i=1}^{N} d_i \, I\left(y_i \neq h(x_i;\tau,p)\right)$$

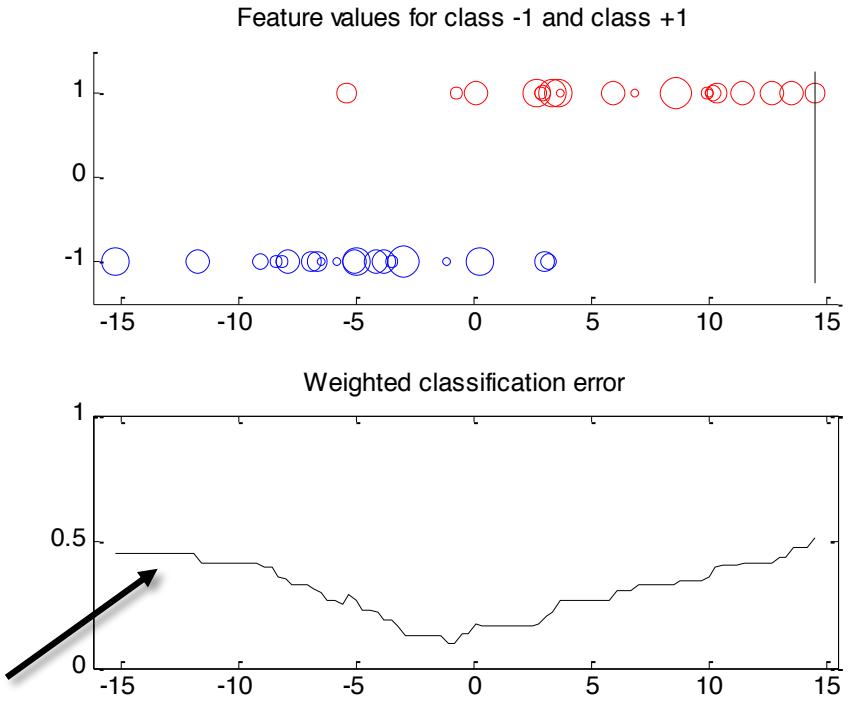
Movie!



# Brute force optimization

$$\min_{\tau,p} \varepsilon(\tau, p) = \sum_{i=1}^{M} d_i \, I\big(y_i \neq h(x_i; \tau, p)\big)$$

Cost-function jumps at the $x_i$:s. Enough to test all thresolds in the set $\tau \in \{x_i\}_{i=1}^{N}$ and see which one gives the smallest error.




Cost function is piece-wise constant!

# Brute force optimization

$$\text{Training samples } \mathbf{x}_i = \begin{pmatrix} x_{1,i} \\ \vdots \\ x_{N,i} \end{pmatrix}, i = 1 \cdots M$$

# training samples

# feature components

Pseudo code:

$\varepsilon_{min} = \inf$
**for all** feature components k = 1:N
  **for all** thresholds $\tau \in \{x_{k,i}\}_{i=1}^{M}$

$$\varepsilon(\tau, p=1) = \sum_{i=1}^{M} d_i \, I(y_i \neq h(x_i; \tau, p=1))$$

    **if** $\varepsilon > 0.5$
      p = -1
      $\varepsilon = 1 - \varepsilon$
    **end**
  **if** $\varepsilon <$ $\varepsilon_{min}$ ..... **end**
  **end**
**end**

# Discrete AdaBoost

Freund & Schapire, 1995

Training data: $\{\mathbf{x}_i, y_i\}_{i=1}^{M}$, $y_i \in \{-1,+1\}$

Initialization: $d_1(i) = \frac{1}{M}$, $T = \#\,\text{base classifiers}$

**for** t = 1 to T

Find weak classifier $h_t(\mathbf{x})$ = {-1,+1} that minimizes the weighted classification error:

$$\varepsilon_t = \sum_{i=1}^{M} d_t(i) I\left(y_i \neq h_t(\mathbf{x}_i)\right)$$

Update weights:

$$d_{t+1}(i) = d_t(i) e^{-\alpha_t y_i h_t(x_i)}, \text{ where } \alpha_t = \frac{1}{2}\ln\frac{1-\varepsilon_t}{\varepsilon_t}$$

and renormalize so that $\sum_{i=1}^{M} d_{t+1}(i) = 1$

**end**

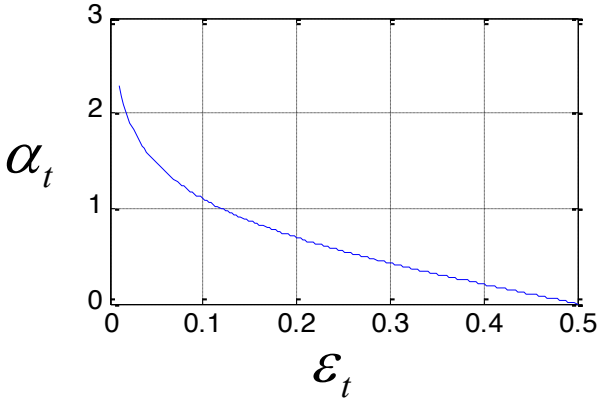
Final strong classifier: $H(\mathbf{x}) = \text{sign}\left(\sum_{t=1}^{T} \alpha_t \mathrm{h}_t(\mathbf{x})\right)$

# Discrete AdaBoost

- Discrete output from the weak classifier $h_t(\mathbf{x}) = \{-1,+1\}$
- Weight update

$$d_{t+1}(i) = d_t(i)e^{-\alpha_t y_i h_t(\mathbf{x}_i)} = \begin{cases} d_t(i)e^{-\alpha_t} & \text{If } \mathbf{x}_i \text{ correctly classified} \\ d_t(i)e^{\alpha_t} & \text{If } \mathbf{x}_i \text{ wrongly classified} \end{cases}$$

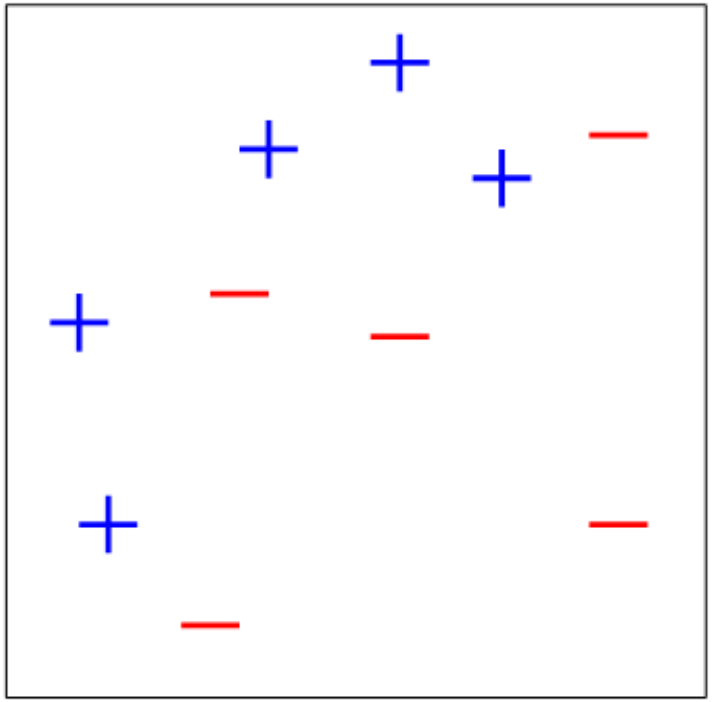
- Performance of weak classifier: $\alpha_t = \frac{1}{2}\ln\frac{1-\varepsilon_t}{\varepsilon_t}$




Final strong classifier: $H(\mathbf{x}) = \text{sign}\left(\sum_{t=1}^{T}\alpha_t h_t(\mathbf{x})\right)$

# Discrete AdaBoost – Toy example

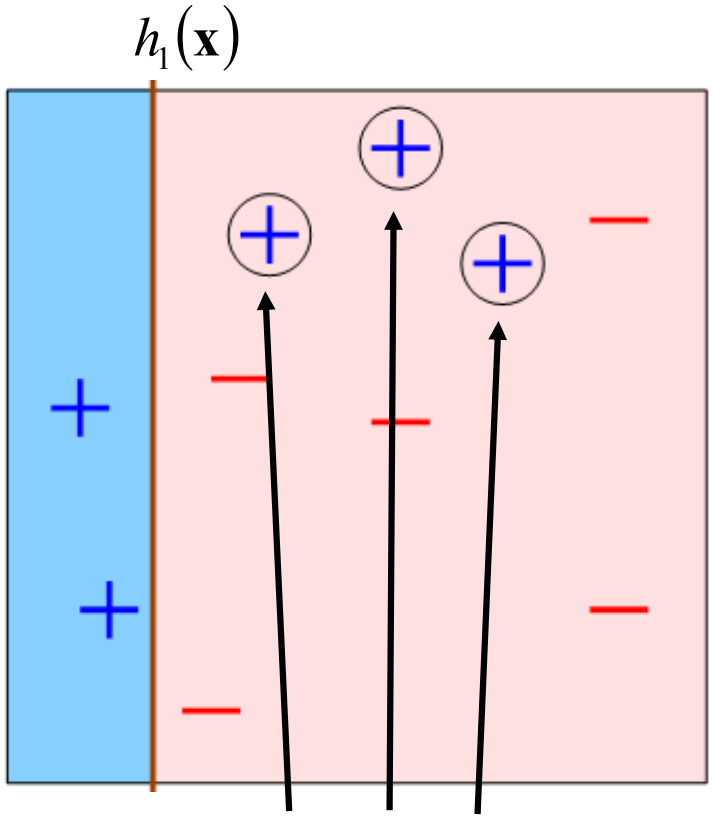
Initial weights $d_1(i) = \frac{1}{10}$, $T = 3$

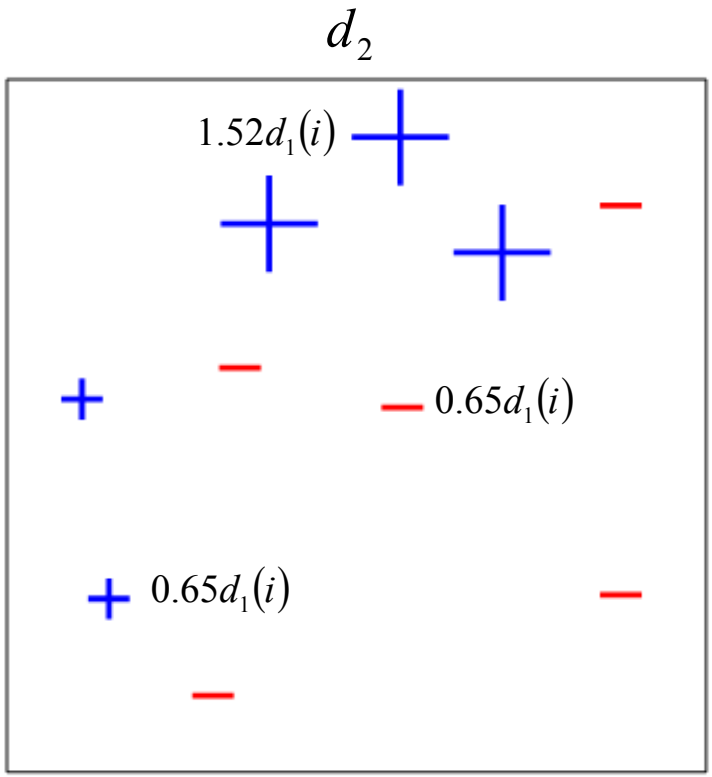


10 training samples $\mathbf{x}_i$, $i = 1 \cdots 10$

Images borrowed from R. Schapire, ”A Boosting Tutorial”

# Discrete AdaBoost – Toy example




$$d_1(i) = \frac{1}{10}$$

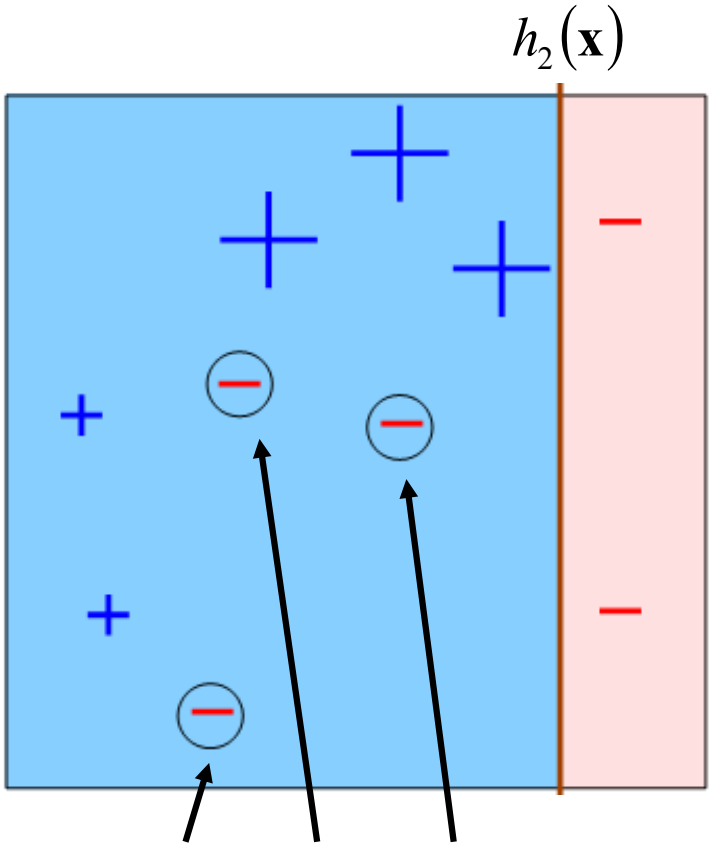
$$\varepsilon_1 = \left(\frac{1}{10} + \frac{1}{10} + \frac{1}{10}\right) = 0.3$$

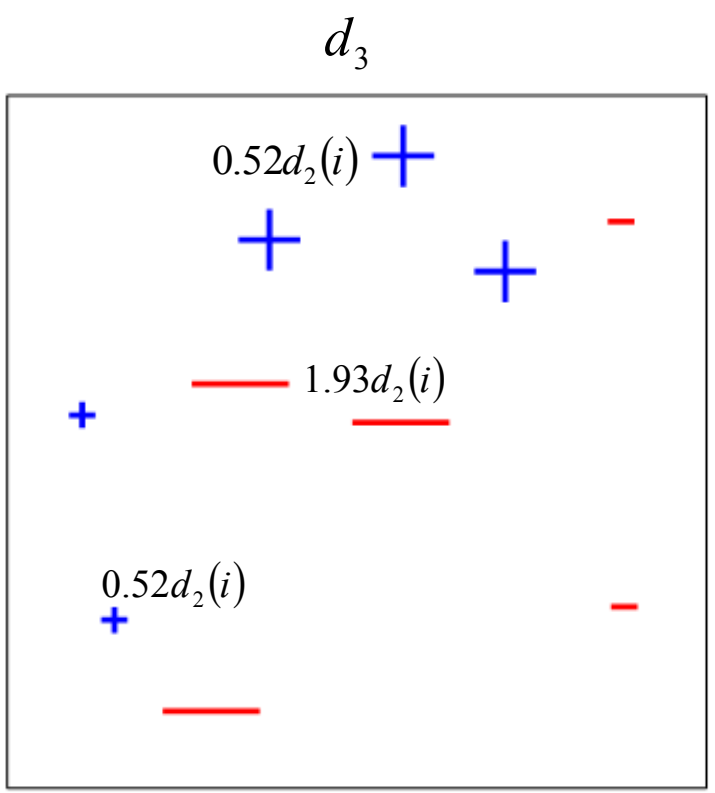
$$\alpha_1 = \frac{1}{2}\ln\left(\frac{1-0.3}{0.3}\right) = 0.42$$

$$d_2(i) = \begin{cases} d_1(i)e^{-0.42} = 0.65d_1(i) & \text{If } \mathbf{x}_i \text{ correctly classified} \\ d_1(i)e^{0.42} = 1.52d_1(i) & \text{If } \mathbf{x}_i \text{ wrongly classified} \end{cases}$$

Normalize $d_2$!

# Discrete AdaBoost – Toy example



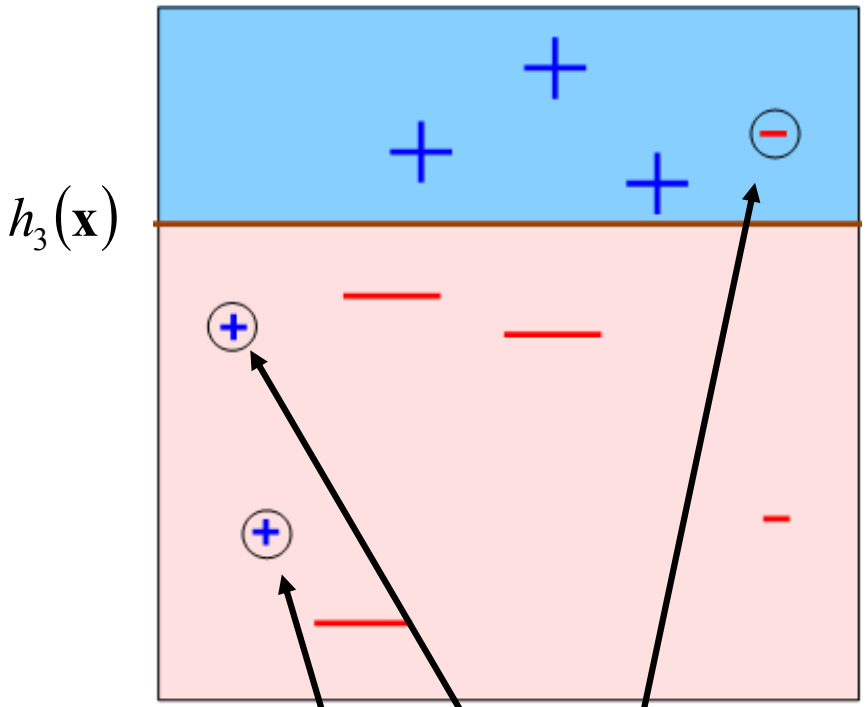

$$\varepsilon_2 = \left(d_2(j) + d_2(k) + d_2(l)\right) = 0.21$$

$$\alpha_2 = \frac{1}{2}\ln\left(\frac{1-0.21}{0.21}\right) = 0.66$$

$$d_3(i) = \begin{cases} d_2(i)e^{-0.66} = 0.52d_2(i) & \text{If } \mathbf{x}_i \text{ correctly classified} \\ d_2(i)e^{0.66} = 1.93d_2(i) & \text{If } \mathbf{x}_i \text{ wrongly classified} \end{cases}$$

Normalize $d_3$!

# Discrete AdaBoost – Toy example



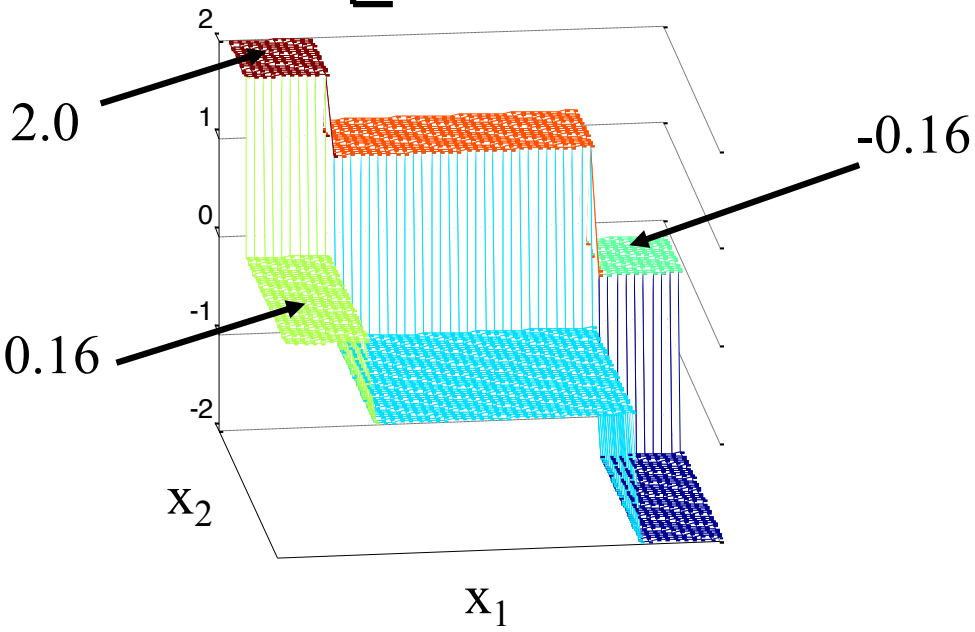

$$\varepsilon_3 = \left(d_3(p) + d_3(q) + d_3(r)\right) = 0.14$$

$$\alpha_3 = \frac{1}{2}\ln\left(\frac{1-0.14}{0.14}\right) = 0.92$$

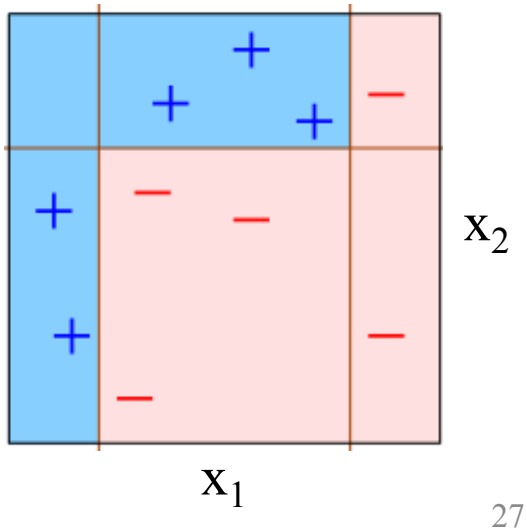
# Discrete AdaBoost – Toy example

Final strong classifier: $H(\mathbf{x}) = \text{sign}\left(\sum_{t=1}^{T} \alpha_t h_t(\mathbf{x})\right)$

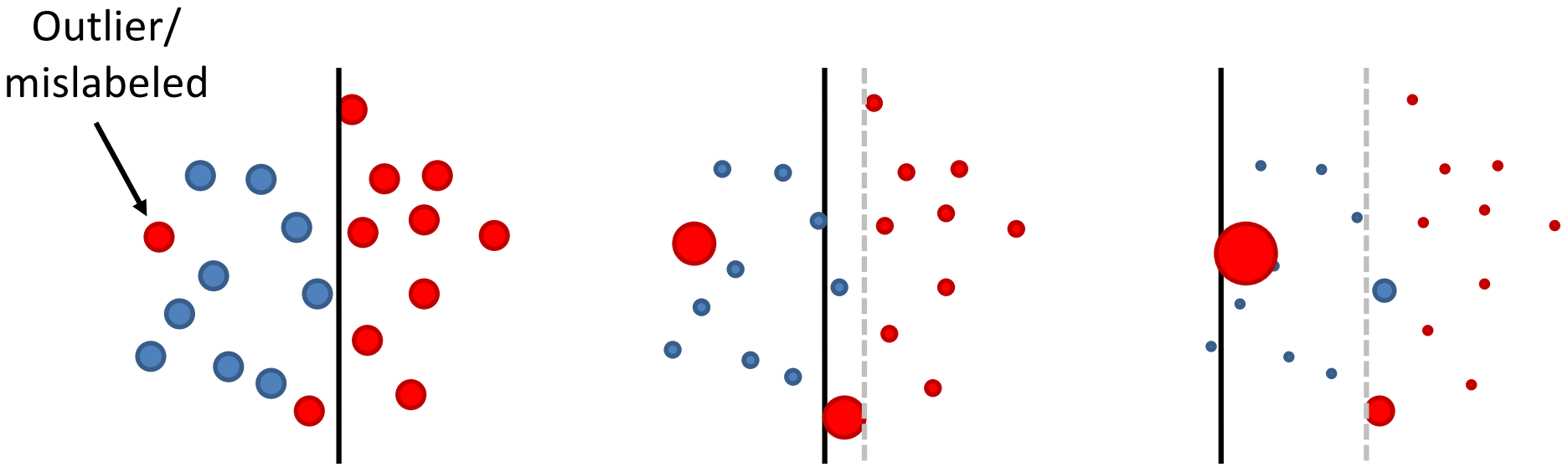
$H(\mathbf{x}) = \text{sign}\left[0.42 + 0.66 + 0.92\right]$




$H(\mathbf{x}) =$



# Problem - Outliers




- Outliers gain weight exponentially
- Will eventually result in bad weak classifiers
- May ruin the final ensemble classifier

# Outlier strategies

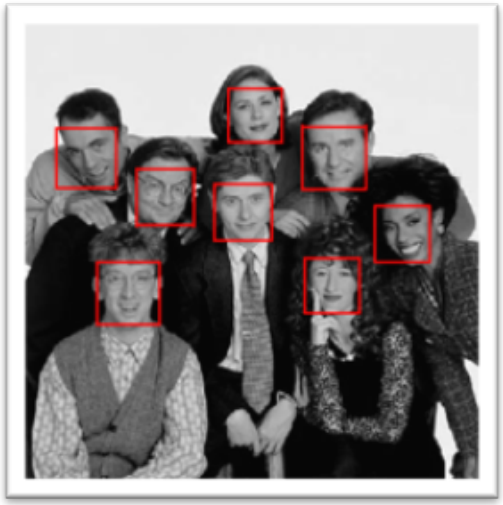
- Keep an eye on the weights (plot them!)
- Weight trimming
  - Don’t allow weights larger than a certain threshold
  - Disregard training samples with too large weights
- Use alternative weight update schemes with less agressive increases for misclassified training data
  - LogitBoost
  - GentleBoost

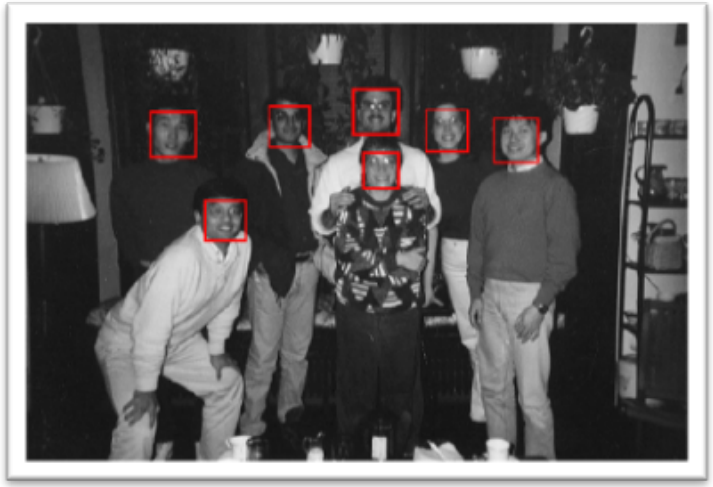
# Summary Ensemble Learning
(using decision trees)

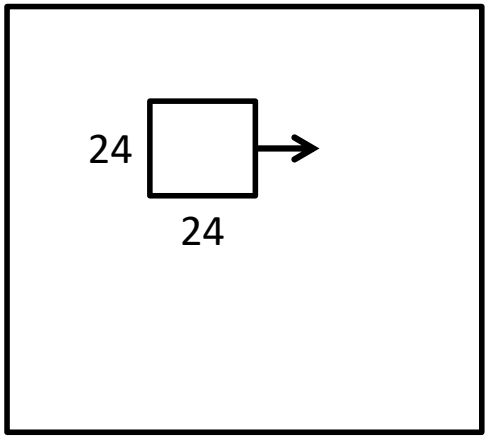
- Nonlinear classifiers that are easy to implement
- Easy to use - just one or a few parameters (T)
- Inherent feature selection
- Slow to train – fast to classify (real-time)
- Look out for outlier problems (AdaBoost)

# Real-time object detection

P. Viola and M. Jones "*Rapid Object Detection using a Boosted Cascade of Simple Features*", 2001



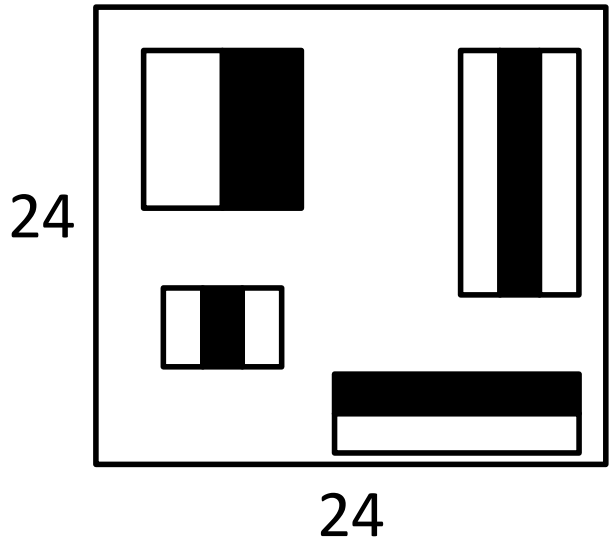





Sweep a sub-window over the image. for each position, determine if the sub-window contains a face or not.

# Haar-features

Rectangle filters



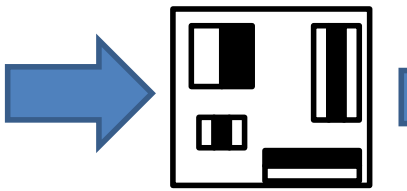

From the sub-window, calculate contrast features (Haar features) at different locations and scales, and in different orientations. LOTS of different combinations, can be several 100,000 features!

# Train detector with AdaBoost

As described previously!

24 x 24 pixels face images

Apply Haar filters to each image



$$\mathbf{x}_i = \begin{pmatrix} x_{1,i} \\ \vdots \\ x_{N,i} \end{pmatrix}, i = 1 \ldots M$$

Similar with non-face images to obtain negative examples.