Carles Sans Fuentes Information sheet 732A62

If it causal it is stationary. It does not hold inversely. AR in one side MA in another. All roots must be $abs > 1$. AR for causal and MA for invertible. Covariance: $cov(X,X) = var(X)$, $cov(X,a) = 0$, $cov(X,Y) = cov(Y,X)$; $cov(aX,bY) = ab * cov(X,Y)$; $cov(X+a,Y+b) = cov(X,Y)$;$Var(X+Y) = Var(X)+Var(Y)+2Cov(X,Y)$ $Var(X-Y) = Var(X)+Var(Y)-2Cov(X,Y)$

$cov(aX+bY,cW+dV) = aw * cov(X,W) + ad * cov(X,V) + bc * cov(Y,W) + bd * cov(Y,V)$

For AR(1): $x_t = \phi x_{t-1} + w_t; \gamma(h) = \frac{\phi^h}{1-\phi^2} * \sigma_w^2$;; AR(2): $x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + w_t$;

MA(2): $x_t = \mu + w_t + \theta_1 w_{t-1} + \theta_2 w_{t-2}$;;

Autocovariance: $\gamma(t,t+h) = \frac{1}{n}\sum_{t=1}^{n}(X_{t+h} - \hat{X})(X_t - \hat{X})$;; Autocorrelation: $\rho(h) = \frac{\gamma(h)}{\gamma(0)}$

Characteristics of different models: MA(1): non-zero correlation only at lag 1- Could be positive but only between -0.5 and 0.5;; MA(2): Has nonzero correlation only at lags 1 and 2;;AR(1) Has exponentially decaying autocorrelations starting from lag 0. If $\phi > 0$, then all autocorrelations are positive, If $\phi < 0$, then autocorrelations alternate negative,positive, negative...;; AR(2) Autocorrelations have different patterns but if roots of the characteristic equations are complex numbers, then the pattern will be a cosine with a decaying magnitude;;ARMA(1,1): Has exponentially decaying autocorrelations starting from lag 1. But Not from lag 0.

For method of moments (Yule-Walker) model until ϕ_2 we have: $\hat{R}_p = \begin{bmatrix} p(1-1) & p(2-1) \\ p(2-1) & p(1-1) \end{bmatrix}$

$\hat{\phi} = \hat{\Gamma_p^{-1}}\hat{\gamma_p}$ and $\hat{\sigma_w^2} = \gamma(\hat{0}) - \hat{\gamma_p'}\Gamma_p^{-1}\hat{\gamma_p}$. Else, divided everything by $\gamma(0)$ we get that

$\hat{\phi} = \hat{R_p^{-1}}\hat{\rho_p}$ and $\hat{\sigma_w^2} = \gamma(\hat{0})(1 - \hat{\rho_p'}R_p^{-1}\hat{\rho_p})$ being $R_p = \rho(k-j), k, j = 1...p$ and $\rho_p = (\rho(1)...\rho(p))$

where $\rho_p = \begin{bmatrix} p(1) \\ p(2) \end{bmatrix}$ A^{-1} of a matrix 2x2 is $\frac{1}{\det(A)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

ARMA truncated Prediction:

$\hat{x}_{n+m}^n = \phi_1 \hat{x}_{n+m-1}^n + ... + \phi_p \hat{x}_{n+m-p}^n + \theta_1 \hat{w}_{n+m-1}^n + ... + \theta_q \hat{w}_{n+m-q}^n$

$\hat{w}_t^n = x_n^t - \phi_1 \hat{x}_{t-1}^n - ... - \phi_p \hat{x}_{t-p}^n - \theta_1 \hat{w}_{t-1}^n - ... - \theta_q \hat{w}_{t-q}^n$ Assumptions: $\hat{x_t^n} = x_n$; $1 \le t \le n$, $\hat{x_t^n} = 0$; $t \le 0$; $\hat{w_t^n} = 0$; $t \le 0$; or $t > n$

AR truncated prediction: $x_{n+10}^n = \mu + \prod_{i=1}^n \phi_i * (x_n - \mu)$

Find the conditional least square estimate of model parameter for MA(1) ($x_t = w_t + \theta w_{t-1}$):

$min_\theta \sum_{t=1}^n w_t^2$. Use Taylor expansions for w_t, and after isolation w_t, (Use back shift operator and put to other side if needed). Remember $\sum_{j=1}^n B^j x_t = x_{t-j}$ and $(1+\theta * B)^{-1}x_t = \sum_{j=0}^{\inf} * (-1)^j * B^j * x_t$

Noise variance: $\hat{Var}(w_t) = \frac{1}{n-1} * \sum_{i=1}^n * (w_i - \hat{w})^2$, and $\hat{w}$ is usually 0.

Coefficient matching: $x_t = \sum_{j=1}^{\inf}(\psi * W_{t-j})$

Difference (Homogeneous) equations: if $0 < h < max(p,q+1) : \gamma(h) - \phi_1 * \gamma(h-1) - ... - \phi_p \gamma(h-p) = \sigma_w^2 * \sum_{j=n}^q \theta_j \psi_{n-j}$. Write Eq for all h > 0.

Right side is 0 if AR. Remember ϕ coefficients must be on "right side" (of original equation) for correct sign.

if $h >= max(p,q+1): \gamma(h) - \phi_1 * \gamma(h-1) - ... - \phi_p * \gamma(h-p) = 0$ For this second case, find z roots. Then pag 91. to find constants, equal it to values found in the part above $(0 < h < max(p,q+1))$

One step prediction: $X_{n+1}^n = \phi_{n1} X_n + ... + \phi_{nn} X_1;; \Gamma_n \phi_n = \gamma_n$

Causal AR(p): for n > p : $X_{n+1}^n = \phi_1 x_n + ... + \phi_n x_{n-p}$ $\sqrt{n} * (\beta - \hat{\beta})$ is $N(o, \sigma_w^2 * \Gamma_p^{-1})$;

$\Gamma = \begin{bmatrix} \gamma(1-1) & \gamma(2-1) \\ \gamma(2-1) & \gamma(2-2) \end{bmatrix}$;;$\sqrt{n}(\hat{\phi} - \hat{\phi} = \sqrt{n}\phi_{nn} \approx AN(\phi, \frac{1-\phi^2}{n})$

ACF or PACF CI: $c\sqrt{Var} = \varepsilon$ where c is usually 2 and ε is the estimation error. : $\hat{\phi} + -2\sqrt{\frac{1}{n} Var(\hat{\phi})}$; CI : $[\frac{-2}{\sqrt{n}}, \frac{+2}{\sqrt{n}}]$

AR(1) prediction with mean: $x_{n+1}^n - \mu = \phi^(x_n - \mu); x_{n+2}^n - \mu = \phi^2 (x_n - \mu)$

AR prediction interval: $P_{n+m}^n = \sigma_w^2 \sum_{r=0}^{m-1} (\psi_j^2)$. Find ψ with coefficient matching. In one step, use $P_{n+m}^n = \sigma_w^2$; CI: $x_{n+m}^n + -c_{\alpha/2} * \sqrt{P_{n+m}^n}; c_{\alpha/2} = 1.96$ usually.

Prediction for $x_1 = 10$ and $x_2 = 12$ for ARMA(1,1): Start with $t = 2$ then $x_t = 12$ and $x_{t-1} = 10$. Write the model $x_t = -0.5 * x_{t-1} + w_{t-1} + w_t$. Isolate w_t. Do backshift to find $w_{t-2} = 0$. With the value of w_{t-1} find w_t Now solve $x_{t+1} = -0.5 * x_t + w_t + w_{t+1}$ being $w_{t+1} = 0$

PACF values :for MA(1) $\phi_{hh} = -\frac{(-\theta)^h * (1-\theta^2)}{1-\theta^{2(n+1)}}$ for AR(p) $\phi_{hh} = \phi_h$

$x_t (1-B)(1+B^{12}) = w_t (1-0.5B)(1-0.5B^{12}) is ARIMA(0,1,1)x(0,1,1)_{12}$

General form: $ARIMA(p,d,q)x(P,D,Q)_s : \theta_P(B^s)\phi(B) \bigtriangledown_s^D \bigtriangledown^d x_t = \delta + \theta_Q(B^s)\theta(B) w_t$, being $\bigtriangledown_s^D = (1-B^s)^D$

$$\begin{bmatrix} & AR(P)_s & MA(Q)_s & ARMA(P,Q)_s \\ ACF & Tailsoffatlagsks, k=1,2... & CutsoffafterlagsQs & Tailsoffatlagsks \\ PACF & CutsoffafterlagsPs & Tailsoffatlagsks, k=1,2... & Tailsoffatlagsks \end{bmatrix}$$

GARCH: $ARMA(max(p,q),p) - GARCH(p,q)$ and p and q is the highest order of GARCH.

Conditional least squares:

Ex. MA(1), $x_t = \theta w_{t-1} + w_t$; $w_t - N(0, \sigma_w^2)$

$x_t = (1-\theta B) w_t \rightarrow w_t = (1+\theta B)^{-1} x_t = \sum_0^\infty (-1)^j \theta^j B^j x_t = \sum_0^\infty (-1)^j \theta^j x_{t-1}$

$w_t = x_t - \theta x_{t-1} + \theta^2 x_{t-2} - \theta^3 x_{t-3}$, it is only to the power of 3 due to that n = 3.

Solve w_1, w_2, w_3

Then do $min_\theta \sum_1^n w_i^2$. Derive the result so we can get θ. Tests:(1)Jarque-Bera test: H_0 residuals are normally ditributed(2) QQ-plot: H_0 data is normally distributed (3) Ljung Box: H_0 The data are independently distributed

GARCH(p,q) model: $\sigma_t^2 = \alpha_0 + \alpha_1 r_{t-1}^2 + .. + \alpha_p r_{t-p}^2 + \beta_1 \sigma_{t-1}^2 + ... + \beta_q \sigma_{t-q}^2$

Spectral Analysis:

Periodogram $P(\frac{j}{n}) = a_j^2 + b_j^2$, $a = 2$, $b = 6$ Ex. $x_t = 2cos(2\pi t \frac{j=10}{100}) + 6sin(2\pi t \frac{j=30}{100})$

When $j = 10 : P(\frac{j=10}{n=100}) = 2^2 + 0^2 = 4$ and the same for $j = 90$ When $j = 30 : P(\frac{j=30}{n=100}) = 0^2 + 6^2 = 36$ and the same for $j = 70$