Text Mining
Statistical Modeling of Textual Data
Lecture 2

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Overview

Text clustering

Hiearchical document clustering

Flat document clustering

Latent variable inference

Evaluation methods

Co-training / Semi-supervision

Section 1

Text clustering

TEXT CLUSTERING

- Unsupervised learning
  - Seldom a lot of training data.
  - More and more data
  - Probabilistic unsupervised learning, modeling our corpus

- Similary/distance based vs. generative models (toolbox vs. model)
- What do we want to cluster?
  - Words (co-occuring)
  - Text segments/documents
- Hiearchical clustering vs. flat clustering

Section 2

HIEARCHICAL DOCUMENT CLUSTERING

HIEARCHICAL DOCUMENT CLUSTERING

- Top-down
  - Start with all documents in one cluster
- Bottom-up
  - Start with each document in its own cluster
- Common approaches
  - Distance based (define different distances such as cosine, KL)
  - See *Mining text data* Ch. 4.

Section 3

Flat document clustering

FLAT DOCUMENT CLUSTERING

- **Problem:** Partion documents into K different clusters
- **Why?** Browse, identify similar documents, partition a corpus, understand.

- **Basic idea:** K-means klustering (**Wikipedia**)
- Basic algorithm:
  - **Assignment:**
    Assign all observations to clusters c_i based on centroid θ_k
  - **Update:**
    Update centroids θ_k based on cluster assignments c_i

PROBABILISTIC K-MEANS CLUSTERING*

- We need a **generative model**
- This is called a **mixture model**

$$p(y|\Theta) = \sum_{k}^{K} \pi_k p(y|C = k, \theta_k)$$

where $p(y|C = k, \theta_k)$ is a probability distribution for cluster k.

- In the context of text? Same idea - but we need a generative model for text...

NAIVE BAYES RECAP

- We model both $\mathbf{x}$ and $\mathbf{s}$ as $p(\mathbf{s}, \mathbf{x})$
- **Multivariate Bernoulli**

$$p(\mathbf{x}|s) = \prod_{j=1}^{n} p(\mathbf{x}_j|s_j)$$
$$= \prod_{j=1}^{n} \prod_{v=1}^{V} p(x_{j,v}|s_j)$$

  where $p(x_{j,v}) \sim Bernoulli(p_{v,s})$
- **Multinomial model**

$$p(\mathbf{w}|s) = \prod_{j=1}^{n} p(\mathbf{w}_j|s_j)$$

  where $p(\mathbf{w}_j|s_j) \sim MN(\theta_{s_j}, n_j)$

The document clustering situation

- **Now:** We do not know **s**, it is a *latent* variable
- **But:** Need to set K
- Choose the generative model:
  - Multinomial
  - Multivariate Bernoulli
  - von Mises Distribution
  - other...

VON MISES DISTRIBUTION

- **von Mises (Fisher) distribution**: random variable on a *circle* $x \in \{-\pi, \pi\}$ or for x^R a *hypersphere*.
  - Think of a Normal distribution on a hypersphere
  - Parameters:
    - $\boldsymbol{\mu}$ (point in sphere) with $\|\boldsymbol{\mu}\| = 1$,
    - κ (variance around the point) where $\kappa > 0$
- **Probability mass function**:

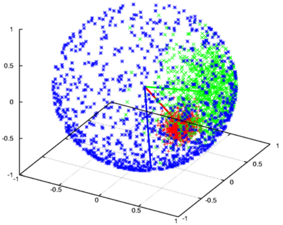
$$p(\mathbf{x}|\boldsymbol{\mu}, \kappa) = \frac{\kappa^{p/2-1}}{(2\pi)^{p/2} I_{p/2-1}(\kappa)} \exp\left(\kappa \boldsymbol{\mu}^T \mathbf{x}\right)$$

  where

$$I_{p/2-1}(\kappa)$$

  is the Bessel function of order $p/2 - 1$
- **Example:**
  - A shotgun shot is $vMF()$ with $p = 3$ and $\boldsymbol{\mu}$ is the aim and κ is the spread of the shot.

VON MISES DISTRIBUTION



FIGUR: Points sampled from three von Mises–Fisher distributions on the sphere (blue: $\kappa = 1$, green: $\kappa = 10$, red: $\kappa = 100$). The mean directions μ are shown with arrows. (Taken from Wikipedia)

VON MISES USAGE IN TEXT

- Cluster **normalized vectors**
- Using TF-IDF vectors (vector space) as data points $\mathbf{x}_d$
- Good clustering performance (?)
- `movMF` R package can fit these models (Python)?

- **Generative model**
  - Generate cluster probability as $\pi \sim \text{Dir}(\alpha)$
  - Generate K mixture components:
    - $\mu_k \sim vMF(\mu_0, \kappa_0)$
    - $\kappa_k \sim logNormal(\mu_\kappa, \sigma_\kappa)$
  - Generate D documents as
    - Generate cluster id $s_d \sim Categorical(\pi)$
    - Generate document vector $\mathbf{x}_d \sim vMF(\mu_{s_d}, \kappa_{s_d})$

GENERATIVE MODEL - MULTINOMIAL EXAMPLE

- **Generative model**
  - Generate cluster probability as $\pi \sim \text{Dir}(\alpha)$
  - Generate K mixture components:
    - $\phi_k \sim \text{Dir}(\beta)$
  - Generate D documents as
    - Generate cluster $s_d \sim \textit{Categorical}(\theta)$
    - Generate document $\mathbf{w}_d \sim \textit{Multinomial}(\phi_{s_d})$

Section 4

Latent variable inference

Expectation - Maximization

- Want to estimate $\phi_1, ..., \phi_k$ using MLE from our data

$$p(\mathbf{w}|\Theta) = \sum_{k}^{K} \pi_k p(\mathbf{w}|s = k, \phi_k)$$

where $\Theta = (\pi_1, ..., \pi_K, \phi_1, ..., \phi_K)$

- Our log likelihood is

$$L(\Theta) = \sum_{d=1}^{D} \log \left\{ \sum_{k}^{K} \pi_k p(\mathbf{w}|s = k, \phi_k) \right\}$$

- This is difficult to optimize!

EXPECTATION - MAXIMIZATION

- Instead... say we introduce $\mathbf{s}$ as a random variable. Then we get the *full observed likelihood*

$$\begin{aligned} L(\Theta, \mathbf{s}) &= \sum_{d=1}^{D} \log \left\{ \sum_{k}^{K} \mathbf{I}(s_d) \pi_k p(\mathbf{w}|s = k, \phi_k) \right\} \\ &= \sum_{d=1}^{D} \log \left\{ \sum_{k}^{K} \mathbf{I}(s_d) \pi_k p(\mathbf{w}|s = k, \phi_k) \right\} \\ &= \sum_{d=1}^{D} \sum_{k}^{K} \mathbf{I}(s_d) \log \left\{ \pi_k p(\mathbf{w}|s = k, \phi_k) \right\} \end{aligned}$$

since where $\mathbf{I}(s_d)$ is an indicator vector with 0 for all k but one that is 1.

EXPECTATION - MAXIMIZATION

- Now, $L(\Theta, \mathbf{s})$ is a random variable.
- We can now estimate Θ in two steps (but multiple iterations):
  1. **Expectation step**
     Compute $E_s(L(\Theta, \mathbf{s}))$ given $\hat{\Phi}$
  2. **Maximazion step**
     Compute $\hat{\Phi}$, a weighted estimate (we now "know" π_k)
- We converge to a local mode of the likelihood

GIBBS SAMPLING (TAKEN FROM BAYESIAN LEARNING)

- Easily implemented methods for sampling from multivariate distributions, $p(\theta_1, ..., \theta_k)$.
- **Requirements:** Easily sampled full conditional posteriors:
  - $p(\theta_1|\theta_2, \theta_3 ..., \theta_k)$
  - $p(\theta_2|\theta_1, \theta_3 ..., \theta_k)$
  - $p(\theta_3|\theta_1, \theta_2 ..., \theta_k)$
  - ...
- Started out in the early 80's in the image analysis literature.
- Gibbs sampling is a Markov Chain Monte Carlo (MCMC) algorithm.
- Generate samples from the posterior distribution $p(\theta|\mathbf{w})$.
- **Straight-forward** for latent variables.

GIBBS SAMPLING (TAKEN FROM BAYESIAN LEARNING)

1. Choose initial values $\theta_1^{(0)}, \theta_2^{(0)}, ..., \theta_k^{(0)}$.
2. Draw
   2.1 $\theta_1^{(1)}$ from $p(\theta_1|\theta_2^{(0)}, ..., \theta_k^{(0)})$
   2.2 $\theta_2^{(1)}$ from $p(\theta_2|\theta_1^{(1)}, \theta_3^{(0)}, ..., \theta_k^{(0)})$
   2.3 $\theta_3^{(1)}$ from $p(\theta_3|\theta_1^{(1)}, \theta_2^{(1)}, ..., \theta_4^{(0)}, ..., \theta_k^{(0)})$
3. Repeat Step 2 N times.

GIBBS SAMPLING (TAKEN FROM BAYESIAN LEARNING)

- The Gibbs draws $\theta^{(0)}, \theta^{(1)}, ..., \theta^{(N)}$ are dependent (autocorrelated), but arithmetic means converge to expected values

$$\frac{1}{N}\sum^{N}\theta_j \to E(\theta_j)$$

$$\frac{1}{N}\sum^{N}g(\theta_j) \to E[g(\theta_j)]$$

- $\theta^{(1)}, ..., \theta^{(N)}$ **converges in distribution** to the target $p(\theta)$
- $\theta^{(1)}, ..., \theta^{(N)}$ converge to the marginal distribution of θ_j , $p(\theta_j)$.
- Dependent draws $\to$ less efficient than iid sampling.

Mixture of Multinomial - Gibbs sampling

- We want the posterior

$$p(\mathbf{s}, \Phi, \theta | \mathbf{w}) \propto p(\mathbf{w} | \mathbf{s}, \Phi, \theta) p(\mathbf{s}, \Phi, \theta)$$

- Need to derive the conditional posteriors (see lab).
- Three step Gibbs sampling
  - Sample $\mathbf{s}_d$ given Φ and θ:

$$p(s_d = k) \propto \theta_k \prod^{V} \phi_k^{n_d^{(w)}}$$

  - Estimate $\Phi | s$:

$$\phi_k \sim Dir(\mathbf{n}_k^{(w)} + \beta)$$

    where $\mathbf{n}^{(w)}$ is the number of **tokens** in each cluster
  - Estimate $\theta | \mathbf{s}$:

$$\theta \sim Dir(\mathbf{n}^{(s)} + \alpha)$$

    where $\mathbf{n}^{(s)}$ is the number of **documents** in each cluster
  - Do this until convergance...

Mixture of Multinomial - Gibbs sampling

- Integrate out Φ and θ can be done so we only sample $\mathbf{s}$.
- Study what the cluster is about by looking at top terms in Φ
- The model has been proposed to clustered small texts such as tweets (KDD2014)

- **Text Mining project:** Compare this model and the von Mises mixture model to cluster tweets

Section 5

Evaluation methods

EVALUATING

- Evaluation is a hard problem
  - As $K \to \infty$ we can fit better and better models
  - No gold standard
- **External** measures :
  - Needs a gold standard (why would we cluster?)
  - **NOT** to be confused with classification
  - confusion matrix, classification accuracy, F1 measure, average purity
- **Internal** measures:
  - Similarity within and between clusters (can be define in various ways)
- **Probabilistic models**
  - Estimate the marginal likelihood on a **test set**

$$p(\mathbf{w}_{test}) = \int p(\mathbf{w}_{test}|\Theta)d\Theta$$

  - AIC, WAIC, BIC, DIC, Perplexity

Section 6

Co-training / Semi-supervision

CO-TRAINING / SEMI-SUPERVISION

- What if we know some **s** but not all?
- **Co-training**: Sample **s** for missing classes
- Can increase accuracy for small training sizes
- **Problem:**
  - Cluster $\neq$ classes (think sentiment in news wire - probably cluster is "content" if not content words are removed)
- **Solutions:**
  - Multiple mixtures/clusters per class

Text mining project idea: But what if?

- Remember the Part-of-Speech tagger with a Hidden Markov Model.
- What if we do not know all $\mathbf{t}$?
  - Treat unknown tags as a parameter!
  - Use a lexicon of tags to restrict the Dirichlet prior distributions (conditional Dirichlet).
- **Text mining project:**
  - Using a Gibbs sampler to combine supervised tags, types and unsupervised data.
  - Will more data improve accuracy? Type supervision, tag supervision?