Assignment 3: Home 2

$\phi = -0'5 \quad \mu = 10 \quad x_n = 12$

a) $\tilde{x}^{n}_{n+10}$

For the first prediction use

AR(1):

$$x_t = \phi x_{t-1} + w_t$$

$$x^{n}_{n+1} = \phi_{11} x_n + \dots + \phi_{pp} x_{n-p+1}$$

In which ϕ_{pp} represents the best prediction possible

$$x^{n}_{n+1} = \phi_1 x_n$$

$$\tilde{x}^{n}_{n+1} - \mu = \phi_1 (x_n - \mu)$$

$$\tilde{x}^{n}_{n+1} = \mu + \phi_1 (x_n - \mu) = 9$$

~~Now apply truncated prediction~~

$$\cancel{\tilde{x}^{n}_{n+2} = \phi \tilde{x}^{n}_{n+1} = \boxed{\phi^2 x_n}}$$

$$\tilde{x}^{n}_{n+3} = \phi \tilde{x}^{n}_{n+2} = \boxed{\phi^3 x_n}$$

$$\cancel{\tilde{x}^{n}_{n+p} = \phi \tilde{x}^{n}_{n+p-1} = \boxed{\phi^p x_n}}$$

~~For this case~~

Now apply truncated prediction

$$\boxed{\tilde{x}^n_{n+m} = \phi_1 \tilde{x}_{n+m-1} + \dots + \phi_p \tilde{x}_{n+m-p}}$$

$$\tilde{x}^n_{n+2} = \phi \tilde{x}^n_{n+1} = \phi^2 x_n$$

$$\tilde{x}^n_{n+3} = \phi \tilde{x}^n_{n+2} = \phi^3 x_n$$

$$\vdots$$

$$x^n_{n+10} = \phi^{10} x_n = \cancel{0'5^{10} \cdot 12}$$

$$\boxed{x^n_{n+10} - \mu = \phi^{10}(x_n - \mu) \Rightarrow x^n_{n+1} = 10 + (0'5)^{10}(12-10) = \cancel{9'998046}\ 10'00195 \simeq 10'002}$$

Assignment 1 : Home 2

Methods of moments estimate:

Use the sample formula

$$\boxed{\begin{aligned} \hat{\mu} &= \sum \bar{x}_n = \frac{1}{n}\sum x_n \\ \hat{\gamma}(h) &= \frac{1}{n}\sum_{t=1}^{n-h} (x_{t+h} - \bar{x})(x_t - x) \\ \hat{\rho}(h) &= \frac{\hat{\gamma}(h)}{\hat{\gamma}(0)} \end{aligned}}$$

$$\hat{\mu} = \frac{1}{5} \cdot (6+5+4+6+4) = 5$$

$$\gamma(0) = \frac{1}{5} \sum_{t=1}^{5} (x_t - \bar{x})(x_t - \bar{x}) = \sum_{t=1}^{5} (x_t - \bar{x})^2 \cdot \frac{1}{5}$$

$$\cancel{(1 \cdot 5) + 0 + (-1 \cdot 1) + (1 \cdot 1) + (-2 \cdot 2)} =$$

$$\frac{1}{5}\left[(1)^2 + (0)^2 + (-1)^2 + (1)^2 (-1)^2\right] = \boxed{\frac{4}{5}}$$

$$\hat{\rho}(h) = \frac{\hat{\gamma}(h)}{\hat{\gamma}(0)} \rightarrow h = 1$$

$$\hat{\gamma}(1) = \frac{1}{5} \sum_{t=1}^{5-|1|=4} (x_{t+1} - \bar{x})(x_t - \bar{x}) =$$

$$\frac{1}{5}\left[(0 \cdot 1) + (-1 \cdot 0) + (1 \cdot -1) + (-1 \cdot 1)\right] = \frac{-2}{5}$$

$$\boxed{\hat{\rho}(1) = \frac{-2/5}{4/5} = \frac{-10}{20} = \frac{-1}{2}}$$

Assignment 2: Home 2

$\hat{\rho}(1) = 0'6$ ~~[illegible]~~ $\hat{\gamma}_0 = 2$

$\hat{\phi}_1 = \hat{R}_p \cdot \hat{\rho}_1 = 0'6$

$\hat{R}_p = \rho(0) = 1$

$\hat{\sigma}_w^2 = 2(1 - 0'6 \cdot 1 \cdot 0'6) = 1'28$

Assignment 4: Home 2

$\phi_1 = -0'5$, $\theta_1 = 0'5$, $x_1 = 10$

$x_2 = 12$

$x_t = -0'5 x_{t-1} + w_t + 0'5 w_{t-1}$

$\tilde{x}^2_{2+1} \rightarrow m = 1 \quad (\tilde{x}^n_{n+m})$

$\boxed{\tilde{x}^2_3 = \phi_1 \tilde{x}_2 + \tilde{\theta}_1 \tilde{w}^2_2 = -6 + 6 = 0}$

$\tilde{w}^2_2 = \tilde{x}^2_2 - \phi_1 \tilde{x}^2_1 - \theta_1 \tilde{w}^2_1 = 12 + 5 - 5 = 12$

$\tilde{w}^2_1 = \tilde{x}^2_1 = 10$

Assignment 5 : Home 2

$x_t = x_{t-1} + x_{t-12} - x_{t-13} + w_t$

$-0'5\, w_{t-1} - 0'5\, w_{t-12} + 0'25\, w_{t-13}$

$x_t - x_{t-1} - x_{t-12} - x_{t-13} = w_t - 0'5\, w_{t-1} - 0'5\, w_{t-12}$

$+ 0'25\, w_{t-13}$

$(1 - B - B^{12} + B^{13})x_t = (1 - 0'5B - 0'5B^{12} + 0'25B^{13})w_t$

$(1 - B^{12} - B(1 - B^{12}))x_t = (1 - 0'5B^{12} - 0'5B(1 - 0'5B^{12}$

$\underbrace{(1 - B^{12})}_{D}\underbrace{(1 - B)}_{d} x_t = \underbrace{(1 - 0'5B^{12})}_{Q}\underbrace{(1 - 0'5B)}_{q} w_t$

$(0,1,1) \times (0,1,1)_{12}$

$\underset{p\ d\ q}{(0,1,1)} \times \underset{P\ D\ Q}{(0,1,1)}_{12 \Rightarrow S}$

Assignment 6: Home 2

$x_t - 0'4\,x_{t-1} - 0'45\,x_{t-2} = W_t$

General Homogeneus Equations:

$\gamma(h) - \phi_1\gamma(h-1) - \dots - \phi_p\gamma(h-p) = 0 \quad h \geq \max(p, q+1$

$\gamma(h) - \phi_1\gamma(h-1) - \dots - \phi_p\gamma(h-p) = \sigma_w^2 \sum_{j=h}^{q} \theta_j \psi_{h-1}$

$\cancel{h\geq}\ 0 \leq h \leq \max(p, q+1)$

$p=2 \quad q=0$

$h=0$

$\gamma(0) - \phi_1\gamma(1) - \phi_2\gamma(2) = 0 \quad (e_1)$

$h=1$

$\gamma(1) - \phi_1\gamma(0) - \phi_2\gamma(1) = 0 \quad (e_2)$

$h \geq 2$

$\gamma(h) - \phi_1\gamma(h-1) - \phi_2\gamma(h-2) = 0 \quad (e_3)$

$$\frac{e_1}{\gamma(0)} = 1 - 0'4\,\rho(1) - 0'45\,\rho(2) = 0 \quad (e1.1)$$

$$\frac{e_2}{\gamma(0)} = \rho(1) - 0'4 - 0'45\,\rho(1) = 0 \quad (e2.2)$$

$$(e2.2) \rightarrow \rho(1)(1-0'45) = 0'4 \Rightarrow \boxed{0'73 = \rho(1)}$$

$$(e1.1) \rightarrow 1 - 0'4(0'73) - 0'45(\rho(2)) = 0$$

$$\boxed{\rho(2) = 1'56}$$

$$\frac{(e3)}{\gamma(0)} = \rho(h) - \phi_1\,\rho(h-1) - \phi_2\,\rho(h-2) = 0$$

$$\Downarrow$$

$$u_n - 0'4\,u_{n-1} - 0'45\,u_{n-2} = 0$$

$$\Downarrow$$

$$1 - 0'4z - 0'45z^2 = 0$$

$$\left(z - \frac{10}{9}\right)(z+2) = 0$$

$z_1 = \frac{10}{9}$ $\cancel{u_n = \left(\frac{10}{9}\right)^{-n}(c_1 + c_2 \cdot n)}$

$\left(+\frac{10}{9}\right)^{-h} P_1(h) + (-2)^{-h} P_2(h) = \rho(h)$

$h=0$

$P_1 + P_2 = 1$

$h=1$

$\frac{9}{10} P_1 - \frac{1}{2} P_2 = 0'73$

$$\boxed{0'88 \left(\frac{10}{9}\right)^{-h} + (-2)^{-h} \cdot 0'12 = \rho(h)}$$

Assignment 7 ; Home 2

$\Gamma_n \phi_n = \gamma(n) \rightarrow$ Find ϕ_{nn}

$$\begin{pmatrix} \gamma(1-1) & \gamma(2-1) \\ \gamma(2-1) & \gamma(2-2) \end{pmatrix} \begin{pmatrix} \phi_{21} \\ \phi_{22} \end{pmatrix} = \begin{pmatrix} \gamma(1) \\ \gamma(2) \end{pmatrix}$$

$$\left.\begin{aligned} \gamma(0)\,\phi_{21} + \gamma(1)\,\phi_{22} &= \gamma(1) \\ \gamma(1)\,\phi_{21} + \gamma(0)\,\phi_{22} &= \gamma(2) \end{aligned}\right\}$$

$$\left.\begin{aligned} 0'5\,\phi_{21} + 0'2\,\phi_{22} &= 0'2 \\ 0'2\,\phi_{21} + 0'5\,\phi_{22} &= 0'1 \end{aligned}\right\} \quad \begin{aligned} \phi_{21} &= \frac{8}{21} \\ \phi_{22} &= \frac{1}{21} \end{aligned}$$

$$\boxed{x_3^2 = \frac{8}{21} \cdot x_2 + \frac{1}{21} x_1 = 0'8}$$

$$x_{n+1}^n = \phi_{n1} x_n + \dots + \phi_{nn} x_1 \rightarrow \text{Prediction}$$

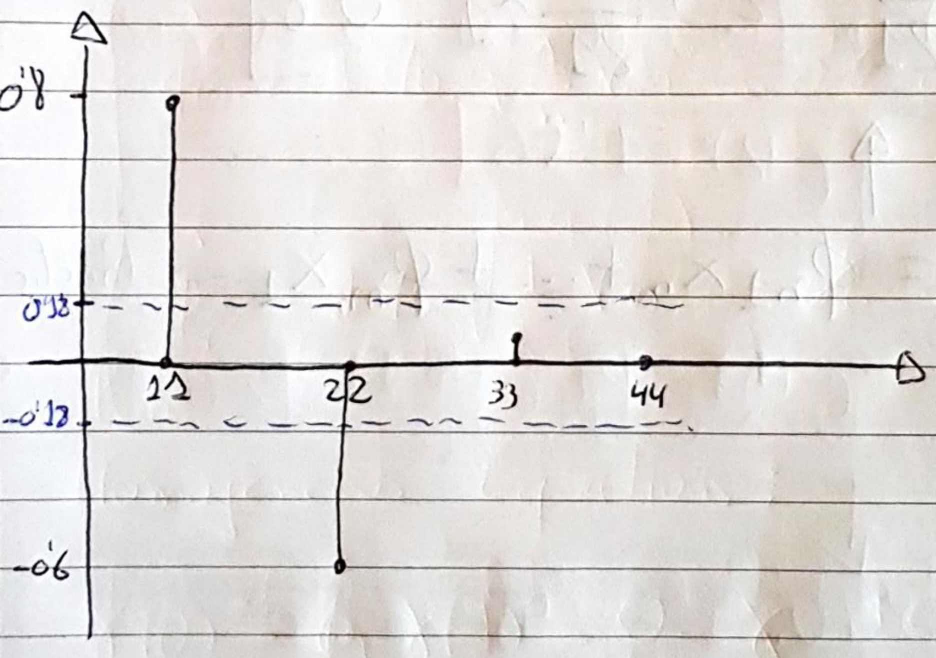
Assignment 8: Home 2

$n = 121$

$\phi_{11} = 0'8$ $\phi_{33} = 0'08$

$\phi_{22} = -0'6$ $\phi_{44} = 0$

$\left[\frac{-2}{\sqrt{n}}, \frac{2}{\sqrt{n}}\right] = [\cancel{-0'18}\ -0'18, 0'18]$



AR (2)

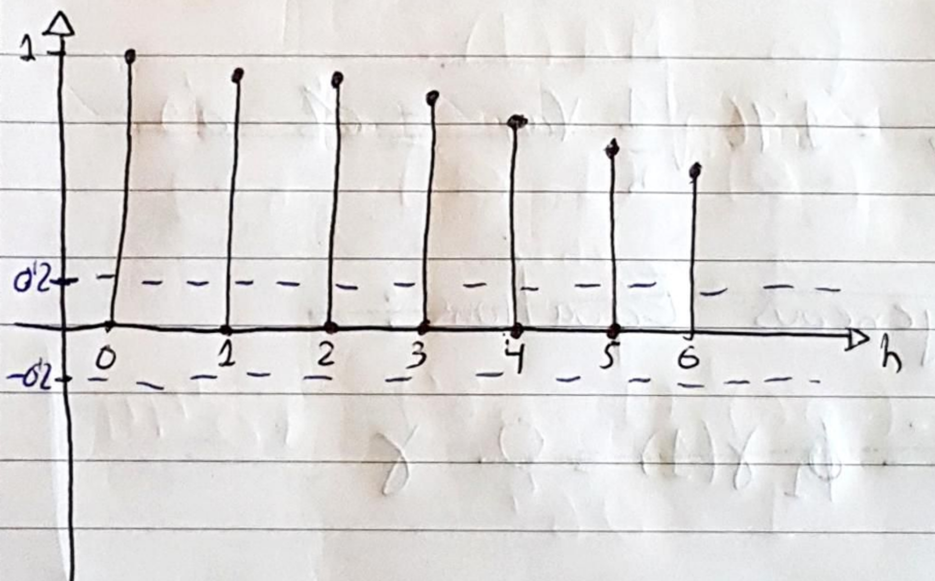
Assignment 9: Home 2

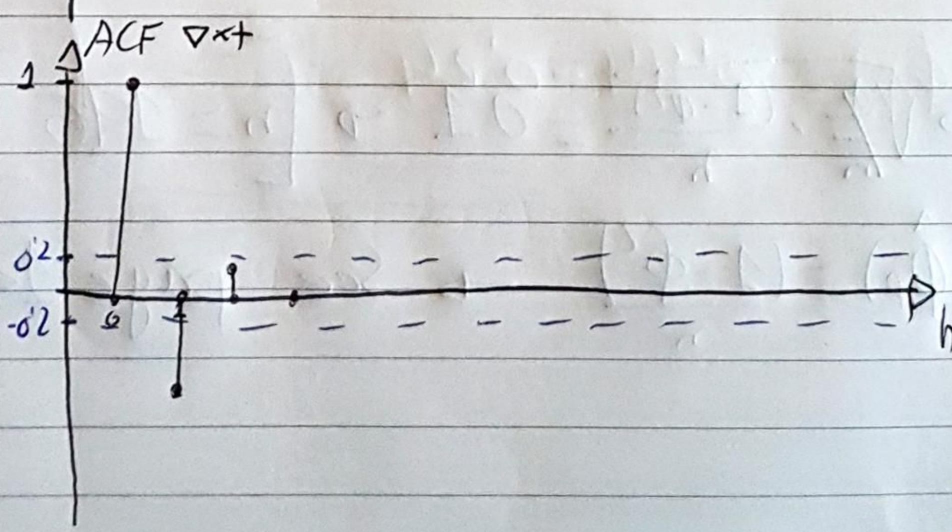
lag:	1	2	3	4	5	6
ACF x_t	0'97	0'97	0'93	0'85	0'80	0'72
ACF ∇x_t	-0'42	0'18	-0'02	0'07	-0'10	-0'09

$n = 100$

Interval: $\left[\frac{-2}{\sqrt{n}}, \frac{2}{\sqrt{n}}\right] = \left[-0'2, 0'2\right]$

ACF x_t





It cuts off after LAG 1 in ACF

ARIMA (0, 1, 1)

<u>Assignment 10: Home 2</u>

$\phi = 0'7$ CI: $[0'7 - 0'1, 0'7 + 0'1]$

CI: $\hat{\phi}_p \pm 1'96 \sqrt{\cancel{\frac{1}{n}} Var(\hat{\phi}_p)}$

$$\begin{cases} \cancel{\hat{\phi}_p + 1'96 \sqrt{\frac{1}{n} Var} = 0'8 \quad 0'1} \\ \cancel{\hat{\phi}_p - 1'96 \sqrt{\frac{1}{n} Var} = 0'6 \quad -0'1} \end{cases}$$

~~Homogeneous Equations:~~

$\cancel{\gamma(0) - \phi_1 \gamma(1) = 0}$ γ

$1'96 \sqrt{\cancel{\frac{1}{n}} \cdot \frac{(1-\phi^2)}{n}} = 0'1 \rightarrow \boxed{n \geq 196}$

$Var(\hat{\phi}_p) = \frac{(1-\phi^2)}{n}$ $\rightarrow$ See page 126

Assignment 11: Home 2

AR(2)

$$x_t = 5 + 1'1\, x_{t-1} - 0'5\, x_{t-2} + w_t$$

$$\sigma_w^2 = 2 \qquad \mu = 5$$

$$x^n_{n+1} = \phi_{11}\, x_n + \phi_{22}\, x_{n-1}$$

$$x^n_{n+1} = 1'1\, x_n - 0'5\, x_{n-1} = 1'1 \cdot 10 - 0'5 \cdot 11 =$$

$$= \boxed{5'5 + 5 = 10'5}$$

$$\cancel{x^n_{n+1} - \mu = 5'5 - 5 = 0'5}$$

$$\boxed{\cancel{2008 = 10 + 0'5 = 10'5}}$$

$$\tilde{x}^n_{n+2} = \phi_1\, \tilde{x}^n_{n+1} + \phi_2\, \tilde{x}_n$$

$$\boxed{\tilde{x}^n_{n+2} = \overset{1'1}{\cancel{\phi_1}} \cdot 10'5 - 0'5 \cdot 10 = 6'55 + 5 = 11'55}$$

$$\cancel{\tilde{x}^n_{n+2} - \mu = 1'55}$$

b) $x_{n+1}^{n} \pm 1'96 \sqrt{P_{n+1}^{n}}$ o $x_{n+1}^{n} \pm 2\sqrt{P_{n+1}^{n}}$

$\cancel{P_{n+1}^{n} = \gamma(0) - \gamma' \Gamma_n^{-1} \gamma_n}$

$P_{n+m}^{n} = \sigma_w^2 \sum_{j=0}^{m-1} \psi_j^2$ if $m=1$ $P_{n+1}^{n} = \sigma_w^2$

$$\boxed{[10'5 - 2\sqrt{2} \,,\, 10'5 + 2\sqrt{2}] = [7'67 \,,\, 13'33]}$$

Assignment 12 : Home 2

PACF for MA(1):

$$\phi_{hh} = \frac{(-\theta)^h (1-\theta^2)}{1-\theta^{2h+2}} \,,\ h \geq 1$$

$$PACF(\phi_{11}) = -\frac{(-0'6)^1 (1-0'6^2)}{1-0'64} = +0'44$$

$$PACF(\phi_{22}) = \cancel{0'44} - 0'24$$

Assignment 13: Home 2

$x_t = 0'8 x_{t-4} + w_t + 0'3 w_{t-1}$ $\qquad \sigma_w^2 = 1$

~~Homogeneous~~ Equations:

$\cancel{\gamma(h) - \phi_1 \gamma(h-1)}$ See page 148

$$\gamma(0) = \frac{1+\theta^2}{1-\Phi^2}$$

Cause x_{t-4}, w_t and w_{t-1} are uncorrelated

$$\gamma(0) = 3'02$$

$$\rho(4h) = \Phi^h = 0'8^h$$

$$\rho(4h \pm 1) = \frac{\theta}{1+\theta^2} \Phi^h \qquad h = 0, 1, 2$$

$$\rho(4h \pm 1) = 0'28 \cdot 0'8^h$$

$$\rho(4h \pm k) = 0 \qquad k \neq 1 \qquad k = 2, 3, 4 \ldots$$