Meeting 10: More on the value of information

Sequential analysis

When the value of sample information concerns the entire sample (sometimes referred to as *single-stage sampling*) the expected net gain of sampling can be written

$$ENGS(n) = EVSI(n) - CS(n)$$

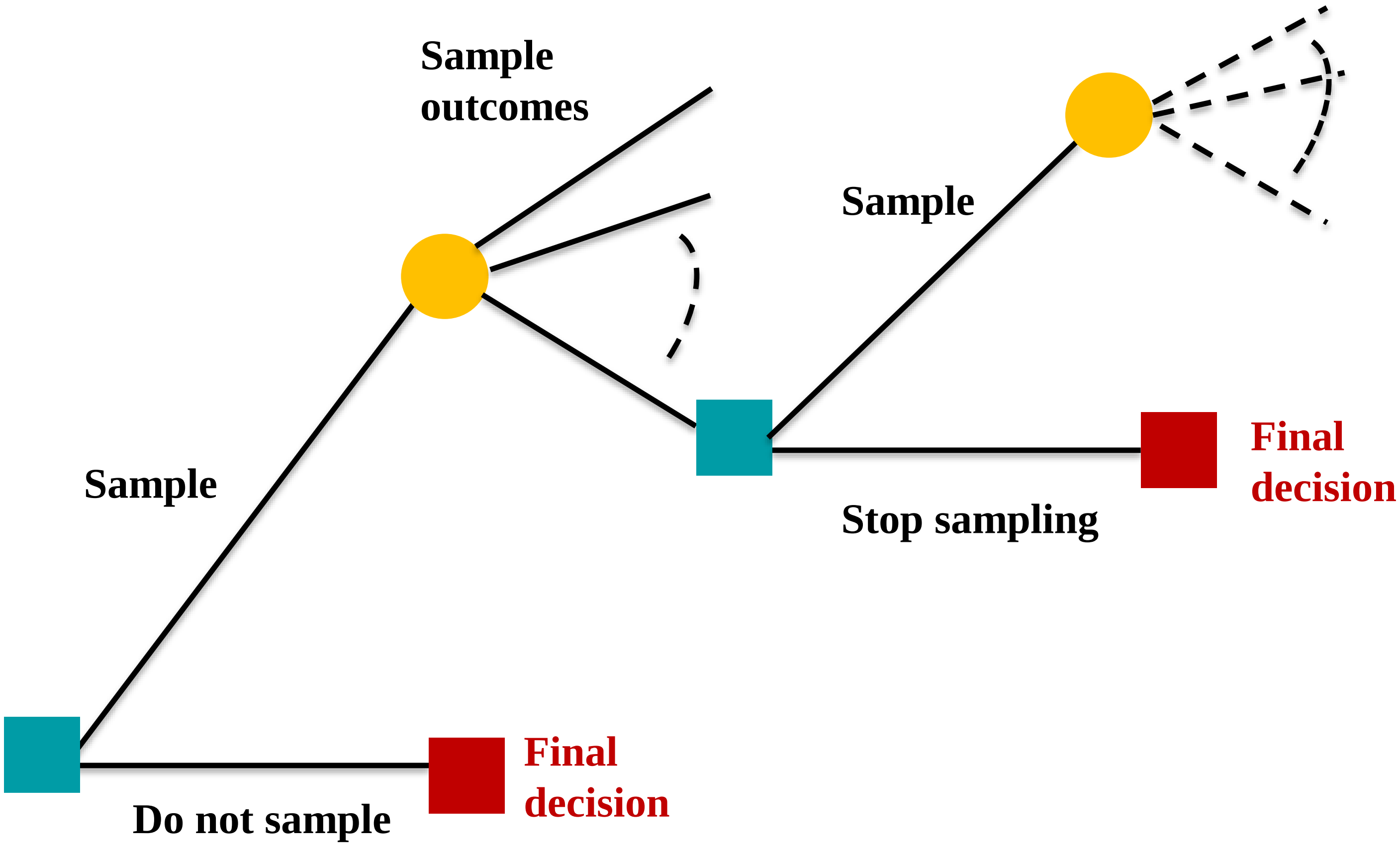
and the optimal sample size n^* satisfies

$$ENGS(n^*) \geq ENGS(n) \quad \text{for} \;\; n = 0,1,2,\ldots$$

However, it is also possible to sample one unit at time and at each step decide whether further sampling should be conducted. This is referred to as *sequential sampling*.

In the textbook there is no attempt to formulate at general description of sequential sampling, since it is a concept closely related to the decision problem at hand.

However, to clarify things it may often be wise to draw decision trees.



Exercise 6.27

27. In Exercise 17, consider a sequential sampling plan with a maximum total sample size of two and analyze the problem as follows.
    (a) Represent the situation in terms of a tree diagram.
    (b) Using backward induction, find the ENGS for the sequential plan.
    (c) Compare the sequential plan with a single-stage plan having $n = 2$.

17. In Exercise 16, suppose that you also want to consider other sample sizes.
    (a) Find EVSI for a sample of size 2.
    (b) Find EVSI for a sample of size 5.
    (c) Find EVSI for a sample of size 10.
    (d) If the cost of sampling is $0.50 per unit sampled, find the expected net gain of sampling (ENGS) for samples of sizes 1, 2, 5, and 10.

16. In Exercise 15, suppose that sample information is available in the form of a random sample of consumers. For a sample of size *one*,
    (a) find the posterior distribution if the one person sampled will purchase the item, and find the value of this sample information;
    (b) find the posterior distribution if the one person sampled will *not* purchase the item, and find the value of this sample information;
    (c) find the expected value of sample information.

15. A store must decide whether or not to stock a new item. The decision depends on the reaction of consumers to the item, and the payoff table (in dollars) is as follows.

		PROPORTION OF CONSUMERS PURCHASING				
		0.10	0.20	0.30	0.40	0.50
DECISION	*Stock 100*	−10	−2	12	22	40
	Stock 50	−4	6	12	16	16
	Do not stock	0	0	0	0	0

If $P(0.10) = 0.2$, $P(0.20) = 0.3$, $P(0.30) = 0.3$, $P(0.40) = 0.1$, and $P(0.50) = 0.1$, what decision maximizes expected payoff? If perfect information is available, find VPI for each of the five possible states of the world and compute EVPI.

DECISION	PROPROTION OF CUSTOMERS BUYING				
	0.10	0.20	0.30	0.40	0.50
Stock 100	–10	–2	12	22	40
Stock 50	–4	6	12	16	16
Do not stock	0	0	0	0	0

$$\mathrm{VPI}(\theta) = R(a_\theta, \theta) - R(a^*, \theta)$$

$$a^* = \arg\max_a \{ER(a)\}$$

$$ER(\text{Stock } 100) = (-10)\cdot 0.2 + (-2)\cdot 0.3 + 12\cdot 0.3 + 22\cdot 0.1 + 40\cdot 0.1 = 7.2$$

$$ER(\text{Stock } 50) = (-4)\cdot 0.2 + 6\cdot 0.3 + 12\cdot 0.3 + 16\cdot 0.1 + 16\cdot 0.1 = 7.8$$

$$ER(\text{Do not stock}) = 0\cdot 0.2 + 0\cdot 0.3 + 0\cdot 0.3 + 0\cdot 0.1 + 0\cdot 0.1 = 0$$

$$\Rightarrow a^* = \text{Stock } 50$$

$$\Rightarrow$$

$$\mathrm{VPI}(0.10) = 0 - (-4) = 4$$

$$\mathrm{VPI}(0.20) = 6 - 6 = 0$$

$$\mathrm{VPI}(0.30) = 12 - 12 = 0$$

$$\mathrm{VPI}(0.40) = 22 - 16 = 6$$

$$\mathrm{VPI}(0.50) = 40 - 16 = 24$$

$$\Rightarrow \mathrm{EVPI} = 4\cdot 0.2 + 0\cdot 0.3 + 0\cdot 0.3 + 6\cdot 0.1 + 24\cdot 0.1 = 3.8$$

16. In Exercise 15, suppose that sample information is available in the form of a random sample of consumers. For a sample of size *one*,

(a) find the posterior distribution if the one person sampled will purchase the item, and find the value of this sample information;

(b) find the posterior distribution if the one person sampled will *not* purchase the item, and find the value of this sample information;

(c) find the expected value of sample information.

Below we have used the word "BUY" instead of "PURCHASE"

(a) Posterior distribution: $P(\theta|\text{BUY}) = \dfrac{P(\text{BUY}|\theta)\cdot P(\theta)}{P(\text{BUY})} = \dfrac{P(\text{BUY}|\theta)\cdot P(\theta)}{\sum_{\lambda} P(\text{BUY}|\lambda)\cdot P(\lambda)}$

$$P(\text{BUY}|\theta) = \theta$$

$$\Rightarrow$$

$$P(\text{BUY}) = 0.10\cdot 0.2 + 0.20\cdot 0.3 + 0.30\cdot 0.3 + 0.40\cdot 0.1 + 0.50\cdot 0.1 = 0.26$$

$$P(0.10|\text{BUY}) = 0.10\cdot 0.2/0.26 \approx 0.0769$$

$$P(0.20|\text{BUY}) = 0.20\cdot 0.3/0.26 \approx 0.2308$$

$$P(0.30|\text{BUY}) = 0.30\cdot 0.3/0.26 \approx 0.3462$$

$$P(0.40|\text{BUY}) = 0.40\cdot 0.1/0.26 \approx 0.1538$$

$$P(0.50|\text{BUY}) = 0.50\cdot 0.1/0.26 \approx 0.1923$$

$$\mathrm{VSI}(\mathrm{BUY}) = E''R(a''|\mathrm{BUY}) - E''R(a'|\mathrm{BUY})$$

$$a' = \langle = a^* \ \text{ from exercise } 6.15 \rangle = \text{Stock } 50$$

$$a'' = \arg\max_a ER(a|\mathrm{BUY})$$

$$ER(a|\mathrm{BUY}) = \sum_\theta R(a,\theta) \cdot P(\theta|\mathrm{BUY})$$

$$\Rightarrow$$

$$ER(\text{Stock } 100|\mathrm{BUY}) = (-10) \cdot 0.0769\ldots + (-2) \cdot 0.2308\ldots + 12 \cdot 0.3462\ldots + 22 \cdot 0.1538\ldots + 40 \cdot 0.1923\ldots = 14.00$$

$$ER(\text{Stock } 50|\mathrm{BUY}) = (-4) \cdot 0.0769\ldots + 6 \cdot 0.2308\ldots + 12 \cdot 0.3462\ldots + 26 \cdot 0.1538\ldots + 16 \cdot 0.1923\ldots \approx 10.77$$

$$ER(\text{Do not stock}|\mathrm{BUY}) = 0 \cdot 0.0769\ldots + 0 \cdot 0.2308\ldots + 0 \cdot 0.3462\ldots + 0 \cdot 0.1538\ldots + 0 \cdot 0.1923\ldots = 0$$

$$\Rightarrow a'' = \text{Stock } 100$$

$$\Rightarrow$$

$$\mathrm{VSI}(\mathrm{BUY}) = E''R(\text{Stock } 100|\mathrm{BUY}) - E''R(\text{Stock } 50|\mathrm{BUY}) = 14.00 - 10.77 = 3.23$$

(b)

$$\text{Posterior distribution}: P(\theta|\text{NOT BUY}) = \frac{P(\text{NOT BUY}|\theta)\cdot P(\theta)}{P(\text{NOT BUY})} = \frac{P(\text{NOT BUY}|\theta)\cdot P(\theta)}{\sum_{\lambda} P(\text{NOT BUY}|\lambda)\cdot P(\lambda)}$$

$$P(\text{NOT BUY}|\theta) = 1-\theta$$

$$\Rightarrow$$

$$P(\text{NOT BUY}) = 0.90\cdot 0.2 + 0.80\cdot 0.3 + 0.70\cdot 0.3 + 0.60\cdot 0.1 + 0.50\cdot 0.1 = 0.74$$

$$P(0.10|\text{NOT BUY}) = 0.90\cdot 0.2/0.74 \approx 0.2432$$

$$P(0.20|\text{NOT BUY}) = 0.80\cdot 0.3/0.74 \approx 0.3243$$

$$P(0.30|\text{NOT BUY}) = 0.70\cdot 0.3/0.74 \approx 0.2838$$

$$P(0.40|\text{NOT BUY}) = 0.60\cdot 0.1/0.74 \approx 0.0811$$

$$P(0.50|\text{NOT BUY}) = 0.50\cdot 0.1/0.74 \approx 0.0676$$

$$\mathrm{VSI}(\text{NOT BUY}) = E''R(a''|\text{NOT BUY}) - E''R(a'|\text{NOT BUY})$$

$$a' = \text{Stock 50} \quad \text{(Same as in (a))}$$

$$a'' = \arg\max_a ER(a|\text{NOT BUY})$$

$$ER(a|\text{NOT BUY}) = \sum_\theta R(a,\theta)\cdot P(\theta|\text{NOT BUY})$$

$$\Rightarrow$$

$$ER(\text{Stock 100}|\text{NOT BUY}) = (-10)\cdot 0.2432\ldots + (-2)\cdot 0.3243\ldots + 12\cdot 0.2838\ldots + 22\cdot 0.0811\ldots + 40\cdot 0.0676\ldots \approx 4.81$$

$$ER(\text{Stock 50}|\text{NOT BUY}) = (-4)\cdot 0.2432\ldots + 6\cdot 0.3243\ldots + 12\cdot 0.2838\ldots + 26\cdot 0.0811\ldots + 16\cdot 0.0676\ldots \approx 6.76$$

$$ER(\text{Do not stock}|\text{NOT BUY}) = 0\cdot 0.2432\ldots + 0\cdot 0.3243\ldots + 0\cdot 0.2838\ldots + 0\cdot 0.0811\ldots + 0\cdot 0.0676\ldots = 0$$

$$\Rightarrow a'' = \text{Stock 50}$$

$$\Rightarrow$$

$$\mathrm{VSI}(\text{NOT BUY}) = E''R(\text{Stock 50}|\text{NOT BUY}) - E''R(\text{Stock 50}|\text{NOT BUY}) = 0$$

(c)

$$\text{EVSI} = \sum_y \text{VSI}(y)P(y) = 3.2 \cdot 0.26 + 0 \cdot 0.74 = 0.832$$

Alternatively:

$$\text{EVSI} = E''R(a'') - E''R(a')$$

$$E''R(a'') = E''R(a''|\text{BUY}) \cdot P(\text{BUY}) + E''R(a''|\text{NOT BUY}) \cdot P(\text{NOT BUY}) =$$
$$= E''R(\text{Stock 100}|\text{BUY}) \cdot P(\text{BUY}) + E''R(\text{Stock 50}|\text{NOT BUY}) \cdot P(\text{NOT BUY}) =$$
$$= 14.00 \cdot 0.26 + 6.76 \cdot 0.74 = 8.64$$

$$E''R(a') = E''R(a'|\text{BUY}) \cdot P(\text{BUY}) + E''R(a'|\text{NOT BUY}) \cdot P(\text{NOT BUY}) =$$
$$= E''R(\text{Stock 50}|\text{BUY}) \cdot P(\text{BUY}) + E''R(\text{Stock 50}|\text{NOT BUY}) \cdot P(\text{NOT BUY}) =$$
$$= 10.77 \cdot 0.26 + 6.76 \cdot 0.74 = 7.80$$

$$\Rightarrow \text{EVSI} = 8.64 - 7.80 = 0.84$$

17. In Exercise 16, suppose that you also want to consider other sample sizes.
  (a) Find EVSI for a sample of size 2.
  (b) Find EVSI for a sample of size 5.
  (c) Find EVSI for a sample of size 10.
  (d) If the cost of sampling is $0.50 per unit sampled, find the expected net gain of sampling (ENGS) for samples of sizes 1, 2, 5, and 10.

(a) Just the case with $n = 2$

We need to consider all possible outcomes in a sample of size 2, i.e.

BUY, BUY
BUY, NOT BUY
NOT BUY, BUY
NOT BUY, NOT BUY

However, the second and third outcome are equal by symmetry

BUY, BUY:

$$\text{Posterior distribution}: P(\theta|\text{BUY},\text{BUY}) = \frac{P(\text{BUY},\text{BUY}|\theta)\cdot P(\theta)}{P(\text{BUY},\text{BUY})} = \frac{P(\text{BUY},\text{BUY}|\theta)\cdot P(\theta)}{\sum_{\lambda} P(\text{BUY},\text{BUY}|\lambda)\cdot P(\lambda)}$$

$$P(\text{BUY},\text{BUY}|\theta) = \theta^2$$

$$\Rightarrow$$

$$P(\text{BUY},\text{BUY}) = 0.10^2\cdot 0.2 + 0.20^2\cdot 0.3 + 0.30^2\cdot 0.3 + 0.40^2\cdot 0.1 + 0.50^2\cdot 0.1 = 0.082$$

$$P(0.10|\text{BUY},\text{BUY}) = 0.10^2\cdot 0.2/0.082 \approx 0.0244$$

$$P(0.20|\text{BUY},\text{BUY}) = 0.20^2\cdot 0.3/0.082 \approx 0.1463$$

$$P(0.30|\text{BUY},\text{BUY}) = 0.30^2\cdot 0.3/0.082 \approx 0.3293$$

$$P(0.40|\text{BUY},\text{BUY}) = 0.40^2\cdot 0.1/0.082 \approx 0.1951$$

$$P(0.50|\text{BUY},\text{BUY}) = 0.50^2\cdot 0.1/0.082 \approx 0.3049$$

$$\mathrm{VSI}(\mathrm{BUY},\mathrm{BUY}) = E''R(a''|\mathrm{BUY},\mathrm{BUY}) - E''R(a'|\mathrm{BUY},\mathrm{BUY})$$

$$a' = \langle = a^{*} \text{ from exercise } 6.15 \rangle = \text{Stock } 50$$

$$a'' = \arg\max_{a} ER(a|\mathrm{BUY},\mathrm{BUY})$$

$$ER(a|\mathrm{BUY},\mathrm{BUY}) = \sum_{\theta} R(a,\theta) \cdot P(\theta|\mathrm{BUY},\mathrm{BUY})$$

$$\Rightarrow$$

$$ER(\text{Stock } 100|\mathrm{BUY},\mathrm{BUY}) = (-10)\cdot 0.0244\ldots + (-2)\cdot 0.1463\ldots + 12\cdot 0.3293\ldots + 22\cdot 0.1951\ldots + 40\cdot 0.3049\ldots \approx 19.90$$

$$ER(\text{Stock } 50|\mathrm{BUY},\mathrm{BUY}) = (-4)\cdot 0.0244\ldots + 6\cdot 0.1463\ldots + 12\cdot 0.3293\ldots + 26\cdot 0.1951\ldots + 16\cdot 0.3049\ldots \approx 12.73$$

$$ER(\text{Do not stock}|\mathrm{BUY},\mathrm{BUY}) = 0\cdot 0.0244\ldots + 0\cdot 0.1463\ldots + 0\cdot 0.3293\ldots + 0\cdot 0.1951\ldots + 0\cdot 0.3049\ldots = 0$$

$$\Rightarrow a'' = \text{Stock } 100$$

$$\Rightarrow$$

$$\mathrm{VSI}(\mathrm{BUY},\mathrm{BUY}) = E''R(\text{Stock } 100|\mathrm{BUY},\mathrm{BUY}) - E''R(\text{Stock } 50|\mathrm{BUY},\mathrm{BUY}) \approx 19.90 - 12.73 = 7.17$$

BUY, NOT BUY or NOT BUY, BUY:

$$\text{Posterior distribution}: P(\theta|\text{BUY, NOT BUY}) = \frac{P(\text{BUY, NOT BUY}|\theta)\cdot P(\theta)}{P(\text{BUY, NOT BUY})} =$$

$$\frac{P(\text{BUY, NOT BUY}|\theta)\cdot P(\theta)}{\sum_{\lambda} P(\text{BUY, NOT BUY}|\lambda)\cdot P(\lambda)}$$

$$P(\text{BUY, NOT BUY}|\theta) = \theta\cdot(1-\theta)$$

$$\Rightarrow$$

$$P(\text{BUY, NOT BUY}) = 0.10\cdot 0.90\cdot 0.2 + 0.20\cdot 0.80\cdot 0.3 + 0.30\cdot 0.70\cdot 0.3 +$$
$$0.40\cdot 0.50\cdot 0.1 + 0.50^2\cdot 0.1 = 0.178$$

$$P(0.10|\text{BUY, NOT BUY}) = 0.10\cdot 0.90\cdot 0.2/0.178 \approx 0.1011$$

$$P(0.20|\text{BUY, NOT BUY}) = 0.20\cdot 0.80\cdot 0.3/0.178 \approx 0.2697$$

$$P(0.30|\text{BUY, NOT BUY}) = 0.30\cdot 0.70\cdot 0.3/0.178 \approx 0.3539$$

$$P(0.40|\text{BUY, NOT BUY}) = 0.40\cdot 0.50\cdot 0.1/0.178 \approx 0.1348$$

$$P(0.50|\text{BUY, NOT BUY}) = 0.50^2\cdot 0.1/0.178 \approx 0.1404$$

$$\mathrm{VSI}\left(\mathrm{BUY},\mathrm{NOT\ \ BUY}\right)=E''R\left(a''\middle|\mathrm{BUY},\mathrm{NOT\ \ BUY}\right)-E''R\left(a'\middle|\mathrm{BUY},\mathrm{NOT\ \ BUY}\right)$$

$$a'=\mathrm{Stock\ \ 50}\quad\text{(as before)}$$

$$a''=\arg\max_{a} ER\left(a\middle|\mathrm{BUY},\mathrm{NOT\ \ BUY}\right)$$

$$ER\left(a\middle|\mathrm{BUY},\mathrm{NOT\ \ BUY}\right)=\sum_{\theta}R\left(a,\theta\right)\cdot P\left(\theta\middle|\mathrm{BUY},\mathrm{NOT\ \ BUY}\right)$$

$$\Rightarrow$$

$$ER\left(\mathrm{Stock\ \ 100}\middle|\mathrm{BUY},\mathrm{NOT\ \ BUY}\right)=\left(-10\right)\cdot 0.1011\ldots+\left(-2\right)\cdot 0.2697\ldots+12\cdot 0.3539\ldots+ \\ 22\cdot 0.1348\ldots+40\cdot 0.1404\ldots\approx 11.28$$

$$ER\left(\mathrm{Stock\ \ 50}\middle|\mathrm{BUY},\mathrm{NOT\ \ BUY}\right)=\left(-4\right)\cdot 0.1011\ldots+6\cdot 0.2697\ldots+12\cdot 0.3539\ldots+ \\ 26\cdot 0.1348\ldots+16\cdot 0.1404\ldots\approx 9.87$$

$$ER\left(\mathrm{Do\ \ not\ \ stock}\middle|\mathrm{BUY},\mathrm{NOT\ \ BUY}\right)=0\cdot 0.1011\ldots+0\cdot 0.2697\ldots+0\cdot 0.3539\ldots+ \\ 0\cdot 0.1348\ldots+0\cdot 0.1404\ldots=0$$

$$\Rightarrow a''=\mathrm{Stock\ \ 100}$$

$$\Rightarrow$$

$$\mathrm{VSI}\left(\mathrm{BUY},\mathrm{NOT\ \ BUY}\right)=E''R\left(\mathrm{Stock\ \ 100}\middle|\mathrm{BUY},\mathrm{NOT\ \ BUY}\right)- \\ E''R\left(\mathrm{Stock\ \ 50}\middle|\mathrm{BUY},\mathrm{NOT\ \ BUY}\right)\approx 11.28-9.87=1.41$$

NOT BUY, NOT BUY:

$$\text{Posterior distribution}: P(\theta \mid \text{NOT BUY}, \text{NOT BUY}) =$$

$$\frac{P(\text{NOT BUY}, \text{NOT BUY} \mid \theta) \cdot P(\theta)}{P(\text{NOT BUY}, \text{NOT BUY})} = \frac{P(\text{NOT BUY}, \text{NOT BUY} \mid \theta) \cdot P(\theta)}{\sum_{\lambda} P(\text{NOT BUY}, \text{NOT BUY} \mid \lambda) \cdot P(\lambda)}$$

$$P(\text{NOT BUY}, \text{NOT BUY} \mid \theta) = (1-\theta)^2$$

$$\Rightarrow$$

$$P(\text{NOT BUY}, \text{NOT BUY}) = 0.90^2 \cdot 0.2 + 0.80^2 \cdot 0.3 + 0.70^2 \cdot 0.3 + 0.60^2 \cdot 0.1 + 0.50^2 \cdot 0.1 = 0.562$$

$$P(0.10 \mid \text{NOT BUY}, \text{NOT BUY}) = 0.90^2 \cdot 0.2 / 0.562 \approx 0.2883$$

$$P(0.20 \mid \text{NOT BUY}, \text{NOT BUY}) = 0.80^2 \cdot 0.3 / 0.562 \approx 0.3416$$

$$P(0.30 \mid \text{NOT BUY}, \text{NOT BUY}) = 0.70^2 \cdot 0.3 / 0.562 \approx 0.2616$$

$$P(0.40 \mid \text{NOT BUY}, \text{NOT BUY}) = 0.60^2 \cdot 0.1 / 0.562 \approx 0.0641$$

$$P(0.50 \mid \text{NOT BUY}, \text{NOT BUY}) = 0.50^2 \cdot 0.1 / 0.562 \approx 0.0445$$

$$\mathrm{VSI}(\text{NOT BUY}, \text{NOT BUY}) = E''R(a'' \mid \text{NOT BUY}, \text{NOT BUY}) -$$
$$E''R(a' \mid \text{NOT BUY}, \text{NOT BUY})$$

$a' = \text{Stock } 50 \quad \text{(as before)} \qquad a'' = \arg\max_{a} ER(a \mid \text{NOT BUY}, \text{NOT BUY})$

$$ER(a \mid \text{NOT BUY}, \text{NOT BUY}) = \sum_{\theta} R(a,\theta) \cdot P(\theta \mid \text{NOT BUY}, \text{NOT BUY})$$

$\Rightarrow$

$$ER(\text{Stock } 100 \mid \text{NOT BUY}, \text{NOT BUY}) = (-10) \cdot 0.2883\ldots + (-2) \cdot 0.3416\ldots + 12 \cdot 0.2616\ldots +$$
$$22 \cdot 0.0641\ldots + 40 \cdot 0.0445\ldots \approx 2.76$$

$$ER(\text{Stock } 50 \mid \text{NOT BUY}, \text{NOT BUY}) = (-4) \cdot 0.2883\ldots + 6 \cdot 0.3416\ldots + 12 \cdot 0.2616\ldots +$$
$$26 \cdot 0.0641\ldots + 16 \cdot 0.0445\ldots \approx 5.77$$

$$ER(\text{Do not stock} \mid \text{NOT BUY}, \text{NOT BUY}) = 0 \cdot 0.2883\ldots + 0 \cdot 0.3416\ldots + 0 \cdot 0.2616\ldots +$$
$$0 \cdot 0.0641\ldots + 0 \cdot 0.0445\ldots = 0$$

$\Rightarrow a'' = \text{Stock } 50$

$\Rightarrow$

$$\mathrm{VSI}(\text{NOT BUY}, \text{NOT BUY}) = E''R(\text{Stock } 50 \mid \text{NOT BUY}, \text{NOT BUY}) -$$
$$E''R(\text{Stock } 50 \mid \text{NOT BUY}, \text{NOT BUY}) = (5.77 - 5.77) = 0$$

$$\mathrm{EVSI} = \sum_y \mathrm{VSI}(y) P(y) =$$

$$= \mathrm{VSI}(\mathrm{BUY},\mathrm{BUY}) \cdot P(\mathrm{BUY},\mathrm{BUY}) + 2 \cdot \mathrm{VSI}(\mathrm{BUY},\mathrm{NOT\ \ BUY}) \cdot P(\mathrm{BUY},\mathrm{NOT\ \ BUY}) +$$

$$\mathrm{VSI}(\mathrm{NOT\ \ BUY},\mathrm{NOT\ \ BUY}) \cdot P(\mathrm{NOT\ \ BUY},\mathrm{NOT\ \ BUY}) =$$

$$= 7.17 \cdot 0.082 + 2 \cdot 1.41 \cdot 0.178 + 0 \cdot 0.652 \approx 1.09$$

(a) Just the cases with $n = 1$ and $n = 2$

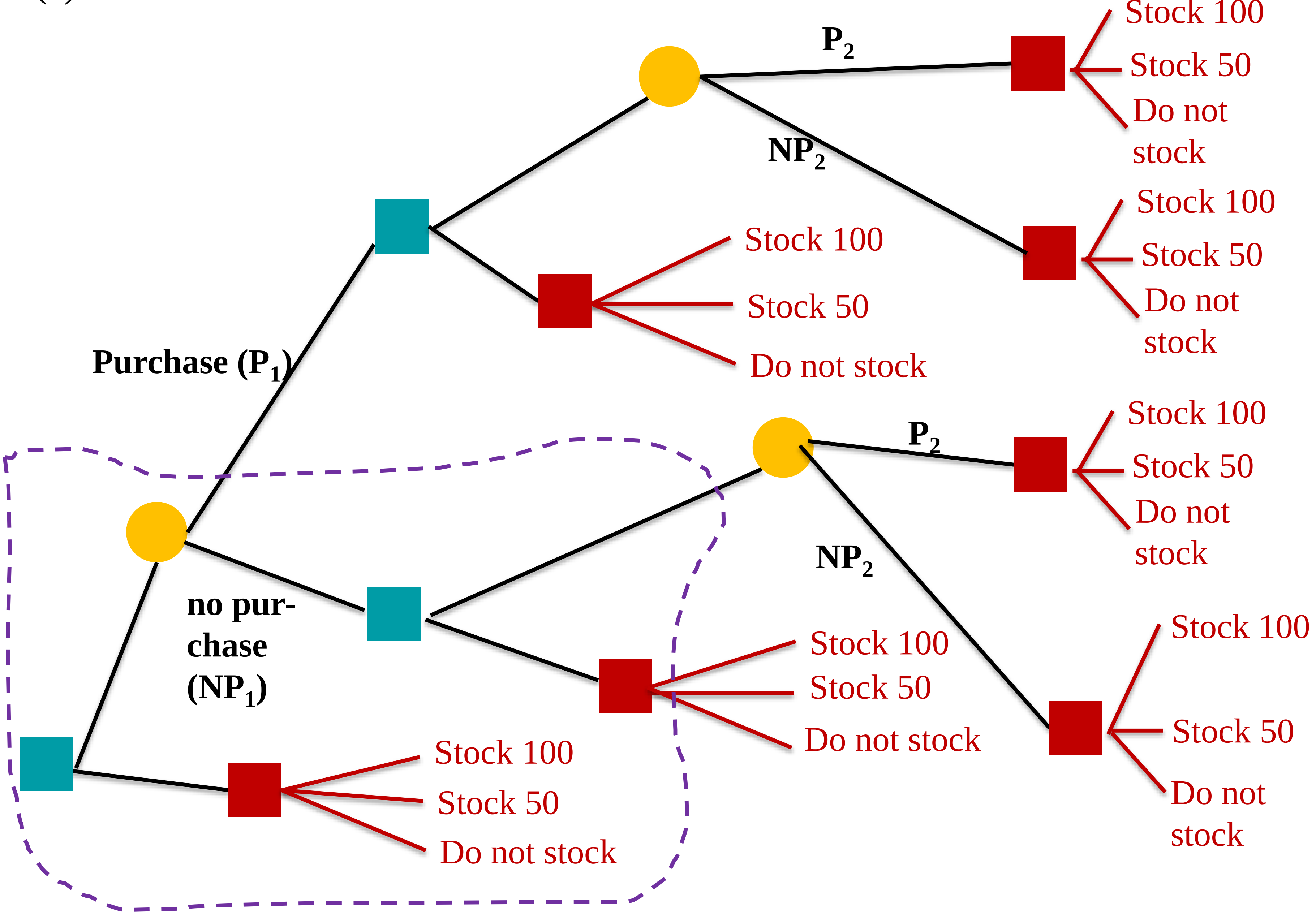
$$\mathrm{ENGS}(1) = \mathrm{EVSI}(1) - \mathrm{CS}(1) \approx 0.84 - 0.50 \cdot 1 = 0.34$$

$$\mathrm{ENGS}(2) = \mathrm{EVSI}(2) - \mathrm{CS}(2) \approx 1.09 - 0.50 \cdot 2 = 0.09$$

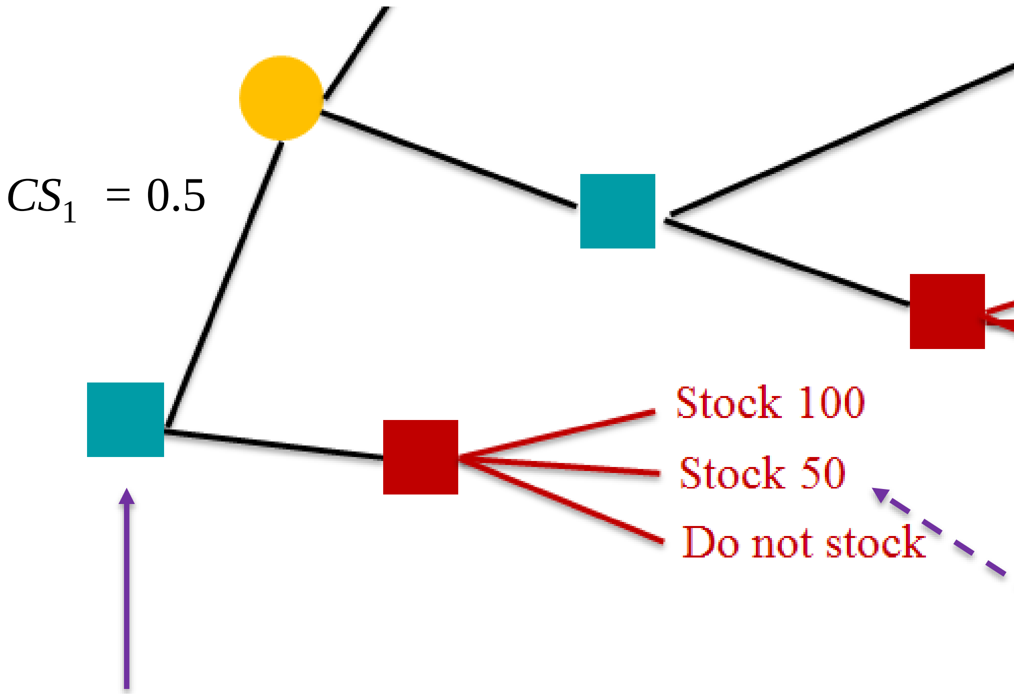
Finally, Exercise 6.27

27. In Exercise 17, consider a sequential sampling plan with a maximum total sample size of two and analyze the problem as follows.
    (a) Represent the situation in terms of a tree diagram.
    (b) Using backward induction, find the ENGS for the sequential plan.
    (c) Compare the sequential plan with a single-stage plan having $n = 2$.

(a)



Purchase (P_1)
no pur-
chase
(NP_1)
P_2
NP_2
P_2
NP_2
Stock 100
Stock 50
Do not
stock
Stock 100
Stock 50
Do not
stock
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not
stock
Stock 100
Stock 50
Do not
stock
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock



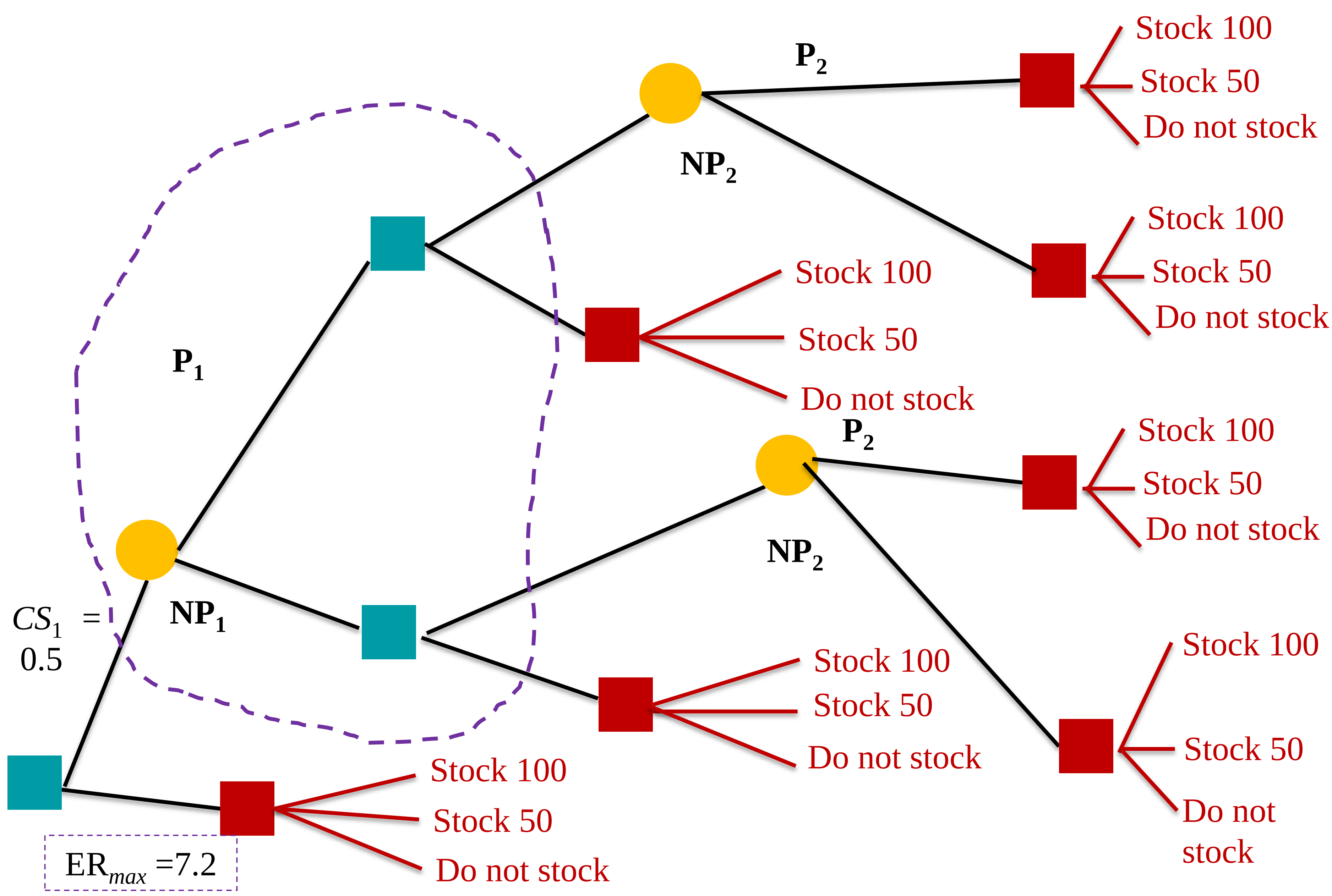
θ

		PROPORTION OF CONSUMERS PURCHASING				
		0.10	0.20	0.30	0.40	0.50
DECISION	*Stock 100*	-10	-2	12	22	40
	Stock 50	-4	6	12	16	16
	Do not stock	0	0	0	0	0

Prior distribution:

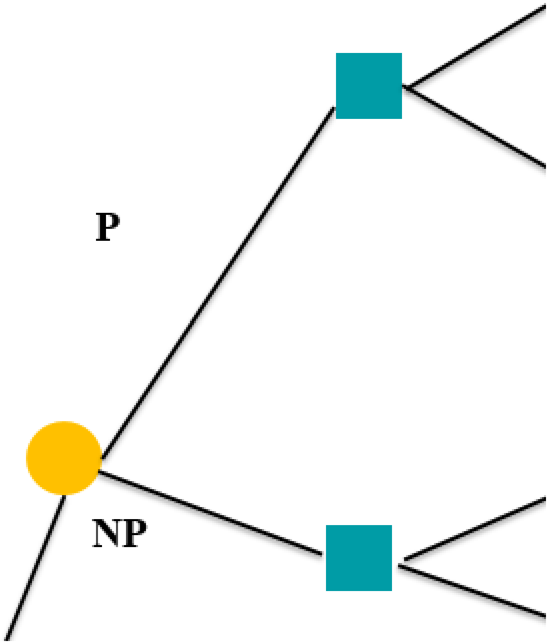
θ	$P(\tilde{\theta} = \theta)$
0.10	0.2
0.20	0.3
0.30	0.3
0.40	0.1
0.50	0.1

$$ER(\text{Stock } 100) =$$
$$(-10)\cdot 0.2 + (-2)\cdot 0.3 + 12\cdot 0.3 + 22\cdot 0.1 + 40\cdot 0.1 = \underline{\underline{7.2}}$$
$$ER(\text{Stock } 50) =$$
$$(-4)\cdot 0.2 + 6\cdot 0.3 + 12\cdot 0.3 + 16\cdot 0.1 + 16\cdot 0.1 = \underline{\underline{7.8}}$$ Max
$$ER(\text{Do not stock}) =$$
$$0\cdot 0.2 + 0\cdot 0.3 + 0\cdot 0.3 + 0\cdot 0.1 + 0\cdot 0.1 = \underline{\underline{0}}$$



P2
NP2
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
P1
P2
Stock 100
Stock 50
Do not stock
NP2
CS1 = 0.5
NP1
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
ERmax =7.2

First sampled consumer



If outcome = Purchase

Posterior probabilities of θ:

$$P\left(\tilde{\theta}=\theta \middle| \mathrm{P}_1\right)=\frac{P\left(\mathrm{P}_1 \middle| \tilde{\theta}=\theta\right)\cdot P\left(\tilde{\theta}=\theta\right)}{\left[\begin{array}{l} P\left(\mathrm{P}_1 \middle| \tilde{\theta}=0.1\right)\cdot P\left(\tilde{\theta}=0.1\right)+P\left(\mathrm{P}_1 \middle| \tilde{\theta}=0.2\right)\cdot P\left(\tilde{\theta}=0.2\right)+ \\ P\left(\mathrm{P}_1 \middle| \tilde{\theta}=0.3\right)\cdot P\left(\tilde{\theta}=0.3\right)+P\left(\mathrm{P}_1 \middle| \tilde{\theta}=0.4\right)\cdot P\left(\tilde{\theta}=0.4\right)+ \\ P\left(\mathrm{P}_1 \middle| \tilde{\theta}=0.5\right)\cdot P\left(\tilde{\theta}=0.5\right) \end{array}\right]}=$$

$$=\frac{P\left(\mathrm{P}_1 \middle| \tilde{\theta}=\theta\right)\cdot P\left(\tilde{\theta}=\theta\right)}{0.1\cdot 0.2+0.2\cdot 0.3+0.3\cdot 0.3+0.4\cdot 0.1+0.5\cdot 0.1}=\frac{P\left(\mathrm{P}_1 \middle| \tilde{\theta}=\theta\right)\cdot P\left(\tilde{\theta}=\theta\right)}{0.26}$$

$$\Rightarrow P\left(\tilde{\theta}=0.1 \middle| \mathrm{P}_1\right)=0.1\cdot 0.2/0.26=2/26\,;\ P\left(\tilde{\theta}=0.2 \middle| \mathrm{P}_1\right)=6/26\,;$$

$$P\left(\tilde{\theta}=0.3 \middle| \mathrm{P}_1\right)=9/26\,;\ P\left(\tilde{\theta}=0.4 \middle| \mathrm{P}_1\right)=4/26\,;\ P\left(\tilde{\theta}=0.2 \middle| \mathrm{P}_1\right)=5/26\,;$$

If outcome = No purchase

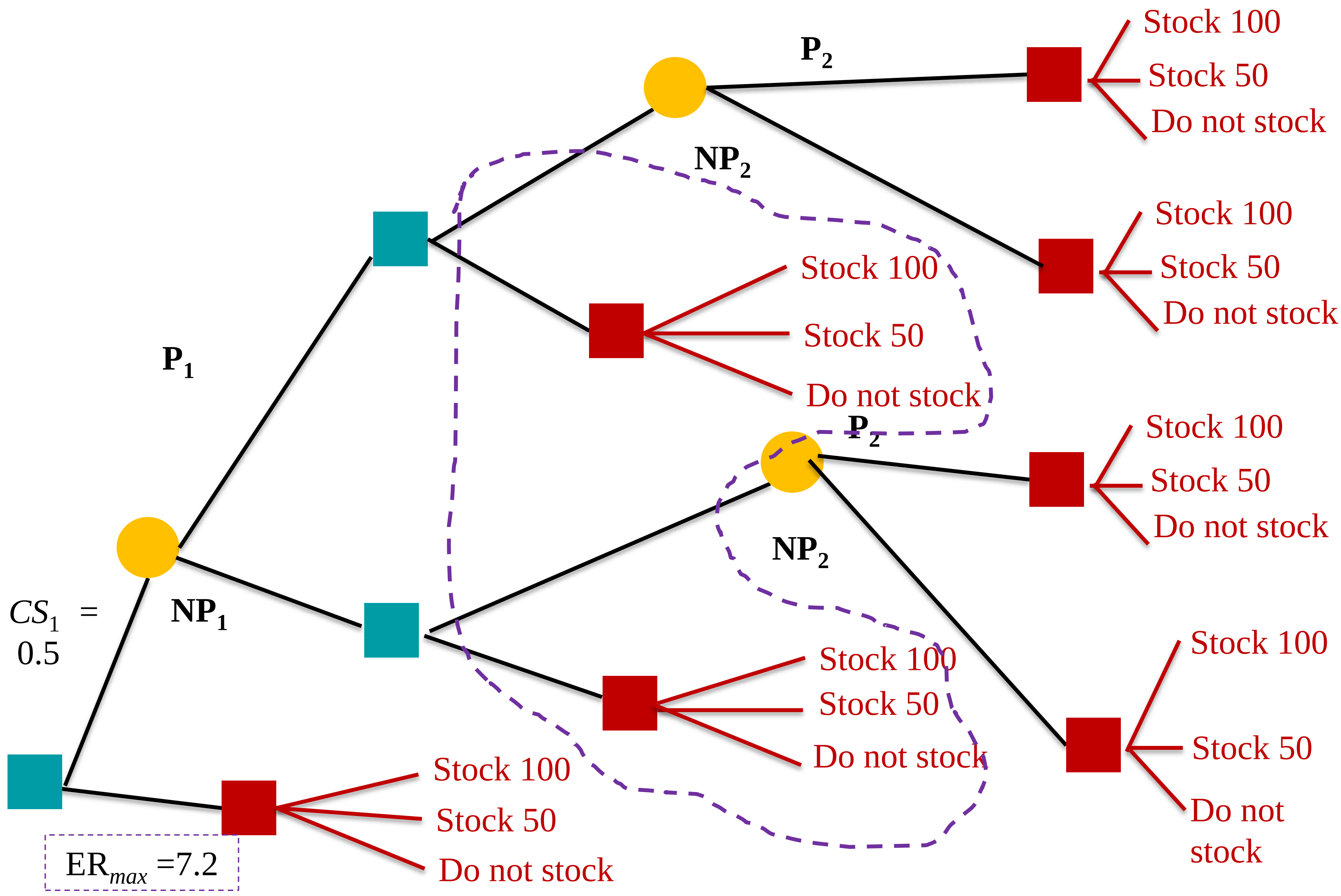
Posterior probabilities of θ:

$$P\left(\tilde{\theta}=\theta \middle| \mathrm{NP}_1\right)=\frac{P\left(\mathrm{NP}_1 \middle| \tilde{\theta}=\theta\right)\cdot P\left(\tilde{\theta}=\theta\right)}{\left[\begin{array}{l} P\left(\mathrm{NP}_1 \middle| \tilde{\theta}=0.1\right)\cdot P\left(\tilde{\theta}=0.1\right)+P\left(\mathrm{NP}_1 \middle| \tilde{\theta}=0.2\right)\cdot P\left(\tilde{\theta}=0.2\right)+ \\ P\left(\mathrm{NP}_1 \middle| \tilde{\theta}=0.3\right)\cdot P\left(\tilde{\theta}=0.3\right)+P\left(\mathrm{NP}_1 \middle| \tilde{\theta}=0.4\right)\cdot P\left(\tilde{\theta}=0.4\right)+ \\ P\left(\mathrm{NP}_1 \middle| \tilde{\theta}=0.5\right)\cdot P\left(\tilde{\theta}=0.5\right) \end{array}\right]}=$$

$$=\frac{P\left(\mathrm{NP}_1 \middle| \tilde{\theta}=\theta\right)\cdot P\left(\tilde{\theta}=\theta\right)}{0.9\cdot 0.2+0.8\cdot 0.3+0.7\cdot 0.3+0.6\cdot 0.1+0.5\cdot 0.1}=\frac{P\left(\mathrm{NP}_1 \middle| \tilde{\theta}=\theta\right)\cdot P\left(\tilde{\theta}=\theta\right)}{0.74}$$

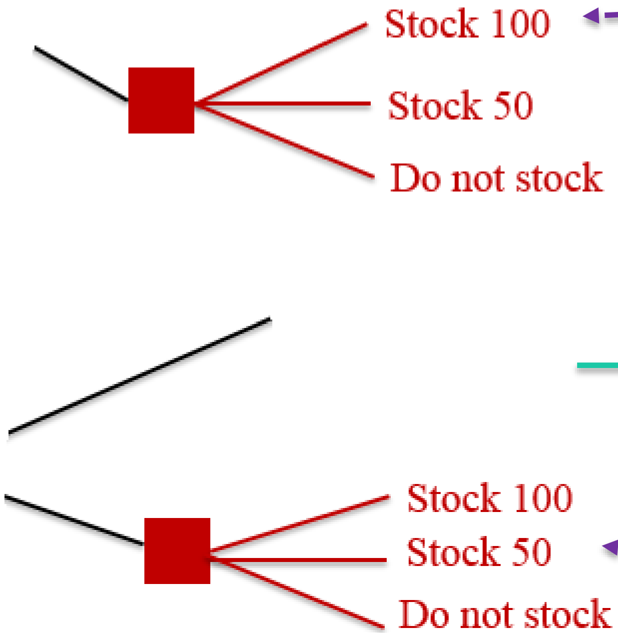
$$\Rightarrow P\left(\tilde{\theta}=0.1 \middle| \mathrm{NP}_1\right)=0.9\cdot 0.2/0.74=18/74\,;\ P\left(\tilde{\theta}=0.2 \middle| \mathrm{NP}_1\right)=24/74\,;$$

$$P\left(\tilde{\theta}=0.3 \middle| \mathrm{NP}_1\right)=21/74\,;\ P\left(\tilde{\theta}=0.4 \middle| \mathrm{NP}_1\right)=6/74\,;\ P\left(\tilde{\theta}=0.2 \middle| \mathrm{NP}_1\right)=5/74\,;$$



$CS_1 = 0.5$
$ER_{max} = 7.2$
P_1
NP_1
P_2
NP_2
P_2
NP_2
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock

θ	0.10	0.20	0.30	0.40	0.50
$P(\tilde{\theta} = \theta \mid \mathrm{P}_1)$	2/26	6/26	9/26	4/26	5/26



$$ER(\text{Stock } 100 \mid \mathrm{P}_1) =$$

$$(-10)\cdot\tfrac{2}{26} + (-2)\cdot\tfrac{6}{26} + 12\cdot\tfrac{9}{26} + 22\cdot\tfrac{4}{26} + 40\cdot\tfrac{5}{26} \approx \underline{\underline{14.0}} \quad \text{Max}$$

$$ER(\text{Stock } 50 \mid \mathrm{P}_1) =$$

$$(-4)\cdot\tfrac{2}{26} + 6\cdot\tfrac{6}{26} + 12\cdot\tfrac{9}{26} + 16\cdot\tfrac{4}{26} + 16\cdot\tfrac{5}{26} \approx \underline{\underline{10.8}}$$

$$ER(\text{Do not stock} \mid \mathrm{P}_1) = \underline{\underline{0}}$$

$$ER(\text{Stock } 100 \mid \mathrm{NP}_1) =$$

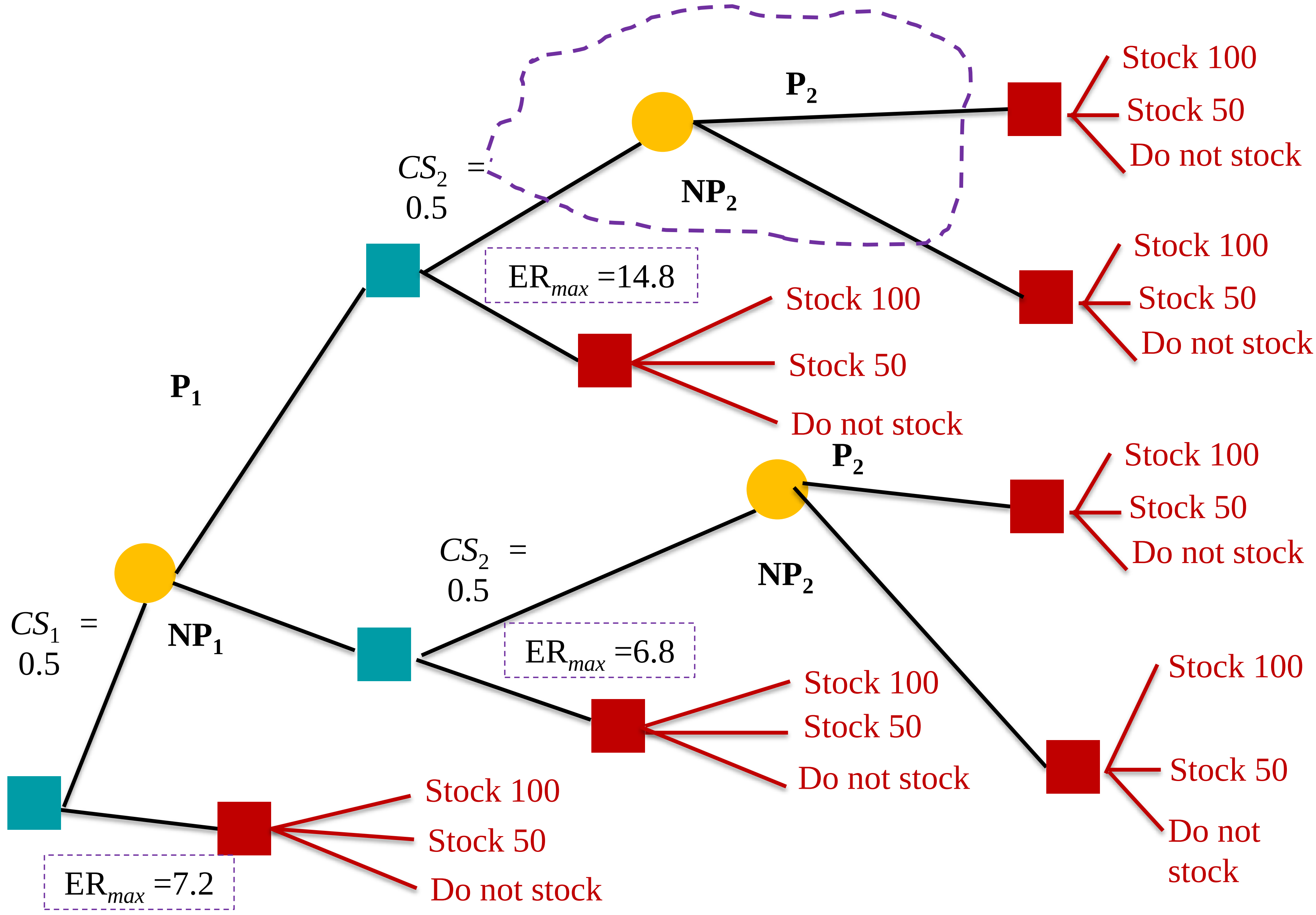
$$(-10)\cdot\tfrac{18}{74} + (-2)\cdot\tfrac{24}{74} + 12\cdot\tfrac{21}{74} + 22\cdot\tfrac{6}{74} + 40\cdot\tfrac{5}{74} \approx \underline{\underline{4.8}}$$

$$ER(\text{Stock } 50 \mid \mathrm{NP}_1) =$$

$$(-4)\cdot\tfrac{18}{74} + 6\cdot\tfrac{24}{74} + 12\cdot\tfrac{21}{74} + 16\cdot\tfrac{6}{74} + 16\cdot\tfrac{5}{74} \approx \underline{\underline{6.8}} \quad \text{Max}$$

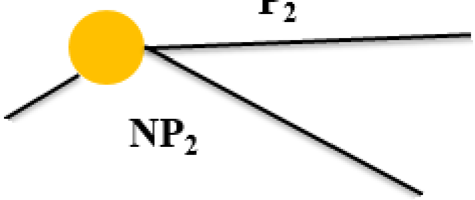
$$ER(\text{Do not stock} \mid \mathrm{NP}_1) = \underline{\underline{0}}$$

θ	0.10	0.20	0.30	0.40	0.50
$P(\tilde{\theta} = \theta \mid \mathrm{NP}_1)$	18/74	24/74	21/74	6/74	5/74



Stock 100
Stock 50
Do not stock
P2
CS2 = 0.5
NP2
ER max =14.8
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
P1
Stock 100
Stock 50
Do not stock
P2
CS2 = 0.5
NP2
Stock 100
Stock 50
Do not stock
CS1 = 0.5
NP1
ER max =6.8
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
ER max =7.2

Second sampled consumer, case 1



If outcome = Purchase

Posterior probabilities of θ:

$$P\left(\tilde{\theta}=\theta \middle| \mathrm{P}_2,\mathrm{P}_1\right)=\frac{P\left(\mathrm{P}_2,\mathrm{P}_1 \middle| \tilde{\theta}=\theta\right)\cdot P\left(\tilde{\theta}=\theta\right)}{\begin{bmatrix} P\left(\mathrm{P}_2,\mathrm{P}_1 \middle| \tilde{\theta}=0.1\right)\cdot P\left(\tilde{\theta}=0.1\right)+P\left(\mathrm{P}_2,\mathrm{P}_1 \middle| \tilde{\theta}=0.2\right)\cdot P\left(\tilde{\theta}=0.2\right)+ \\ P\left(\mathrm{P}_2,\mathrm{P}_1 \middle| \tilde{\theta}=0.3\right)\cdot P\left(\tilde{\theta}=0.3\right)+P\left(\mathrm{P}_2,\mathrm{P}_1 \middle| \tilde{\theta}=0.4\right)\cdot P\left(\tilde{\theta}=0.4\right)+ \\ P\left(\mathrm{P}_2,\mathrm{P}_1 \middle| \tilde{\theta}=0.5\right)\cdot P\left(\tilde{\theta}=0.5\right) \end{bmatrix}}=$$

$$=\frac{P\left(\mathrm{P}_2,\mathrm{P}_1 \middle| \tilde{\theta}=\theta\right)\cdot P\left(\tilde{\theta}=\theta\right)}{0.1^2\cdot 0.2+0.2^2\cdot 0.3+0.3^2\cdot 0.3+0.4^2\cdot 0.1+0.5^2\cdot 0.1}=\frac{P\left(\mathrm{P}_2,\mathrm{P}_1 \middle| \tilde{\theta}=\theta\right)\cdot P\left(\tilde{\theta}=\theta\right)}{0.082}$$

$$\Rightarrow P\left(\tilde{\theta}=0.1 \middle| \mathrm{P}_2,\mathrm{P}_1\right)=0.1^2\cdot 0.2/0.26=2/82\,;\; P\left(\tilde{\theta}=0.2 \middle| \mathrm{P}_2,\mathrm{P}_1\right)=12/82\,;$$

$$P\left(\tilde{\theta}=0.3 \middle| \mathrm{P}_2,\mathrm{P}_1\right)=27/82\,;\; P\left(\tilde{\theta}=0.4 \middle| \mathrm{P}_2,\mathrm{P}_1\right)=16/82\,;\; P\left(\tilde{\theta}=0.5 \middle| \mathrm{P}_2,\mathrm{P}_1\right)=25/82\,;$$

If outcome = No purchase

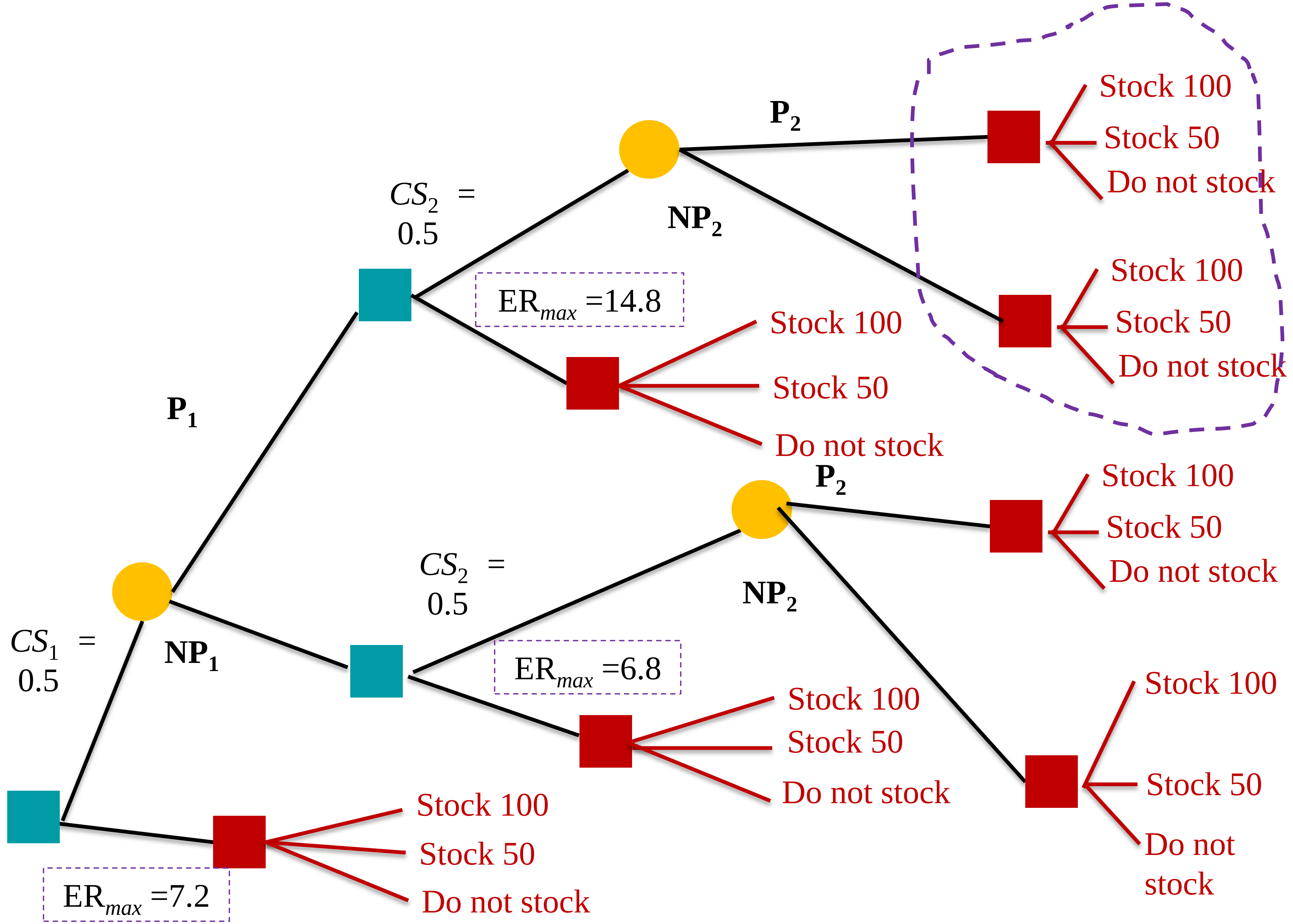
Posterior probabilities of θ:

$$P\left(\tilde{\theta}=\theta \middle| \mathrm{NP}_2,\mathrm{P}_1\right)=\frac{P\left(\mathrm{NP}_2,\mathrm{P}_1 \middle| \tilde{\theta}=\theta\right)\cdot P\left(\tilde{\theta}=\theta\right)}{\begin{bmatrix} P\left(\mathrm{NP}_2,\mathrm{P}_1 \middle| \tilde{\theta}=0.1\right)\cdot P\left(\tilde{\theta}=0.1\right)+P\left(\mathrm{NP}_2,\mathrm{P}_1 \middle| \tilde{\theta}=0.2\right)\cdot P\left(\tilde{\theta}=0.2\right)+ \\ P\left(\mathrm{NP}_2,\mathrm{P}_1 \middle| \tilde{\theta}=0.3\right)\cdot P\left(\tilde{\theta}=0.3\right)+P\left(\mathrm{NP}_2,\mathrm{P}_1 \middle| \tilde{\theta}=0.4\right)\cdot P\left(\tilde{\theta}=0.4\right)+ \\ P\left(\mathrm{NP}_2,\mathrm{P}_1 \middle| \tilde{\theta}=0.5\right)\cdot P\left(\tilde{\theta}=0.5\right) \end{bmatrix}}=$$

$$=\frac{P\left(\mathrm{NP}_2,\mathrm{P}_1 \middle| \tilde{\theta}=\theta\right)\cdot P\left(\tilde{\theta}=\theta\right)}{0.1\cdot 0.9\cdot 0.2+0.2\cdot 0.8\cdot 0.3+0.3\cdot 0.7\cdot 0.3+0.4\cdot 0.6\cdot 0.1+0.5^2\cdot 0.1}=\frac{P\left(\mathrm{NP}_2,\mathrm{P}_1 \middle| \tilde{\theta}=\theta\right)\cdot P\left(\tilde{\theta}=\theta\right)}{0.178}$$

$$\Rightarrow P\left(\tilde{\theta}=0.1 \middle| \mathrm{NP}_2,\mathrm{P}_1\right)=0.1\cdot 0.9\cdot 0.2/0.26=18/178\,;\; P\left(\tilde{\theta}=0.2 \middle| \mathrm{NP}_2,\mathrm{P}_1\right)=48/178\,;$$

$$P\left(\tilde{\theta}=0.3 \middle| \mathrm{NP}_2,\mathrm{P}_1\right)=63/178\,;\; P\left(\tilde{\theta}=0.4 \middle| \mathrm{NP}_2,\mathrm{P}_1\right)=24/178\,;\; P\left(\tilde{\theta}=0.5 \middle| \mathrm{NP}_2,\mathrm{P}_1\right)=25/178\,;$$



Stock 100
Stock 50
Do not stock
P2
NP2
Stock 100
Stock 50
Do not stock
CS2 = 0.5
ERmax =14.8
Stock 100
Stock 50
Do not stock
P1
P2
Stock 100
Stock 50
Do not stock
CS2 = 0.5
NP2
CS1 = 0.5
NP1
ERmax =6.8
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
ERmax =7.2

θ	0.10	0.20	0.30	0.40	0.50
$P(\tilde{\theta} = \theta \mid \mathrm{P}_2, \mathrm{P}_1)$	2/82	12/82	27/82	16/82	25/82

$$ER(\text{Stock } 100 \mid \mathrm{P}_1) =$$

$$(-10)\cdot\frac{2}{82} + (-2)\cdot\frac{12}{82} + 12\cdot\frac{27}{82} + 22\cdot\frac{16}{82} + 40\cdot\frac{25}{82} \approx \underline{\underline{19.9}}$$ Max

$$ER(\text{Stock } 50 \mid \mathrm{P}_1) =$$

$$(-4)\cdot\frac{2}{82} + 6\cdot\frac{12}{82} + 12\cdot\frac{27}{82} + 16\cdot\frac{16}{82} + 16\cdot\frac{25}{892} \approx \underline{\underline{12.7}}$$

$$ER(\text{Do not stock} \mid \mathrm{P}_1) = \underline{\underline{0}}$$

$$ER(\text{Stock } 100 \mid \mathrm{NP}_2, \mathrm{P}_1) =$$

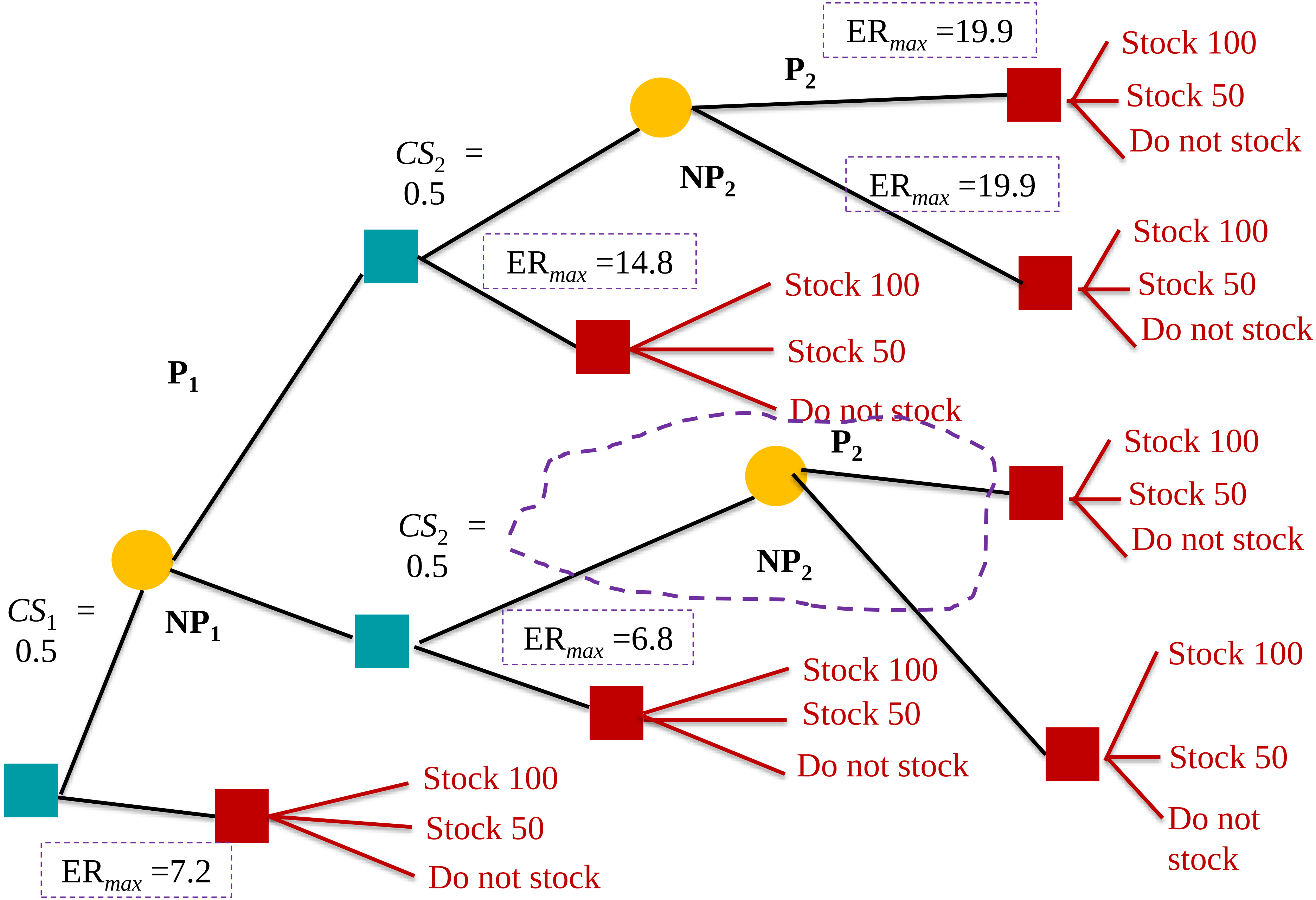
$$(-10)\cdot\frac{18}{178} + (-2)\cdot\frac{48}{178} + 12\cdot\frac{63}{178} + 22\cdot\frac{24}{178} + 40\cdot\frac{25}{178} \approx \underline{\underline{11.3}}$$ Max

$$ER(\text{Stock } 50 \mid \mathrm{NP}_2, \mathrm{P}_1) =$$

$$(-4)\cdot\frac{18}{178} + 6\cdot\frac{48}{178} + 12\cdot\frac{63}{178} + 16\cdot\frac{24}{178} + 16\cdot\frac{25}{178} \approx \underline{\underline{9.9}}$$

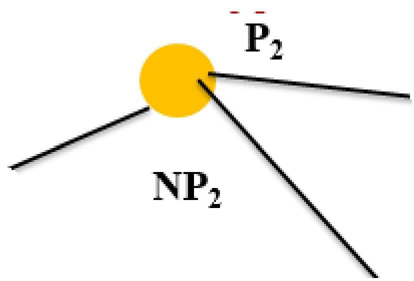
$$ER(\text{Do not stock} \mid \mathrm{NP}_2, \mathrm{P}_1) = \underline{\underline{0}}$$

θ	0.10	0.20	0.30	0.40	0.50
$P(\tilde{\theta} = \theta \mid \mathrm{NP}_2, \mathrm{P}_1)$	18/178	48/178	63/178	24/178	25/178



ER_{max} =19.9
P_2
Stock 100
Stock 50
Do not stock
CS_2 =
0.5
NP_2
ER_{max} =19.9
ER_{max} =14.8
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
P_1
P_2
Stock 100
Stock 50
Do not stock
CS_2 =
0.5
NP_2
CS_1 =
0.5
NP_1
ER_{max} =6.8
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not
stock
Stock 100
Stock 50
Do not stock
ER_{max} =7.2

Second sampled consumer, case 2



If outcome = Purchase

Posterior probabilities of θ:

$$P\left(\tilde{\theta}=\theta \middle| \mathrm{P}_2,\mathrm{NP}_1\right)=\left\langle \begin{array}{l}\text{Independent}\\ \text{samples (Bernoulli}\\ \text{trials)}\end{array}\right\rangle = P\left(\tilde{\theta}=\theta \middle| \mathrm{NP}_2,\mathrm{P}_1\right)=\frac{P\left(\mathrm{NP}_2,\mathrm{P}_1 \middle| \tilde{\theta}=\theta\right)\cdot P\left(\tilde{\theta}=\theta\right)}{0.178}$$

$$\Rightarrow P\left(\tilde{\theta}=0.1 \middle| \mathrm{P}_2,\mathrm{NP}_1\right)=18/178\,;\ P\left(\tilde{\theta}=0.2 \middle| \mathrm{P}_2,\mathrm{NP}_1\right)=48/178\,;$$

$$P\left(\tilde{\theta}=0.3 \middle| \mathrm{P}_2,\mathrm{NP}_1\right)=63/178\,;\ P\left(\tilde{\theta}=0.4 \middle| \mathrm{P}_2,\mathrm{NP}_1\right)=24/178\,;\ P\left(\tilde{\theta}=0.5 \middle| \mathrm{P}_2,\mathrm{NP}_1\right)=25/178\,;$$

If outcome = No purchase

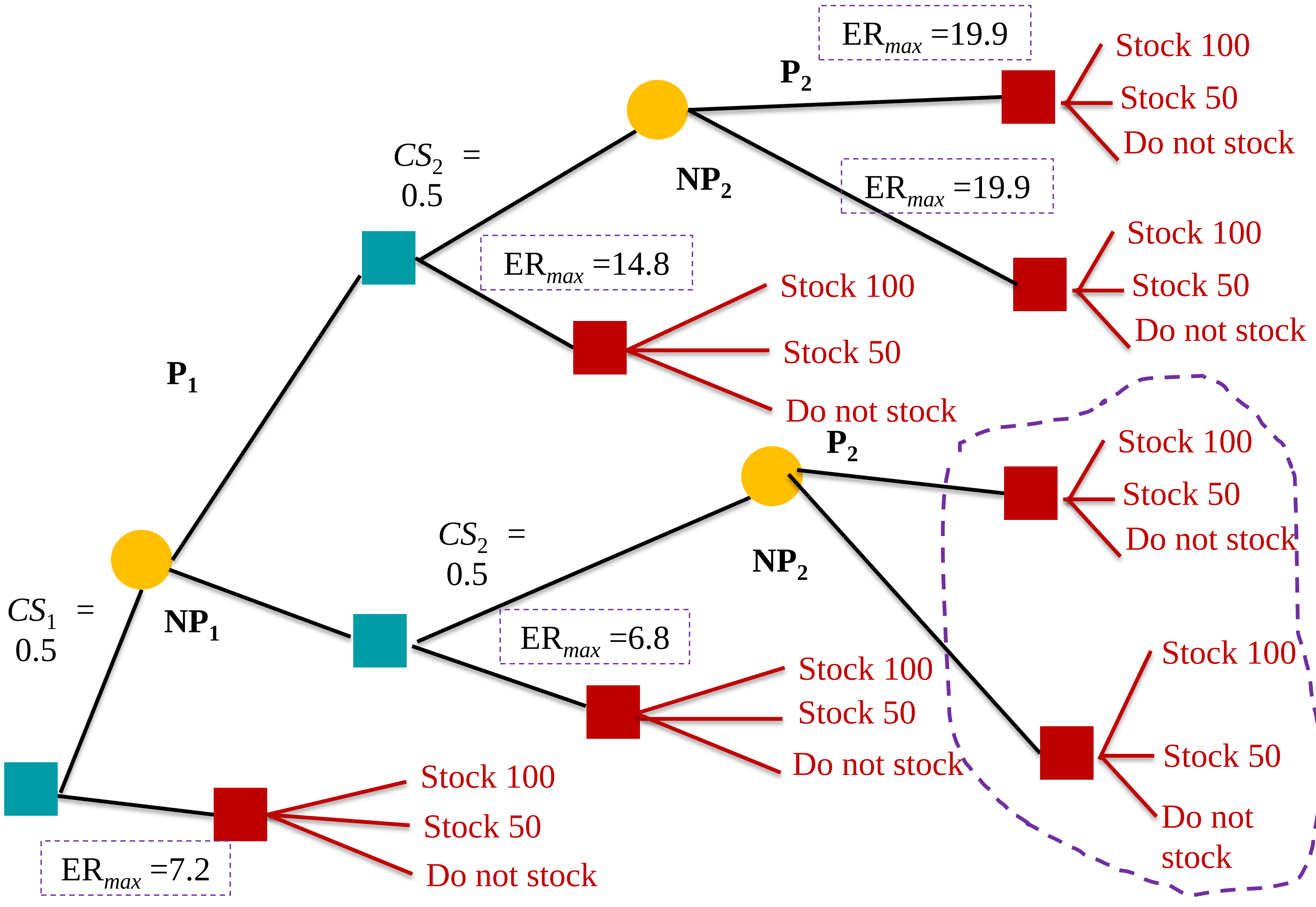
Posterior probabilities of θ:

$$P\left(\tilde{\theta}=\theta \middle| \mathrm{NP}_2,\mathrm{NP}_1\right)=\frac{P\left(\mathrm{NP}_2,\mathrm{NP}_1 \middle| \tilde{\theta}=\theta\right)\cdot P\left(\tilde{\theta}=\theta\right)}{\left[\begin{array}{l} P\left(\mathrm{NP}_2,\mathrm{NP}_1 \middle| \tilde{\theta}=0.1\right)\cdot P\left(\tilde{\theta}=0.1\right)+P\left(\mathrm{NP}_2,\mathrm{NP}_1 \middle| \tilde{\theta}=0.2\right)\cdot P\left(\tilde{\theta}=0.2\right)+\\ P\left(\mathrm{NP}_2,\mathrm{NP}_1 \middle| \tilde{\theta}=0.3\right)\cdot P\left(\tilde{\theta}=0.3\right)+P\left(\mathrm{NP}_2,\mathrm{NP}_1 \middle| \tilde{\theta}=0.4\right)\cdot P\left(\tilde{\theta}=0.4\right)+\\ P\left(\mathrm{NP}_2,\mathrm{P}_1 \middle| \tilde{\theta}=0.5\right)\cdot P\left(\tilde{\theta}=0.5\right)\end{array}\right]}=$$

$$=\frac{P\left(\mathrm{NP}_2,\mathrm{NP}_1 \middle| \tilde{\theta}=\theta\right)\cdot P\left(\tilde{\theta}=\theta\right)}{0.9^2\cdot 0.2+0.8^2\cdot 0.3+0.7^2\cdot 0.3+0.6^2\cdot 0.1+0.5^2\cdot 0.1}=\frac{P\left(\mathrm{NP}_2,\mathrm{P}_1 \middle| \tilde{\theta}=\theta\right)\cdot P\left(\tilde{\theta}=\theta\right)}{0.562}$$

$$\Rightarrow P\left(\tilde{\theta}=0.1 \middle| \mathrm{NP}_2,\mathrm{P}_1\right)=0.9^2\cdot 0.2/0.562=162/562\,;\ P\left(\tilde{\theta}=0.2 \middle| \mathrm{NP}_2,\mathrm{P}_1\right)=192/562\,;$$

$$P\left(\tilde{\theta}=0.3 \middle| \mathrm{NP}_2,\mathrm{P}_1\right)=147/562\,;\ P\left(\tilde{\theta}=0.4 \middle| \mathrm{NP}_2,\mathrm{P}_1\right)=36/562\,;\ P\left(\tilde{\theta}=0.2 \middle| \mathrm{NP}_2,\mathrm{P}_1\right)=25/562\,;$$



ER_{max} =19.9
P_2
Stock 100
Stock 50
Do not stock
CS_2 = 0.5
NP_2
ER_{max} =19.9
Stock 100
Stock 50
Do not stock
ER_{max} =14.8
Stock 100
Stock 50
Do not stock
P_1
P_2
Stock 100
Stock 50
Do not stock
CS_2 = 0.5
NP_2
CS_1 = 0.5
NP_1
ER_{max} =6.8
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
ER_{max} =7.2

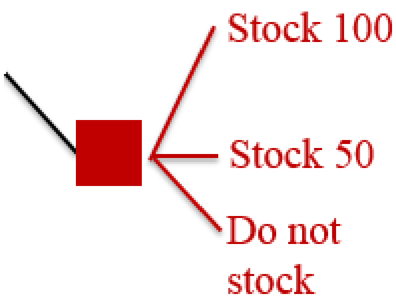
θ	0.10	0.20	0.30	0.40	0.50
$P(\tilde{\theta} = \theta \mid \mathrm{P}_2, \mathrm{NP}_1)$	18/178	48/178	63/178	24/178	25/178

$$ER(\text{Stock } 100 \mid \mathrm{P}_2, \mathrm{NP}_1) = ER(\text{Stock } 100 \mid \mathrm{NP}_2, \mathrm{P}_1) \approx \underline{\underline{11.3}}$$

Max

$$ER(\text{Stock } 50 \mid \mathrm{P}_2, \mathrm{NP}_1) = ER(\text{Stock } 50 \mid \mathrm{NP}_2, \mathrm{P}_1) \approx \underline{\underline{9.9}}$$

$$ER(\text{Do not stock} \mid \mathrm{P}_2, \mathrm{NP}_1) = \underline{\underline{0}}$$



$$ER(\text{Stock } 100 \mid \mathrm{NP}_2, \mathrm{NP}_1) =$$

$$(-10) \cdot \frac{162}{562} + (-2) \cdot \frac{192}{562} + 12 \cdot \frac{147}{562} + 22 \cdot \frac{36}{562} + 40 \cdot \frac{25}{562} \approx \underline{\underline{2.8}}$$

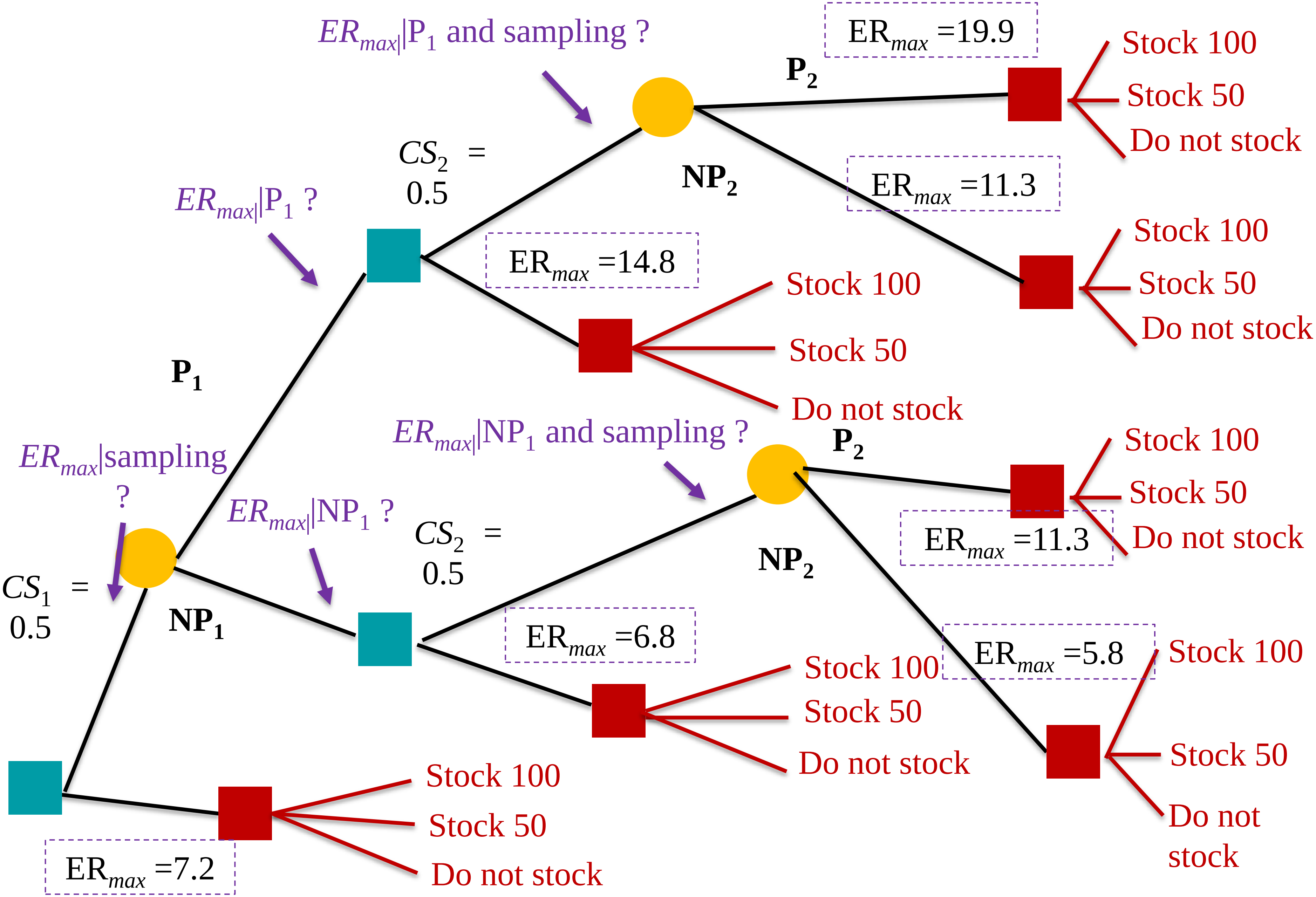
$$ER(\text{Stock } 50 \mid \mathrm{NP}_2, \mathrm{NP}_1) =$$

$$(-4) \cdot \frac{162}{562} + 6 \cdot \frac{192}{562} + 12 \cdot \frac{147}{562} + 16 \cdot \frac{36}{562} + 16 \cdot \frac{25}{562} \approx \underline{\underline{5.8}}$$

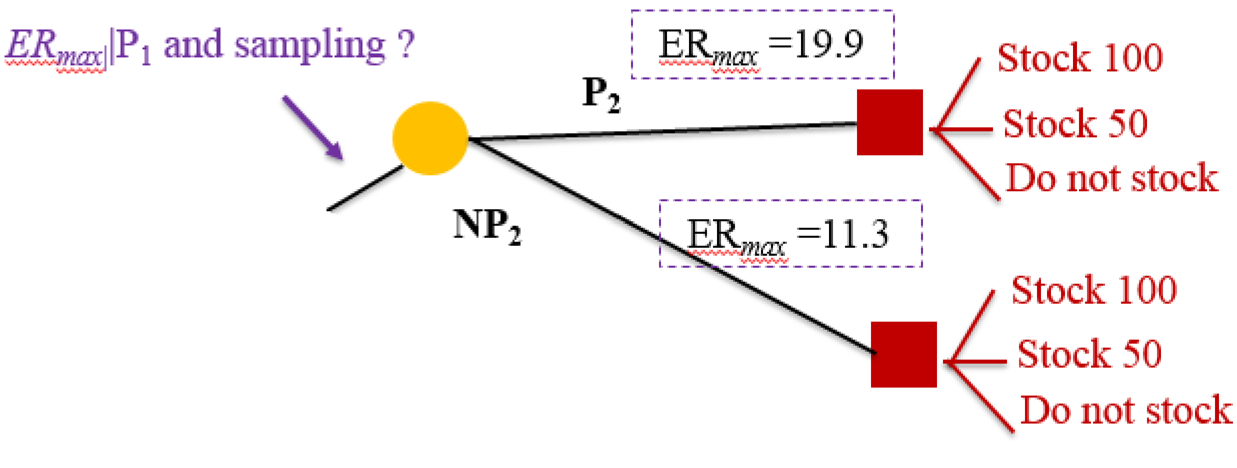
Max

$$ER(\text{Do not stock} \mid \mathrm{NP}_2, \mathrm{NP}_1) = \underline{\underline{0}}$$

θ	0.10	0.20	0.30	0.40	0.50
$P(\tilde{\theta} = \theta \mid \mathrm{NP}_2, \mathrm{P}_1)$	162/562	192/562	147/562	36/562	25/562



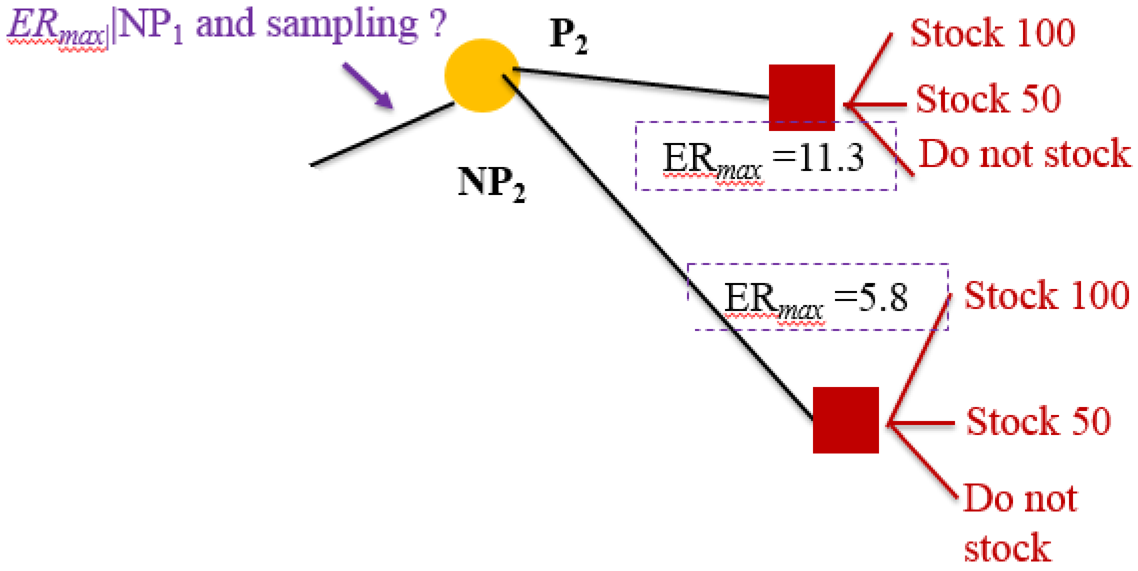
$ER_{max}|P_1$ and sampling ?
ER_{max} =19.9
P_2
Stock 100
Stock 50
Do not stock
CS_2 = 0.5
NP_2
ER_{max} =11.3
Stock 100
Stock 50
Do not stock
$ER_{max}|P_1$?
ER_{max} =14.8
Stock 100
Stock 50
Do not stock
P_1
$ER_{max}|NP_1$ and sampling ?
P_2
Stock 100
Stock 50
Do not stock
ER_{max}|sampling ?
$ER_{max}|NP_1$?
CS_2 = 0.5
ER_{max} =11.3
NP_2
CS_1 = 0.5
NP_1
ER_{max} =6.8
ER_{max} =5.8
Stock 100
Stock 100
Stock 50
Do not stock
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
ER_{max} =7.2



$$ER_{\max}\left|\mathrm{P}_1\right. \text{ and sampling} = 19.9\cdot P\left(\mathrm{P}_2\left|\mathrm{P}_1\right.\right) + 11.3\cdot P\left(\mathrm{NP}_2\left|\mathrm{P}_1\right.\right) = \left\langle \begin{array}{l} \text{Sampling in stage 2} \\ \text{is assumed to be} \\ \text{independent of} \\ \text{sampling in stage 1} \end{array} \right\rangle =$$

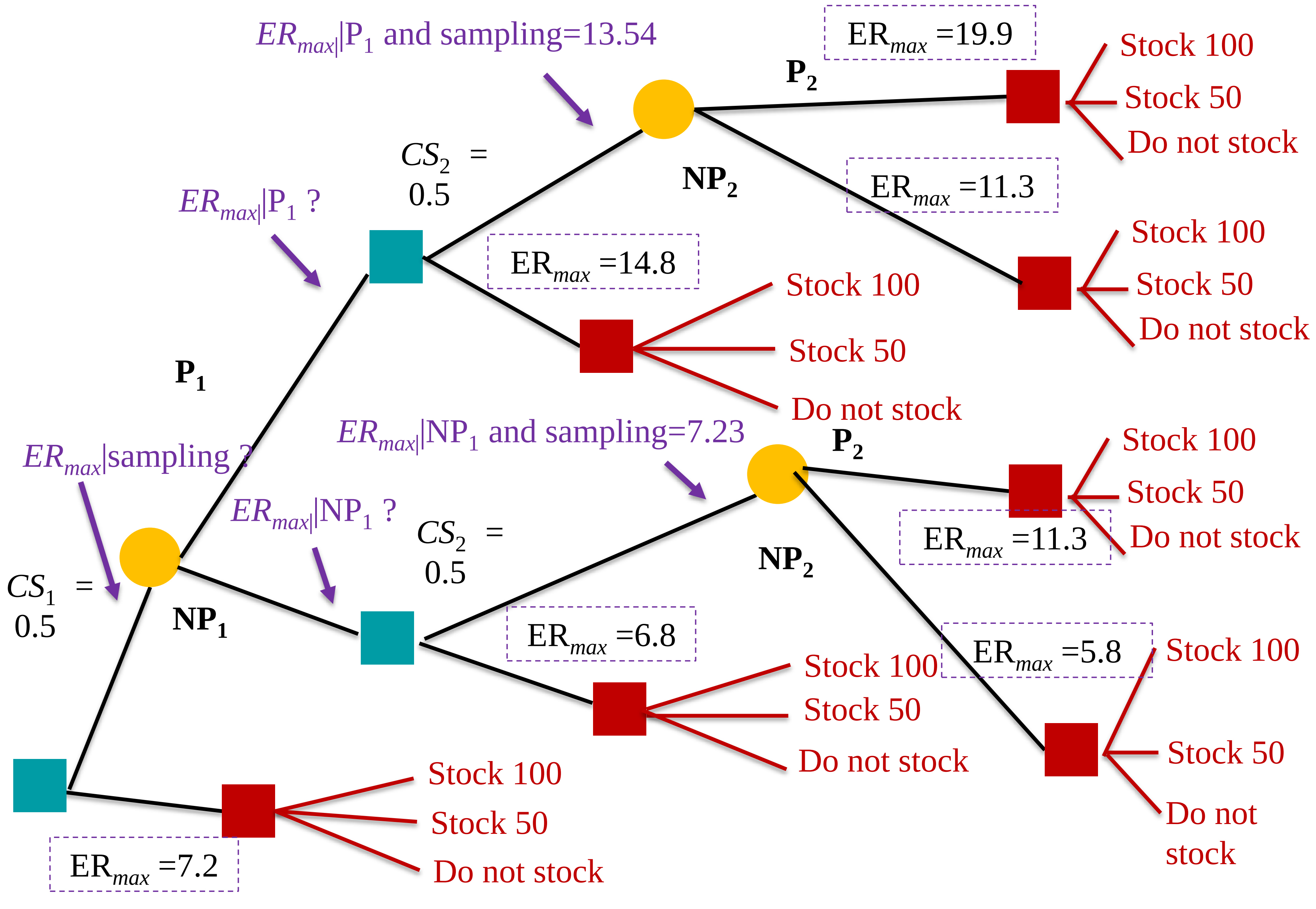
$$= 19.9\cdot P\left(\mathrm{P}_2\right) + 11.3\cdot P\left(\mathrm{NP}_2\right) = 19.9\cdot \sum_{\theta} P\left(\mathrm{P}_2\left|\tilde{\theta} = \theta\right.\right)\cdot P\left(\tilde{\theta} = \theta\right) +$$

$$+ 11.3\cdot \sum_{\theta} P\left(\mathrm{NP}_2\left|\tilde{\theta} = \theta\right.\right)\cdot P\left(\tilde{\theta} = \theta\right) = 19.9\cdot 0.26 + 11.3\cdot 0.74 \approx 13.54$$

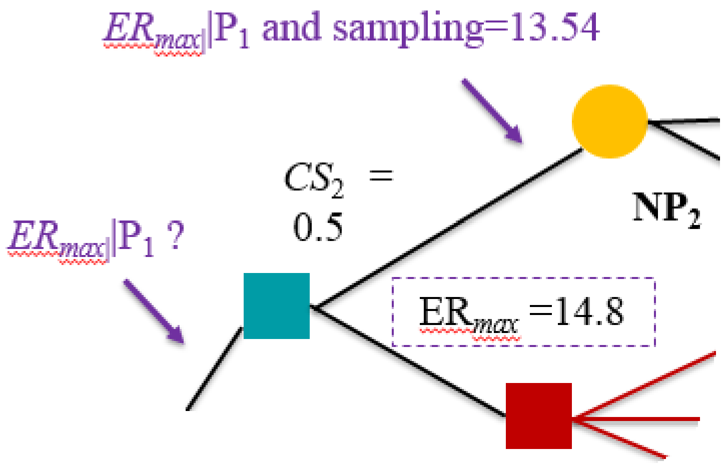


$$ER_{\max}\left|\mathrm{NP}_1 \text{ and sampling} = 11.3\cdot P\left(\mathrm{P}_2\middle|\mathrm{NP}_1\right) + 5.8\cdot P\left(\mathrm{NP}_2\middle|\mathrm{NP}_1\right) = \right.$$

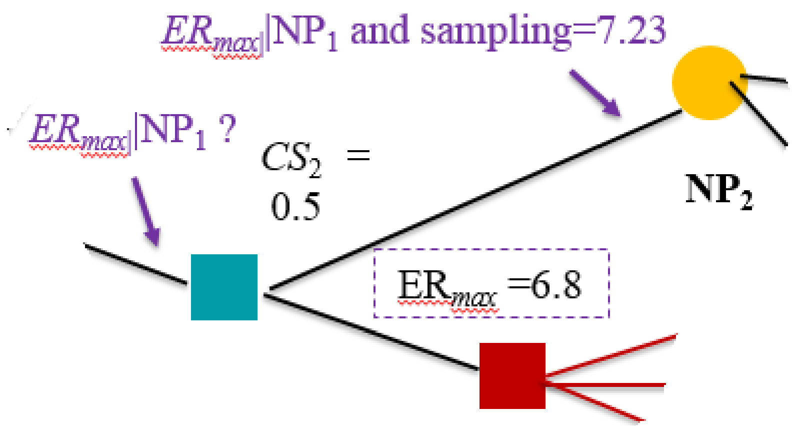
$$= 11.3\cdot P(\mathrm{P}_2) + 5.8\cdot P(\mathrm{NP}_2) = 11.3\cdot 0.26 + 5.8\cdot 0.74 \approx 7.23$$



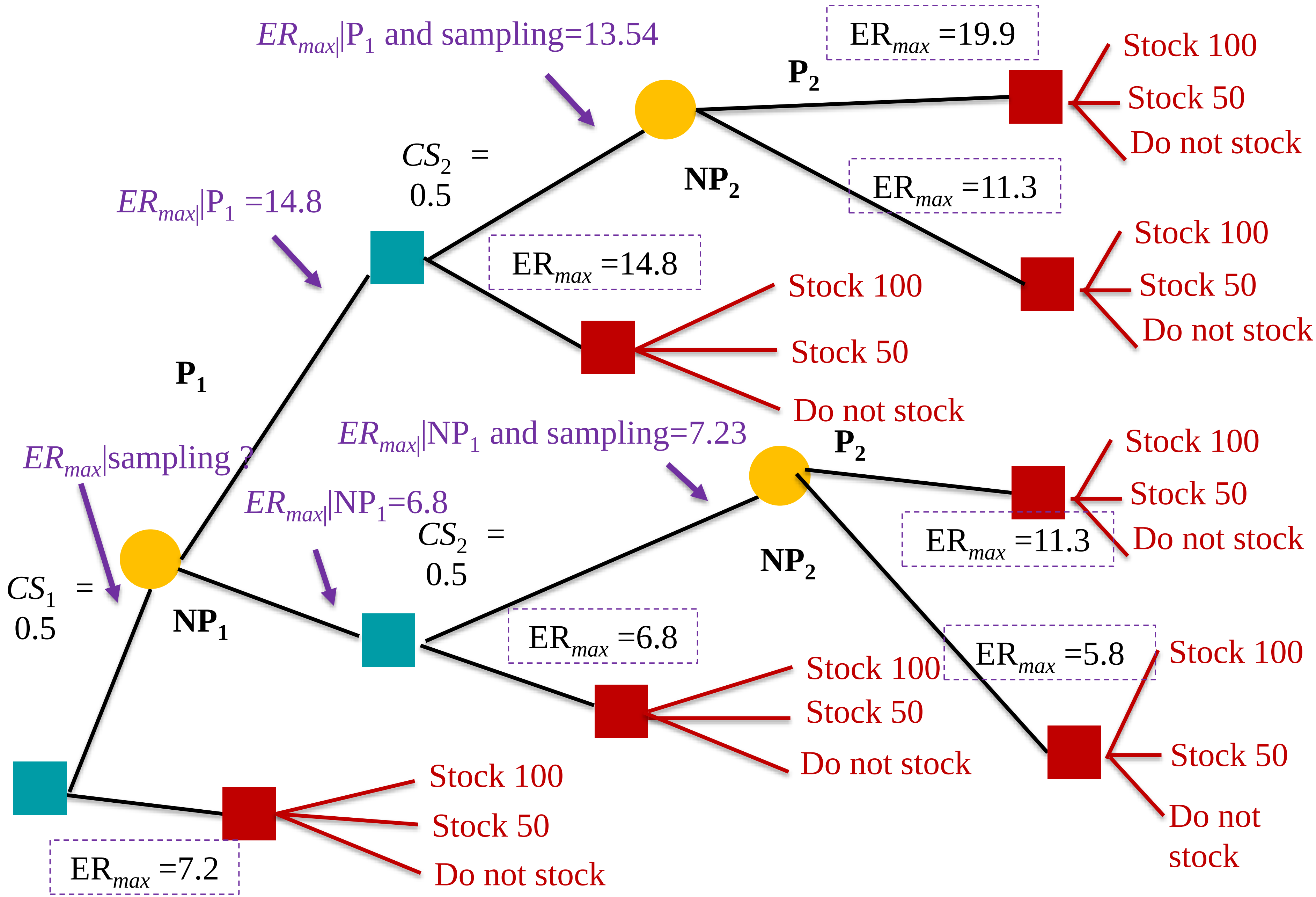
$ER_{max|}|P_1$ and sampling=13.54
ER_{max} =19.9
Stock 100
Stock 50
Do not stock
P_2
CS_2 =
0.5
NP_2
ER_{max} =11.3
$ER_{max|}|P_1$?
Stock 100
Stock 50
Do not stock
ER_{max} =14.8
Stock 100
Stock 50
Do not stock
P_1
$ER_{max|}|NP_1$ and sampling=7.23
P_2
Stock 100
Stock 50
Do not stock
$ER_{max}|$sampling ?
$ER_{max|}|NP_1$?
CS_2 =
0.5
NP_2
ER_{max} =11.3
CS_1 =
0.5
NP_1
ER_{max} =6.8
ER_{max} =5.8
Stock 100
Stock 100
Stock 50
Do not stock
Stock 50
Do not
stock
Stock 100
Stock 50
Do not stock
ER_{max} =7.2



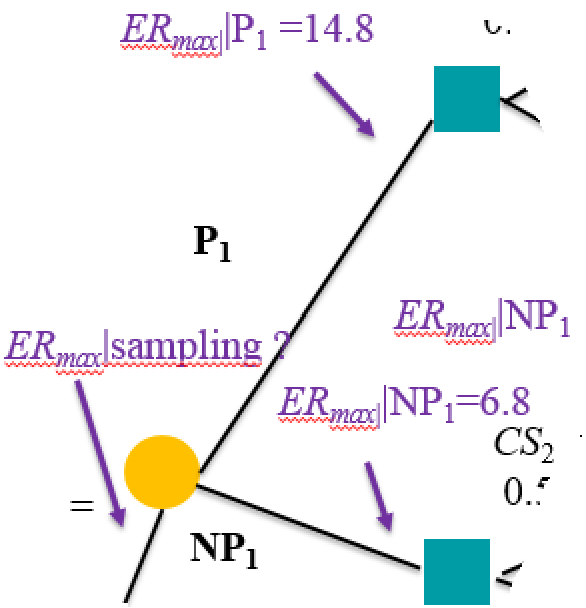
$$ER_{\max}\big|P_1 = \max(13.54 - 0.5, 14.8) = 14.8$$



$$ER_{\max}\big|NP_1 = \max(7.23 - 0.5, 6.8) = 6.8$$

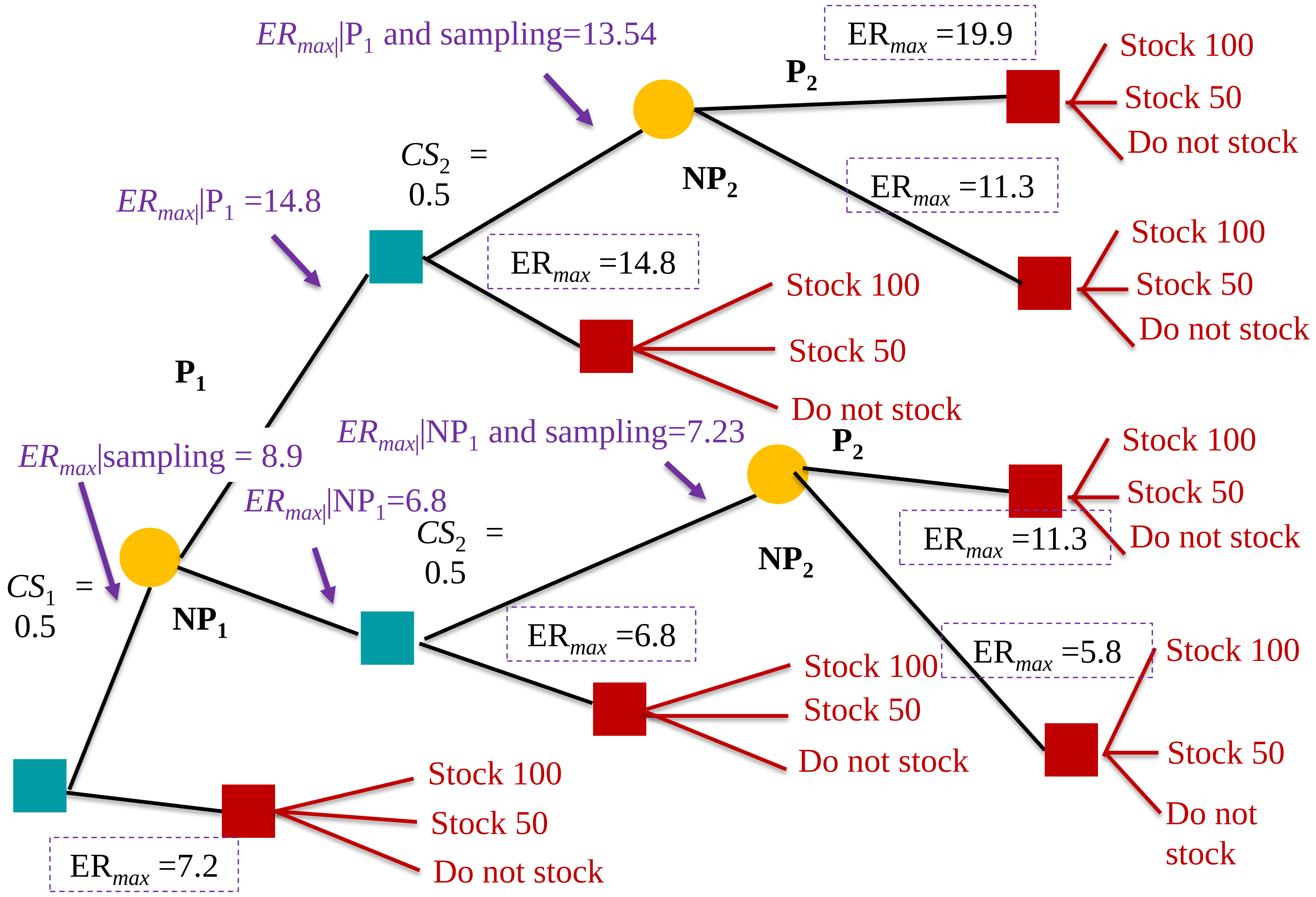


$ER_{max|}|P_1$ and sampling=13.54
ER_{max} =19.9
P_2
Stock 100
Stock 50
Do not stock
CS_2 = 0.5
NP_2
ER_{max} =11.3
$ER_{max|}|P_1$ =14.8
Stock 100
Stock 50
Do not stock
ER_{max} =14.8
Stock 100
Stock 50
Do not stock
P_1
$ER_{max|}|NP_1$ and sampling=7.23
$ER_{max}|$sampling ?
P_2
Stock 100
Stock 50
Do not stock
ER_{max} =11.3
$ER_{max|}|NP_1$=6.8
CS_2 = 0.5
NP_2
CS_1 = 0.5
NP_1
ER_{max} =6.8
ER_{max} =5.8
Stock 100
Stock 100
Stock 50
Stock 50
Do not stock
Do not stock
Stock 100
Stock 50
Do not stock
ER_{max} =7.2

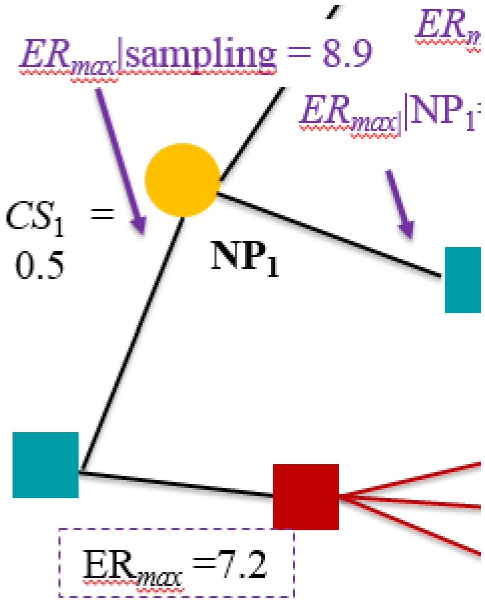


$$ER_{\max}\left|\text{sampling} = 14.8 \cdot P\left(\mathrm{P}_1\right) + 6.8 \cdot P\left(\mathrm{NP}_1\right) = \right.$$

$$= 14.8 \cdot 0.26 + 6.8 \cdot 0.74 \approx 8.9$$



$ER_{max|}|P_1$ and sampling=13.54
ER_{max} =19.9
P_2
Stock 100
Stock 50
Do not stock
CS_2 = 0.5
NP_2
ER_{max} =11.3
$ER_{max|}|P_1$ =14.8
Stock 100
Stock 50
Do not stock
ER_{max} =14.8
Stock 100
Stock 50
Do not stock
P_1
$ER_{max|}|NP_1$ and sampling=7.23
P_2
Stock 100
Stock 50
Do not stock
$ER_{max}|$sampling = 8.9
$ER_{max|}|NP_1$=6.8
CS_2 = 0.5
NP_2
ER_{max} =11.3
CS_1 = 0.5
NP_1
ER_{max} =6.8
ER_{max} =5.8
Stock 100
Stock 100
Stock 50
Do not stock
Stock 50
Do not stock
Stock 100
Stock 50
Do not stock
ER_{max} =7.2



$$ER_{\max} \mid \text{optimal} = ER_{\max} \left| \text{sampling} = 8.9 \right.$$

$$\Rightarrow \text{ENGS} = 8.9 - 7.2 = 1.7$$

Single-stage plan (from Exercise 6.17): $\text{ENGS}(2) = 0.09$