LINKÖPINGS UNIVERSITET
Institutionen för datavetenskap
Avdelningen för statistik
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2016-02-26
Time Series Analysis, 6 hp
732A34

Exam in Time Series Analysis, 6 credits

Exam time:	8-12
Allowed:	Pocket calculator. Text book (Cryer & Chan: "Time Series Analysis- with applications in R"), handwritten notes and page markers allowed in the book.
Assisting teacher:	Per Sidén. *phone 0704-977175*
Grades:	Grades: Maximum is 20 points. A=19-20 points B=17-18.5 points C=14-16.5 points D=12-13.5 points E=10-11.5 points F=0-9.5 points

- Provide a detailed report that shows motivation of the results.

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1. For the processes below, assume e_t is zero mean white noise with variance $\sigma_e^2 = 1$.

   (a) Is the process $Y_t = Y_{t-1} + e_t$ weakly stationary? 1p.

   (b) Consider the process $Y_t = e_t - \frac{3}{2}e_{t-1}$. Is it invertible? 1p.

   (c) Suppose $X_t = 1 + aX_{t-1} + e_t$ and $Y_t = 1 - bX_t$. For which real values of a and b is Y_t weakly stationary? 2p.

2. For the stationary process $Y_t = 1 - \frac{1}{4}Y_{t-2} + e_t + \frac{1}{2}e_{t-1}$, compute the mean and the covariance function. e_t is white noise with mean 0 and variance 1. 3p.

3. Below you find three model specifications (1,2,3), three simulated time series (X,Y,Z) and three sample ACF functions (A,B,C). The noise in all models is white with zero mean and unit variance. Your job is to connect each simulated time series with the model that generated it and also with the corresponding sample ACF function. Motivate your choices. 3p.

   Model 1: AR(1), $\phi_1 = 0.8$.
   Model 2: AR(2), $\phi_1 = 0.7$, $\phi_2 = 0.4$.
   Model 3: MA(2), $\theta_1 = 0.7$, $\theta_2 = 0.4$.

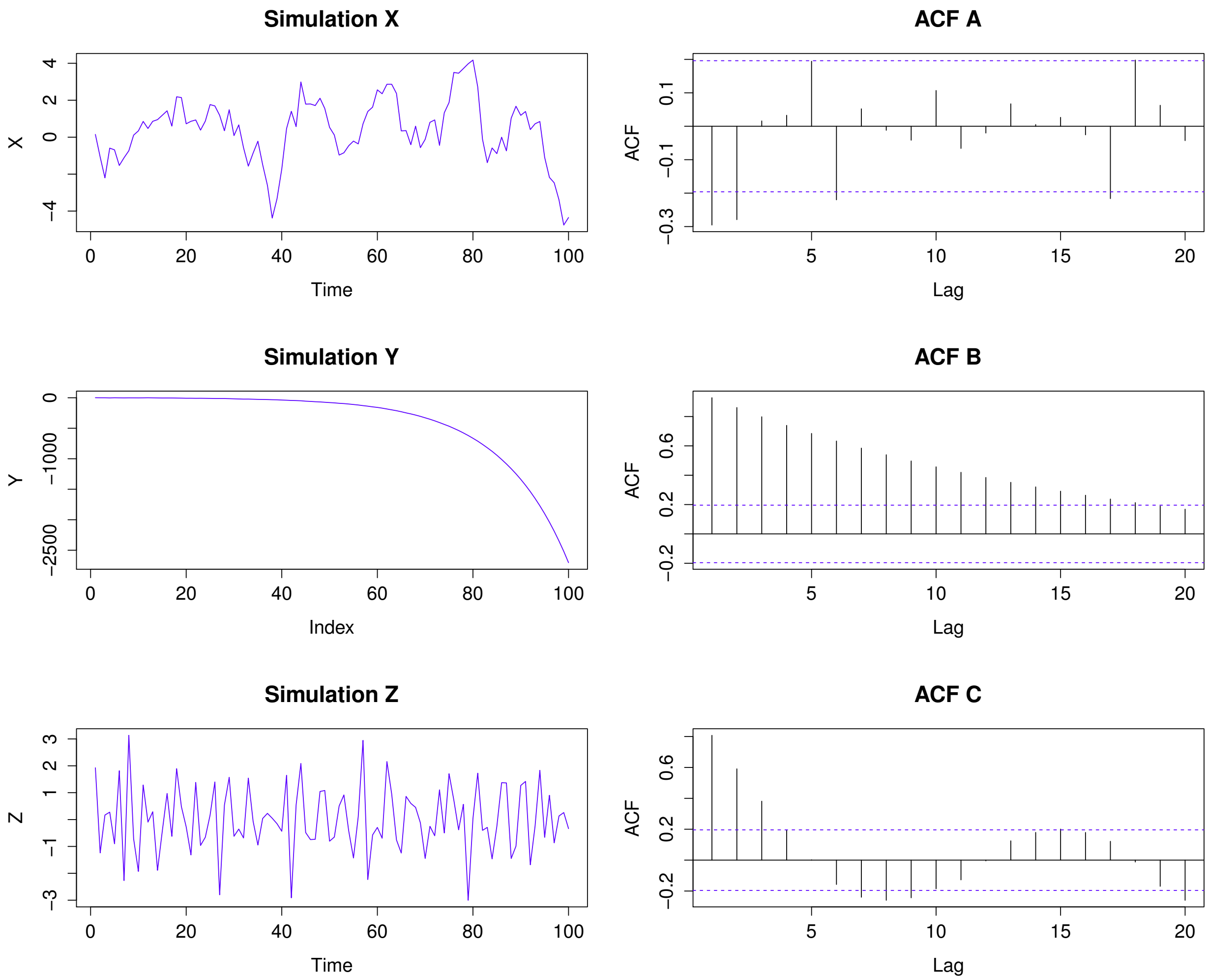

Figure 1: Simulated time series (X,Y,Z) and sample ACF (A,B,C)

4. In the figures below you can see the time series Y_t with 200 observations together with the sample ACF. Also numerical values of the sample ACF are shown, as well as the quantile of the χ^2-distribution corresponding to the test significance level 5% for different degrees of freedom and the sample skewness g_1 and kurtosis g_2 of the time series.

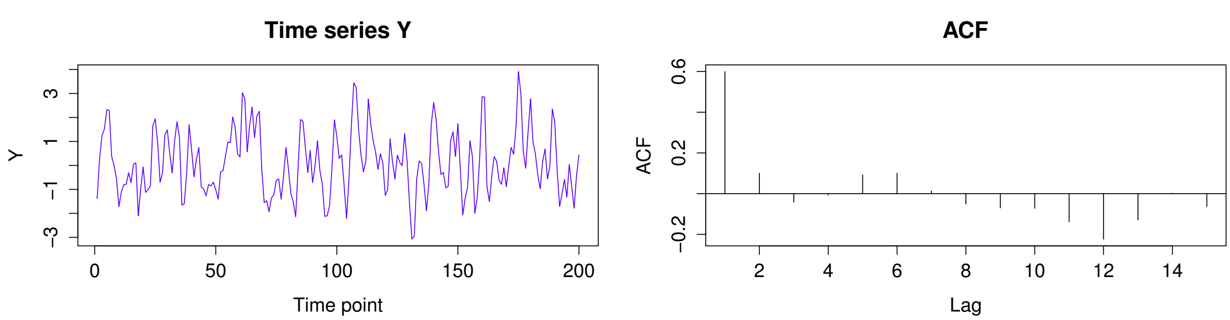

Figure 2: Time series Y_t and the sample ACF.

Lag	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
ACF	0.599	0.100	-0.042	-0.007	0.092	0.100	0.013	-0.051	-0.070	-0.072	-0.138	-0.223	-0.129	0.003	-0.063

df	1	2	3	4	5	6	7	8	9	10
$\chi^2_{0.05}$	3.841	5.991	7.815	9.488	11.070	12.592	14.067	15.507	16.919	18.307

g_1	g_2
0.368	-0.240

(a) Compute approximate 95% confidence bands for the sample ACF when Y_t is assumed to be white noise. Which spikes are significant? 1p.

(b) Perform a Ljung-Box test on the time series, based on the first 5 lags. Again use the assumption that Y_t is white noise. What are the conclusions from the test? 1p.

(c) Perform a Jarque-Bera test on the time series. Also perform two tests for the normality assumption based on the skewness and kurtosis separately. What are the conclusions from the tests? 2p.

5. Assume that r_t follows a heteroscedastic process according to

$$\begin{aligned} r_t &= \sigma_{t|t-1}\varepsilon_t \\ \sigma^2_{t|t-1} &= 0.1 + 0.6r^2_{t-1} + 0.2r^2_{t-2}, \end{aligned}$$

where ε_t is Gaussian zero mean white noise with variance $\sigma^2_\varepsilon = 1$.

(a) Identify the process as a certain GARCH(p, q) model. Specify the values of p and q and of the model parameters. 1p.

(b) Show that $\eta_t = r_t^2 - \sigma^2_{t|\text{t-1}}$ is serially uncorrelated with mean zero. Also show that η_t is uncorrelated with squared past values of r_t, that is show that $Corr\left(n_t, r^2_{t-k}\right) = 0$ for $k > 0$. 2p.

(c) Show that r_t^2 can be written as a certain ARMA-process. You may use the results in (b) even if you did not show them. 1p.

(d) Assume that we have a realization of the process of length 500 and also conditional variances for previous time points according to the table below. Construct forecasts for $\sigma^2_{t|500}$ for $t = 501, 502, 503$ and 1000. 2p.

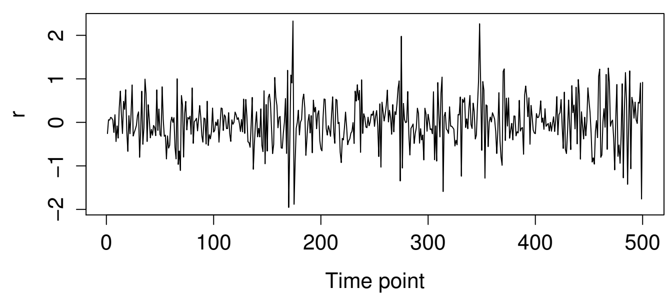

Figure 3: Time series r_t.

t	496	497	498	499	500
r_t	0.462	0.442	0.905	-1.760	0.921
$\sigma^2_{t\|t-1}$	0.101	0.228	0.260	0.631	2.120

Good luck!
Per