Decision Theory Lab2

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6 de diciembre de 2017

R Markdown

Question 1:

This is essentially Exercises 12 and 13 in Chapter 5 of "Winkler: An Introduction to Bayesian inference and decision, 2nd ed."One nonprobabilistic decision-making criterion involves the consideration of a weighted average of the highest and lowest payoffs for each action. The weights, which must sum to 1, can be thought of as an optimism-pessimism index. The action with the highest weighted average of the highest and lowest payoffs is the action chosen by this criterion.

```
PayTab <- matrix(c(-50, 30, 10, -10, 80, 40, 30, -50, 20, 70,
    -30, -70, 100, 20, 10, -20, 0, 50, 40, 200), 4, 5)
rownames(PayTab) <- 1:4
colnames(PayTab) <- LETTERS[1:5]
PayTab
```

```
##     A   B   C   D   E
## 1 -50  80  20 100   0
## 2  30  40  70  20  50
## 3  10  30 -30  10  40
## 4 -10 -50 -70 -20 200
```

a) Comment on this decision-making criterion and use it for payoff table (i) below with the highest payoff in each row receiving a weight of 0.4 and the lowest payoff receiving a weight of 0.6

Given the table we have been provided with (shown), the maximum and the lowest payoffs have been selected, and the following calculus have been done analogoulsy as shown below:

So for instance, the first case the expected reward for action one will be:

$$ER(Action1) = min(Action1) * 0.6 + max(Action1) * 0.4 =$$

$$A * 0.6 + D * 0.4 = -50 * 0.6 + 100 * 0.4 = 30 - 40 = 10$$

Analogously, and using R software, we get the following Payoffs

```
Payoffs <- data.frame(ExpectedPayoff = apply(PayTab, 1, function(x) {
    min(x) * 0.6 + max(x) * 0.4
}))
Payoffs
```

```
##   ExpectedPayoff
## 1             10
## 2             40
## 3             -2
## 4             38
```

Action 2 is therefore the most preferable one.

Use the decision-making criterion described above for payoff table (ii) below, with the highest payoff in each row receiving a weight of 0.8 and the lowest payoff receiving a weight of 0.2. For payoff table (ii) the ER criterion would also involve a weighted average of the two payoffs in each row. Compare the criterion described above with the ER criterion.

```
Payoff <- matrix(c(10, 7, 4, 9), 2, 2)
rownames(Payoff) <- 1:2
colnames(Payoff) <- c("I", "II")
Payoff
```

```
##    I II
## 1 10  4
## 2  7  9
```

Doing the same as before we do the following as an example for action 1:

$$ER(Action1) = min(Action1) * 0.2 + max(Action1) * 0.8 =$$

$$II * 0.2 + I * 0.8 = 4 * 0.2 + 10 * 0.8 = 0. + 8 = 8.8$$

Analogously, we get by using the software and inputting the rules the following results:

```
Payoffs2 <- data.frame(ExpectedPayoff = apply(Payoff, 1, function(x) {
    min(x) * 0.2 + max(x) * 0.8
}))
Payoffs2
```

```
##   ExpectedPayoff
## 1            8.8
## 2            8.6
```

The first action is the best one to do.

Question 2:

This is essentially Exercises 35ab, 38 and 39 in Chapter 5 of "Winkler: An Introduction to Bayesian inference and decision, 2nd ed." Suppose that a person's utility function for total assets (not changes in assets) is

$$U(A) = 200A - A^2$$

for

$$0 <= A <= 100$$

,

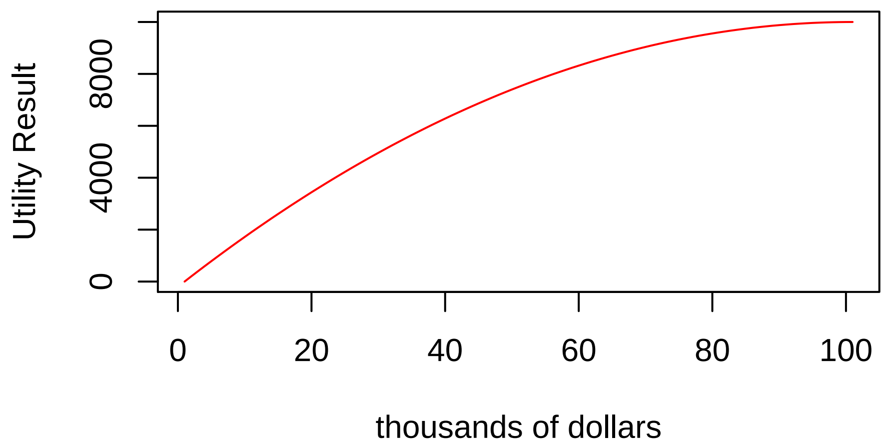
where A represents total assets in thousands of dollars.

```
Utilityfunc <- function(A) {
    if (A >= 0 && A <= 100) {
        return(200 * A - A^2)
    } else {
        stop("A not between 0 and 100")
    }
}
```

a) Graph this utility function. How could you classify this person with regard to their attitude towards risk?

The plot is visible below using the function written I have written above.

```
vectorResult <- Utilityfunc(0:100)  # the utlitiy function between 0 and 100
plot(vectorResult, type = "l", col = "red", xlab = "thousands of dollars",
    ylab = "Utility Result")
```



I would classify this person as a risk avoider person because for the same point of Reward of a risk neutral person (linear utily instead of concave) this person has a such lower utility for the same outcome, making him or her not to take the bet if the risk neutral person feels indiferent from taking it or not.

b) If the person's total assets are currently \$10 000, should they take a bet in which they will win \$10 000 with probability 0.6 and loose \$10 000with probability 0.4? Given that A is in thousand of dollars, 10000 will be 10 in our utility function. Therefore, the expected payoff and the following reward are the following ones:

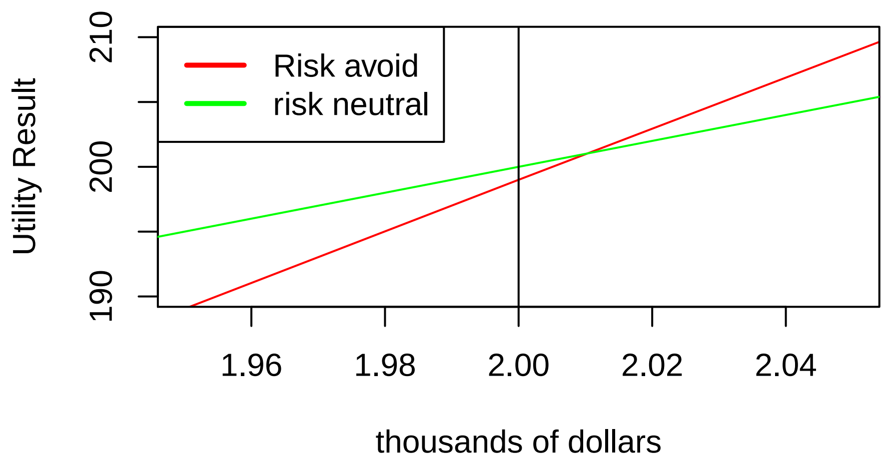
```
dollars <- 10000/1000
Expay <- dollars*0.6-dollars*0.4 #Expected payoff from the activity
Expay
```

```
## [1] 2
```

```
Data<- data.frame(y = vectorResult, x = 0:100)
model <-lm(y~x, Data)
linear<-model$coefficients[2]*0:100

## Getting the the Utility for the linear function, which is when A = 2, therefore the 3rd number

linear[3] #Utility = 200
```

```
## [1] 200
```

```
{plot(vectorResult, type = "l", col = "red", xlab = "thousands of dollars", ylab =  "Utility Result", x
lines(x = 0:100, linear, col ="green")
abline(v=2, col= "black")
legend("topleft", c("Risk avoid ", "risk neutral"), # puts text in the legend
       lty=c(1,1), # gives the legend appropriate symbols (lines)
       lwd=c(2.5,2.5),col=c("Red", "Green")) # gives the legend lines the correct color and width
}
```



It will be taking it by a really small difference.

c)A function that is often used to measure the degree of risk aversion in a given utility function is the Pratt-Arrow risk-aversion function. This function is of the form

$$r(A) = -U''(A)/U'(A)$$

, where U(A) represents the utility function for total assets, and where the primes denote differentiation (first and second derivatives, i.e.

$$U'(A) = dU/dA$$

and

$$U''(A) = d^2U/dA^2$$

[Primes have otherwise related to prior and posterior function in the textbook]. Find r(A) for the utility function given above.

Given that

$$U(A) = 200A - A^2$$

,

$$r(A) = -U''(A)/U'(A) = -(-2)/(-2A + 200) = 2/(-2A + 200)$$

d)Find the Pratt-Arrow risk-aversion functions for $0 < A < 100$ **for the following utility functions, where A represents total assets in thousands of dollars:** (i)

$$U(A) = 1 - \exp^(-0.05 * A)$$

(ii)

$$U(A) = ln(A)$$

The Pratt-Arrow risk aversion result for this ones are:

i)

$$r(A) = -U''(A)/U'(A) = -(-0.05^2) * e^(- 0.05 * A)/(0.05) * \exp^(-0.05 * A) =$$

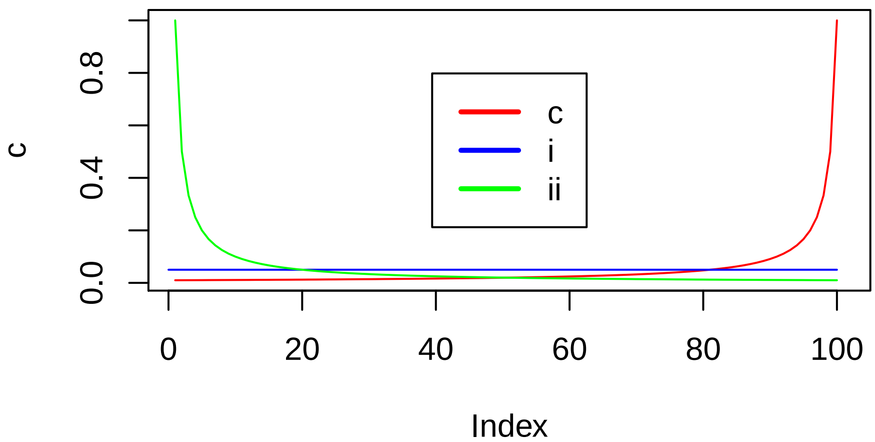
$$\frac{0.05^2 * e^(- 0.05 * A)}{(0.05) * e^(- 0.05 * A)} = 0.05$$

ii)

$$r(A) = -U''(A)/U'(A) = \frac{-(-1/A^2)}{1/A} = 1/A$$

** Graph these risk-aversion functions and the risk-aversion function in subtask above and compare them in terms of how the risk aversion changes as A increases** The plot can be seen below:

```
A<-0:100
c<-2/(-2*A+200)
i<-rep(0.05,101)
ii<-1/A
{plot(c, col = "red", type = "l")
lines(x = A, i, col = "blue")
lines(x= A, ii, col ="green")
legend("center", c("c ", "i", "ii"), # puts text in the legend
       lty=c(1,1), # gives the legend appropriate symbols (lines)
       lwd=c(2.5,2.5),col=c("Red","Blue", "Green")) # gives the legend lines the correct color and widt
}
```



The function from activity part c is risk avoidance, the i is meant to be risk neutral and the ii is mean to be risk lover.

Question 3

In the slides to Meeting 8 (12 October) (which you find under Work plan and material on the course web) Exercise 36 and Exercise 45a in Chapter 5 of the textbook are solved. For each of the gambles in Exercise 45, show the risk premium graphically when the utility function for monetary payoffs is

$$U(R) = 4000 - (200 - R)^2$$

for

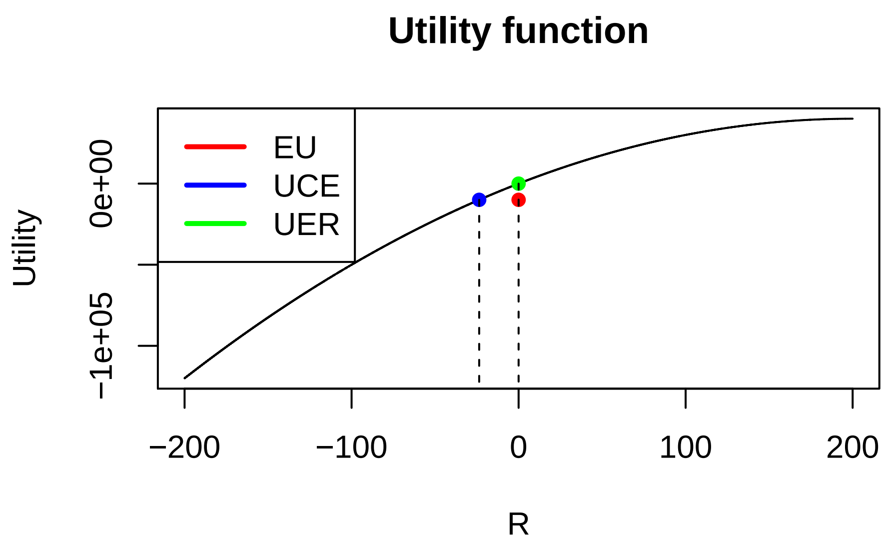
$$-200 <= R <= 200$$

a) 100*0.5-100*0.5
b) 100*0.4-50*0.6

c) 70*0.3-30*0.7
d) 200*0.5-50*0.5

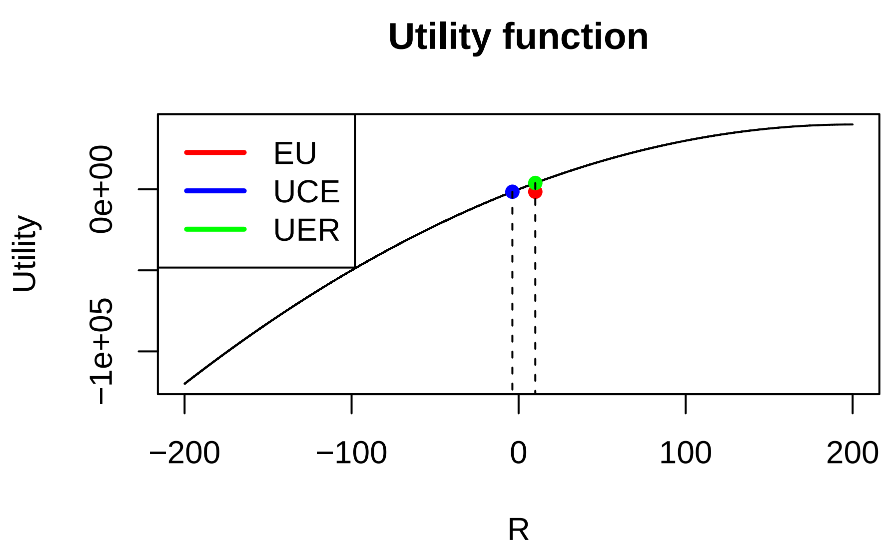
The code for the activity has been done as following, doing four plots for each of the cases

```
Utilityfunc2<- function(R){
  if(R>=-200 && R<=200){
    return(40000-(200-R)^2)
  }
  else{
    stop("A not between -200 and 200")
  }
}


ce <- function(r) {
  return(200 - sqrt(40000 - r))
}

plot_RE <- function(utility, inverse_utility, probs=c(0.5, 0.5), values=c(100, -100)) {
  prob_win <- probs[1]
  prob_loose <- probs[2]
  value_win <- values[1]
  value_loose <- values[2]

  ER <- prob_win*value_win + prob_loose*value_loose
  UER <- utility(ER)
  EU <- prob_win*utility(value_win) + prob_loose*utility(value_loose)
  CE <- ce(EU)
  UCE <- utility(CE)
  RP <-ER-CE
  x <- seq(-200, 200, 0.01)
  plot(x, sapply(x, utility), type="l",
       main="Utility function", xlab="R", ylab="Utility")

  print(c("ER"=ER, "EU"=EU, "CE"=CE, "UCE" = UCE, "RP" = RP))

  points(ER, EU, col="red", pch=16)
  points(CE, UCE, col="blue", pch=16)
  points(ER, UER, col="green", pch=16)

  lines(c(ER, ER), c(UER, -10^12), lty=2)
  lines(c(CE, UCE), c(UCE, -10^12), lty=2)
  legend("topleft", c("EU", "UCE", "UER"), # puts text in the legend
       lty=c(1,1), # gives the legend appropriate symbols (lines)
       lwd=c(2.5,2.5),col=c("Red","Blue", "Green")) # gives the legend lines the correct color and widt
}
par(mfrow=c(1,1))
#a
plot_RE(Utilityfunc2, ce, probs=c(0.5, 0.5), values=c(100, -100))
```



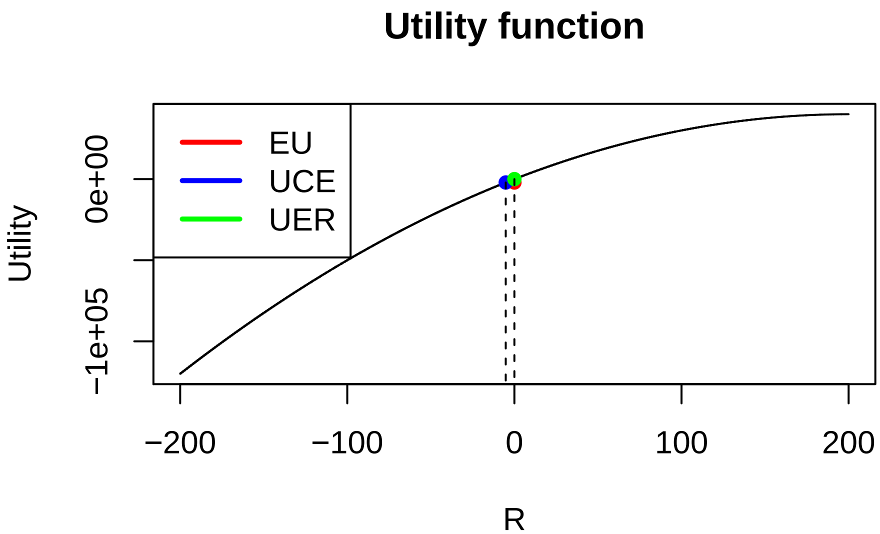

```
##          ER          EU          CE         UCE          RP
##      0.0000 -10000.0000    -23.6068 -10000.0000     23.6068
```

```r
#b
plot_RE(Utilityfunc2, ce, probs=c(0.4, 0.6), values=c(100, -50))
```



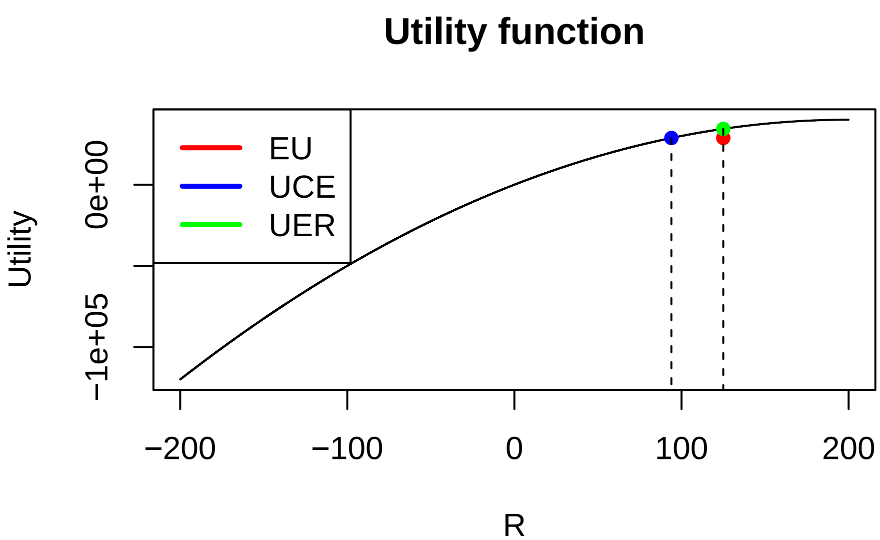

```
##          ER           EU           CE          UCE           RP
##   10.000000 -1500.000000    -3.715488 -1500.000000    13.715488
```

```r
#c
plot_RE(Utilityfunc2, ce, probs=c(0.3, 0.7), values=c(70, -30))
```




```
##          ER          EU          CE         UCE          RP
##    0.000000 -2100.000000   -5.182845 -2100.000000    5.182845
```

```r
#d
plot_RE(Utilityfunc2, ce, probs=c(0.5, 0.5), values=c(200, 50))
```




```
##         ER          EU          CE         UCE          RP
##   125.00000 28750.00000    93.93398 28750.00000    31.06602
```