**Note: there might be some mistakes in the text. Ask me if you are confused by some sentence or you can not get the right answer.**

### Assignment 1

Suppose $E(X) = 2$, $Var(X) = 9$, $E(Y) = 0$, $Var(Y) = 4$, and $Corr(X,Y) = 0.25$. Find:
**(a)** $Corr(X + Y, X - Y)$.

### Assignment 2

Let $\{w_t\}$ be a zero mean white noise process. Suppose that the observed process is $x_t = w_t + \theta w_{t-1}$, where $\theta$ is either 3 or 1/3.

**(a)** Find the autocorrelation function for $\{x_t\}$ both when $\theta = 3$ and when $\theta = 1/3$.

**(b)** You should have discovered that the time series is stationary regardless of the value of $\theta$ and that the autocorrelation functions are the same for $\theta = 3$ and $\theta = 1/3$. For simplicity, suppose that the process mean is known to be zero and the variance of $y_t$ is known to be 1. You observe the series $\{y_t\}$ for $t = 1, 2, ..., n$ and suppose that you can produce good estimates of the autocorrelations $\rho_k$. Do you think that you could determine which value $\theta$ is correct (3 or 1/3) based on the estimate of $\rho_k$? Why or why not?

### Assignment 3

Let $\{x_t\}$ be a zero-mean, unit-variance stationary process with autocorrelation function $\rho_h$. Suppose that $\mu_t$ is a nonconstant function and that $\sigma_t$ is a positive-valued nonconstant function. The observed series is formed as $y_t = \mu_t + \sigma_t x_t$.

**(a)** Find the mean and covariance function for the $\{y_t\}$ process.
**(b)** Show that the autocorrelation function for the $\{y_t\}$ process depends only on the time lag. Is the $\{y_t\}$ process stationary?
**(c)** Is it possible to have a time series with a constant mean and with $Corr(y_t, y_{t+h})$ free of $t$ but with $\{y_t\}$ *not* stationary?

### Assignment 4

Suppose that $x$ is a random variable with zero mean. Define a time series by $y_t = (-1)^t x$
**(a)** Find the mean function for $\{y_t\}$.
**(b)** Find the autocovariance function for $\{y_t\}$.
**(c)** Is $\{y_t\}$ stationary?

### Assignment 5

Suppose $x_t = \mu + w_t + w_{t-1}$. Find $var(\bar{x})$. Note any unusual results. In particular, compare your answer to what would have been obtained if $x_t = \mu + w_t$.

### Assignment 6

Calculate and sketch the autocorrelation function for MA(2) model with $\theta_1 = 0.5$. and $\theta_2 = 0.4$

### Assignment 7

Describe the important characteristics of the autocorrelation function for the following models: (a) MA(1), (b) MA(2), (c) AR(1), (d) AR(2), and (e) ARMA(1,1).

### Assignment 8

Suppose that $\{x_t\}$ is an AR(1) process with $-1 < \phi < +1$.

**(a)** Find the autocovariance function for $y_t = \nabla x_t = x_t - x_{t-1}$ in terms of $\phi$ and $\sigma_w^2$

**(b)** In particular, show that $var(y_t) = \frac{2\sigma_w^2}{1+\phi}$

### Assignment 9

For each of the following ARMA models, find the roots of the AR and MA polynomials, identify the values of *p* and *q* for which they are ARMA(p,q) (be careful of parameter redundancy), determine whether they are causal, and determine whether they are invertible. In each case, $w_t \sim wn(0,1)$.
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a) $x_t - 3x_{t-1} = w_t - -2w_{t-1} - 8w_{t-2}$
b) $x_t - 2x_{t-1} + 2x_{t-2} = w_t - \frac{8}{9}w_{t-1}$
c) $x_t - 4x_{t-2} = w_t - w_{t-1} + 0.5w_{t-2}$
d) $x_t - \frac{9}{4}x_{t-1} - \frac{9}{4}x_{t-2} = w_t$

### Assignment 10

For the following models, compute the first four coefficients $\psi_0, \ldots \psi_3$ in the causal linear process representation $x_t = \sum_{j=0}^{\infty} \psi_j w_{t-j}$

a) $x_t - 2x_{t-1} + 2x_{t-2} = w_t - \frac{8}{9}w_{t-1}$
b) $x_t - \frac{9}{4}x_{t-1} - \frac{9}{4}x_{t-2} = w_t - 3w_{t-1} + \frac{1}{9}w_{t-2} - \frac{1}{3}w_{t-3}$

# Answers:

### Assignment 1

Approximately 0.39

### Assignment 2

$\gamma(0) = 1, \gamma(1) = 0.3, \gamma(h) = 0\ otherwise$

### Assignment 3

a)$Ex_t = \mu_t, \gamma = \sigma_t \sigma_{t+h} \rho_h$

b) Not necessarily stationary

c) yes

### Assignment 4

a) 0
b) $(-1)^h \sigma_x^2$
c) Yes

### Assignment 5

$var(\bar{x}) = \frac{2(2n-1)}{n^2}\sigma_w^2$

**Assignment 6**

$\rho_1 \approx 0.5, \rho_2 \approx 0.28, \rho_i = 0, i > 2$

**Assignment 8**

a) $-\frac{1-\phi}{1+\phi}\phi^{h-1}\sigma_w^2$

**Assignment 9**

a) p=1,q=2, neither causial or invertible
b) p=2,q=1, invertible, but not causial
c) p=2,q=2, invertible, but not causial
d) p=2, q=0, invertible, not causial

**Assignment 10**

a)1, -0.5, -0.5, 0

2b) 1, -3/4, 1/9+9/16, -1/12+27/64