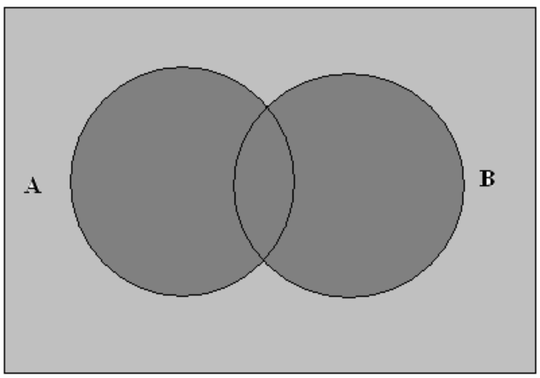
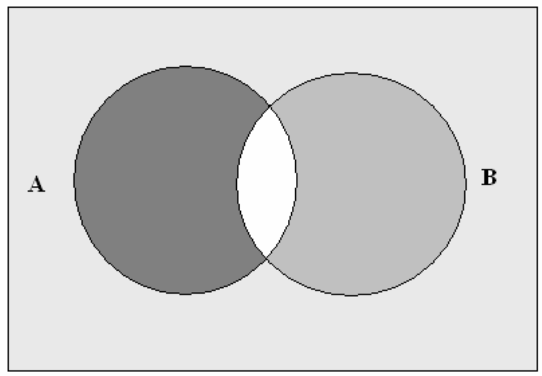
Chapter 2: Probability

2.1 A = {FF}, B = {MM}, C = {MF, FM, MM}. Then, $A \cap B$ = null set, $B \cap C$ = {MM}, $C \cap \overline{B}$ = {MF, FM}, $A \cup B$ ={FF,MM}, $A \cup C = S$, $B \cup C = C$.

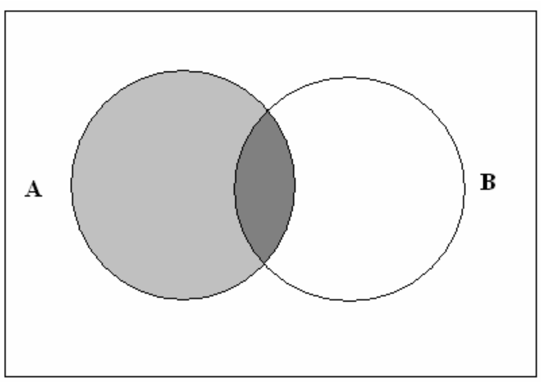
2.2 **a.** $A \cap B$ **b.** $A \cup B$ **c.** $\overline{A \cup B}$ **d.** $(A \cap \overline{B}) \cup (\overline{A} \cap B)$

2.3

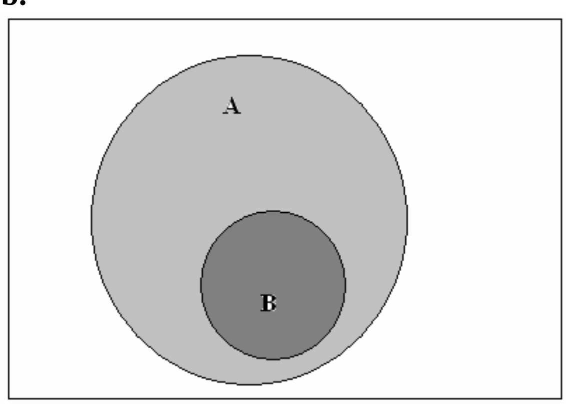




2.4 **a.**



b.



2.5 **a.** $(A \cap B) \cup (A \cap \overline{B}) = A \cap (B \cup \overline{B}) = A \cap S = A$.
b. $B \cup (A \cap \overline{B}) = (B \cap A) \cup (B \cap \overline{B}) = (B \cap A) = A$.
c. $(A \cap B) \cap (A \cap \overline{B}) = A \cap (B \cap \overline{B}) =$ null set. The result follows from part a.
d. $B \cap (A \cap \overline{B}) = A \cap (B \cap \overline{B}) =$ null set. The result follows from part b.

2.6 **a.** 36 + 6 = 42 **b.** 33 **c.** 18

2.7 A = {two males} = {M_1, M_2), (M_1,M_3), (M_2,M_3)}
B = {at least one female} = {(M_1,W_1), (M_2,W_1), (M_3,W_1), (M_1,W_2), (M_2,W_2), (M_3,W_2), {W_1,W_2)}
$\overline{B}$ = {no females} = A $A \cup B = S$ $A \cap B =$ null $A \cap \overline{B} = A$

2.8 *A* = {(1,2), (2,2), (3,2), (4,2), (5,2), (6,2), (1,4), (2,4), (3,4), (4,4), (5,4), (6,4), (1,6), (2,6), (3,6), (4,6), (5,6), (6,6)}
$\overline{C}$ = {(2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6)}
$A \cap B$ = {(2,2), (4,2), (6,2), (2,4), (4,4), (6,4), (2,6), (4,6), (6,6)}
$A \cap \overline{B}$ = {(1,2), (3,2), (5,2), (1,4), (3,4), (5,4), (1,6), (3,6), (5,6)}
$\overline{A} \cup B$ = everything but {(1,2), (1,4), (1,6), (3,2), (3,4), (3,6), (5,2), (5,4), (5,6)}
$\overline{A} \cap C = \overline{A}$

2.9 *S* = {A+, B+, AB+, O+, A-, B-, AB-, O-}

2.10 **a.** *S* = {A, B, AB, O}
b. *P*({A}) = 0.41, *P*({B}) = 0.10, *P*({AB}) = 0.04, *P*({O}) = 0.45.
c. *P*({A} or {B}) = *P*({A}) + *P*({B}) = 0.51, since the events are mutually exclusive.

2.11 **a.** Since $P(S) = P(E_1) + \ldots + P(E_5) = 1$, $1 = .15 + .15 + .40 + 3P(E_5)$. So, $P(E_5) = .10$ and $P(E_4) = .20$.
b. Obviously, $P(E_3) + P(E_4) + P(E_5) = .6$. Thus, they are all equal to .2

2.12 **a.** Let L = {left tern}, R = {right turn}, C = {continues straight}.
b. *P*(vehicle turns) = *P*(L) + *P*(R) = 1/3 + 1/3 = 2/3.

2.13 **a.** Denote the events as VL, SL, U, O.
b. Not equally likely: *P*(VL) = .24, *P*(SL) = .24, *P*(U) = .40, *P*(O) = .12.
c. *P*(at least SL) = *P*(SL) + *P*(VL) = .48.

2.14 **a.** *P*(needs glasses) = .44 + .14 = .48
b. *P*(needs glasses but doesn't use them) = .14
c. *P*(uses glasses) = .44 + .02 = .46

2.15 **a.** Since the events are M.E., $P(S) = P(E_1) + \ldots + P(E_4) = 1$. So, $P(E_2) = 1 - .01 - .09 - .81 = .09$.
b. *P*(at least one hit) = $P(E_1) + P(E_2) + P(E_3) = .19$.

2.16 **a.** 1/3 **b.** 1/3 + 1/15 = 6/15 **c.** 1/3 + 1/16 = 19/48 **d.** 49/240

2.17 Let B = bushing defect, SH = shaft defect.
a. P(B) = .06 + .02 = .08
b. P(B or SH) = .06 + .08 + .02 = .16
c. P(exactly one defect) = .06 + .08 = .14
d. P(neither defect) = 1 – P(B or SH) = 1 - .16 = .84

2.18 **a.** S = {HH, TH, HT, TT}
b. if the coin is fair, all events have probability .25.
c. A = {HT, TH}, B = {HT, TH, HH}
d. $P(A) = .5$, $P(B) = .75$, $P(A \cap B) = P(A) = .5$, $P(A \cup B) = P(B) = .75$, $P(\overline{A} \cup B) = 1$.

2.19 **a.** (V_1, V_1), (V_1, V_2), (V_1, V_3), (V_2, V_1), (V_2, V_2), (V_2, V_3), (V_3, V_1), (V_3, V_2), (V_3, V_3)
b. if equally likely, all have probability of 1/9.
c. A = {same vendor gets both} = $\{(V_1, V_1), (V_2, V_2), (V_3, V_3)\}$
B = {at least one V2} = $\{(V_1, V_2), (V_2, V_1), (V_2, V_2), (V_2, V_3), (V_3, V_2)\}$
So, $P(A) = 1/3$, $P(B) = 5/9$, $P(A \cup B) = 7/9$, $P(A \cap B) = 1/9$.

2.20 **a.** $P(G) = P(D_1) = P(D_2) = 1/3$.
b. i. The probability of selecting the good prize is 1/3.
ii. She will get the other dud.
iii. She will get the good prize.
iv. Her probability of winning is now 2/3.
v. The best strategy is to switch.

2.21 $P(A) = P((A \cap B) \cup (A \cap \overline{B})) = P(A \cap B) + P(A \cap \overline{B})$ since these are M.E. by Ex. 2.5.

2.22 $P(A) = P(B \cup (A \cap \overline{B})) = P(\text{B}) + P(A \cap \overline{B})$ since these are M.E. by Ex. 2.5.

2.23 All elements in B are in A, so that when B occurs, A must also occur. However, it is possible for A to occur and B *not* to occur.

2.24 From the relation in Ex. 2.22, $P(A \cap \overline{B}) \geq 0$, so $P(B) \leq P(A)$.

2.25 Unless exactly 1/2 of all cars in the lot are Volkswagens, the claim is not true.

2.26 **a.** Let w1 denote the first wine, w2 the second, and w3 the third. Each sample point is an ordered triple indicating the ranking.
b. triples: (w1,w2,w3), (w1,w3,w2), (w2,w1,w3), (w2,w3,w1), (w3,w1,w2), (w3,w2,w1)
c. For each wine, there are 4 ordered triples where it is <u>not</u> last. So, the probability is 2/3.

2.27 **a.** S = {CC, CR. CL, RC, RR, RL, LC, LR, LL}
b. 5/9
c. 5/9

2.28 **a.** Denote the four candidates as A_1, A_2, A_3, and M. Since order is not important, the outcomes are $\{A_1A_2, A_1A_3, A_1M, A_2A_3, A_2M, A_3M\}$.
b. assuming equally likely outcomes, all have probability 1/6.
c. P(minority hired) = $P(A_1M) + P(A_2M) + P(A_3M) = .5$

2.29 **a.** The experiment consists of randomly selecting two jurors from a group of two women and four men.
b. Denoting the women as w1, w2 and the men as m1, m2, m3, m4, the sample space is

w1,m1	w2,m1	m1,m2	m2,m3	m3,m4
w1,m2	w2,m2	m1,m3	m2,m4	
w1,m3	w2,m3	m1,m4		
w1,m4	w2,m4			w1,w2

c. P(w1,w2) = 1/15

2.30 **a.** Let N_1, N_2 denote the empty cans and W_1, W_2 denote the cans filled with water. Thus, $S = \{N_1N_2, N_1W_2, N_2W_2, N_1W_1, N_2W_1, W_1W_2\}$
b. If this a merely a guess, the events are equally likely. So, $P(W_1W_2) = 1/6$.

2.31 **a.** Define the events: G = family income is greater than \$43,318, N otherwise. The points are

E1: GGGG	E2: GGGN	E3: GGNG	E4: GNGG
E5: NGGG	E6: GGNN	E7: GNGN	E8: NGGN
E9: GNNG	E10: NGNG	E11: NNGG	E12: GNNN
E13: NGNN	E14: NNGN	E15: NNNG	E16: NNNN

b. A = {E1, E2, …, E11} B = {E6, E7, …, E11} C = {E2, E3, E4, E5}
c. If $P(E) = P(N) = .5$, each element in the sample space has probability 1/16. Thus,
$P(A) = 11/16$, $P(B) = 6/16$, and $P(C) = 4/16$.

2.32 **a.** Three patients enter the hospital and randomly choose stations 1, 2, or 3 for service. Then, the sample space S contains the following 27 three-tuples:
111, 112, 113, 121, 122, 123, 131, 132, 133, 211, 212, 213, 221, 222, 223,
231, 232, 233, 311, 312, 313, 321, 322, 323, 331, 332, 333
b. A = {123, 132, 213, 231, 312, 321}
c. If the stations are selected at random, each sample point is equally likely. $P(A) = 6/27$.

2.33 **a.** There are four "good" systems and two "defactive" systems. If two out of the six systems are chosen randomly, there are 15 possible unique pairs. Denoting the systems as g1, g2, g3, g4, d1, d2, the sample space is given by S = {g1g2, g1g3, g1g4, g1d1, g1d2, g2g3, g2g4, g2d1, g2d2, g3g4, g3d1, g3d2, g4g1, g4d1, d1d2}. Thus:
P(at least one defective) = 9/15 P(both defective) = P(d1d2) = 1/15
b. If four are defective, P(at least one defective) = 14/15. P(both defective) = 6/15.

2.34 **a.** Let "1" represent a customer seeking style 1, and "2" represent a customer seeking style 2. The sample space consists of the following 16 four-tuples:
1111, 1112, 1121, 1211, 2111, 1122, 1212, 2112, 1221, 2121,

2211, 2221, 2212, 2122, 1222, 2222

b. If the styles are equally in demand, the ordering should be equally likely. So, the probability is 1/16.

c. $P(A) = P(1111) + P(2222) = 2/16$.

2.35 The total number of flights is 6*7 = 42.

2.36 There are 3! = 6 orderings.

2.37 **a.** There are 6! = 720 possible itineraries.

b. In the 720 orderings, exactly 360 have Denver before San Francisco and 360 have San Francisco before Denver. So, the probability is .5.

2.38 By the *mn* rule, 4*3*4*5 = 240.

2.39 **a.** By the *mn* rule, there are 6*6 = 36 possible roles.

b. Define the event A = {(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)}. Then, $P(A) = 6/36$.

2.40 **a.** By the *mn* rule, the dealer must stock 5*4*2 = 40 autos.

b. To have each of these in every one of the eight colors, he must stock 8*40 = 320 autos.

2.41 If the first digit cannot be zero, there are 9 possible values. For the remaining six, there are 10 possible values. Thus, the total number is $9*10*10*10*10*10*10 = 9*10^6$.

2.42 There are three *different* positions to fill using ten engineers. Then, there are $P_3^{10} = 10!/3!$ = 720 different ways to fill the positions.

2.43 $\binom{9}{3}\binom{6}{5}\binom{1}{1}$ = 504 ways.

2.44 **a.** The number of ways the taxi needing repair can be sent to airport C is $\binom{8}{5}\binom{5}{5}$ = 56.

So, the probability is 56/504 = 1/9.

b. $3\binom{6}{2}\binom{4}{4}$ = 45, so the probability that every airport receives one of the taxis requiring repair is 45/504.

2.45 $\binom{17}{2\ 7\ 10}$ = 408,408.

2.46 There are $\binom{10}{2}$ ways to chose two teams for the first game, $\binom{8}{2}$ for second, etc. So, there are $\binom{10}{2}\binom{8}{2}\binom{6}{2}\binom{4}{2}\binom{2}{2} = \frac{10!}{2^5}$ = 113,400 ways to assign the ten teams to five games.

2.47 There are $\binom{2n}{2}$ ways to chose two teams for the first game, $\binom{2n-2}{2}$ for second, etc. So, following Ex. 2.46, there are $\frac{2n!}{2^n}$ ways to assign $2n$ teams to n games.

2.48 Same answer: $\binom{8}{5} = \binom{8}{3}$ = 56.

2.49 **a.** $\binom{130}{2}$ = 8385.

b. There are 26*26 = 676 two-letter codes and 26*26*26 = 17,576 three-letter codes. Thus, 18,252 total major codes.
c. 8385 + 130 = 8515 required.
d. Yes.

2.50 Two numbers, 4 and 6, are possible for each of the three digits. So, there are 2*2*2 = 8 potential winning three-digit numbers.

2.51 There are $\binom{50}{3}$ = 19,600 ways to choose the 3 winners. Each of these is equally likely.

a. There are $\binom{4}{3}$ = 4 ways for the organizers to win all of the prizes. The probability is 4/19600.

b. There are $\binom{4}{2}\binom{46}{1}$ = 276 ways the organizers can win two prizes and one of the other 46 people to win the third prize. So, the probability is 276/19600.

c. $\binom{4}{1}\binom{46}{2}$ = 4140. The probability is 4140/19600.

d. $\binom{46}{3}$ = 15,180. The probability is 15180/19600.

2.52 The *mn* rule is used. The total number of experiments is 3*3*2 = 18.

2.53 **a.** In choosing three of the five firms, order is important. So $P_3^5 = 60$ sample points.

b. If F_3 is awarded a contract, there are $P_2^4 = 12$ ways the other contracts can be assigned. Since there are 3 possible contracts, there are 3*12 = 36 total number of ways to award F_3 a contract. So, the probability is 36/60 = 0.6.

2.54 There are $\binom{8}{4} = 70$ ways to chose four students from eight. There are $\binom{3}{2}\binom{5}{2} = 30$ ways to chose exactly 2 (of the 3) undergraduates and 2 (of the 5) graduates. If each sample point is equally likely, the probability is 30/70 = 0.7.

2.55 **a.** $\binom{90}{10}$ **b.** $\binom{20}{4}\binom{70}{6}\bigg/\binom{90}{10} = 0.111$

2.56 The student can solve all of the problems if the teacher selects 5 of the 6 problems that the student can do. The probability is $\binom{6}{5}\bigg/\binom{10}{5} = 0.0238$.

2.57 There are $\binom{52}{2} = 1326$ ways to draw two cards from the deck. The probability is 4*12/1326 = 0.0362.

2.58 There are $\binom{52}{5} = 2{,}598{,}960$ ways to draw five cards from the deck.

a. There are $\binom{4}{3}\binom{4}{2} = 24$ ways to draw three Aces and two Kings. So, the probability is 24/2598960.

b. There are 13*12 = 156 types of "full house" hands. From part a. above there are 24 different ways each type of full house hand can be made. So, the probability is 156*24/2598960 = 0.00144.

2.59 There are $\binom{52}{5} = 2{,}598{,}960$ ways to draw five cards from the deck.

a. $\binom{4}{1}\binom{4}{1}\binom{4}{1}\binom{4}{1}\binom{4}{1} = 4^5 = 1024$. So, the probability is 1024/2598960 = 0.000394.

b. There are 9 different types of "straight" hands. So, the probability is $9*4^5/2598960 =$ 0.00355. Note that this also includes "straight flush" and "royal straight flush" hands.

2.60 **a.** $\dfrac{365(364)(363)\cdots(365-n+1)}{365^n}$ **b.** With $n = 23$, $1-\dfrac{365(364)\cdots(343)}{365^{23}} = 0.507$.

2.61 **a.** $\frac{364(364)(364)\cdots(364)}{365^n} = \frac{364^n}{365^n}$. **b.** With $n = 253$, $1 - \left(\frac{364}{365}\right)^{253} = 0.5005$.

2.62 The number of ways to divide the 9 motors into 3 groups of size 3 is $\binom{9!}{3!\,3!\,3!} = 1680$. If both motors from a particular supplier are assigned to the first line, there are only 7 motors to be assigned: one to line 1 and three to lines 2 and 3. This can be done $\binom{7!}{1!\,3!\,3!}$ = 140 ways. Thus, 140/1680 = 0.0833.

2.63 There are $\binom{8}{5} = 56$ sample points in the experiment, and only one of which results in choosing five women. So, the probability is 1/56.

2.64 $6!\left(\frac{1}{6}\right)^6 = 5/324$.

2.65 $5!\left(\frac{2}{6}\right)^6\left(\frac{1}{6}\right)^4 = 5/162$.

2.66 **a.** After assigning an ethnic group member to each type of job, there are 16 laborers remaining for the other jobs. Let n_a be the number of ways that one ethnic group can be assigned to each type of job. Then:

$$n_a = \binom{4}{1\ 1\ 1\ 1}\binom{16}{5\ 3\ 4\ 4}.$$ The probability is n_a/N = 0.1238.

b. It doesn't matter how the ethnic group members are assigned to jobs type 1, 2, and 3. Let n_a be the number of ways that no ethnic member gets assigned to a type 4 job. Then:

$$n_a = \binom{4}{0}\binom{16}{5}.$$ The probability is $\binom{4}{0}\binom{16}{5}\Big/\binom{20}{5} = 0.2817$.

2.67 As shown in Example 2.13, N = 10^7.

a. Let A be the event that all orders go to different vendors. Then, A contains n_a = 10*9*…*4 = 604,800 sample points. Thus, $P(A)$ = 604,800/10^7 = 0.0605.

b. The 2 orders assigned to Distributor I can be chosen from the 7 in $\binom{7}{2}$ = 21 ways.

The 3 orders assigned to Distributor II can be chosen from the remaining 5 in $\binom{5}{3}$ = 10 ways. The final 2 orders can be assigned to the remaining 8 distributors in 8^2 ways. Thus, there are 21*10*8^2 = 13,440 possibilities so the probability is 13440/10^7 = 0.001344.

c. Let A be the event that Distributors I, II, and III get exactly 2, 3, and 1 order(s) respectively. Then, there is one remaining unassigned order. Thus, A contains $\binom{7}{2}\binom{5}{3}\binom{2}{1}7 = 2940$ sample points and $P(A) = 2940/10^7 = 0.00029$.

2.68 **a.** $\binom{n}{n} = \dfrac{n!}{n!(n-n)!} = 1$. There is only one way to choose all of the items.

b. $\binom{n}{0} = \dfrac{n!}{0!(n-0)!} = 1$. There is only one way to chose none of the items.

c. $\binom{n}{r} = \dfrac{n!}{r!(n-r)!} = \dfrac{n!}{(n-r)!(n-(n-r))!} = \binom{n}{n-r}$. There are the same number of ways to choose r out of n objects as there are to choose $n - r$ out of n objects.

d. $2^n = (1+1)^n = \sum_{i=1}^{n}\binom{n}{i}1^{n-i}1^i = \sum_{i=1}^{n}\binom{n}{i}$.

2.69 $\binom{n}{k}+\binom{n}{k-1} = \dfrac{n!}{k!(n-k)!}+\dfrac{n!}{(k-1)!(n-k+1)!} = \dfrac{n!(n-k+1)}{k!(n-k+1)!}+\dfrac{n!k}{k!(n-k+1)!} = \dfrac{(n+1)!}{k!(n+1-k)!}$

2.70 From Theorem 2.3, let $y_1 = y_2 = \ldots = y_k = 1$.

2.71 **a.** $P(A|B) = .1/.3 = 1/3$.
b. $P(B|A) = .1/.5 = 1/5$.
c. $P(A|\,A\cup B\,) = .5/(.5+.3-.1) = 5/7$
d. $P(A|A\cap B) = 1$, since A has occurred.
e. $P(A\cap B|\,A\cup B\,) = .1(.5+.3-.1) = 1/7$.

2.72 Note that $P(A) = 0.6$ and $P(A|M) = .24/.4 = 0.6$. So, A and M are independent. Similarly, $P(\,\overline{A}\,|\,F\,) = .24/.6 = 0.4 = P(\,\overline{A}\,)$, so $\overline{A}$ and F are independent.

2.73 **a.** P(at least one R) = P(Red) 3/4. **b.** P(at least one r) = 3/4.
c. P(one r | Red) = .5/.75 = 2/3.

2.74 **a.** $P(A) = 0.61$, $P(D) = .30$. $P(A\cap D) = .20$. Dependent.
b. $P(B) = 0.30$, $P(D) = .30$. $P(B\cap D) = 0.09$. Independent.
c. $P(C) = 0.09$, $P(D) = .30$. $P(C\cap D) = 0.01$. Dependent.

2.75 **a.** Given the first two cards drawn are spades, there are 11 spades left in the deck. Thus, the probability is $\dfrac{\binom{11}{3}}{\binom{50}{3}}$ = 0.0084. Note: this is also equal to $P(S_3S_4S_5|S_1S_2)$.

b. Given the first three cards drawn are spades, there are 10 spades left in the deck. Thus, the probability is $\dfrac{\binom{10}{2}}{\binom{49}{2}}$ = 0.0383. Note: this is also equal to $P(S_4S_5|S_1S_2S_3)$.

c. Given the first four cards drawn are spades, there are 9 spades left in the deck. Thus, the probability is $\dfrac{\binom{9}{1}}{\binom{48}{1}}$ = 0.1875. Note: this is also equal to $P(S_5|S_1S_2S_3S_4)$

2.76 Define the events: U: job is unsatisfactory A: plumber A does the job
a. $P(U|A) = P(A\cap U)/P(A) = P(A|U)P(U)/P(A) = .5*.1/.4 = 0.125$
b. From part a. above, $1 - P(U|A) = 0.875$.

2.77

a. 0.40	**b.** 0.37	**c.** 0.10	**d.** 0.40 + 0.37 – 0.10 = 0.67
e. 1 – 0.4 = 0.6	**f.** 1 – 0.67 = 0.33	**g.** 1 – 0.10 = 0.90	
h. .1/.37 = 0.27	**i.** 1/.4 = 0.25		

2.78 **1.** Assume $P(A|B) = P(A)$. Then:
$P(A\cap B) = P(A|B)P(B) = P(A)P(B)$. $P(B|A) = P(B\cap A)/P(A) = P(A)P(B)/P(A) = P(B)$.
2. Assume $P(B|A) = P(B)$. Then:
$P(A\cap B) = P(B|A)P(A) = P(B)P(A)$. $P(A|B) = P(A\cap B)/P(B) = P(A)P(B)/P(B) = P(A)$.
3. Assume $P(A\cap B) = P(B)P(A)$. The results follow from above.

2.79 Given $P(A) < P(A|B) = P(A\cap B)/P(B) = P(B|A)P(A)/P(B)$, solve for $P(B|A)$ in the inequality.

2.80 If $A \subset B$, $P(A\cap B) = P(A) \neq P(A)P(B)$, unless $B = S$ (in which case $P(B) = 1$).

2.81 If A and B are M.E., $P(A\cap B) = 0$. But, $P(A)P(B) > 0$. So they are not independent

2.82 $P(B|A) = P(B\cap A)/P(A) = P(A)/P(A) = 1$
$P(A|B) = P(A\cap B)/P(B) = P(A)/P(B)$.

2.83 $P(A \mid A \cup B) = P(A)/P(A \cup B) = \dfrac{P(A)}{P(A)+P(B)}$, since A and B are M.E. events.

2.84 Note that if $P(A_2 \cap A_3) = 0$, then $P(A_1 \cap A_2 \cap A_3)$ also equals 0. The result follows from Theorem 2.6.

2.85 $P(A \mid \overline{B}) = P(A \cap \overline{B})/P(\overline{B}) = \dfrac{P(\overline{B} \mid A)P(A)}{P(\overline{B})} = \dfrac{[1-P(B \mid A)]P(A)}{P(\overline{B})} = \dfrac{[1-P(B)]P(A)}{P(\overline{B})} =$

$\dfrac{P(\overline{B})P(A)}{P(\overline{B})} = P(A)$. So, A and $\overline{B}$ are independent.

$P(\overline{B} \mid \overline{A}) = P(\overline{B} \cap \overline{A})/P(\overline{A}) = \dfrac{P(\overline{A} \mid \overline{B})P(\overline{B})}{P(\overline{A})} = \dfrac{[1-P(A \mid \overline{B})]P(\overline{B})}{P(\overline{A})}$. From the above,

A and $\overline{B}$ are independent. So $P(\overline{B} \mid \overline{A}) = \dfrac{[1-P(A)]P(\overline{B})}{P(\overline{A})} = \dfrac{P(\overline{A})P(\overline{B})}{P(\overline{A})} = P(\overline{B})$. So,

$\overline{A}$ and $\overline{B}$ are independent

2.86 **a.** No. It follows from $P(A \cup B) = P(A) + P(B) - P(A \cap B) \le 1$.
b. $P(A \cap B) \ge 0.5$
c. No.
d. $P(A \cap B) \le 0.70$.

2.87 **a.** $P(A) + P(B) - 1$.
b. the smaller of $P(A)$ and $P(B)$.

2.88 **a.** Yes.
b. 0, since they could be disjoint.
c. No, since $P(A \cap B)$ cannot be larger than either of $P(A)$ or $P(B)$.
d. $0.3 = P(A)$.

2.89 **a.** 0, since they could be disjoint.
b. the smaller of $P(A)$ and $P(B)$.

2.90 **a.** (1/50)*(1/50) = 0.0004.
b. P(at least one injury) = 1 – P(no injuries in 50 jumps) = 1 = $(49/50)^{50}$ = 0.636. Your friend is not correct.

2.91 If A and B are M.E., $P(A \cup B) = P(A) + P(B)$. This value is greater than 1 if $P(A) = 0.4$ and $P(B) = 0.7$. So they cannot be M.E. It is possible if $P(A) = 0.4$ and $P(B) = 0.3$.

2.92 **a.** The three tests are independent. So, the probability in question is $(.05)^3 = 0.000125$.
b. P(at least one mistake) = 1 – P(no mistakes) = 1 – $(.95)^3$ = 0.143.

2.93 Part a is found using the Addition Rule. Parts b and c use DeMorgan's Laws.
a. 0.2 + 0.3 – 0.4 = 0.1

b. 1 – 0.1 = 0.9
c. 1 – 0.4 = 0.6
d. $P(\overline{A} \mid B) = \dfrac{P(\overline{A} \cap B)}{P(B)} = \dfrac{P(B) - P(A \cap B)}{P(B)} = \dfrac{.3 - .1}{.3}$ = 2/3.

2.94 Define the events *A*: device *A* detects smoke *B*: device *B* detects smoke
a. $P(A \cup B)$ = .95 + .90 - .88 = 0.97.
b. *P*(smoke is undetected) = 1 - $P(A \cup B)$ = 1 – 0.97 = 0.03.

2.95 Let H denote a hit and let M denote a miss. Then, she wins the game in three trials with the events HHH, HHM, and MHH. If she begins with her right hand, the probability she wins the game, assuming independence, is (.7)(.4)(.7) + (.7)(.4)(.3) + (.3)(.4)(.7) = 0.364.

2.96 Using the results of Ex. 2.95:
a. 0.5 + 0.2 – (0.5)(0.2) = 0.6.
b. 1 – 0.6 = 0.4.
c. 1 – 0.1 = 0.9.

2.97 **a.** *P*(current flows) = 1 – *P*(all three relays are open) = 1 – $(.1)^3$ = 0.999.
b. Let *A* be the event that current flows and *B* be the event that relay 1 closed properly. Then, $P(B|A) = P(B \cap A)/P(A) = P(B)/P(A)$ = .9/.999 = 0.9009. Note that $B \subset A$.

2.98 Series system: *P*(both relays are closed) = (.9)(.9) = 0.81
Parallel system: *P*(at least one relay is closed) = .9 + .9 – .81 = 0.99.

2.99 Given that $P(\overline{A \cup B}) = a$, $P(B) = b$, and that *A* and *B* are independent. Thus $P(A \cup B)$ = 1 – *a* and $P(B \cap A) = bP(A)$. Thus, $P(A \cup B) = P(A) + b - bP(A) = 1 - a$. Solve for *P*(*A*).

2.100 $P(A \cup B \mid C) = \dfrac{P((A \cup B) \cap C)}{P(C)} = \dfrac{P((A \cap C) \cup (B \cap C))}{P(C)} =$

$$\frac{P(A \cap C) + P(B \cap C) - P(A \cap B \cap C)}{P(C)} = P(A|C) + P(B|C) + P(A \cap B|C).$$

2.101 Let A be the event the item gets past the first inspector and B the event it gets past the second inspector. Then, *P*(*A*) = 0.1 and *P*(*B*|*A*) = 0.5. Then $P(A \cap B)$ = .1(.5) = 0.05.

2.102 Define the events: I: disease I us contracted II: disease II is contracted. Then, *P*(I) = 0.1, *P*(II) = 0.15, and $P(\text{I} \cap \text{II})$ = 0.03.
a. $P(\text{I} \cup \text{II})$ = .1 + .15 – .03 = 0.22
b. $P(\text{I} \cap \text{II} \mid \text{I} \cup \text{II})$ = .03/.22 = 3/22.

2.103 Assume that the two state lotteries are independent.
a. *P*(666 in CT|666 in PA) = *P*(666 in CT) = 0.001
b. *P*(666 in CT∩666 in PA) = *P*(666 in CT)*P*(666 in PA) = .001(1/8) = 0.000125.

2.104 By DeMorgan's Law, $P(A\cap B) = 1 - P(\overline{A\cap B}) = 1 - P(\overline{A}\cup\overline{B})$. Since $P(\overline{A}\cup\overline{B}) \le P(\overline{A}) + P(\overline{B})$, $P(A\cap B) \ge 1 - P(\overline{A}) - P(\overline{B})$.

2.105 P(landing safely on both jumps) ≥ – 0.05 – 0.05 = 0.90.

2.106 Note that it must be also true that $P(\overline{A}) = P(\overline{B})$. Using the result in Ex. 2.104,

$P(A\cap B) \ge 1 - 2P(\overline{A}) \ge 0.98$, so $P(A) \ge 0.99$.

2.107 (Answers vary) Consider flipping a coin twice. Define the events:
A: observe at least one tail *B*: observe two heads or two tails *C*: observe two heads

2.108 Let *U* and *V* be two events. Then, by Ex. 2.104, $P(U\cap V) \ge 1 - P(\overline{U}) - P(\overline{V})$. Let $U = A\cap B$ and $V = C$. Then, $P(A\cap B\cap C) \ge 1 - P(\overline{A\cap B}) - P(\overline{C})$. Apply Ex. 2.104 to $P(\overline{A\cap B})$ to obtain the result.

2.109 This is similar to Ex. 2.106. Apply Ex. 2.108: $0.95 \le 1 - P(\overline{A}) - P(\overline{B}) - P(\overline{C}) \le P(A\cap B\cap C)$. Since the events have the same probability, $0.95 \le 1 - 3P(\overline{A})$. Thus, $P(A) \ge 0.9833$.

2.110 Define the events:
I: item is from line *I* *II*: item is from line *II* *N*: item is not defective
Then, $P(N) = P(N\cap(I\cup II)) = P(N\cap I) + P(N\cap II) = .92(.4) + .90(.6) = 0.908$.

2.111 Define the following events:
A: buyer sees the magazine ad
B: buyer sees the TV ad
C: buyer purchases the product
The following are known: $P(A) = .02$, $P(B) = .20$, $P(A\cap B) = .01$. Thus $P(A\cap B) = .21$. Also, *P*(buyer sees no ad) = $P(\overline{A}\cap\overline{B}) = 1 - P(A\cup B) = 1 - 0.21 = 0.79$. Finally, it is known that $P(C\,|\,A\cup B) = 0.1$ and $P(C\,|\,\overline{A}\cap\overline{B}) = 1/3$. So, we can find $P(C)$ as $P(C) = P(C\cap(A\cup B)) + P(C\cap(\overline{A}\cap\overline{B})) = (1/3)(.21) + (.1)(.79) = 0.149$.

2.112 **a.** P(aircraft undetected) = P(all three fail to detect) = (.02)(.02)(.02) = $(.02)^3$.
b. P(all three detect aircraft) = $(.98)^3$.

2.113 By independence, (.98)(.98)(.98)(.02) = $(.98)^3(.02)$.

2.114 Let *T* = {detects truth} and *L* = {detects lie}. The sample space is *TT*, *TL*, *LT*, *LL*. Since one suspect is guilty, assume the guilty suspect is questioned first:
a. *P*(*LL*) = .95(.10) = 0.095 **b.** *P*(*LT*) = ..95(.9) = 0.885

b. $P(TL) = .05(.10) = 0.005$ **d.** $1 - (.05)(.90) = 0.955$

2.115 **a.** From the description of the problem, there is a 50% chance a car will be rejected. To find the probability that three out of four will be rejected (i.e. the drivers chose team 2), note that there are $\binom{4}{3} = 4$ ways that three of the four cars are evaluated by team 2. Each one has probability (.5)(.5)(.5)(.5) of occurring, so the probability is $4(.5)^4 = 0.25$.
b. The probability that all four pass (i.e. all four are evaluated by team 1) is $(.5)^4 = 1/16$.

2.116 By the complement rule, P(system works) = 1 – P(system fails) = $1 - (.01)^3$.

2.117 By independence, $(.75)(.75)(.75)(.75) = (.75)^4$.

2.118 If the victim is to be saved, a proper donor must be found within eight minutes. The patient will be saved if the proper donor is found on the 1st, 2nd, 3rd, or 4th try. But, if the donor is found on the 2nd try, that implies he/she wasn't found on the 1st try. So, the probability of saving the patient is found by, letting A = {correct donor is found}:

$$P(\text{save}) = P(A) + P(\bar{A}A) + P(\bar{A}\bar{A}A) + P(\bar{A}\bar{A}\bar{A}A).$$

By independence, this is $.4 + .6(.4) + (.6)^2(.4) + (.6)^3(.4) = 0.8704$

2.119 **a.** Define the events: A: obtain a sum of 3 B: do not obtain a sum of 3 or 7
Since there are 36 possible rolls, $P(A) = 2/36$ and $P(B) = 28/36$. Obtaining a sum of 3 before a sum of 7 can happen on the 1st roll, the 2nd roll, the 3rd roll, etc. Using the events above, we can write these as A, BA, BBA, $BBBA$, etc. The probability of obtaining a sum of 3 before a sum of 7 is given by $P(A) + P(B)P(A) + [P(B)]^2P(A) + [P(B)]^3P(A) + \dots$. (Here, we are using the fact that the rolls are independent.) This is an infinite sum, and it follows as a geometric series. Thus, $2/36 + (28/36)(2/36) + (28/36)^2(2/26) + \dots = 1/4$.

b. Similar to part a. Define C: obtain a sum of 4 D: do not obtain a sum of 4 or 7
Then, $P(C) = 3/36$ and $P(D) = 27/36$. The probability of obtaining a 4 before a 7 is 1/3.

2.120 Denote the events G: good refrigerator D: defective refrigerator
a. If the last defective refrigerator is found on the 4th test, this means the first defective refrigerator was found on the 1st, 2nd, or 3rd test. So, the possibilities are DGGD, GDGD, and GGDD. So, the probability is $\left(\frac{2}{6}\right)\left(\frac{4}{5}\right)\left(\frac{3}{4}\right)\frac{1}{3}$. The probabilities associated with the other two events are identical to the first. So, the desired probability is $3\left(\frac{2}{6}\right)\left(\frac{4}{5}\right)\left(\frac{3}{4}\right)\frac{1}{3} = \frac{1}{5}$.

b. Here, the second defective refrigerator must be found on the 2nd, 3rd, or 4th test.
Define: A_1: second defective found on 2nd test
A_2: second defective found on 3rd test
A_3: second defective found on 4th test
Clearly, $P(A_1) = \left(\frac{2}{6}\right)\left(\frac{1}{5}\right) = \frac{1}{15}$. Also, $P(A_3) = \frac{1}{5}$ from part a. Note that $A_2 = \{DGD, GDD\}$.
Thus, $P(A_2) = 2\left(\frac{2}{6}\right)\left(\frac{4}{5}\right)\left(\frac{1}{4}\right) = \frac{2}{15}$. So, $P(A_1) + P(A_2) + P(A_3) = 2/5$.

c. Define: B_1: second defective found on 3rd test

B_2: second defective found on 4th test

Clearly, $P(B_1) = 1/4$ and $P(B_2) = (3/4)(1/3) = 1/4$. So, $P(B_1) + P(B_2) = 1/2$.

2.121 **a.** $1/n$
b. $\frac{n-1}{n}\cdot\frac{1}{n-1} = 1/n$. $\frac{n-1}{n}\cdot\frac{n-2}{n-1}\cdot\frac{1}{n-2} = 1/n$.
c. P(gain access) = P(first try) + P(second try) + P(third try) = 3/7.

2.122 Applet exercise (answers vary).

2.123 Applet exercise (answers vary).

2.124 Define the events for the voter: D: democrat R: republican F: favors issue

$$P(D \mid F) = \frac{P(F \mid D)P(D)}{P(F \mid D)P(D) + P(F \mid R)P(R)} = \frac{.7(.6)}{.7(.6) + .3(.4)} = 7/9$$

2.125 Define the events for the person: D: has the disease H: test indicates the disease
Thus, $P(H|D) = .9$, $P(\overline{H} \mid \overline{D}) = .9$, $P(D) = .01$, and $P(\overline{D}) = .99$. Thus,

$$P(D \mid H) = \frac{P(H \mid D)P(D)}{P(H \mid D)P(D) + P(H \mid \overline{D})P(\overline{D})} = 1/12.$$

2.126 **a.** (.95*.01)/(.95*.01 + .1*.99) = 0.08756.
b. .99*.01/(.99*.01 + .1*.99) = 1/11.
c. Only a small percentage of the population has the disease.
d. If the specificity is .99, the positive predictive value is .5.
e. The sensitivity and specificity should be as close to 1 as possible.

2.127 **a.** .9*.4/(.9*.4 + .1*.6) = 0.857.
b. A larger proportion of the population has the disease, so the numerator and denominator values are closer.
c. No; if the sensitivity is 1, the largest value for the positive predictive value is .8696.
d. Yes, by increasing the specificity.
e. The specificity is more important with tests used for rare diseases.

2.128 For i = 1, 2, 3, let F_i represent the event that the plane is found in region i and N_i be the complement. Also R_i is the event the plane is in region i. Then $P(F_i|R_i) = 1 - \alpha_i$ and $P(R_i) = 1/3$ for all i. Then,

a. $$P(R_1 \mid N_1) = \frac{P(N_1 \mid R_1)P(R_1)}{P(N_1 \mid R_1)P(R_1) + P(N_1 \mid R_2)P(R_2) + P(N_1 \mid R_3)P(R_3)} = \frac{\alpha_1 \frac{1}{3}}{\alpha_1 \frac{1}{3} + \frac{1}{3} + \frac{1}{3}}$$
$$= \frac{\alpha_1}{\alpha_1 + 2}.$$

b. Similarly, $P(R_2 \mid N_1) = \dfrac{1}{\alpha_1 + 2}$ and c. $P(R_3 \mid N_1) = \dfrac{1}{\alpha_1 + 2}$.

2.129 Define the events: P: positive response M: male respondent F: female respondent
$P(P|F) = .7$, $P(P|M) = .4$, $P(M) = .25$. Using Bayes' rule,

$$P(M\mid\overline{P}) = \frac{P(\overline{P}\mid M)P(M)}{P(\overline{P}\mid M)P(M) + P(\overline{P}\mid F)P(F)} = \frac{.6(.25)}{.6(.25)+.3(.75)} = 0.4.$$

2.130 Define the events: C: contract lung cancer S: worked in a shipyard
Thus, $P(S|C) = .22$, $P(S\mid\overline{C}) = .14$, and $P(C) = .0004$. Using Bayes' rule,

$$P(C\mid S) = \frac{P(S\mid C)P(C)}{P(S\mid C)P(C) + P(S\mid\overline{C})P(\overline{C})} = \frac{.22(.0004)}{.22(.0004)+.14(.9996)} = 0.0006.$$

2.131 The additive law of probability gives that $P(A\Delta B) = P(A\cap\overline{B}) + P(\overline{A}\cap B)$. Also, A and B can be written as the union of two disjoint sets: $A = (A\cap\overline{B})\cup(A\cap B)$ and $B = (\overline{A}\cap B)\cup(A\cap B)$. Thus, $P(A\cap\overline{B}) = P(A) - P(A\cap B)$ and $P(\overline{A}\cap B) = P(B) - P(A\cap B)$. Thus, $P(A\Delta B) = P(A) + P(B) - 2P(A\cap B)$.

2.132 **a.** Let $P(A\mid B) = P(A\mid\overline{B}) = p$. By the Law of Total Probability,

$$P(A) = P(A\mid B)P(B) + P(A\mid\overline{B})P(B) = p\left(P(B) + P(\overline{B})\right) = p.$$

Thus, A and B are independent.

b. $P(A) = P(A\mid C)P(C) + P(A\mid\overline{C})P(\overline{C}) > P(B\mid C)P(C) + P(B\mid\overline{C})P(\overline{C}) = P(B)$.

2.133 Define the events: G: student guesses C: student is correct

$$P(\overline{G}\mid C) = \frac{P(C\mid\overline{G})P(\overline{G})}{P(C\mid\overline{G})P(\overline{G}) + P(C\mid G)P(G)} = \frac{1(.8)}{1(.8)+.25(.2)} = 0.9412.$$

2.134 Define F as "failure to learn. Then, $P(F|A) = .2$, $P(F|B) = .1$, $P(A) = .7$, $P(B) = .3$. By Bayes' rule, $P(A|F) = 14/17$.

2.135 Let M = major airline, P = private airline, C = commercial airline, B = travel for business
a. $P(B) = P(B|M)P(M) + P(B|P)P(P) + P(B|C)P(C) = .6(.5) + .3(.6) + .1(.9) = 0.57$.
b. $P(B\cap P) = P(B|P)P(P) = .3(.6) = 0.18$.
c. $P(P|B) = P(B\cap P)/P(B) = .18/.57 = 0.3158$.
d. $P(B|C) = 0.90$.

2.136 Let A = woman's name is selected from list 1, B = woman's name is selected from list 2.
Thus, $P(A) = 5/7$, $P(\overline{B}\mid A) = 2/3$, $P(\overline{B}\mid\overline{A}) = 7/9$.

$$P(A\mid\overline{B}) = \frac{P(\overline{B}\mid A)P(A)}{P(\overline{B}\mid A)P(A) + P(\overline{B}\mid\overline{A})P(\overline{A})} = \frac{\frac{2}{3}\left(\frac{5}{7}\right)}{\frac{2}{3}\left(\frac{5}{7}\right)+\frac{7}{9}\left(\frac{2}{7}\right)} = \frac{30}{44}.$$

2.137 Let A = {both balls are white}, and for i = 1, 2, … 5
A_i = both balls selected from bowl i are white. Then $\bigcup A_i = A$.
B_i = bowl i is selected. Then, $P(B_i) = .2$ for all i.

a. $P(A) = \sum P(A_i \mid B_i)P(B_i) = \frac{1}{5}\left[0 + \frac{2}{5}\left(\frac{1}{4}\right) + \frac{3}{5}\left(\frac{2}{4}\right) + \frac{4}{5}\left(\frac{3}{4}\right) + 1\right] = 2/5$.

b. Using Bayes' rule, $P(B_3|A) = \dfrac{\frac{3}{50}}{\frac{2}{50}} = 3/20$.

2.138 Define the events:

A: the player wins
B_i: a sum of i on first toss
C_k: obtain a sum of k before obtaining a 7

Now, $P(A) = \sum_{i=1}^{12} P(A \cap B_i)$. We have that $P(A \cap B_2) = P(A \cap B_3) = P(A \cap B_{12}) = 0$.
Also, $P(A \cap B_7) = P(B_7) = \frac{6}{36}$, $P(A \cap B_{11}) = P(B_{11}) = \frac{2}{36}$.
Now, $P(A \cap B_4) = P(C_4 \cap B_7) = P(C_4)P(B_7) = \frac{1}{3}\left(\frac{3}{36}\right) = \frac{3}{36}$ (using independence Ex. 119).
Similarly, $P(C_5) = P(C_9) = \frac{4}{10}$, $P(C_6) = P(C_8) = \frac{5}{11}$, and $P(C_{10}) = \frac{3}{9}$.
Thus, $P(A \cap B_5) = P(A \cap B_9) = \frac{2}{45}$, $P(A \cap B_6) = P(A \cap B_8) = \frac{25}{396}$, $P(A \cap B_{10}) = \frac{1}{36}$.
Putting all of this together, $P(A) = 0.493$.

2.139 From Ex. 1.112, $P(Y = 0) = (.02)^3$ and $P(Y = 3) = (.98)^3$. The event $Y = 1$ are the events *FDF*, *DFF*, and *FFD*, each having probability $(.02)^2(.98)$. So, $P(Y = 1) = 3(.02)^2(.98)$. Similarly, $P(Y = 2) = 3(.02)^2(.98)$.

2.140 The total number of ways to select 3 from 6 refrigerators is $\binom{6}{3} = 20$. The total number of ways to select y defectives and $3 - y$ nondefectives is $\binom{2}{y}\binom{4}{3-y}$, $y = 0, 1, 2$. So,

$P(Y = 0) = \dfrac{\binom{2}{0}\binom{4}{3}}{20} = 4/20$, $P(Y = 1) = 4/20$, and $P(Y = 2) = 12/20$.

2.141 The events $Y = 2$, $Y = 3$, and $Y = 4$ were found in Ex. 2.120 to have probabilities 1/15, 2/15, and 3/15 (respectively). The event $Y = 5$ can occur in four ways:

DGGGD *GDGGD* *GGDGD* *GGGDD*

Each of these possibilities has probability 1/15, so that $P(Y = 5) = 4/15$. By the complement rule, $P(Y = 6) = 5/15$.

2.142 Each position has probability 1/4, so every ordering of two positions (from two spins) has probability 1/16. The values for Y are 2, 3. $P(Y = 2) = \binom{4}{2}\frac{1}{16} = 3/4$. So, $P(Y = 3) = 1/4$.

2.143 Since $P(B) = P(B \cap A) + P(B \cap \overline{A})$, $1 = \dfrac{P(B \cap A)}{P(B)} + \dfrac{P(B \cap \overline{A})}{P(B)} = P(A \mid B) + P(\overline{A} \mid B)$.

2.144 **a.** S = {16 possibilities of drawing 0 to 4 of the sample points}

b. $\binom{4}{0} + \binom{4}{1} + \binom{4}{2} + \binom{4}{3} + \binom{4}{4} = 1 + 4 + 6 + 4 + 1 = 16 = 2^4$.

c. $A \cup B = \{E_1, E_2, E_3, E_4\}$, $A \cap B = \{E_2\}$, $\overline{A} \cap \overline{B} = \emptyset$, $\overline{A} \cup B = \{E_2, E_4\}$.

2.145 All 18 orderings are possible, so the total number of orderings is 18!

2.146 There are $\binom{52}{5}$ ways to draw 5 cards from the deck. For each suit, there are $\binom{13}{5}$ ways to select 5 cards. Since there are 4 suits, the probability is $4\binom{13}{5} \Big/ \binom{52}{5}$ = 0.00248.

2.147 The gambler will have a full house if he is dealt {two kings} or {an ace and a king} (there are 47 cards remaining in the deck, two of which are aces and three are kings). The probabilities of these two events are $\binom{3}{2} \Big/ \binom{47}{2}$ and $\binom{3}{1}\binom{2}{1} \Big/ \binom{47}{2}$, respectively. So, the probability of a full house is $\binom{3}{2} \Big/ \binom{47}{2} + \binom{3}{1}\binom{2}{1} \Big/ \binom{47}{2}$ = 0.0083.

2.148 Note that $\binom{12}{4} = 495$. P(each supplier has at least one component tested) is given by

$$\frac{\binom{3}{2}\binom{4}{1}\binom{5}{1} + \binom{3}{1}\binom{4}{2}\binom{5}{1} + \binom{3}{1}\binom{4}{1}\binom{5}{2}}{495} = 270/475 = 0.545.$$

2.149 Let A be the event that the person has symptom A and define B similarly. Then

a. $P(\overline{A \cup B}) = P(\overline{A} \cap \overline{B}) = 0.4$

b. $P(A \cup B) = 1 - P(\overline{A \cup B}) = 0.6$.

c. $P(A \cap B \mid B) = P(A \cap B)/P(B) = .1/.4 = 0.25$

2.150 $P(Y = 0) = 0.4$, $P(Y = 1) = 0.2 + 0.3 = 0.5$, $P(Y = 2) = 0.1$.

2.151 The probability that team A wins in 5 games is $p^4(1 - p)$ and the probability that team B wins in 5 games is $p(1 - p)^4$. Since there are 4 ways that each team can win in 5 games, the probability is $4[p^4(1 - p) + p(1 - p)^4]$.

2.152 Let *R* denote the event that the specimen turns red and *N* denote the event that the specimen contains nitrates.
a. $P(R) = P(R \mid N)P(N) + P(R \mid \overline{N})P(\overline{N})$ = .95(.3) + .1(.7) = 0.355.
b. Using Bayes' rule, $P(N|R)$ = .95(.3)/.355 = 0.803.

2.153 Using Bayes' rule,

$$P(I_1 \mid H) = \frac{P(H \mid I_1)P(I_1)}{P(H \mid I_1)P(I_1) + P(H \mid I_2)P(I_2) + P(H \mid I_3)P(I_3)} = 0.313.$$

2.154 Let *Y* = the number of pairs chosen. Then, the possible values are 0, 1, and 2.
a. There are $\binom{10}{4}$ = 210 ways to choose 4 socks from 10 and there are $\binom{5}{4}2^4$ = 80 ways to pick 4 non-matching socks. So, $P(Y = 0)$ = 80/210.

b. Generalizing the above, the probability is $\binom{n}{2r}2^{2r}\Big/\binom{2n}{2r}$.

2.155 **a.** $P(A)$ = .25 + .1 + .05 + .1 = .5
b. $P(A\cap B)$ = .1 + .05 = 0.15.
c. 0.10
d. Using the result from Ex. 2.80, $\frac{.25+.25-.15}{.4}$ = 0.875.

2.156 **a.** i. 1 – 5686/97900 = 0.942 ii. (97900 – 43354)/97900 = 0.557
ii. 10560/14113 = 0.748 iv. (646+375+568)/11533 = 0.138

b. If the US population in 2002 was known, this could be used to divide into the total number of deaths in 2002 to give a probability.

2.157 Let *D* denote death due to lung cancer and *S* denote being a smoker. Thus:
$P(D) = P(D \mid S)P(S) + P(D \mid \overline{S})P(\overline{S}) = 10P(D \mid \overline{S})(.2) + P(D \mid \overline{S})(.8)$ = 0.006. Thus, $P(D \mid S) = 0.021$.

2.158 Let *W* denote the even that the first ball is white and *B* denote the event that the second ball is black. Then:

$$P(W \mid B) = \frac{P(B \mid W)P(W)}{P(B \mid W)P(W) + P(B \mid \overline{W})P(\overline{W})} = \frac{\frac{b}{w+b+n}\left(\frac{w}{w+b}\right)}{\frac{b}{w+b+n}\left(\frac{w}{w+b}\right) + \frac{b+n}{w+b+n}\left(\frac{b}{w+b}\right)} = \frac{w}{w+b+n}$$

2.159 Note that $S = S \cup \emptyset$, and *S* and $\emptyset$ are disjoint. So, 1 = $P(S) = P(S) + P(\emptyset)$. So, $P(\emptyset) = 0$.

2.160 There are 10 nondefective and 2 defective tubes that have been drawn from the machine, and number of distinct arrangements is $\binom{12}{2}$ = 66.

a. The probability of observing the specific arrangement is 1/66.

b. There are two such arrangements that consist of "runs." In addition to what was given in part a, the other is *DDNNNNNNNNNNNN*. Thus, the probability of two runs is 2/66 = 1/33.

2.161 We must find $P(R \le 3) = P(R = 3) + P(R = 2)$, since the minimum value for R is 2. Id the two D's occurs on consecutive trials (but not in positions 1 and 2 or 11 and 12), there are 9 such arrangements. The only other case is a defective in position 1 and 12, so that (combining with Ex. 2.160 with $R = 2$), there are 12 possibilities. So, $P(R \le 3) = 12/66$.

2.162 There are 9! ways for the attendant to park the cars. There are 3! ways to park the expensive cars together and there are 7 ways the expensive cars can be next to each other in the 9 spaces. So, the probability is 7(3!)/9! = 1/12.

2.163 Let A be the event that current flows in design A and let B be defined similarly. Design A will function if (1 or 2) & (3 or 4) operate. Design B will function if (1 & 3) or (2 & 4) operate. Denote the event R_i = {relay i operates properly}, i = 1, 2, 3, 4. So, using independence and the addition rule,

$$P(A) = (R_1 \cup R_2) \cap (R_3 \cup R_4) = (.9 + .9 - .9^2)(.9 + .9 - .9^2) = 0.9801.$$

$$P(B) = (R_1 \cap R_3) \cup (R_2 \cap R_4) = .9^2 + .9^2 - (.9^2)^2 = .9639.$$

So, design A has the higher probability.

2.164 Using the notation from Ex. 2.163, $P(R_1 \cap R_4 \mid A) = P(R_1 \cap R_4 \cap A)/P(A)$.
Note that $R_1 \cap R_4 \cap A = R_1 \cap R_4$, since the event $R_1 \cap R_4$ represents a path for the current to flow. The probability of this above event is $.9^2 = .81$, and the conditional probability is in question is .81/.9801 = 0.8264.

2.165 Using the notation from Ex. 2.163, $P(R_1 \cap R_4 \mid B) = P(R_1 \cap R_4 \cap B)/P(B)$.
$R_1 \cap R_4 \cap B = (R_1 \cap R_4) \cap (R_1 \cap R_3) \cup (R_2 \cap R_4) = (R_1 \cap R_4 \cap R_3) \cup (R_2 \cap R_4)$. The probability of the above event is $.9^3 + .9^2 - .9^4 = 0.8829$. So, the conditional probability in question is .8829/.9639 = 0.916.

2.166 There are $\binom{8}{4}$ = 70 ways to choose the tires. If the best tire the customer has is ranked #3, the other three tires are from ranks 4, 5, 6, 7, 8. There are $\binom{5}{3}$ = 10 ways to select three tires from these five, so that the probability is 10/70 = 1/7.

2.167 If $Y = 1$, the customer chose the best tire. There are $\binom{7}{3} = 35$ ways to choose the remaining tires, so $P(Y = 1) = 35/70 = .5$.
If $Y = 2$, the customer chose the second best tire. There are $\binom{6}{3} = 20$ ways to choose the remaining tires, so $P(Y = 2) = 20/70 = 2/7$. Using the same logic, $P(Y = 4) = 4/70$ and so $P(Y = 5) = 1/70$.

2.168 **a.** The two other tires picked by the customer must have ranks 4, 5, or 6. So, there are $\binom{3}{2} = 3$ ways to do this. So, the probability is 3/70.

b. There are four ways the range can be 4: #1 to #5, #2 to #6, #3 to #7, and #4 to #8. Each has probability 3/70 (as found in part **a**). So, $P(R = 4) = 12/70$.

c. Similar to parts a and b, $P(R = 3) = 5/70$, $P(R = 5) = 18/70$, $P(R = 6) = 20/70$, and $P(R = 7) = 15/70$.

2.169 **a.** For each beer drinker, there are $4! = 24$ ways to rank the beers. So there are $24^3 = 13{,}824$ total sample points.

b. In order to achieve a combined score of 4 our less, the given beer may receive at most one score of two and the rest being one. Consider brand A. If a beer drinker assigns a one to A there are still $3! = 6$ ways to rank the other brands. So, there are 6^3 ways for brand A to be assigned all ones. Similarly, brand A can be assigned two ones and one two in $3(3!)^3$ ways. Thus, some beer may earn a total rank less than or equal to four in $4[6^3 + 3(3!)^3] = 3456$ ways. So, the probability is $3456/13824 = 0.25$.

2.170 There are $\binom{7}{3} = 35$ ways to select three names from seven. If the first name on the list is included, the other two names can be picked $\binom{6}{2} = 15$ ways. So, the probability is $15/35 = 3/7$.

2.171 It is stated that the probability that Skylab will hit someone is (unconditionally) 1/150, without regard to where that person lives. If one wants to know the probability condition on living in a certain area, it is not possible to determine.

2.172 Only $P(A \mid B + P(\overline{A} \mid B) = 1$ is <u>true</u> for any events A and B.

2.173 Define the events: D: item is defective C: item goes through inspection
Thus $P(D) = .1$, $P(C|D) = .6$, and $P(C\,|\,\overline{D}) = .2$. Thus,

$$P(D\,|\,C) = \frac{P(C\,|\,D)P(D)}{P(C\,|\,D)P(D) + P(C\,|\,\overline{D})P(\overline{D})} = .25.$$

2.174 Let A = athlete disqualified previously B = athlete disqualified next term
Then, we know $P(B\,|\,\overline{A}) = .15, P(B\,|\,A) = .5, P(A) = .3$. To find $P(B)$, use the law of total probability: $P(B) = .3(.5) + .7(.15) = 0.255$.

2.175 Note that $P(A) = P(B) = P(C) = .5$. But, $P(A \cap B \cap C) = P(HH) = .25 \neq (.5)^3$. So, they are not mutually independent.

2.176 **a.** $P[(A \cup B) \cap C)] = P[(A \cap C) \cup (B \cap C)] = P(A \cap C) + P(B \cap C) - P(A \cap B \cap C) =$
$P(A)P(C) + P(B)P(C) - P(A)P(B)P(C) = [P(A) + P(B) - P(A)P(B)]P(C) = P(A \cap B)P(C)$
b. Similar to part a above.

2.177 **a.** P(no injury in 50 jumps) = $(49/50)^{50}$ = 0.364.
b. P(at least one injury in 50 jumps) = 1 – P(no injury in 50 jumps) = 0.636.
c. P(no injury in n jumps) = $(49/50)^n \geq 0.60$, so n is at most 25.

2.178 Define the events: E: person is exposed to the flu F: person gets the flu
Consider two employees, one of who is inoculated and one not. The probability of interest is the probability that at least one contracts the flu. Consider the complement:
P(at least one gets the flu) = 1 – P(neither employee gets the flu).
For the inoculated employee: $P(\overline{F}) = P(\overline{F} \cap E) + P(\overline{F} \cap \overline{E})$ = .8(.6) + 1(.4) = 0.88.
For the non-inoculated employee: $P(\overline{F}) = P(\overline{F} \cap E) + P(\overline{F} \cap \overline{E})$ = .1(.6) + 1(.4) = 0.46.
So, P(at least one gets the flu) = 1 – .88(.46) = 0.5952

2.179 **a.** The gamblers break even if each win three times and lose three times. Considering the possible sequences of "wins" and "losses", there are $\binom{6}{3}$ = 20 possible orderings. Since each has probability $\left(\frac{1}{2}\right)^6$, the probability of breaking even is $20\left(\frac{1}{2}\right)^6$ = 0.3125.
b. In order for this event to occur, the gambler Jones must have \$11 at trial 9 and must win on trial 10. So, in the nine remaining trials, seven "wins" and two "losses" must be placed. So, there are $\binom{9}{2}$ = 36 ways to do this. However, this includes cases where Jones would win before the 10$^{\text{th}}$ trial. Now, Jones can only win the game on an even trial (since he must gain \$6). Included in the 36 possibilities, there are three ways Jones could

win on trial 6: *WWWWWWWWLL*, *WWWWWWWLLW*, *WWWWWWWLWL*, and there are six ways Jones could win on trial 8: *LWWWWWWWWL*, *WLWWWWWWWL*, *WWLWWWWWWL*, *WWWLWWWWWL*, *WWWWLWWWWL*, *WWWWWLWWWL*. So, these nine cases must be removed from the 36. So, the probability is $27\left(\frac{1}{2}\right)^{10}$.

2.180 **a.** If the patrolman starts in the center of the 16x16 square grid, there are 4^8 possible paths to take. Only four of these will result in reaching the boundary. Since all possible paths are equally likely, the probability is $4/4^8 = 1/4^7$.

b. Assume the patrolman begins by walking north. There are nine possible paths that will bring him back to the starting point: *NNSS*, *NSNS*, *NSSN*, *NESW*, *NWSE*, *NWES*, *NEWS*, *NSEW*, *NSWE*. By symmetry, there are nine possible paths for each of north, south, east, and west as the starting direction. Thus, there are 36 paths in total that result in returning to the starting point. So, the probability is $36/4^8 = 9/4^7$.

2.181 We will represent the n balls as 0's and create the N boxes by placing bars (|) between the 0's. For example if there are 6 balls and 4 boxes, the arrangement

0|00||000

represents one ball in box 1, two balls in box 2, no balls in box 3, and three balls in box 4. Note that six 0's were need but only 3 bars. In general, n 0's and $N - 1$ bars are needed to represent each possible placement of n balls in N boxes. Thus, there are $\binom{N+n-1}{N-1}$ ways to arrange the 0's and bars. Now, if no two bars are placed next to each other, no box will be empty. So, the $N - 1$ bars must be placed in the $n - 1$ spaces between the 0's. The total number of ways to do this is $\binom{n-1}{N-1}$, so that the probability is as given in the problem.