LINKÖPINGS UNIVERSITET
Institutionen för datavetenskap
Avdelningen för statistik
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2015-04-21
Time Series Analysis, 6 hp
732A34

Exam in Time Series Analysis, 6 credits

Exam time:	8-12
Allowed:	Pocket calculator. Text book (Cryer & Chan: “Time Series Analysis- with applications in R”), notes allowed.
Examinator:	Lotta Hallberg.
Assisting teacher:	Per Sidén.
Grades:	Grades: Maximum is 20 points. A=19-20 points B=17-18.5 points C=14-16.5 points D=12-13.5 points E=10-11.5 points F=0-9.5 points

- Provide a detailed report that shows motivation of the results.

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1. For the processes below, assume e_t is zero mean white noise with variance $\sigma_e^2 = 1$.

   (a) Show that the process $Y_t = Y_{t-1} + e_t$ is not weakly stationary. 1p.

   (b) Consider the process $Y_t = e_t - \frac{1}{4}e_{t-2}$. Is it invertible? 1p.

   (c) Suppose $Y_t = \beta_0 + \beta_1 \cdot t + e_t$ and $W_t = Y_t - Y_{t-1}$. For which real values of β_0 and β_1 is Y_t weakly stationary? For which real values of β_0 and β_1 is W_t weakly stationary? 2p.

2. For the stationary process $Y_t = 2 + \frac{1}{2}Y_{t-1} + e_t - \frac{1}{3}e_{t-1}$, compute the mean and the covariance function. e_t is white noise with variance 1. 3p.

3. Below you find three model specifications (a,b,c), three simulated time series (X,Y,Z) and three SAC/SPAC functions (1,2,3). e_t is white noise with unit variance. Your job is to connect each simulated time series with the model that generated it and also with the corresponding SAC/SPAC functions. Motivate your choices. 3p.

   (a) AR(1), $(1 + 0.8B)\,W_t = e_t$.

   (b) MA(1), $W_t = (1 + 0.8B)e_t$.

   (c) ARMA(1,1), $(1 - 0.8B)\,W_t = (1 - 0.4B)\,e_t$

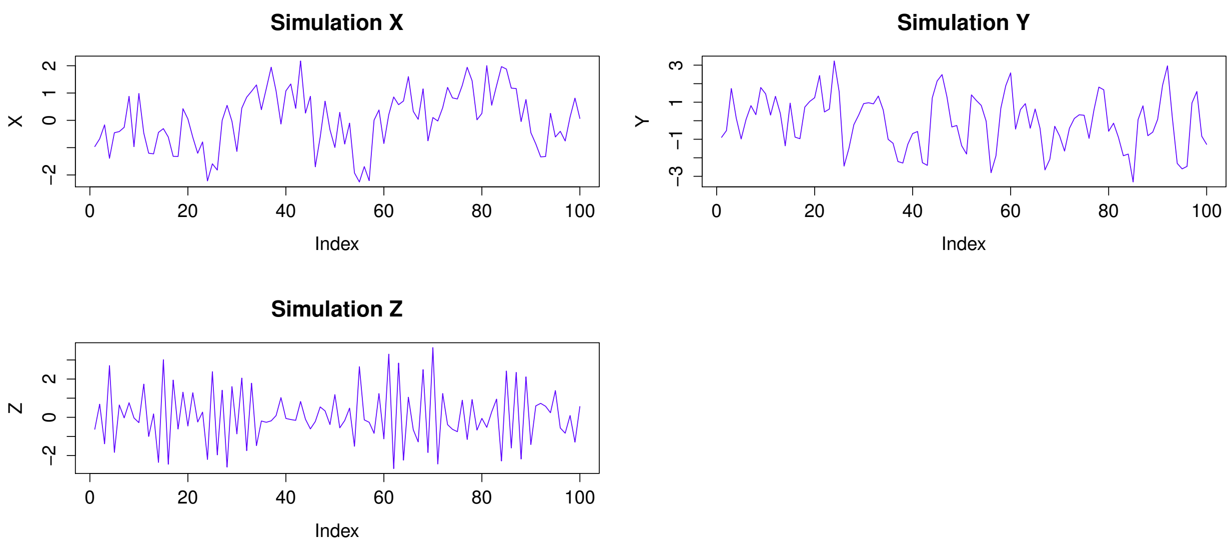

Figure 1: Simulated time series (X,Y,Z).

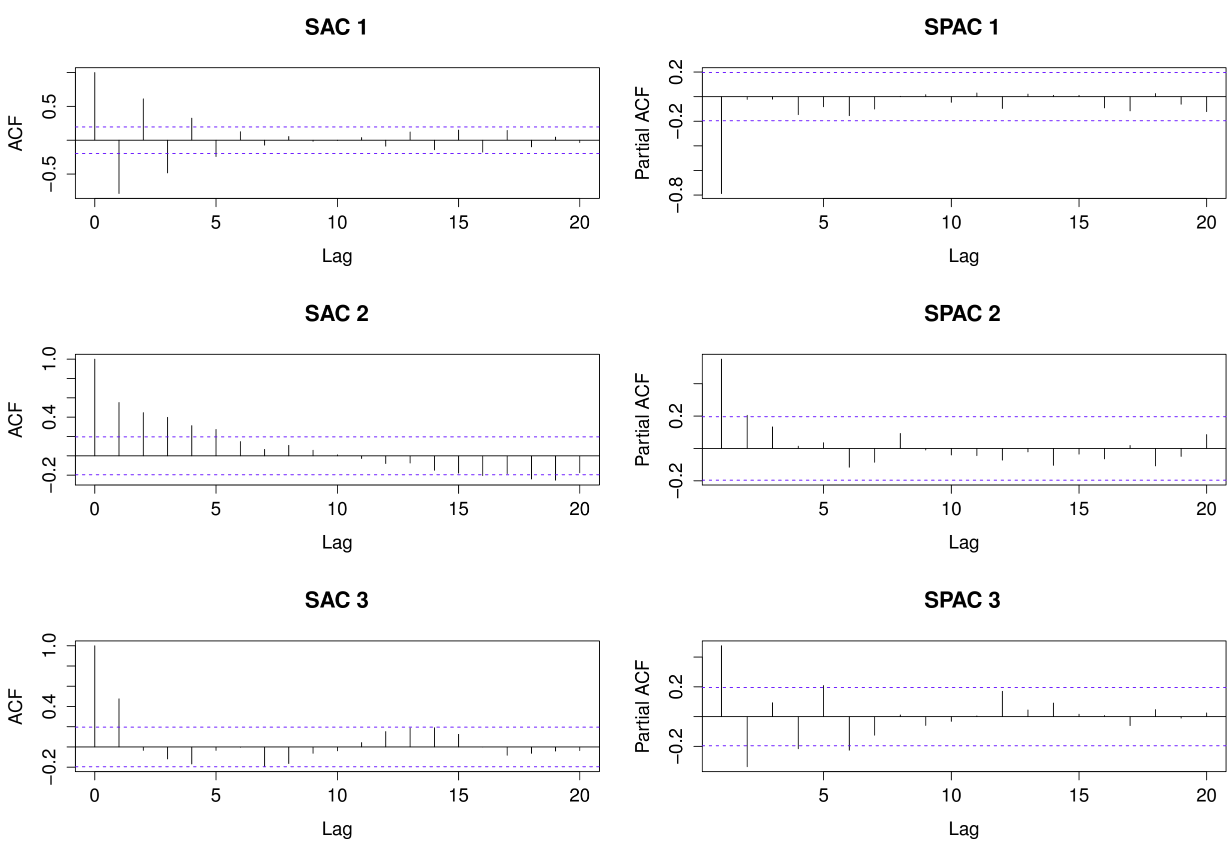

Figure 2: SAC/SPAC graphs (1,2,3).

4. The water-level Y_t in a lake has been measured once a year for 50 years. The level has been modelled with two different models. In the models below, e_t is white noise. Calculate the forecasts for Y_t the next two years. Two years for each of the models. 4p.

Model 1: $Y_t = \beta_0 + \beta_1 \cdot t + N_t$, where $N_t = \phi_1 N_{t-1} + \phi_2 N_{t-2} + e_t$.

Model 2: $(1 - B)\, Y_t = W_t$, where $W_t = \rho_1 W_{t-1} + \rho_2 W_{t-2} + e_t$.

t	Y_t	Residuals model 1	Residuals model 2
47		0.099	0.554
48	7.31	-0.376	0.159
49	8.43	0.724	1.406
50	6.67	-1.797	-1.686

Parameter	Estimate
β_0	9.32
β_1	-0.027
ϕ_1	0.771
ϕ_2	-0.239
ρ_1	0.035
ρ_2	-0.329

5. Consider the model $Y_t = 4 - \frac{1}{2} Y_{t-1} + e_t + \frac{1}{4} e_{t-1}$, where e_t is normally distributed white noise with variance 1. A simulation of the model with 200 values is shown in the graph below.

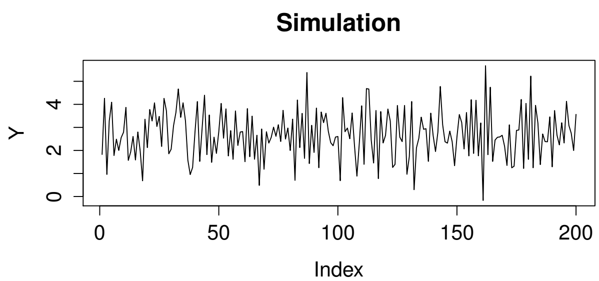

Figure 3: Simulation from the model.

(a) Express the process as a MA(∞)-process. 2p.

We use this simulated series to estimate the parameters. The result is given in the table below:

Parameter	Estimate	SE of estimate
μ	2.699	0.061
θ	-0.330	0.302
ϕ	-0.531	0.270
σ_e^2	0.996	

(b) Estimate the constant δ in the process using the fitted values. Use the method of moments. True value is 4. 1p.

(c) Use the fitted model to calculate 95% forecast-intervals for the time points $t = 201, 202$ and 203. The last four simulated values are $Y_{197} = 2.34$, $Y_{198} = 3.58$, $Y_{199} = 1.84$, $Y_{200} = 3.26$. The last four residuals are $\hat{e}_{197} = -0.157$, $\hat{e}_{198} = 0.738$, $\hat{e}_{199} = -0.635$, $\hat{e}_{200} = 0.319$. 3p.

Good luck!