Note: there might be some mistakes in the text. Ask me if you are confused by some sentence or you can not get the right answer.

Assignment 1

Assuming that the following data arise from a stationary process, calculate method-of-moments estimates of $\mu, \gamma(0)$ and $\rho(1)$: 6, 5, 4, 6, 4.

Assignment 2

From a given time series, the following sample autocorrelation was computed $\rho(1) = 0.6$, and the sample variance was equal to 2. Assuming that AR(1) model is appropriate, estimate the coefficients of this model and the error variance by using the method of moments (Yule-Walker equations). What are PACF values for this model?

Assignment 3

Assume that the fitted AR(1) model have the following estimated parameters: $\phi = -0.5, \mu = 10$. Assuming also that $x_n = 12$, compute the following forecasts:

a) $\tilde{x}_{n+10}^{n}$

Assignment 4

Assume that the fitted ARMA(1,1) model have the following estimated parameters: $\phi = -0.5, \theta = 0.5$. Assuming also that $x_1 = 10, x_2 = 12$, compute the following forecasts:

a) $\tilde{x}_3^2$

Assignment 5

Identify the following model as a certain multiplicative seasonal ARIMA:

$$x_t = x_{t-1} + x_{t-12} - x_{t-13} + w_t - 0.5w_{t-1} - 0.5w_{t-12} + 0.25w_{t-13}$$

Assignment 6

Compute $\rho(h)$ for the following model by using general homogeneous equations:

$$x_t - 0.4x_{t-1} - 0.45x_{t-2} = w_t$$

Assignment 7

Compute the best linear one-step ahead prediction x_{n+1}^{n} from the data $x_1 = 1, x_2 = 2, n = 2$. Autocovariances were estimated from the previous studies as $\gamma(0) = 0.5, \gamma(1) = 0.2$ and $\gamma(2) = 0.1$

Assignment 8

A stationary time series of length 121 produced sample partial autocorrelation of $\phi_{11} = 0.8, \phi_{22} = -0.6, \phi_{33} = 0.08, \phi_{44} = 0$. Based on this information alone, what model would we tentatively specify for the series?

Assignment 9

The sample ACF for a series and its first difference are given in the following table. Here *n* = 100.

lag	1	2	3	4	5	6
ACF for x_t	0.97	0.97	0.93	0.85	0.80	0.71
ACF for ∇x_t	−0.42	0.18	−0.02	0.07	−0.10	−0.09

Based on this information alone, which ARIMA model(s) would we consider for the series?

Assignment 10

If $\{x_t\}$ satisfies an AR(1) model with ϕ of about 0.7, how long of a series do we need to estimate ϕ with 95% confidence that our estimation error is no more than ± 0.1?

Assignment 11

Suppose that annual sales (in millions of dollars) of some company follow the AR(2) model $z_t = 5 + 1.1x_{t-1} - 0.5x_{t-2} + w_t$ with $\sigma_w^2 = 2$.
(a) If sales for 2005, 2006, and 2007 were \$9 million, \$11 million, and \$10 million, respectively, forecast sales for 2008 and 2009.
(b) Calculate 95% prediction limits for your forecast in part (a) for 2006.

Assignment 12

Compute PACF values ϕ_{11} and ϕ_{22} of MA(1) model with parameter $\theta = 0.6$

Assignment 13

For a seasonal model $x_t = 0.8x_{t-4} + w_t + 0.3w_{t-1}$ with $\sigma_w^2 = 1$, compute $\gamma(0)$ and $\rho(h), h = 1,2, \ldots$

Answers:

Assignment 1

$\mu = 5, \gamma(0) = \frac{4}{5}, \rho(1) = -1/2$

Assignment 2

$0.6 \; and \; 1.28$

Assignment 3

Approximately 10.801

Assignment 4

0

Assignment 5

$ARIMA(0,1,1)\times(0,1,1)_{12}$

Assignment 6

$\rho(h) = 0.88 \cdot 9^h - 0.12 \cdot 5^h$

Assignment 7

0.81

Assignment 8

AR(2)

Assignment 9

IMA(1,1)

Assignment 10

196 or 204 (if you take 1.96 or 2, respectively, as 95% normal quantile)

Assignment 11

a) 10.5 and 11.55
b) [7.67,11.33]

Assignment 12

0.34 and -0.22

Assignment 13

$\gamma(0) = 0.66, \rho(4h) = 0.8^h, \ \rho(4h \pm 1) \approx 0.83 \cdot 0.8^h, \rho(4h+2) = 0$