732A96: Lab 2
Advanced Machine Learning

Carles Sans Fuentes

September 21, 2017

Assignment

Main information about the lab : You do not have direct observation of the robot. However, the robot is equipped with a tracking device that you can access. The device is not very accurate though: If the robot is in the sector i, then the device will report that the robot is in the sectors $[i-2, i+2]$ with equal probability.

Question 1

Build a HMM for the scenario described above. The HMM has been built according to the description above. THe information about my HMM is shown below:

```
> myhmm
$States
 [1] "1"  "2"  "3"  "4"  "5"  "6"  "7"  "8"  "9"  "10"

$Symbols
 [1] "1"  "2"  "3"  "4"  "5"  "6"  "7"  "8"  "9"  "10"

$startProbs
  1   2   3   4   5   6   7   8   9  10
0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1

$transProbs
    to
from   1   2   3   4   5   6   7   8   9  10
  1  0.5 0.5 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
  2  0.0 0.5 0.5 0.0 0.0 0.0 0.0 0.0 0.0 0.0
  3  0.0 0.0 0.5 0.5 0.0 0.0 0.0 0.0 0.0 0.0
  4  0.0 0.0 0.0 0.5 0.5 0.0 0.0 0.0 0.0 0.0
  5  0.0 0.0 0.0 0.0 0.5 0.5 0.0 0.0 0.0 0.0
  6  0.0 0.0 0.0 0.0 0.0 0.5 0.5 0.0 0.0 0.0
  7  0.0 0.0 0.0 0.0 0.0 0.0 0.5 0.5 0.0 0.0
  8  0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.5 0.5 0.0
  9  0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.5 0.5
  10 0.5 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.5

$emissionProbs
      symbols
states   1   2   3   4   5   6   7   8   9  10
    1  0.2 0.2 0.2 0.0 0.0 0.0 0.0 0.0 0.2 0.2
    2  0.2 0.2 0.2 0.2 0.0 0.0 0.0 0.0 0.0 0.2
    3  0.2 0.2 0.2 0.2 0.2 0.0 0.0 0.0 0.0 0.0
    4  0.0 0.2 0.2 0.2 0.2 0.2 0.0 0.0 0.0 0.0
    5  0.0 0.0 0.2 0.2 0.2 0.2 0.2 0.0 0.0 0.0
    6  0.0 0.0 0.0 0.2 0.2 0.2 0.2 0.2 0.0 0.0
    7  0.0 0.0 0.0 0.0 0.2 0.2 0.2 0.2 0.2 0.0
    8  0.0 0.0 0.0 0.0 0.0 0.2 0.2 0.2 0.2 0.2
    9  0.2 0.0 0.0 0.0 0.0 0.0 0.2 0.2 0.2 0.2
    10 0.2 0.2 0.0 0.0 0.0 0.0 0.0 0.2 0.2 0.2
```

Question 2 & 3

We have Simulated the HMM for 100 time steps and discarded the hidden states from the sample obtained above. We have used the remaining observations to compute the filtered and smoothed probability distributions for each of the 100 time points. Compute also the most probable path. Here below I show the result.

```
> marginalFilter
     1         2     3   4   5         6         7         8         9 10 11  12   13         14
1  0.0 0.0000000 0.000 0.0 0.0 0.0000000 0.0000000 0.0000000 0.0000000  0  0 0.0 0.00 0.0000000
2  0.2 0.1111111 0.125 0.2 0.1 0.0000000 0.0000000 0.0000000 0.0000000  0  0 0.0 0.00 0.0000000
3  0.2 0.2222222 0.375 0.8 0.5 0.3157895 0.1578947 0.1090909 0.1818182  0  0 0.0 0.00 0.0000000
4  0.2 0.2222222 0.500 0.0 0.4 0.4736842 0.3947368 0.3818182 0.8181818  0  0 0.0 0.00 0.0000000
5  0.2 0.2222222 0.000 0.0 0.0 0.2105263 0.3421053 0.5090909 0.0000000  1  0 0.0 0.00 0.0000000
6  0.2 0.2222222 0.000 0.0 0.0 0.0000000 0.1052632 0.0000000 0.0000000  0  1 0.5 0.25 0.1428571
7  0.0 0.0000000 0.000 0.0 0.0 0.0000000 0.0000000 0.0000000 0.0000000  0  0 0.5 0.50 0.4285714
8  0.0 0.0000000 0.000 0.0 0.0 0.0000000 0.0000000 0.0000000 0.0000000  0  0 0.0 0.25 0.4285714
9  0.0 0.0000000 0.000 0.0 0.0 0.0000000 0.0000000 0.0000000 0.0000000  0  0 0.0 0.00 0.0000000
10 0.0 0.0000000 0.000 0.0 0.0 0.0000000 0.0000000 0.0000000 0.0000000  0  0 0.0 0.00 0.0000000
           15 16  17   18   19    20         21         22         23         24         25
1  0.00000000  0 0.0 0.25 0.00 0.375 0.53846154 0.46153846 0.40000000 0.28089888 0.18539326
2  0.00000000  0 0.0 0.00 0.00 0.000 0.00000000 0.26923077 0.42222222 0.41573034 0.34831461
3  0.00000000  0 0.0 0.00 0.00 0.000 0.00000000 0.00000000 0.00000000 0.21348315 0.31460674
4  0.00000000  0 0.0 0.00 0.00 0.000 0.00000000 0.00000000 0.00000000 0.00000000 0.10674157
5  0.00000000  0 0.0 0.00 0.00 0.000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000
6  0.09090909  0 0.0 0.00 0.00 0.000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000
7  0.36363636  0 0.0 0.00 0.00 0.000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000
8  0.54545455  0 0.0 0.00 0.00 0.000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000
9  0.00000000  1 0.5 0.25 0.25 0.125 0.07692308 0.03846154 0.02222222 0.00000000 0.00000000
10 0.00000000  0 0.5 0.50 0.75 0.500 0.38461538 0.23076923 0.15555556 0.08988764 0.04494382
           26        27         28         29         30         31         32          33
                         34
1  0.00000000 0.0000000 0.00000000 0.00000000 0.00000000 0.00000000 0.01790763 0.098020735
     0.0000000
2  0.30944625 0.0000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.008953817
     0.0000000
3  0.38436482 0.0000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.000000000
     0.0000000
4  0.24429967 0.6307190 0.31535948 0.00000000 0.00000000 0.00000000 0.00000000 0.000000000
     0.0000000
5  0.06188925 0.3071895 0.46895425 0.00000000 0.00000000 0.00000000 0.00000000 0.000000000
     0.0000000
6  0.00000000 0.0620915 0.18464052 0.72595281 0.36297641 0.00000000 0.00000000 0.000000000
     0.0000000
7  0.00000000 0.0000000 0.03104575 0.23956443 0.48275862 0.00000000 0.00000000 0.000000000
     0.0000000
8  0.00000000 0.0000000 0.00000000 0.03448276 0.13702359 0.78325688 0.00000000 0.000000000
     0.0000000
9  0.00000000 0.0000000 0.00000000 0.00000000 0.01724138 0.19495413 0.80395853 0.401979265
     0.3104076
10 0.00000000 0.0000000 0.00000000 0.00000000 0.00000000 0.02178899 0.17813384 0.491046183
     0.6895924
          35         36         37         38         39       40        41        42        43
1  0.3447962 0.42239811 0.37500000 0.33585485 0.00000000 0.000000 0.0000000 0.0000000 0.0000000
2  0.0000000 0.17239811 0.29739811 0.39097545 0.43675896 0.351519 0.1757595 0.0000000 0.0000000
3  0.0000000 0.00000000 0.08619905 0.22304803 0.36897231 0.648481 0.5000000 0.3704333 0.0000000
4  0.0000000 0.00000000 0.00000000 0.05012167 0.16415016 0.000000 0.3242405 0.4518267 0.0000000
5  0.0000000 0.00000000 0.00000000 0.00000000 0.03011857 0.000000 0.0000000 0.1777400 0.7798358
6  0.0000000 0.00000000 0.00000000 0.00000000 0.00000000 0.000000 0.0000000 0.0000000 0.2201642
7  0.0000000 0.00000000 0.00000000 0.00000000 0.00000000 0.000000 0.0000000 0.0000000 0.0000000
8  0.0000000 0.00000000 0.00000000 0.00000000 0.00000000 0.000000 0.0000000 0.0000000 0.0000000
9  0.1552038 0.07760189 0.03880095 0.00000000 0.00000000 0.000000 0.0000000 0.0000000 0.0000000
10 0.5000000 0.32760189 0.20260189 0.00000000 0.00000000 0.000000 0.0000000 0.0000000 0.0000000
          44         45        46        47         48        49        50        51         52
1  0.0000000 0.00000000 0.0000000 0.0000000 0.00000000 0.0000000 0.0000000 0.1114899 0.23495034
2  0.0000000 0.00000000 0.0000000 0.0000000 0.00000000 0.0000000 0.0000000 0.0000000 0.00000000
3  0.0000000 0.00000000 0.0000000 0.0000000 0.00000000 0.0000000 0.0000000 0.0000000 0.00000000
4  0.0000000 0.00000000 0.0000000 0.0000000 0.00000000 0.0000000 0.0000000 0.0000000 0.00000000
5  0.3899179 0.19495896 0.1002381 0.0000000 0.00000000 0.0000000 0.0000000 0.0000000 0.00000000
6  0.5000000 0.44495896 0.3290136 0.0000000 0.00000000 0.0000000 0.0000000 0.0000000 0.00000000
7  0.1100821 0.30504104 0.3856122 0.4859713 0.24298567 0.1502614 0.0751307 0.0000000 0.00000000
8  0.0000000 0.05504104 0.1851361 0.3881294 0.43705039 0.4205317 0.2853965 0.1872996 0.09917849
9  0.0000000 0.00000000 0.0000000 0.1258992 0.25701433 0.4292069 0.4248693 0.3689943 0.29456761
10 0.0000000 0.00000000 0.0000000 0.0000000 0.06294961 0.0000000 0.2146035 0.3322162 0.37130356
          53        54         55        56        57        58        59        60         61
1  0.3434770 0.3603652 0.36128948 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.00000000
2  0.0000000 0.1717385 0.29104355 0.6914874 0.4088044 0.2044022 0.0000000 0.0000000 0.00000000
3  0.0000000 0.0000000 0.00000000 0.3085126 0.5911956 0.5000000 0.3922940 0.0000000 0.00000000
4  0.0000000 0.0000000 0.00000000 0.0000000 0.0000000 0.2955978 0.4430824 0.5196077 0.00000000
5  0.0000000 0.0000000 0.00000000 0.0000000 0.0000000 0.0000000 0.1646236 0.3779957 0.60632805
6  0.0000000 0.0000000 0.00000000 0.0000000 0.0000000 0.0000000 0.0000000 0.1023966 0.32450340
7  0.0000000 0.0000000 0.00000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.06916855
8  0.0561902 0.0280951 0.01536711 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.00000000
9  0.2230793 0.1396348 0.09174282 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.00000000
10 0.3772535 0.3001664 0.24055704 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.00000000
```

```
            62         63          64          65          66          67          68         69
                  70
1   0.00000000 0.0000000 0.00000000 0.00000000 0.00000000 0.04777084 0.16397296 0.2903185
    0.7192699
2   0.00000000 0.0000000 0.00000000 0.00000000 0.00000000 0.00000000 0.03609363 0.0000000
    0.2807301
3   0.00000000 0.0000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.0000000
    0.0000000
4   0.00000000 0.0000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.0000000
    0.0000000
5   0.30316403 0.1542493 0.07712466 0.00000000 0.00000000 0.00000000 0.00000000 0.0000000
    0.0000000
6   0.46541572 0.3910520 0.27265066 0.18190223 0.09236510 0.00000000 0.00000000 0.0000000
    0.0000000
7   0.19683597 0.3369525 0.36400224 0.33109421 0.26048590 0.18496778 0.00000000 0.0000000
    0.0000000
8   0.03458428 0.1177462 0.22734934 0.30753506 0.32427889 0.30653916 0.00000000 0.0000000
    0.0000000
9   0.00000000 0.0000000 0.05887310 0.14885128 0.23174080 0.29147071 0.45183108 0.2561633
    0.0000000
10  0.00000000 0.0000000 0.00000000 0.03061722 0.09112931 0.16925152 0.34810232 0.4535182
    0.0000000
           71  72  73   74  75   76          77          78   79   80      81  82  83   84   85          86
                 87
1   0.3596349  0   0 0.0   0 0.0 0.0000000 0.0000000 0.2 0.5 0.5625   1   1 0.5 0.25 0.0000000
    0.0000000
2   0.5000000  0   0 0.0   0 0.0 0.0000000 0.0000000 0.0 0.1 0.3750   0   0 0.5 0.50 0.4285714
    0.2307692
3   0.1403651  0   0 0.0   0 0.0 0.0000000 0.0000000 0.0 0.0 0.0625   0   0 0.0 0.25 0.4285714
    0.4615385
4   0.0000000  1   0 0.0   0 0.0 0.0000000 0.0000000 0.0 0.0 0.0000   0   0 0.0 0.00 0.1428571
    0.3076923
5   0.0000000  0   1 0.5   0 0.0 0.0000000 0.0000000 0.0 0.0 0.0000   0   0 0.0 0.00 0.0000000
    0.0000000
6   0.0000000  0   0 0.5   0 0.0 0.0000000 0.0000000 0.0 0.0 0.0000   0   0 0.0 0.00 0.0000000
    0.0000000
7   0.0000000  0   0 0.0   1 0.5 0.0000000 0.0000000 0.0 0.0 0.0000   0   0 0.0 0.00 0.0000000
    0.0000000
8   0.0000000  0   0 0.0   0 0.5 0.6666667 0.3333333 0.0 0.0 0.0000   0   0 0.0 0.00 0.0000000
    0.0000000
9   0.0000000  0   0 0.0   0 0.0 0.3333333 0.5000000 0.0 0.0 0.0000   0   0 0.0 0.00 0.0000000
    0.0000000
10  0.0000000  0   0 0.0   0 0.0 0.0000000 0.1666667 0.8 0.4 0.0000   0   0 0.0 0.00 0.0000000
    0.0000000
           88         89          90          91          92          93         94         95
          96
1   0.0000000 0.0000000 0.00000000 0.00000000 0.00000000 0.00000000 0.0000000 0.0000000
    0.00000000
2   0.1153846 0.0000000 0.00000000 0.00000000 0.00000000 0.00000000 0.0000000 0.0000000
    0.00000000
3   0.3461538 0.0000000 0.00000000 0.00000000 0.00000000 0.00000000 0.0000000 0.0000000
    0.00000000
4   0.3846154 0.5135135 0.25675676 0.00000000 0.00000000 0.00000000 0.0000000 0.0000000
    0.00000000
5   0.1538462 0.3783784 0.44594595 0.40310078 0.20155039 0.10156250 0.2007722 0.1003861
    0.05019305
6   0.0000000 0.1081081 0.24324324 0.39534884 0.39922481 0.30273438 0.7992278 0.5000000
    0.30019305
7   0.0000000 0.0000000 0.05405405 0.17054264 0.28294574 0.34375000 0.0000000 0.3996139
    0.44980695
8   0.0000000 0.0000000 0.00000000 0.03100775 0.10077519 0.19335937 0.0000000 0.0000000
    0.19980695
9   0.0000000 0.0000000 0.00000000 0.00000000 0.01550388 0.05859375 0.0000000 0.0000000
    0.00000000
10  0.0000000 0.0000000 0.00000000 0.00000000 0.00000000 0.00000000 0.0000000 0.0000000
    0.00000000
           97         98          99        100
1   0.0000000 0.00000000 0.05711921 0.1837807
2   0.0000000 0.00000000 0.00000000 0.0000000
3   0.0000000 0.00000000 0.00000000 0.0000000
4   0.0000000 0.00000000 0.00000000 0.0000000
5   0.0000000 0.00000000 0.00000000 0.0000000
6   0.1797030 0.00000000 0.00000000 0.0000000
7   0.3846535 0.00000000 0.00000000 0.0000000
8   0.3331683 0.57154119 0.00000000 0.0000000
9   0.1024752 0.34686638 0.64293598 0.3309189
10  0.0000000 0.08159243 0.29994481 0.4853004
> smoothing
      index
states          1         2         3         4         5         6         7         8 9 10 11
       12
     1  0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0  0  0
        0.0
     2  0.8148148 0.6296296 0.4444444 0.2592593 0.1111111 0.0000000 0.0000000 0.0000000 0  0  0
        0.0
     3  0.1851852 0.3703704 0.5555556 0.7407407 0.7407407 0.6666667 0.4444444 0.2222222 0  0  0
        0.0
```

```
     4  0.0000000 0.0000000 0.0000000 0.0000000 0.1481481 0.3333333 0.5555556 0.7777778 1  0  0 0.0
     5  0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0  1  0 0.0
     6  0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0  0  1 0.5
     7  0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0  0  0 0.5
     8  0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0  0  0 0.0
     9  0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0  0  0 0.0
     10 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0  0  0 0.0
      index
states        13  14  15  16         17        18        19         20         21          22
     1  0.0000000 0.0   0   0 0.0000000 0.000000 0.0000000 0.28611141 0.66759329 0.633197262
     2  0.0000000 0.0   0   0 0.0000000 0.000000 0.0000000 0.00000000 0.00000000 0.298228218
     3  0.0000000 0.0   0   0 0.0000000 0.000000 0.0000000 0.00000000 0.00000000 0.000000000
     4  0.0000000 0.0   0   0 0.0000000 0.000000 0.0000000 0.00000000 0.00000000 0.000000000
     5  0.0000000 0.0   0   0 0.0000000 0.000000 0.0000000 0.00000000 0.00000000 0.000000000
     6  0.1666667 0.0   0   0 0.0000000 0.000000 0.0000000 0.00000000 0.00000000 0.000000000
     7  0.6666667 0.5   0   0 0.0000000 0.000000 0.0000000 0.00000000 0.00000000 0.000000000
     8  0.1666667 0.5   1   0 0.0000000 0.000000 0.0000000 0.00000000 0.00000000 0.000000000
     9  0.0000000 0.0   0   1 0.7452445 0.490489 0.2357335 0.07634843 0.01233386 0.001085725
     10 0.0000000 0.0   0   0 0.2547555 0.509511 0.7642665 0.63754016 0.32007285 0.067488795
      index
states          23          24          25         26         27         28          29          30
     1  0.182923331 0.02714313 0.00000000 0.0000000 0.0000000 0.0000000 0.00000000 0.00000000
     2  0.809476593 0.33583724 0.06731496 0.0000000 0.0000000 0.0000000 0.00000000 0.00000000
     3  0.000000000 0.63701963 0.44749358 0.1281156 0.0000000 0.0000000 0.00000000 0.00000000
     4  0.000000000 0.00000000 0.48519145 0.5178924 0.2095450 0.0000000 0.00000000 0.00000000
     5  0.000000000 0.00000000 0.00000000 0.3539921 0.5470336 0.3116031 0.00000000 0.00000000
     6  0.000000000 0.00000000 0.00000000 0.0000000 0.2434214 0.5349173 0.43429008 0.00000000
     7  0.000000000 0.00000000 0.00000000 0.0000000 0.0000000 0.1534796 0.48154340 0.57760581
     8  0.000000000 0.00000000 0.00000000 0.0000000 0.0000000 0.0000000 0.08416652 0.38691196
     9  0.000000000 0.00000000 0.00000000 0.0000000 0.0000000 0.0000000 0.00000000 0.03548224
     10 0.007600076 0.00000000 0.00000000 0.0000000 0.0000000 0.0000000 0.00000000 0.00000000
      index
states          31         32         33         34          35          36          37         38
     1  0.000000000 0.0000000 0.0000000 0.0000000 0.36216976 0.54445735 0.50117641 0.2756079
     2  0.000000000 0.0000000 0.0000000 0.0000000 0.00000000 0.13995424 0.36920910 0.5777995
     3  0.000000000 0.0000000 0.0000000 0.0000000 0.00000000 0.00000000 0.03294120 0.1465927
     4  0.000000000 0.0000000 0.0000000 0.0000000 0.00000000 0.00000000 0.00000000 0.0000000
     5  0.000000000 0.0000000 0.0000000 0.0000000 0.00000000 0.00000000 0.00000000 0.0000000
     6  0.000000000 0.0000000 0.0000000 0.0000000 0.00000000 0.00000000 0.00000000 0.0000000
     7  0.000000000 0.0000000 0.0000000 0.0000000 0.00000000 0.00000000 0.00000000 0.0000000
     8  0.741550314 0.0000000 0.0000000 0.0000000 0.00000000 0.00000000 0.00000000 0.0000000
     9  0.251022958 0.9261236 0.5927026 0.2592816 0.08888475 0.01851422 0.00000000 0.0000000
     10 0.007426728 0.0738764 0.4072974 0.7407184 0.54894549 0.29707419 0.09667329 0.0000000
      index
states        39        40        41        42        43         44         45        46
           47
     1  0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.00000000 0.00000000 0.0000000 0.00000000
     2  0.5964486 0.1187578 0.0000000 0.0000000 0.0000000 0.00000000 0.00000000 0.0000000 0.00000000
     3  0.4035514 0.8812422 0.3378417 0.0000000 0.0000000 0.00000000 0.00000000 0.0000000 0.00000000
     4  0.0000000 0.0000000 0.6621583 0.5569255 0.0000000 0.00000000 0.00000000 0.0000000 0.00000000
     5  0.0000000 0.0000000 0.0000000 0.4430745 0.7760094 0.30962274 0.08077435 0.0000000 0.00000000
     6  0.0000000 0.0000000 0.0000000 0.0000000 0.2239906 0.59805752 0.52230553 0.2651274 0.00000000
     7  0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.09231974 0.37166247 0.5696354 0.57586333
     8  0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.00000000 0.02525765 0.1652372 0.38319954
     9  0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.00000000 0.00000000 0.0000000 0.04093713
     10 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.00000000 0.00000000 0.0000000 0.00000000
      index
states        48         49         50         51         52         53        54        55
     1  0.0000000 0.00000000 0.00000000 0.04715039 0.18764856 0.48419879 0.6221480 0.4184326
     2  0.0000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.1877031 0.5815674
     3  0.0000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.0000000 0.0000000
     4  0.0000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.0000000 0.0000000
     5  0.0000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.0000000 0.0000000
     6  0.0000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.0000000 0.0000000
     7  0.2540821 0.09031835 0.02123451 0.00000000 0.00000000 0.00000000 0.0000000 0.0000000
     8  0.5787774 0.45832030 0.26242651 0.10189735 0.01779764 0.00000000 0.0000000 0.0000000
     9  0.1671405 0.45136135 0.53542156 0.45235491 0.24978248 0.07065798 0.0000000 0.0000000
     10 0.0000000 0.00000000 0.18091742 0.39859734 0.54477132 0.44514323 0.1901490 0.0000000
      index
```

```
states         56         57          58         59         60          61          62          63
          64
      1  0.0000000 0.0000000 0.00000000 0.0000000 0.0000000 0.00000000 0.00000000 0.00000000
          0.0000000
      2  0.7555088 0.2075164 0.03479317 0.0000000 0.0000000 0.00000000 0.00000000 0.00000000
          0.0000000
      3  0.2444912 0.7924836 0.42250832 0.1199028 0.0000000 0.00000000 0.00000000 0.00000000
          0.0000000
      4  0.0000000 0.0000000 0.54269851 0.5368674 0.2553288 0.00000000 0.00000000 0.00000000
          0.0000000
      5  0.0000000 0.0000000 0.00000000 0.3432299 0.5505936 0.44107119 0.11026780 0.01575254
          0.0000000
      6  0.0000000 0.0000000 0.00000000 0.0000000 0.1940776 0.46368702 0.50784753 0.23961455
          0.0556883
      7  0.0000000 0.0000000 0.00000000 0.0000000 0.0000000 0.09524179 0.34774139 0.51616355
          0.3717333
      8  0.0000000 0.0000000 0.00000000 0.0000000 0.0000000 0.00000000 0.03414328 0.22846936
          0.4643561
      9  0.0000000 0.0000000 0.00000000 0.0000000 0.0000000 0.00000000 0.00000000 0.00000000
          0.1082223
      10 0.0000000 0.0000000 0.00000000 0.0000000 0.0000000 0.00000000 0.00000000 0.00000000
          0.0000000
     index
states          65         66          67         68  69  70  71  72  73  74  75         76         77
          78
      1  0.00000000 0.0000000 0.07048501 0.3202126   1   0   0   0   0   0   0 0.0000000 0.0000000
          0.0000000
      2  0.00000000 0.0000000 0.00000000 0.0000000   0   1   0   0   0   0   0 0.0000000 0.0000000
          0.0000000
      3  0.00000000 0.0000000 0.00000000 0.0000000   0   0   1   0   0   0   0 0.0000000 0.0000000
          0.0000000
      4  0.00000000 0.0000000 0.00000000 0.0000000   0   0   0   1   0   0   0 0.0000000 0.0000000
          0.0000000
      5  0.00000000 0.0000000 0.00000000 0.0000000   0   0   0   0   1   0   0 0.0000000 0.0000000
          0.0000000
      6  0.00000000 0.0000000 0.00000000 0.0000000   0   0   0   0   0   1   0 0.0000000 0.0000000
          0.0000000
      7  0.13003496 0.0000000 0.00000000 0.0000000   0   0   0   0   0   0   1 0.2222222 0.0000000
          0.0000000
      8  0.48312907 0.2508172 0.00000000 0.0000000   0   0   0   0   0   0   0 0.7777778 0.4444444
          0.0000000
      9  0.35076186 0.5377277 0.43005981 0.0000000   0   0   0   0   0   0   0 0.0000000 0.5555556
          0.6666667
      10 0.03607411 0.2114550 0.49945518 0.6797874   0   0   0   0   0   0   0 0.0000000 0.0000000
          0.3333333
     index
states        79         80 81 82 83        84         85          86         87         88
       89
      1  0.1111111 0.5555556  1  1  1 0.4310467 0.1264025 0.00000000 0.0000000 0.0000000
          0.00000000
      2  0.0000000 0.0000000  0  0  0 0.5689533 0.6092885 0.37920750 0.1026843 0.0000000
          0.00000000
      3  0.0000000 0.0000000  0  0  0 0.0000000 0.2643090 0.53472518 0.5530464 0.3080528
          0.00000000
      4  0.0000000 0.0000000  0  0  0 0.0000000 0.0000000 0.08606732 0.3442693 0.5794631
          0.65033376
      5  0.0000000 0.0000000  0  0  0 0.0000000 0.0000000 0.00000000 0.0000000 0.1124840
          0.33205511
      6  0.0000000 0.0000000  0  0  0 0.0000000 0.0000000 0.00000000 0.0000000 0.0000000
          0.01761113
      7  0.0000000 0.0000000  0  0  0 0.0000000 0.0000000 0.00000000 0.0000000 0.0000000
          0.00000000
      8  0.0000000 0.0000000  0  0  0 0.0000000 0.0000000 0.00000000 0.0000000 0.0000000
          0.00000000
      9  0.0000000 0.0000000  0  0  0 0.0000000 0.0000000 0.00000000 0.0000000 0.0000000
          0.00000000
      10 0.8888889 0.4444444  0  0  0 0.0000000 0.0000000 0.00000000 0.0000000 0.0000000
          0.00000000
     index
states          90        91         92         93          94          95          96         97
          98
      1  0.00000000 0.000000 0.0000000 0.0000000 0.00000000 0.00000000 0.00000000 0.0000000
          0.0000000
      2  0.00000000 0.000000 0.0000000 0.0000000 0.00000000 0.00000000 0.00000000 0.0000000
          0.0000000
      3  0.00000000 0.000000 0.0000000 0.0000000 0.00000000 0.00000000 0.00000000 0.0000000
          0.0000000
      4  0.28334043 0.000000 0.0000000 0.0000000 0.00000000 0.00000000 0.00000000 0.0000000
          0.0000000
      5  0.63740946 0.775458 0.5465133 0.3175685 0.08862378 0.01477063 0.00000000 0.0000000
          0.0000000
      6  0.07925011 0.224542 0.4534867 0.6824315 0.91137622 0.36784548 0.08833972 0.0000000
          0.0000000
      7  0.00000000 0.000000 0.0000000 0.0000000 0.00000000 0.61738389 0.52947025 0.2207073
          0.0000000
      8  0.00000000 0.000000 0.0000000 0.0000000 0.00000000 0.00000000 0.38219003 0.5734981
          0.4118733
```

```
     9  0.00000000 0.000000 0.0000000 0.0000000 0.00000000 0.00000000 0.00000000 0.2057946 0.4999290
     10 0.00000000 0.000000 0.0000000 0.0000000 0.00000000 0.00000000 0.00000000 0.0000000 0.0881977
        index
states           99       100
     1  0.02939923 0.1837807
     2  0.00000000 0.0000000
     3  0.00000000 0.0000000
     4  0.00000000 0.0000000
     5  0.00000000 0.0000000
     6  0.00000000 0.0000000
     7  0.00000000 0.0000000
     8  0.00000000 0.0000000
     9  0.66183781 0.3309189
     10 0.30876296 0.4853004
> Viterbi
  [1] "2"  "2"  "2"  "2"  "2"  "3"  "3"  "3"  "4"  "5"  "6"  "6"  "6"  "7"  "8"  "9"  "9"  "9"  "10"
 [20] "1"  "1"  "1"  "1"  "1"  "2"  "3"  "4"  "5"  "6"  "7"  "8"  "9"  "9"  "10" "1"  "1"  "1"  "1"
 [39] "2"  "2"  "3"  "4"  "5"  "5"  "5"  "6"  "7"  "8"  "9"  "10" "1"  "1"  "1"  "1"  "1"  "2"  "2"
 [58] "2"  "3"  "4"  "5"  "6"  "7"  "8"  "9"  "10" "1"  "1"  "1"  "2"  "3"  "4"  "5"  "6"  "7"  "8"
 [77] "9"  "10" "1"  "1"  "1"  "1"  "1"  "1"  "1"  "2"  "2"  "3"  "4"  "4"  "5"  "5"  "5"  "6"  "7"
 [96] "8"  "9"  "10" "1"  "1"
```

Question 4 & 5

Compute the accuracy of the filtered and smoothed probability distributions, and of the most probable path. That is, compute the percentage of the true hidden states that are guessed by each method with different samples.

As the hint was telling us,the forward function in the HMM package returns probabilities in log scale so we needed to use the functions exp and prop.table in order to obtain a normalized probability distribution. Then, we also have used the functions apply and which.max to find out the most probable states. I have repeated the procedure 5 times for a simulation of length 100 leading to the following results:

```
> Simulations$PercentageSIm
[[1]]
      smoothingresult Filteringresult Viterbiresult
FALSE            0.32            0.54          0.39
TRUE             0.68            0.46          0.61

[[2]]
      smoothingresult Filteringresult Viterbiresult
FALSE            0.23            0.51          0.35
TRUE             0.77            0.49          0.65

[[3]]
      smoothingresult Filteringresult Viterbiresult
FALSE            0.42            0.51          0.44
TRUE             0.58            0.49          0.56

[[4]]
      smoothingresult Filteringresult Viterbiresult
FALSE             0.2             0.4          0.35
TRUE              0.8             0.6          0.65

[[5]]
      smoothingresult Filteringresult Viterbiresult
FALSE            0.36            0.46          0.61
TRUE             0.64            0.54          0.39
```

In general,the smoothed algorithm is more accurate than the filtered distributions because it is using future values to predict previous values as well, giving to the predicted one a better prediction from the future. Moreover, the smoothed distribution is also more accurate than the most probable path because it has more constraints that needs to be taken into account. Those constraints might be wrong at some point because you are never sure of where your real robot is (e.g. it can be between $[i-2, i+2]$), so marking a probable path might make you choose not the most optimal place in general but according to the constraint.

Question 6

In order to answer whether it is true that the more observations you have the better you know where the robot is, it is necessary to understand what entropy in statistics is. Entropy is a measure of uncertainty which goes up when the uncertainty is high and goes down when the uncertainty goes down. By plotting the entropy for each point of the observations, if that statement was correct, we should see a decreasing trend of uncertainty in our model that should go asymptotically until 0. Nevertheless that is not the case, and it can be seen in the figure below. This makes sense since the model is randomly transitioned and no matter the more observations you have, you still have the same probability transition for it and the same information about the model for a particular point.

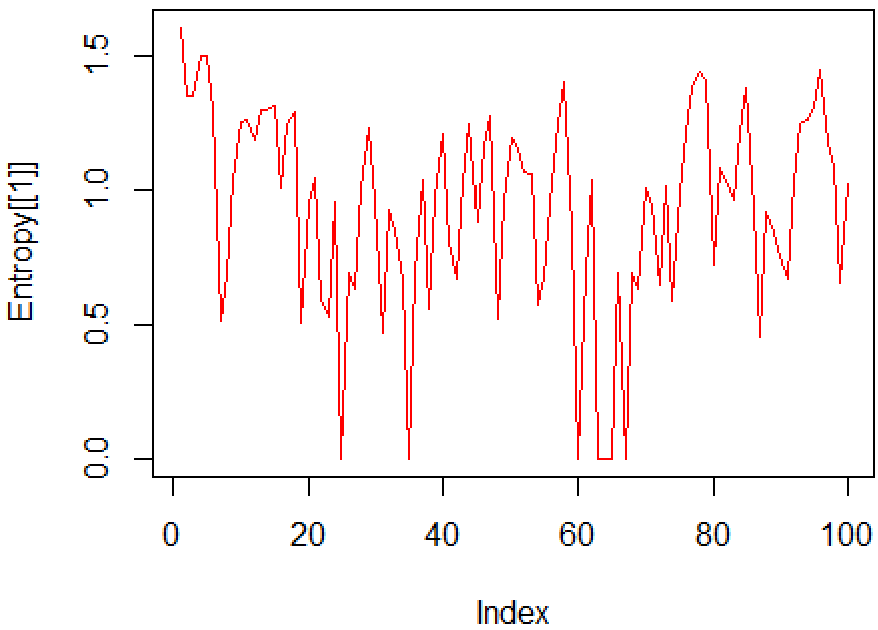

Figure 1: Graphical representation of the entropy for one of the simulations

Question 7

Consider any of the samples above of length 100. Compute the probabilities of the hidden states for the time step 101.

In order to do so, we just need as likelihood the posterior of our data and take the last distribution of the value and then multiplied by our prior which is our transition distribution, giving the following possible distribution:

```
> result
        [,1]
 [1,] 0.1875
 [2,] 0.5000
 [3,] 0.3125
 [4,] 0.0000
 [5,] 0.0000
 [6,] 0.0000
```

```
 [7,] 0.0000
 [8,] 0.0000
 [9,] 0.0000
[10,] 0.0000
```

Contributions

All results and comments presented have been developed and discussed together by the members of the group.

Appendix

Poisson regression-the MCMC way

```r
###################Lab1

set.seed(12345)
#install.packages("HMM")
library(HMM)

##QUestion1
# Initialise HMM
states          <- as.character(1:10)
symbols         <- as.character(1:10)
startProbs      <- rep(0.1,10)
transProbs      <- diag(x = 0.5, 10)
transProbs[10,1]<- 0.5 #writing the ones left
emissionProbs   <- diag(x = 0.2, 10)
emissionProbs[1,9:10]<-rep(0.2,2)
emissionProbs[2,10]<-rep(0.2,1)
emissionProbs[9,1]<-rep(0.2,1)
emissionProbs[10,1:2]<-rep(0.2,1)
for(i in 1:ncol(transProbs)){
  for(j in 1:nrow(transProbs)){
    if(j == i+1){
      transProbs[i,j]<-0.5
      emissionProbs[i,j]<-0.2}
    if(j == i+2){ emissionProbs[i,j]<-0.2}
    if(j == i-1){ emissionProbs[i,j]<-0.2}
    if(j == i-2){ emissionProbs[i,j]<-0.2}
}}


myhmm = initHMM(States = states,
              Symbols = symbols,
              startProbs = startProbs,
              transProbs= transProbs,
              emissionProbs=emissionProbs)

##2
length<-100

my100sim<-simHMM(myhmm, length)


#3

myobs<-my100sim$observation # Taking just the real observations (Z)
###Filtering
filtering<-exp(forward(myhmm, observation=myobs))#A matrix containing the forward probabilities given on a logarithmic scale (natural logarithm).
marginalFilter<-apply(as.data.frame(filtering),2,  FUN = function(x){prop.table(x)})##prop.table already calculated the % on 1.


###Smoothing
smoothing<-posterior(myhmm, observation=myobs)

##Most probable path

Viterbi<-viterbi(myhmm, observation=myobs)

#4
mystates<-my100sim$states


smoothingMostProb   <- sapply(as.data.frame(smoothing), which.max)
FilteringMostProb   <- sapply(as.data.frame(marginalFilter),which.max)
Viterbi

smoothingresult <-table(mystates == smoothingMostProb)
Filteringresult <-table(mystates == FilteringMostProb)
Viterbiresult   <-table(mystates == Viterbi)

ResultTable     <-cbind(smoothingresult, Filteringresult, Viterbiresult)

###5


Comparingsimuations<-function(HMM, length){
  ResultTable<-list()
  FilterEntropy<-list()
  library(entropy)
```

```r
  for(i in 1:length(length)){
    my100sim              <-simHMM(myhmm, length[i])
    myobs                 <-my100sim$observation # Taking just the real observations (Z)
    ###Filtering
    filtering             <-exp(forward(myhmm, observation=myobs))#A matrix containing the forward probabilities given on a logarithmic scale (natural logarithm).
    marginalFilter        <-apply(as.data.frame(filtering),2,  FUN = function(x){prop.table(x)})##prop.table already calculated the % on 1.
    ###Smoothing
    smoothing             <-posterior(myhmm, observation=myobs)


    #Getting just the Z states for different models
    mystates              <-my100sim$states
    smoothingMostProb     <- sapply(as.data.frame(smoothing), which.max)
    FilteringMostProb     <- sapply(as.data.frame(marginalFilter),which.max)

    smoothingresult       <-table(mystates == smoothingMostProb)
    Filteringresult       <-table(mystates == FilteringMostProb)
    Viterbi               <-viterbi(myhmm, observation=myobs)##Most probable path

    Viterbiresult         <-table(mystates == Viterbi)
    ResultTable[[i]]      <-cbind(smoothingresult, Filteringresult, Viterbiresult)/length[i]


    FilterEntropy[[i]]    <- apply(marginalFilter,2,entropy.empirical)

  }
 return(list(PercentageSIm=ResultTable, Entropy=FilterEntropy))
}


length  <-rep(100,5)
myhmm = initHMM(States = states,
                Symbols = symbols,
                startProbs = startProbs,
                transProbs= transProbs,
                emissionProbs=emissionProbs)

Simulations<-Comparingsimuations(myhmm, length = length)

Simulations$PercentageSIm

#6
#install.packages("entropy")

Entropy<-Simulations$Entropy

plot(Entropy[[1]], col = "red", type = "l")
lines(Entropy[[2]], col = "blue")
lines(Entropy[[3]], col = "green")
lines(Entropy[[4]], col = "purple")
lines(Entropy[[5]], col = "black")

##7
likelihood<-posterior(myhmm, myobs)
prior<-transProbs
result<-prior%*%likelihood[,100]
```