

# 732A96: Labs

## Advanced Machine Learning

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---

```
1
2 trans_probs <- diag(1/2, 10) +
3   diag(1/2, 10)[, c(10, 1:9)]
4 emission_probs <-
5   diag(1/5, 10)[, c(3:10, 1:2)] +
6   diag(1/5, 10)[, c(2:10, 1)] +
7   diag(1/5, 10) +
8   diag(1/5, 10)[, c(10, 1:9)] +
9   diag(1/5, 10)[, c(9:10, 1:8)]
10
11 emission_density <- function(x, z) {
12   return(emission_probs[z, x])
13 }
14
15 transition_density <- function(z, previous_z) {
16   return(trans_probs[previous_z, z])
17 }
18
19 transition_density2 <- function(z, previous_z){
20   if( z == zt){
21
22     return(0.5)
23
24   } else if( (z + 1) == zt){
25     return(0.5)
26   } else return(0)
27 }
28 }
29
30 get_alpha_scalar <- function(zt, xt, previous_alpha, previous_z) {
31   # Args:
32   #   zt Scalar, hidden state at which to compute alpha.
33   #   xt Scalar, observed state.
34   #   previous_alpha Vector, alpha for all z_{t-1}.
35   #   previous_z      Vector, all z_{t-1}.
36
37   summation_term <- 0
38   for (i in 1:length(previous_z)) {
39     summation_term <- summation_term +
40       previous_alpha[i] * transition_density(zt, previous_z[i])
41   }
42
43   alpha <- emission_density(xt, zt) * sum(summation_term)
44   return(alpha)
45 }
46
47 get_alpha <- function(Zt, xt, previous_alpha, previous_z) {
48   # Args:
49   #   Zt Vector, hidden states at which to compute alpha.
50   #   xt Scalar, observed state.
51   #   previous_alpha Vector, alpha for all z_{t-1}.
52   #   previous_z      Vector, all z_{t-1}.
53
54   alpha <- sapply(Zt, function(zt) {
55     get_alpha_scalar(zt, xt, previous_alpha, previous_z)
56   })
57
58   return(alpha)
59 }
60
61 get_beta_scalar <- function(zt, next_x, next_beta, next_z) {
62   # Args:
63   #   zt      Scalar, hidden state at which to compute alpha.
64   #   next_x  Scalar, observed next state.
65   #   next_beta Vector, alpha for all z_{t+1}.
66   #   next_z  Vector, all z_{t+1}.
```

```

67
68 summation_term <- 0
69 for (i in 1:length(next_z)) {
70   summation_term <- summation_term +
71     next_beta[i] * emission_density(next_x, next_z[i]) * transition_density(next_z[i], zt)
72 }
73
74 #  $P(z_{(t+1)} | z_t) =$ 
75 # 0.5 if  $z_t = z_{(t+1)}$ 
76 # 0.5 if  $z_t = z_{(t+1)} + 1$ 
77 # 0 otherwise
78
79
80 return(summation_term)
81 }
82
83 get_beta <- function(Zt, next_x, next_beta, next_z) {
84   # Args:
85   #   Zt      Vector, hidden states at which to compute alpha.
86   #   next_x   Scalar, observed next state.
87   #   next_beta Vector, alpha for all  $z_{\{t+1\}}$ .
88   #   next_z   Vector, all  $z_{\{t+1\}}$ .
89
90   beta <- sapply(Zt, function(zt) {
91     get_beta_scalar(zt, next_x, next_beta, next_z)
92   })
93
94   return(beta)
95 }
96
97 fb_algorithm <- function(
98   observations,
99   emission_density,
100   transition_density,
101   possible_states,
102   initial_density) {
103
104   t_total <- length(observations)
105   cardinality <- length(possible_states)
106
107   # Alpha
108   alpha <- matrix(NA, ncol=cardinality, nrow=t_total)
109
110   for (i in 1:cardinality) {
111     alpha[1, i] <-
112       emission_density(observations[1], possible_states[i]) * initial_density[i]
113   }
114
115   for (t in 2:t_total) {
116     alpha[t, ] <- get_alpha(possible_states, observations[t], alpha[t - 1, ], possible_states)
117   }
118
119   # Beta
120   beta <- matrix(NA, ncol=cardinality, nrow=t_total)
121
122   beta[t_total, ] <- 1
123
124   for (t in (t_total - 1):1) {
125     beta[t, ] <- get_beta(possible_states, observations[t + 1], beta[t + 1, ], possible_states)
126   }
127
128   return(list(alpha = alpha, beta = beta))
129 }
130
131 filtering <- function(alpha) {
132   alpha / rowSums(alpha)
133 }
134
135 smoothing <- function(alpha, beta) {
136   alpha * beta / rowSums(alpha * beta)
137 }
138
139
140
141
142
143
144
145
146 robotHMM <- HMM::initHMM(
147   States = 1:10,
148   Symbols = 1:10,
149   transProbs = trans_probs,
150   emissionProbs = emission_probs
151 )
152
153 # Create a wrapper for simHMM to assign class to the output

```

```

154 simHMM <- function(hmm, length) {
155   simulation <- HMM::simHMM(hmm, length)
156   return(structure(simulation, class="HMMSimulation"))
157 }
158
159 # Simulate
160 nSim <- 100
161 robotSimulation <- simHMM(hmm=robotHMM, length=nSim)
162
163 #debugonce(fb_algorithm)
164
165 alphabeta <- fb_algorithm(observations = robotSimulation$observation,
166                           emission_density = emission_density,
167                           transition_density = transition_density,
168                           possible_states = 1:10,
169                           initial_density = rep(0.1, 10))
170
171 filtering(alphabeta$alpha)
172 smoothing(alphabeta$alpha, alphabeta$beta)
173
174
175 plot(apply(filtering(alphabeta$alpha), 1, which.max), type = "l")
176 plot(apply(smoothing(alphabeta$alpha, alphabeta$beta), 1, which.max), type = "l")
177 lines(x = 1:100, robotSimulation$states, type = "l", col = "green")
178
179
180
181
182 # # # Test
183 # zt <- 5
184 # xt <- 6
185 # previous_alpha <- rep(0.1, 10)
186 # previous_z <- 1:10
187 # transition_density(zt, previous_z[5])
188 # get_alpha_scalar(zt, xt, previous_alpha, previous_z)
189
190
191 # # Test
192 # zt <- 1:10
193 # xt <- 6
194 # previous_alpha <- rep(0.1, 10)
195 # previous_z <- 1:10
196 # transition_density(zt, previous_z[5])
197 # get_alpha(zt, xt, previous_alpha, previous_z)
198 #
199
200
201 # # Test
202 # Zt <- 1:10
203 # next_x <- 6
204 # next_beta <- rep(0.1, 10)
205 # next_z <- 1:10
206 # transition_density(zt, previous_z[5])
207 # get_beta_scalar(5, next_x, next_beta, next_z)
208 # get_beta(zt, next_x, next_beta, next_z)
209
210
211 # Define the transition, emission and initialization probabilities -----
212
213 emission_probs <- matrix(c(.2, .2, .2, 0, 0, 0, 0, 0, .2, .2,
214                             .2, .2, .2, .2, 0, 0, 0, 0, 0, .2,
215                             .2, .2, .2, .2, .2, 0, 0, 0, 0, 0,
216                             0, .2, .2, .2, .2, .2, 0, 0, 0, 0,
217                             0, 0, .2, .2, .2, .2, .2, 0, 0, 0,
218                             0, 0, 0, .2, .2, .2, .2, .2, 0, 0,
219                             0, 0, 0, 0, .2, .2, .2, .2, .2, 0,
220                             0, 0, 0, 0, 0, .2, .2, .2, .2, .2,
221                             .2, 0, 0, 0, 0, 0, .2, .2, .2, .2,
222                             .2, .2, 0, 0, 0, 0, 0, .2, .2, .2), byrow=TRUE, nrow=10)
223
224 transition_probs <- matrix(c(.5, .5, 0, 0, 0, 0, 0, 0, 0, 0,
225                              0, .5, .5, 0, 0, 0, 0, 0, 0, 0,
226                              0, 0, .5, .5, 0, 0, 0, 0, 0, 0,
227                              0, 0, 0, .5, .5, 0, 0, 0, 0, 0,
228                              0, 0, 0, 0, .5, .5, 0, 0, 0, 0,
229                              0, 0, 0, 0, 0, .5, .5, 0, 0, 0,
230                              0, 0, 0, 0, 0, 0, .5, .5, 0, 0,
231                              0, 0, 0, 0, 0, 0, 0, .5, .5, 0,
232                              0, 0, 0, 0, 0, 0, 0, 0, .5, .5,
233                              .5, 0, 0, 0, 0, 0, 0, 0, 0, .5), byrow=TRUE, nrow=10)
234
235 tProbDensity <- function(zt, zt_1) {
236   return(transition_probs[zt_1, zt])
237 }
238
239 eProbDensity <- function(xt, zt) {
240   return(emission_probs[zt, xt])

```

```

241 }
242
243 initProbDensity <- function(z0) {
244   return(dunif(z0, min=1, max=10))
245 }
246
247
248 # Simulate data -----
249
250 library(HMM)
251
252 robotHmm <- HMM::initHMM(
253   States = 1:10,
254   Symbols = 1:10,
255   transProbs = transition_probs,
256   emissionProbs = emission_probs
257 )
258
259 simHMM <- function(hmm, length) {
260   simulation <- HMM::simHMM(hmm, length)
261   return(structure(simulation, class="HmmSimulation"))
262 }
263
264 nSim <- 100
265 robotSimulation <- simHMM(hmm=robotHmm, length=nSim)
266
267 X <- robotSimulation$observation
268 Z <- robotSimulation$states
269
270 # Implement Viterbi -----
271
272 possibleStates <- 1:10
273 get_omega <- function(Z, Omega, Z_next, x_next) {
274   sapply(Z_next, function(z_next) {
275     term1 <- log(eProbDensity(x_next, z_next))
276
277     term2 <- sapply(Z, function(z) {
278       log(tProbDensity(z_next, z))
279     }) + Omega
280
281     return(term1+ max(term2))
282   })
283 }
284
285 get_phi <- function(Z, Z_next, Omega) {
286   sapply(Z_next, function(z_next) {
287     term <- sapply(Z, function(z) {
288       log(tProbDensity(z_next, z))
289     }) + Omega
290     return(Z[which.max(term)])
291   })
292 }
293
294 viterbi <- function(observations, possibleStates) {
295   cardinality <- length(possibleStates)
296   t_total <- length(observations)
297
298   omega_0 <- vector("numeric", length = cardinality)
299   for (i in 1:cardinality) {
300     omega_0[i] <- log(initProbDensity(possibleStates[i])) + log(eProbDensity(observations[i],
301       possibleStates[i]))
302   }
303
304   omega <- matrix(NA, nrow=t_total, ncol=cardinality)
305   phi <- matrix(NA, nrow=t_total, ncol=cardinality)
306   omega[1, ] <- omega_0
307
308   for (i in 1:(t_total-1)) {
309     omega[i+1, ] <- get_omega(possibleStates, omega[i, ], possibleStates, observations[i+1])
310     phi[i+1, ] <- get_phi(possibleStates, possibleStates, omega[i, ])
311   }
312
313   mpp <- rep(NA, t_total)
314   mpp[t_total] <- possibleStates[which.max(omega[t_total, ])]
315   for (t in (t_total - 1):1) {
316     mpp[t] <- phi[t + 1, possibleStates[mpp[t + 1]] == possibleStates]
317   }
318
319   return(list(path = mpp, omega = omega, phi = phi))
320 }
321
322
323 results <- viterbi(X, possibleStates)
324 results$path
325
326 results_HMM <- HMM::viterbi(robotHmm, X)

```

```
327  
328 cbind(results$path, results_HMM)
```

# 732A96: Lab 1

## Advanced Machine Learning

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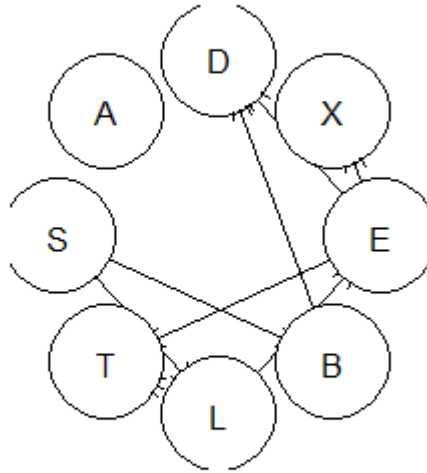
### Assignment

#### Question 1

In this question we are asked to show that multiple runs of the hill-climbing algorithm can return non-equivalent graphs. The hill-climbing algorithm is a local greedy search optimization technique which iteratively starts with an arbitrary solution to a problem, and then it tries to find a better solution by incrementally changing a single element of the solution, such that if the change produces a better solution, then this new one is taken for the next incremental change until no further improvements are found. Intuitively, it can be understood that different ending outcomes can be found from different initial points if the function is complex enough. In order to prove this, the asia dataset from the bnlearn package in R has been taken. Two different starts have been tried, specifying in one an initial connection to exist and in the second one just totally random. It can be seen in the information and graphs below (table and 1 that the graphs are not equivalent.

```
1 > mygraph
2 Bayesian network learned via Score-based methods
3 model:
4   [partially directed graph]
5 nodes: 8
6 arcs: 7
7   undirected arcs: 2
8   directed arcs: 5
9 average markov blanket size: 2.25
10 average neighbourhood size: 1.75
11 average branching factor: 0.62
12 learning algorithm: Hill-Climbing
13 score: BIC (disc.)
14 penalization coefficient: 4.258597
15 tests used in the learning procedure: 77
16 optimized: TRUE
17 > cmygraph
18 Bayesian network learned via Score-based methods model:
19   [partially directed graph]
20 nodes: 8
21 arcs: 8
22   undirected arcs: 6
23   directed arcs: 2
24 average markov blanket size: 2.25
25 average neighbourhood size: 2.00
26 average branching factor: 0.25
27 learning algorithm: Hill-Climbing
28 score: BIC (disc.)
29 penalization coefficient: 4.258597
30 tests used in the learning procedure: 82
31 optimized: TRUE
32 > all.equal(mygraph, cmygraph)
33 [1] "Different number of directed/undirected arcs"
```

**HC with initial constraint**



**HC without constraint**

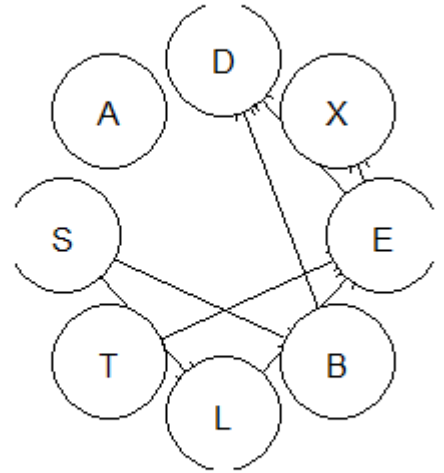


Figure 1: Graphical illustration of HC optimization for two different set ups

ALTERNATIVE SHOWING:

In order to show it in another way, I am going to use the data set alarm. I will be simulating from hill climbing as many times until it creates different graphs. And I will show this last ones graphically. This is what it can be seen in the following graph:

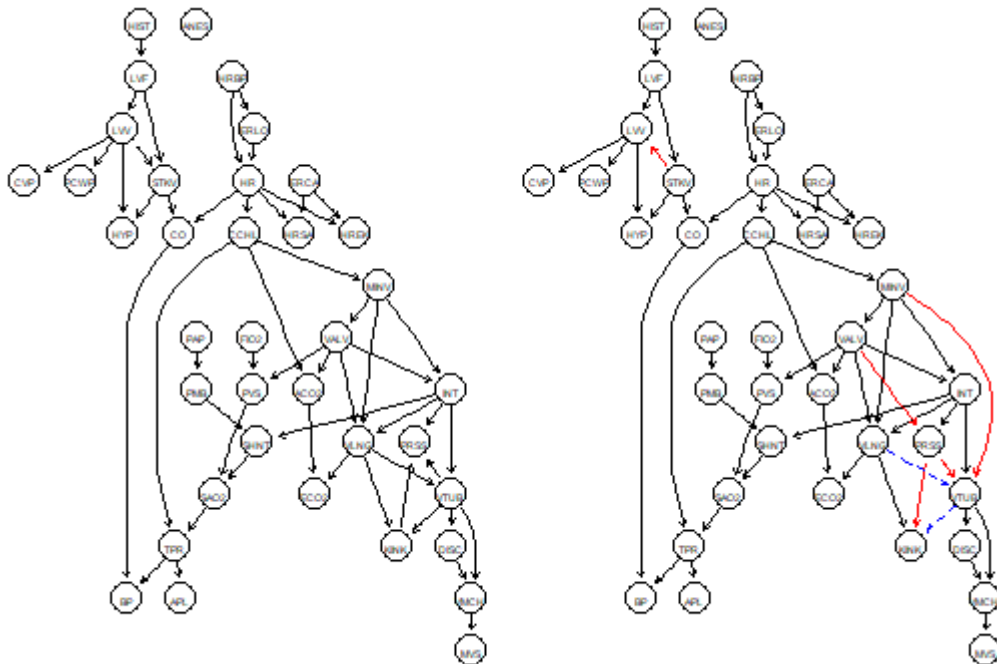


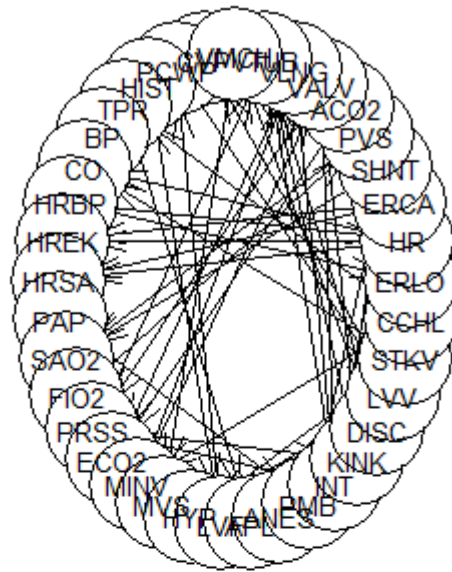
Figure 2: Graphical representation of the alarm data with different hc ending points

As explained previously, the  $hc()$  is a greedy algorithm which ends up in a local optima, so it tries to change arcs in order to get a better optima.

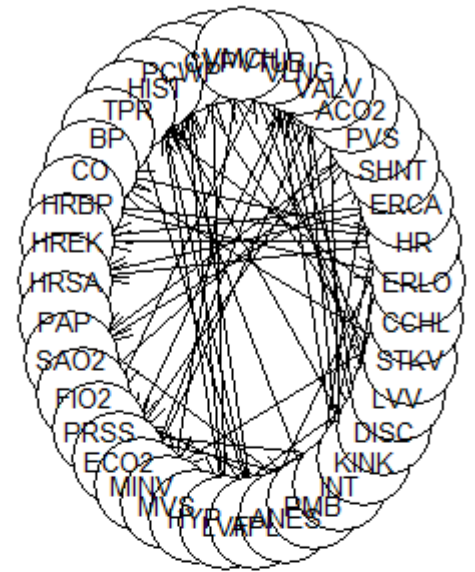
## **Question 2**

In figure 3 it can be found the graphical proof that increasing the sample size the BDeu decreases regularization.

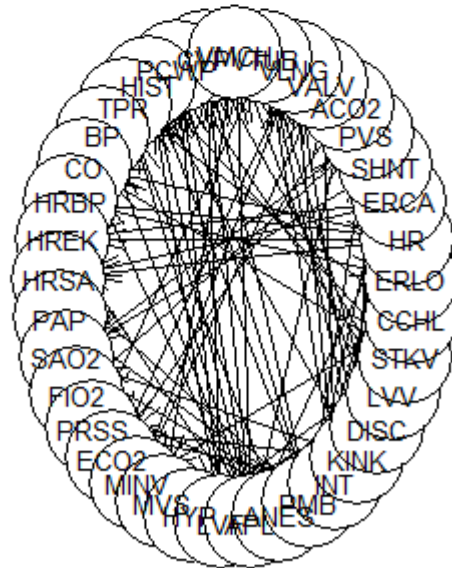




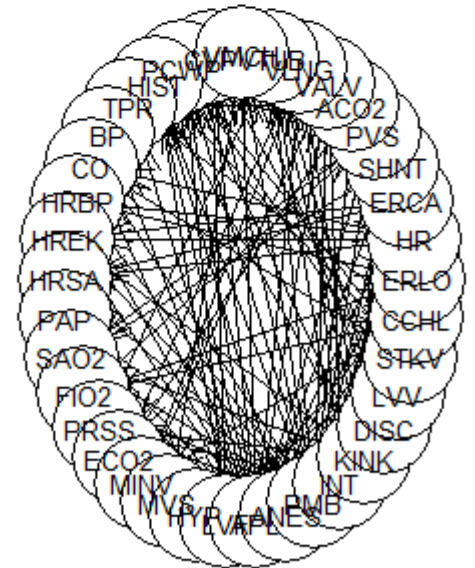
iss = 1



iss = 25



iss = 50



iss = 100

Figure 3: Graphical representation of the alarm data with different iss scores

The alpha.star results are the following ones:

```
1 [1] 6.779801 #for iss = 1
2 [1] 7.439901 #for iss = 25
3 [1] 10.34823 #for iss = 50
4 [1] 16.62416 #for iss = 100
```

The score got is lower because the more arc a graph has, the more likely to represent a graph from the data but the more complex it is so the more penalized the model is. Also, it is much probable that there will be a lot of parameters (e.g. arcs that do not exist in the real data) which will worsen the result.

### Question 3

2nd rewritten EXPLANATION

Now we are asked to compare the answers given some queries by exact and approximate inference algorithms.

The approximate algorithm uses samples from the distribution while it discards the samples that are not consistent with the conditioning set. Now we will iterate 4 times on both algorithm in order to compare how well they perform. For that, I have used the dataset called data.learning and I have compared the LS and the approximate algorithm in order to find the conditional distribution of A, E given  $B = b$  and  $F = b$ . and then further below it can be found the conditional distribution of A, E given  $B = b$  and  $F = b$ ,  $D = b$  and  $C = b$ .

```
1 > res2
2 [[1]]
3      A A_Approx.Freq      E E_Approx.Freq
4 a 0.07394366 0.06010929 0.3167939 0.3333333
5 b 0.64964789 0.65391621 0.3664122 0.3679417
6 c 0.27640845 0.28597450 0.3167939 0.2987250
7
8 [[2]]
9      A A_Approx.Freq      E E_Approx.Freq
10 a 0.07394366 0.08791209 0.3167939 0.3150183
11 b 0.64964789 0.64285714 0.3664122 0.3791209
12 c 0.27640845 0.26923077 0.3167939 0.3058608
13
14 [[3]]
15      A A_Approx.Freq      E E_Approx.Freq
16 a 0.07394366 0.0752 0.3167939 0.2992
17 b 0.64964789 0.6352 0.3664122 0.3840
18 c 0.27640845 0.2896 0.3167939 0.3168
19
20 [[4]]
21      A A_Approx.Freq      E E_Approx.Freq
22 a 0.07394366 0.08448276 0.3167939 0.2948276
23 b 0.64964789 0.66724138 0.3664122 0.3862069
24 c 0.27640845 0.24827586 0.3167939 0.3189655
```

As previously mentioned, it can be found below the conditional distribution of A, E given  $B = b$  and  $F = b$ ,  $D = b$  and  $C = b$

```
1 > res5
2 [[1]]
3      A A_Approx.Freq      E E_Approx.Freq
4 a 0.05115203 0.07142857 0.3167939 0.4285714
5 b 0.23753656 0.07142857 0.3664122 0.3214286
6 c 0.71131142 0.85714286 0.3167939 0.2500000
7
8 [[2]]
9      A A_Approx.Freq      E E_Approx.Freq
10 a 0.05115203 0.00000000 0.3167939 0.3529412
11 b 0.23753656 0.1764706 0.3664122 0.2352941
12 c 0.71131142 0.8235294 0.3167939 0.4117647
13
14 [[3]]
15      A A_Approx.Freq      E E_Approx.Freq
16 a 0.05115203 0.08333333 0.3167939 0.3333333
17 b 0.23753656 0.16666667 0.3664122 0.3333333
18 c 0.71131142 0.75000000 0.3167939 0.3333333
19
```

```

20 [[4]]
21      A A_Approx.Freq      E E_Approx.Freq
22 a 0.05115203      0.05 0.3167939      0.40
23 b 0.23753656      0.15 0.3664122      0.25
24 c 0.71131142      0.80 0.3167939      0.35

```

Answer: As a result, the LS algorithm gives the same results every time (being exact) whereas the approximate one gives every time different result.

Comparing both algorithms, the approximate one performs worse when the conditioning set is large because it is based on first obtaining a sample from the distribution and then discarding the samples that are not consistent with the conditioning set (the less samples the higher variance). So, when the conditioning set is large, most of the samples are discarded leading to a poor approximation of the exact result.

In other words, the variance of the output prediction is greater the more nodes are observed because there is less data to calculate it, leading to a worse prediction of the approximate model compared to the exact one. Nevertheless, the goodness of the model is higher since the prediction will be more accurate.

## Question 4

In this section We are asked to compute approximately the fraction of the 29281 DAGs that represent different independence models. In order to do so, I have used the `random.graph function()` sampling 10000 graphs. From these ones, I have check the unique ones and transformed them into DAGS. After that, I accounted for the the fraction of graphs resulted for different "burn.in" and "every" parameters getting the following results:

```

1 > result
2      burn in = 1 burn in = 100 burn in = 10000 burn in = 1e+06
3 every = 2      0.6178384      0.6073096      0.6229086      0.6252186
4 every = 20     0.5471494      0.5539439      0.5519067      0.5567743
5 every = 100    0.5531273      0.5551819      0.5510988      0.5536594
6 every = 200    0.5454000      0.5534984      0.5536512      0.5552473

```

Results seem to show that the true proportion of essential graphs is approximately 0.55-0.6 of the DAGS. In light of the results, it is preferable to perform structure learning in the space of essential graphs for computation simplicity given that the result must be quite well accurate. The "Burn in" in this case does not change the result. These might mean that the process is directly stationary from the first observations. On the other side, the "every" parameter affects more to the output. By doing each observation less correlated with the previous ones (e.g. increasing the "every" parameter) the existent correlation through data tends to a certain level (e.g. in our case with 20 it already converges). leading to a more accurate fraction of the number of unique elements graphs.

## Contributions

All results and comments presented have been developed and discussed together by the members of the group.

# Appendix

## Poisson regression-the MCMC way

```
1
2 ##Advanced Machine Learning lab 1
3
4
5 # source("http://bioconductor.org/biocLite.R")
6 # biocLite(c("graph", "RBGL", "Rgraphviz"))
7 #install.packages("gRain", dependencies=TRUE)
8 #install.packages("bnlearn")
9 #install.packages("gRbase")
10 #install.packages("gRain")
11 library(bnlearn)
12 library(gRain)
13 library(gRbase)
14
15 ###1
16 data(asia)
17
18 plot(asia)
19
20 par(mfrow=c(1,2))
21
22 w1 = matrix(c("E", "T"), ncol = 2, byrow = TRUE,
23             dimnames = list(NULL, c("from", "to")))
24
25 plot(hc(asia, whitelist = w1), main="HC with initial constraint")
26 plot(hc(asia, whitelist = NULL),main="HC without constraint")
27
28 mygraph<-cpdag(hc(asia, whitelist = NULL, restart = 10))
29 cmygraph<-cpdag(hc(asia, whitelist = w1))
30 mygraph
31 cmygraph
32 all.equal(mygraph, cmygraph)
33
34 #####Alternative
35 set.seed(12345)
36 countinue <- TRUE
37 while(countinue){
38   hc1 <- hc(alarm, restart = 10)
39   hc2 <- hc(alarm, restart = 10)
40   continue <- ifelse(all.equal(vstructs(hc1), vstructs(hc2)) == TRUE, TRUE, FALSE)
41   if (continue !=TRUE){
42     par(mfrow = c(1,2))
43     graphviz.compare(hc1, hc2)
44     par(mfrow = c(1,1))
45     break
46   }
47 }
48
49 ###2
50 data("alarm")
51 n<-4
52 iss<- c(1,25,50,100)
53 mydag<-list()
54 par(mfrow=c(2,2))
55 for(i in 1:n){
56   mydag[[i]]<- hc(alarm, restart = n,
57                   score = "bde", iss = iss[i])
58   plot(mydag[[i]], sub = paste0("iss = ", iss[i] ))
59   print(alpha.star(mydag[[i]], alarm, debug = FALSE))
60 }
61
62
63 par(mfrow=c(1,1))
64
65 ###3
66 data(learning.test)
67 pdag = iamb(learning.test)
68 pdag
69 plot(pdag)
70 dag = set.arc(pdag, from = "B", to = "A")
71 dag = pdag2dag(pdag, ordering = c("A", "B", "C", "D", "E", "F"))
72 plot(dag)
73 LS<- as.grain(fit)## it creates LS
74 plot(LS)
75 fit = bn.fit(dag, learning.test)##It creates all conditional tables from one edge with another
76   with his possible outcomes
77
78 MM<- compile(LS)##it triangulates and moralizes
79 plot(MM)
80 basic<-querygrain(MM, nodes = c("A","E"))
```

```

81 #####
82 #####comparison of results for one change
83 ## I want the conditional distribution of A, E given B = b
84
85 ##Exact one
86 ChangeEvidence<-setEvidence(MM, c("B"), c("b"))
87 finalstate<-querygrain(ChangeEvidence, nodes = c("A","E"))
88
89 ##Approximate algorithm
90 rsample<-cpdist(fit, nodes = c("A", "E"), evidence= (B=="b"), method = "ls")
91 condprob<-lapply(rsample, FUN = function(x){table(x)/length(x)})
92
93 res1<-cbind(A_LS= as.data.frame(finalstate)[1], A_Greedy= condprob$A,
94           E_LS= as.data.frame(finalstate)[2], E_Greedy= condprob$E)
95 res1<-res1[,c(1,3,4,6)]
96 colnames(res1)<- c("A_LS", "A_Greedy","E_LS", "E_Greedy" )
97 res1
98
99 #####comparison of results for two changes
100 ## I want the conditional distribution of A, E given B = b and F = b
101 ChangeEvidence2<-setEvidence(MM, c("B","F"), c("b", "b"))
102 finalstate2<-querygrain(ChangeEvidence2, nodes = c("A","E"))
103
104
105 res2<- list()
106 for(i in 1:4){
107   ##Exact one
108   ChangeEvidence2<-setEvidence(MM, c("B","F"), c("b", "b"))
109   finalstate2<-querygrain(ChangeEvidence2, nodes = c("A","E"))
110   ##Greedy algorithm
111   rsample2<-cpdist(fit, nodes = c("A", "E"), evidence= (B=="b")&(F=="b"), method = "ls")
112   condprob2<-lapply(rsample2, FUN = function(x){table(x)/length(x)})
113
114   res2[[i]]<-cbind(A_LS= as.data.frame(finalstate2)[1], A_Approx= condprob2$A,
115                 E_LS= as.data.frame(finalstate2)[2], E_Approx= condprob2$E)
116   res2[[i]]<-res2[[i]][,c(1,3,4,6)]
117 }
118
119 res2
120
121
122 ###WITHOUT EVIDENCE
123 ## I want the conditional distribution of A, E
124
125 ##Exact one
126 finalstate3<-querygrain(MM, nodes = c("A","E"))
127
128
129 #####comparison of results for two changes
130 ## I want the conditional distribution of A, E given B = b and F = b, D=d
131 ChangeEvidence4<-setEvidence(MM, c("B","F","D", "A"), c("b", "b", "b", "b"))
132 finalstate4<-querygrain(ChangeEvidence4, nodes = c("A","E"))
133
134 i<-1
135 res5<- list()
136 for(i in 1:4){
137   ##Exact one
138   ChangeEvidence5<-setEvidence(MM, c("B","F","D", "C"), c("b", "b", "b", "b"))
139   finalstate5<-querygrain(ChangeEvidence5, nodes = c("A","E"))
140   ##Greedy algorithm
141   rsample5<-cpdist(fit, nodes = c("A", "E"), evidence= (B=="b")&(F=="b")&(D=="b")&(C=="b"),
142                 method = "ls")
143   condprob5<-lapply(rsample5, FUN = function(x){table(x)/length(x)})
144
145   res5[[i]]<-cbind(A_LS= data.frame(finalstate5)[1], A_Approx= data.frame(condprob5$A),
146                 E_LS= data.frame(finalstate5)[2], E_Approx= data.frame(condprob5$E))
147   res5[[i]]<-res5[[i]][,c(1,3,4,6)]
148 }
149 res5
150
151
152
153 ###4
154 burn_in<- c(1,100, 10000, 1000000)
155 every<-c(2,20,100, 200)
156
157 nodes<- LETTERS[1:5]
158 num <- 1000
159
160
161 checkingDags<- function(burnin, every, num, nodes){
162   mymat<- matrix(ncol=length(burnin), nrow=length(every))
163   for(i in 1:length(burnin)){
164     for(j in 1:length(every)){
165       z<-random.graph(nodes = nodes, num = num, method = "ic-dag",
166                     every = every[j], burn.in =burnin[i])

```

```

167
168     m<-unique(z)
169     print(length(m))# This gives the proportion the number of repeated graphs out my
170
171     dagGraph<-lapply(m, FUN= function(x){cpdag(x)})
172     mymat[j,i]<- length(unique(dagGraph))/length(m)
173   }
174 }
175 return(mymat)
176 }
177
178 result<-checkingDags(burnin = burn_in, every = every, num = num, nodes = nodes)
179 colnames(result)<- paste0("burn in = " , burn_in)
180 rownames(result)<- paste0("every = " , every)
181 result

```

# 732A96: Lab 2

## Advanced Machine Learning

Carles Sans Fuentes

September 21, 2017

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### Assignment

Main information about the lab : You do not have direct observation of the robot. However, the robot is equipped with a tracking device that you can access. The device is not very accurate though: If the robot is in the sector  $i$ , then the device will report that the robot is in the sectors  $[i - 2, i + 2]$  with equal probability.

### Question 1

Build a HMM for the scenario described above. The HMM has been built according to the description above. The information about my HMM is shown below:

```
1 > myhmm
2 $States
3 [1] "1" "2" "3" "4" "5" "6" "7" "8" "9" "10"
4
5 $Symbols
6 [1] "1" "2" "3" "4" "5" "6" "7" "8" "9" "10"
7
8 $startProbs
9 1 2 3 4 5 6 7 8 9 10
10 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1
11
12 $transProbs
13 to
14 from 1 2 3 4 5 6 7 8 9 10
15 1 0.5 0.5 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
16 2 0.0 0.5 0.5 0.0 0.0 0.0 0.0 0.0 0.0 0.0
17 3 0.0 0.0 0.5 0.5 0.0 0.0 0.0 0.0 0.0 0.0
18 4 0.0 0.0 0.0 0.5 0.5 0.0 0.0 0.0 0.0 0.0
19 5 0.0 0.0 0.0 0.0 0.5 0.5 0.0 0.0 0.0 0.0
20 6 0.0 0.0 0.0 0.0 0.0 0.5 0.5 0.0 0.0 0.0
21 7 0.0 0.0 0.0 0.0 0.0 0.0 0.5 0.5 0.0 0.0
22 8 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.5 0.5 0.0
23 9 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.5 0.5
24 10 0.5 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.5
25
26 $emissionProbs
27 symbols
28 states 1 2 3 4 5 6 7 8 9 10
29 1 0.2 0.2 0.2 0.0 0.0 0.0 0.0 0.2 0.2
30 2 0.2 0.2 0.2 0.2 0.0 0.0 0.0 0.0 0.2
31 3 0.2 0.2 0.2 0.2 0.2 0.0 0.0 0.0 0.0
32 4 0.0 0.2 0.2 0.2 0.2 0.2 0.0 0.0 0.0
33 5 0.0 0.0 0.2 0.2 0.2 0.2 0.2 0.0 0.0
34 6 0.0 0.0 0.0 0.2 0.2 0.2 0.2 0.2 0.0
35 7 0.0 0.0 0.0 0.0 0.2 0.2 0.2 0.2 0.0
36 8 0.0 0.0 0.0 0.0 0.0 0.2 0.2 0.2 0.2
37 9 0.2 0.0 0.0 0.0 0.0 0.0 0.2 0.2 0.2
38 10 0.2 0.2 0.0 0.0 0.0 0.0 0.0 0.2 0.2
```



## Question 2 & 3

We have Simulated the HMM for 100 time steps and discarded the hidden states from the sample obtained above. We have used the remaining observations to compute the filtered and smoothed probability distributions for each of the 100 time points. Compute also the most probable path. Here below I show the result.

```

1 > marginalFilter
2      1      2      3      4      5      6      7      8      9 10 11 12 13      14
3 1 0.0 0.0000000 0.000 0.0 0.0 0.0000000 0.0000000 0.0000000 0.0000000 0 0 0.0 0.00 0.0000000
4 2 0.2 0.1111111 0.125 0.2 0.1 0.0000000 0.0000000 0.0000000 0.0000000 0 0 0.0 0.00 0.0000000
5 3 0.2 0.2222222 0.375 0.8 0.5 0.3157895 0.1578947 0.1090909 0.1818182 0 0 0.0 0.00 0.0000000
6 4 0.2 0.2222222 0.500 0.0 0.4 0.4736842 0.3947368 0.3818182 0.8181818 0 0 0.0 0.00 0.0000000
7 5 0.2 0.2222222 0.000 0.0 0.0 0.2105263 0.3421053 0.5090909 0.0000000 1 0 0.0 0.00 0.0000000
8 6 0.2 0.2222222 0.000 0.0 0.0 0.0000000 0.1052632 0.0000000 0.0000000 0 1 0.5 0.25 0.1428571
9 7 0.0 0.0000000 0.000 0.0 0.0 0.0000000 0.0000000 0.0000000 0.0000000 0 0 0.5 0.50 0.4285714
10 8 0.0 0.0000000 0.000 0.0 0.0 0.0000000 0.0000000 0.0000000 0.0000000 0 0 0.0 0.25 0.4285714
11 9 0.0 0.0000000 0.000 0.0 0.0 0.0000000 0.0000000 0.0000000 0.0000000 0 0 0.0 0.00 0.0000000
12 10 0.0 0.0000000 0.000 0.0 0.0 0.0000000 0.0000000 0.0000000 0.0000000 0 0 0.0 0.00 0.0000000
13      15 16 17      18 19      20      21      22      23      24      25
14 1 0.00000000 0 0.0 0.25 0.00 0.375 0.53846154 0.46153846 0.40000000 0.28089888 0.18539326
15 2 0.00000000 0 0.0 0.00 0.00 0.000 0.00000000 0.26923077 0.42222222 0.41573034 0.34831461
16 3 0.00000000 0 0.0 0.00 0.00 0.000 0.00000000 0.00000000 0.00000000 0.21348315 0.31460674
17 4 0.00000000 0 0.0 0.00 0.00 0.000 0.00000000 0.00000000 0.00000000 0.00000000 0.10674157
18 5 0.00000000 0 0.0 0.00 0.00 0.000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000
19 6 0.09090909 0 0.0 0.00 0.00 0.000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000
20 7 0.36363636 0 0.0 0.00 0.00 0.000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000
21 8 0.54545455 0 0.0 0.00 0.00 0.000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000
22 9 0.00000000 1 0.5 0.25 0.25 0.125 0.07692308 0.03846154 0.02222222 0.00000000 0.00000000
23 10 0.00000000 0 0.5 0.50 0.75 0.500 0.38461538 0.23076923 0.15555556 0.08988764 0.04494382
24      26      27      28      29      30      31      32      33
25 1 0.00000000 0.0000000 0.00000000 0.00000000 0.00000000 0.00000000 0.01790763 0.098020735
26 2 0.30944625 0.0000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.008953817
27 3 0.38436482 0.0000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000
28 4 0.24429967 0.6307190 0.31535948 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000
29 5 0.06188925 0.3071895 0.46895425 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000
30 6 0.00000000 0.0620915 0.18464052 0.72595281 0.36297641 0.00000000 0.00000000 0.00000000
31 7 0.00000000 0.0000000 0.03104575 0.23956443 0.48275862 0.00000000 0.00000000 0.00000000
32 8 0.00000000 0.0000000 0.00000000 0.03448276 0.13702359 0.78325688 0.00000000 0.00000000
33 9 0.00000000 0.0000000 0.00000000 0.00000000 0.01724138 0.19495413 0.80395853 0.401979265
34 10 0.00000000 0.0000000 0.00000000 0.00000000 0.00000000 0.02178899 0.17813384 0.491046183
35      35      36      37      38      39      40      41      42      43
36 1 0.3447962 0.42239811 0.37500000 0.33585485 0.00000000 0.0000000 0.0000000 0.0000000 0.0000000
37 2 0.0000000 0.17239811 0.29739811 0.39097545 0.43675896 0.351519 0.1757595 0.0000000 0.0000000
38 3 0.0000000 0.00000000 0.08619905 0.22304803 0.36897231 0.648481 0.5000000 0.3704333 0.0000000
39 4 0.0000000 0.00000000 0.00000000 0.05012167 0.16415016 0.000000 0.3242405 0.4518267 0.0000000
40 5 0.0000000 0.00000000 0.00000000 0.00000000 0.03011857 0.000000 0.0000000 0.1777400 0.7798358
41 6 0.0000000 0.00000000 0.00000000 0.00000000 0.00000000 0.000000 0.0000000 0.0000000 0.2201642
42 7 0.0000000 0.00000000 0.00000000 0.00000000 0.00000000 0.000000 0.0000000 0.0000000 0.0000000
43 8 0.0000000 0.00000000 0.00000000 0.00000000 0.00000000 0.000000 0.0000000 0.0000000 0.0000000
44 9 0.1552038 0.07760189 0.03880095 0.00000000 0.00000000 0.000000 0.0000000 0.0000000 0.0000000
45 10 0.5000000 0.32760189 0.20260189 0.00000000 0.00000000 0.000000 0.0000000 0.0000000 0.0000000
46      44      45      46      47      48      49      50      51      52
47 1 0.0000000 0.00000000 0.0000000 0.0000000 0.00000000 0.0000000 0.0000000 0.1114899 0.23495034
48 2 0.0000000 0.00000000 0.0000000 0.0000000 0.00000000 0.0000000 0.0000000 0.0000000 0.0000000
49 3 0.0000000 0.00000000 0.0000000 0.0000000 0.00000000 0.0000000 0.0000000 0.0000000 0.0000000
50 4 0.0000000 0.00000000 0.0000000 0.0000000 0.00000000 0.0000000 0.0000000 0.0000000 0.0000000
51 5 0.3899179 0.19495896 0.1002381 0.0000000 0.00000000 0.0000000 0.0000000 0.0000000 0.0000000
52 6 0.5000000 0.44495896 0.3290136 0.0000000 0.00000000 0.0000000 0.0000000 0.0000000 0.0000000
53 7 0.1100821 0.30504104 0.3856122 0.4859713 0.24298567 0.1502614 0.0751307 0.0000000 0.0000000
54 8 0.0000000 0.05504104 0.1851361 0.3881294 0.43705039 0.4205317 0.2853965 0.1872996 0.09917849
55 9 0.0000000 0.00000000 0.0000000 0.1258992 0.25701433 0.4292069 0.4248693 0.3689943 0.29456761
56 10 0.0000000 0.00000000 0.0000000 0.0000000 0.06294961 0.0000000 0.2146035 0.3322162 0.37130356
57      53      54      55      56      57      58      59      60      61
58 1 0.3434770 0.3603652 0.36128948 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000
59 2 0.0000000 0.1717385 0.29104355 0.6914874 0.4088044 0.2044022 0.0000000 0.0000000 0.0000000
60 3 0.0000000 0.0000000 0.00000000 0.3085126 0.5911956 0.5000000 0.3922940 0.0000000 0.0000000
61 4 0.0000000 0.0000000 0.00000000 0.0000000 0.0000000 0.2955978 0.4430824 0.5196077 0.0000000
62 5 0.0000000 0.0000000 0.00000000 0.0000000 0.0000000 0.0000000 0.1646236 0.3779957 0.60632805
63 6 0.0000000 0.0000000 0.00000000 0.0000000 0.0000000 0.0000000 0.1023966 0.32450340
64 7 0.0000000 0.0000000 0.00000000 0.0000000 0.0000000 0.0000000 0.0000000 0.06916855
65 8 0.0561902 0.0280951 0.01536711 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000
66 9 0.2230793 0.1396348 0.09174282 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000
67 10 0.3772535 0.3001664 0.24055704 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000

```

```

68          62          63          64          65          66          67          68          69
69 1  0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.04777084 0.16397296 0.2903185
    0.7192699
70 2  0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.03609363 0.00000000
    0.2807301
71 3  0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000
    0.00000000
72 4  0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000
    0.00000000
73 5  0.30316403 0.1542493 0.07712466 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000
    0.00000000
74 6  0.46541572 0.3910520 0.27265066 0.18190223 0.09236510 0.00000000 0.00000000 0.00000000
    0.00000000
75 7  0.19683597 0.3369525 0.36400224 0.33109421 0.26048590 0.18496778 0.00000000 0.00000000
    0.00000000
76 8  0.03458428 0.1177462 0.22734934 0.30753506 0.32427889 0.30653916 0.00000000 0.00000000
    0.00000000
77 9  0.00000000 0.00000000 0.05887310 0.14885128 0.23174080 0.29147071 0.45183108 0.2561633
    0.00000000
78 10 0.00000000 0.00000000 0.00000000 0.03061722 0.09112931 0.16925152 0.34810232 0.4535182
    0.00000000
79          71 72 73 74 75 76          77          78 79 80          81 82 83 84 85          86
    87
80 1  0.3596349 0 0 0.0 0 0.0 0.00000000 0.00000000 0.2 0.5 0.5625 1 1 0.5 0.25 0.00000000
    0.00000000
81 2  0.50000000 0 0 0.0 0 0.0 0.00000000 0.00000000 0.0 0.1 0.3750 0 0 0.5 0.50 0.4285714
    0.2307692
82 3  0.1403651 0 0 0.0 0 0.0 0.00000000 0.00000000 0.0 0.0 0.0625 0 0 0.0 0.25 0.4285714
    0.4615385
83 4  0.00000000 1 0 0.0 0 0.0 0.00000000 0.00000000 0.0 0.0 0.0000 0 0 0.0 0.00 0.1428571
    0.3076923
84 5  0.00000000 0 1 0.5 0 0.0 0.00000000 0.00000000 0.0 0.0 0.0000 0 0 0.0 0.00 0.00000000
    0.00000000
85 6  0.00000000 0 0 0.5 0 0.0 0.00000000 0.00000000 0.0 0.0 0.0000 0 0 0.0 0.00 0.00000000
    0.00000000
86 7  0.00000000 0 0 0.0 1 0.5 0.00000000 0.00000000 0.0 0.0 0.0000 0 0 0.0 0.00 0.00000000
    0.00000000
87 8  0.00000000 0 0 0.0 0 0.5 0.6666667 0.3333333 0.0 0.0 0.0000 0 0 0.0 0.00 0.00000000
    0.00000000
88 9  0.00000000 0 0 0.0 0 0.0 0.3333333 0.5000000 0.0 0.0 0.0000 0 0 0.0 0.00 0.00000000
    0.00000000
89 10 0.00000000 0 0 0.0 0 0.0 0.00000000 0.1666667 0.8 0.4 0.0000 0 0 0.0 0.00 0.00000000
    0.00000000
90          88          89          90          91          92          93          94          95
    96
91 1  0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000
    0.00000000
92 2  0.1153846 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000
    0.00000000
93 3  0.3461538 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000
    0.00000000
94 4  0.3846154 0.5135135 0.25675676 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000
    0.00000000
95 5  0.1538462 0.3783784 0.44594595 0.40310078 0.20155039 0.10156250 0.2007722 0.1003861
    0.05019305
96 6  0.00000000 0.1081081 0.24324324 0.39534884 0.39922481 0.30273438 0.7992278 0.5000000
    0.30019305
97 7  0.00000000 0.00000000 0.05405405 0.17054264 0.28294574 0.34375000 0.00000000 0.3996139
    0.44980695
98 8  0.00000000 0.00000000 0.00000000 0.03100775 0.10077519 0.19335937 0.00000000 0.00000000
    0.19980695
99 9  0.00000000 0.00000000 0.00000000 0.00000000 0.01550388 0.05859375 0.00000000 0.00000000
    0.00000000
100 10 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000
    0.00000000
101          97          98          99          100
102 1  0.00000000 0.00000000 0.05711921 0.1837807
103 2  0.00000000 0.00000000 0.00000000 0.00000000
104 3  0.00000000 0.00000000 0.00000000 0.00000000
105 4  0.00000000 0.00000000 0.00000000 0.00000000
106 5  0.00000000 0.00000000 0.00000000 0.00000000
107 6  0.1797030 0.00000000 0.00000000 0.00000000
108 7  0.3846535 0.00000000 0.00000000 0.00000000
109 8  0.3331683 0.57154119 0.00000000 0.00000000
110 9  0.1024752 0.34686638 0.64293598 0.3309189
111 10 0.00000000 0.08159243 0.29994481 0.4853004
112 > smoothing
113     index
114 states      1      2      3      4      5      6      7      8 9 10 11
    12
115 1  0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0.00000000 0 0 0
    0.0
116 2  0.8148148 0.6296296 0.4444444 0.2592593 0.1111111 0.0000000 0.0000000 0.0000000 0 0 0
    0.0
117 3  0.1851852 0.3703704 0.5555556 0.7407407 0.7407407 0.6666667 0.4444444 0.2222222 0 0 0
    0.0

```

118	4	0.00000000	0.00000000	0.00000000	0.00000000	0.1481481	0.33333333	0.55555556	0.77777778	1	0	0
119	5	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0	1	0
120	6	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0	0	1
121	7	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0	0	0
122	8	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0	0	0
123	9	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0	0	0
124	10	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0	0	0
125	index											
126	states	13	14	15	16	17	18	19	20	21	22	
127	1	0.00000000	0.0	0	0	0.00000000	0.00000000	0.00000000	0.28611141	0.66759329	0.633197262	
128	2	0.00000000	0.0	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.298228218	
129	3	0.00000000	0.0	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	
130	4	0.00000000	0.0	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	
131	5	0.00000000	0.0	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	
132	6	0.16666667	0.0	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	
133	7	0.66666667	0.5	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	
134	8	0.16666667	0.5	1	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	
135	9	0.00000000	0.0	0	1	0.7452445	0.490489	0.2357335	0.07634843	0.01233386	0.001085725	
136	10	0.00000000	0.0	0	0	0.2547555	0.509511	0.7642665	0.63754016	0.32007285	0.067488795	
137	index											
138	states	23	24	25	26	27	28	29	30			
139	1	0.182923331	0.02714313	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	
140	2	0.809476593	0.33583724	0.06731496	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	
141	3	0.000000000	0.63701963	0.44749358	0.1281156	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	
142	4	0.000000000	0.00000000	0.48519145	0.5178924	0.2095450	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	
143	5	0.000000000	0.00000000	0.00000000	0.3539921	0.5470336	0.3116031	0.00000000	0.00000000	0.00000000	0.00000000	
144	6	0.000000000	0.00000000	0.00000000	0.00000000	0.2434214	0.5349173	0.43429008	0.00000000	0.00000000	0.00000000	
145	7	0.000000000	0.00000000	0.00000000	0.00000000	0.00000000	0.1534796	0.48154340	0.57760581			
146	8	0.000000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.08416652	0.38691196			
147	9	0.000000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.03548224			
148	10	0.007600076	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	
149	index											
150	states	31	32	33	34	35	36	37	38			
151	1	0.000000000	0.00000000	0.00000000	0.00000000	0.36216976	0.54445735	0.50117641	0.2756079			
152	2	0.000000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.13995424	0.36920910	0.5777995		
153	3	0.000000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.03294120	0.1465927			
154	4	0.000000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000		
155	5	0.000000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000		
156	6	0.000000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000		
157	7	0.000000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000		
158	8	0.741550314	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000		
159	9	0.251022958	0.9261236	0.5927026	0.2592816	0.08888475	0.01851422	0.00000000	0.00000000	0.00000000		
160	10	0.007426728	0.0738764	0.4072974	0.7407184	0.54894549	0.29707419	0.09667329	0.00000000			
161	index											
162	states	39	40	41	42	43	44	45	46			
163	1	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000		
164	2	0.5964486	0.1187578	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000		
165	3	0.4035514	0.8812422	0.3378417	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000		
166	4	0.00000000	0.00000000	0.6621583	0.5569255	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000		
167	5	0.00000000	0.00000000	0.00000000	0.4430745	0.7760094	0.30962274	0.08077435	0.00000000			
168	6	0.00000000	0.00000000	0.00000000	0.00000000	0.2239906	0.59805752	0.52230553	0.2651274			
169	7	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.09231974	0.37166247	0.5696354			
170	8	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.02525765	0.1652372			
171	9	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000		
172	10	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000		
173	index											
174	states	48	49	50	51	52	53	54	55			
175	1	0.00000000	0.00000000	0.00000000	0.04715039	0.18764856	0.48419879	0.6221480	0.4184326			
176	2	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.1877031	0.5815674			
177	3	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000		
178	4	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000		
179	5	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000		
180	6	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000		
181	7	0.2540821	0.09031835	0.02123451	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000		
182	8	0.5787774	0.45832030	0.26242651	0.10189735	0.01779764	0.00000000	0.00000000	0.00000000	0.00000000		
183	9	0.1671405	0.45136135	0.53542156	0.45235491	0.24978248	0.07065798	0.00000000	0.00000000	0.00000000		
184	10	0.00000000	0.00000000	0.18091742	0.39859734	0.54477132	0.44514323	0.1901490	0.00000000			
185	index											

186	states	56	57	58	59	60	61	62	63					
		64												
187	1	0.0000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000					
		0.0000000												
188	2	0.7555088	0.2075164	0.03479317	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000					
		0.0000000												
189	3	0.2444912	0.7924836	0.42250832	0.1199028	0.00000000	0.00000000	0.00000000	0.00000000					
		0.0000000												
190	4	0.0000000	0.00000000	0.54269851	0.5368674	0.2553288	0.00000000	0.00000000	0.00000000					
		0.0000000												
191	5	0.0000000	0.00000000	0.00000000	0.3432299	0.5505936	0.44107119	0.11026780	0.01575254					
		0.0000000												
192	6	0.0000000	0.00000000	0.00000000	0.00000000	0.1940776	0.46368702	0.50784753	0.23961455					
		0.0556883												
193	7	0.0000000	0.00000000	0.00000000	0.00000000	0.00000000	0.09524179	0.34774139	0.51616355					
		0.3717333												
194	8	0.0000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.03414328	0.22846936					
		0.4643561												
195	9	0.0000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000					
		0.1082223												
196	10	0.0000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000					
		0.0000000												
197		index												
198	states	65	66	67	68	69	70	71	72	73	74	75	76	77
		78												
199	1	0.00000000	0.00000000	0.07048501	0.3202126	1	0	0	0	0	0	0	0.00000000	0.00000000
		0.0000000												
200	2	0.00000000	0.00000000	0.00000000	0.00000000	0	1	0	0	0	0	0	0.00000000	0.00000000
		0.0000000												
201	3	0.00000000	0.00000000	0.00000000	0.00000000	0	0	1	0	0	0	0	0.00000000	0.00000000
		0.0000000												
202	4	0.00000000	0.00000000	0.00000000	0.00000000	0	0	0	1	0	0	0	0.00000000	0.00000000
		0.0000000												
203	5	0.00000000	0.00000000	0.00000000	0.00000000	0	0	0	0	1	0	0	0.00000000	0.00000000
		0.0000000												
204	6	0.00000000	0.00000000	0.00000000	0.00000000	0	0	0	0	0	1	0	0.00000000	0.00000000
		0.0000000												
205	7	0.13003496	0.00000000	0.00000000	0.00000000	0	0	0	0	0	0	1	0.22222222	0.00000000
		0.0000000												
206	8	0.48312907	0.2508172	0.00000000	0.00000000	0	0	0	0	0	0	0	0.77777778	0.44444444
		0.0000000												
207	9	0.35076186	0.5377277	0.43005981	0.00000000	0	0	0	0	0	0	0	0.00000000	0.55555556
		0.6666667												
208	10	0.03607411	0.2114550	0.49945518	0.6797874	0	0	0	0	0	0	0	0.00000000	0.00000000
		0.3333333												
209		index												
210	states	79	80	81	82	83	84	85	86	87	88			
		89												
211	1	0.1111111	0.5555556	1	1	1	0.4310467	0.1264025	0.00000000	0.00000000	0.00000000			
		0.0000000												
212	2	0.00000000	0.00000000	0	0	0	0.5689533	0.6092885	0.37920750	0.1026843	0.00000000			
		0.0000000												
213	3	0.00000000	0.00000000	0	0	0	0.00000000	0.2643090	0.53472518	0.5530464	0.3080528			
		0.0000000												
214	4	0.00000000	0.00000000	0	0	0	0.00000000	0.00000000	0.08606732	0.3442693	0.5794631			
		0.65033376												
215	5	0.00000000	0.00000000	0	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.1124840			
		0.33205511												
216	6	0.00000000	0.00000000	0	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000			
		0.01761113												
217	7	0.00000000	0.00000000	0	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000			
		0.0000000												
218	8	0.00000000	0.00000000	0	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000			
		0.0000000												
219	9	0.00000000	0.00000000	0	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000			
		0.0000000												
220	10	0.8888889	0.4444444	0	0	0	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000			
		0.0000000												
221		index												
222	states	90	91	92	93	94	95	96	97					
		98												
223	1	0.00000000	0.0000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000					
		0.0000000												
224	2	0.00000000	0.0000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000					
		0.0000000												
225	3	0.00000000	0.0000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000					
		0.0000000												
226	4	0.28334043	0.0000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	0.00000000					
		0.0000000												
227	5	0.63740946	0.775458	0.5465133	0.3175685	0.08862378	0.01477063	0.00000000	0.00000000					
		0.0000000												
228	6	0.07925011	0.224542	0.4534867	0.6824315	0.91137622	0.36784548	0.08833972	0.00000000					
		0.0000000												
229	7	0.00000000	0.0000000	0.00000000	0.00000000	0.00000000	0.61738389	0.52947025	0.2207073					
		0.0000000												
230	8	0.00000000	0.0000000	0.00000000	0.00000000	0.00000000	0.00000000	0.38219003	0.5734981					
		0.4118733												

```

231      9  0.00000000 0.000000 0.0000000 0.0000000 0.00000000 0.00000000 0.00000000 0.00000000 0.2057946
232      10 0.00000000 0.000000 0.0000000 0.0000000 0.00000000 0.00000000 0.00000000 0.00000000 0.0000000
233      index
234 states      99      100
235      1  0.02939923 0.1837807
236      2  0.00000000 0.0000000
237      3  0.00000000 0.0000000
238      4  0.00000000 0.0000000
239      5  0.00000000 0.0000000
240      6  0.00000000 0.0000000
241      7  0.00000000 0.0000000
242      8  0.00000000 0.0000000
243      9  0.66183781 0.3309189
244     10 0.30876296 0.4853004
245 > Viterbi
246 [1] "2" "2" "2" "2" "2" "3" "3" "3" "4" "5" "6" "6" "6" "7" "8" "9" "9" "9"
247      "10"
248 [20] "1" "1" "1" "1" "1" "2" "3" "4" "5" "6" "7" "8" "9" "9" "10" "1" "1" "1"
249      "1"
250 [39] "2" "2" "3" "4" "5" "5" "5" "6" "7" "8" "9" "10" "1" "1" "1" "1" "1" "2"
251      "2"
252 [58] "2" "3" "4" "5" "6" "7" "8" "9" "10" "1" "1" "1" "2" "3" "4" "5" "6" "7"
253      "8"
254 [77] "9" "10" "1" "1" "1" "1" "1" "1" "1" "2" "2" "3" "4" "4" "5" "5" "5" "6"
255      "7"
256 [96] "8" "9" "10" "1" "1"

```

## Question 4 & 5

Compute the accuracy of the filtered and smoothed probability distributions, and of the most probable path. That is, compute the percentage of the true hidden states that are guessed by each method with different samples.

As the hint was telling us, the forward function in the HMM package returns probabilities in log scale so we needed to use the functions `exp` and `prop.table` in order to obtain a normalized probability distribution. Then, we also have used the functions `apply` and `which.max` to find out the most probable states. I have repeated the procedure 5 times for a simulation of length 100 leading to the following results:

```

1 > Simulations$PercentageSim
2 [[1]]
3      smoothingresult Filteringresult Viterbiresult
4 FALSE              0.32             0.54          0.39
5 TRUE               0.68             0.46          0.61
6
7 [[2]]
8      smoothingresult Filteringresult Viterbiresult
9 FALSE              0.23             0.51          0.35
10 TRUE              0.77             0.49          0.65
11
12 [[3]]
13      smoothingresult Filteringresult Viterbiresult
14 FALSE              0.42             0.51          0.44
15 TRUE              0.58             0.49          0.56
16
17 [[4]]
18      smoothingresult Filteringresult Viterbiresult
19 FALSE              0.2             0.4           0.35
20 TRUE              0.8             0.6           0.65
21
22 [[5]]
23      smoothingresult Filteringresult Viterbiresult
24 FALSE              0.36             0.46          0.61
25 TRUE              0.64             0.54          0.39

```

In general, the smoothed algorithm is more accurate than the filtered distributions because it is using future values to predict previous values as well, giving to the predicted one a better prediction from the future. Moreover, the smoothed distribution is also more accurate than the most probable path because it has more constraints that needs to be taken into account. Those constraints might be wrong at some point because you are never sure of where your real robot is (e.g. it can be

between  $[i - 2, i + 2]$ ), so marking a probable path might make you choose not the most optimal place in general but according to the constraint.

## Question 6

In order to answer whether it is true that the more observations you have the better you know where the robot is, it is necessary to understand what entropy in statistics is. Entropy is a measure of uncertainty which goes up when the uncertainty is high and goes down when the uncertainty goes down. By plotting the entropy for each point of the observations, if that statement was correct, we should see a decreasing trend of uncertainty in our model that should go asymptotically until 0. Nevertheless that is not the case, and it can be seen in the figure below. This makes sense since the model is randomly transitioned and no matter the more observations you have, you still have the same probability transition for it and the same information about the model for a particular point.

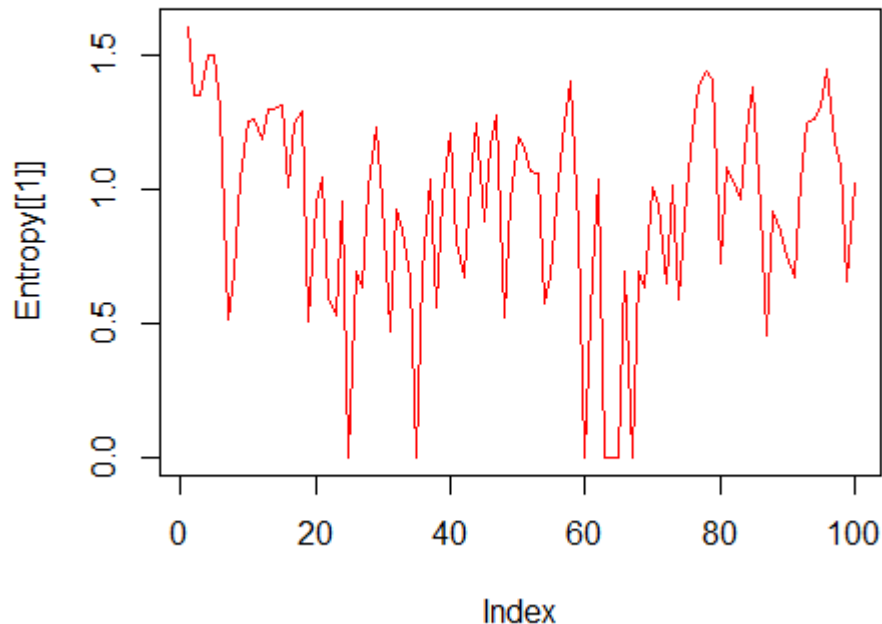


Figure 1: Graphical representation of the entropy for one of the simulations

## Question 7

Consider any of the samples above of length 100. Compute the probabilities of the hidden states for the time step 101.

In order to do so, we just need as likelihood the posterior of our data and take the last distribution of the value and then multiplied by our prior which is our transition distribution, giving the following possible distribution:

```
1 > result
2      [,1]
3 [1,] 0.1875
4 [2,] 0.5000
5 [3,] 0.3125
6 [4,] 0.0000
7 [5,] 0.0000
8 [6,] 0.0000
```

```
9 [7,] 0.0000
10 [8,] 0.0000
11 [9,] 0.0000
12 [10,] 0.0000
```

## Contributions

All results and comments presented have been developed and discussed together by the members of the group.



# Appendix

## Poisson regression-the MCMC way

```
1
2
3 #####Lab1
4
5 set.seed(12345)
6 #install.packages("HMM")
7 library(HMM)
8
9 ##Quesion1
10 # Initialise HMM
11 states <- as.character(1:10)
12 symbols <- as.character(1:10)
13 startProbs <- rep(0.1,10)
14 transProbs <- diag(x = 0.5, 10)
15 transProbs[10,1]<- 0.5 #writing the ones left
16 emissionProbs <- diag(x = 0.2, 10)
17 emissionProbs[1,9:10]<-rep(0.2,2)
18 emissionProbs[2,10]<-rep(0.2,1)
19 emissionProbs[9,1]<-rep(0.2,1)
20 emissionProbs[10,1:2]<-rep(0.2,1)
21 for(i in 1:ncol(transProbs)){
22   for(j in 1:nrow(transProbs)){
23     if(j == i+1){
24       transProbs[i,j]<-0.5
25       emissionProbs[i,j]<-0.2}
26     if(j == i+2){ emissionProbs[i,j]<-0.2}
27     if(j == i-1){ emissionProbs[i,j]<-0.2}
28     if(j == i-2){ emissionProbs[i,j]<-0.2}
29   }}
30
31
32 myhmm = initHMM(States = states,
33                 Symbols = symbols,
34                 startProbs = startProbs,
35                 transProbs= transProbs,
36                 emissionProbs=emissionProbs)
37
38 ##2
39 length<-100
40
41 my100sim<-simHMM(myhmm, length)
42
43
44 #3
45
46 myobs<-my100sim$observation # Taking just the real observations (Z)
47 ###Filtering
48 filtering<-exp(forward(myhmm, observation=myobs))#A matrix containing the forward probabilities
49   given on a logarithmic scale (natural logarithm).
50 marginalFilter<-apply(as.data.frame(filtering),2, FUN = function(x){prop.table(x)}##prop.
51   table already calculated the % on 1.
52
53
54
55 ###Smoothing
56 smoothing<-posterior(myhmm, observation=myobs)
57
58
59 ##Most probable path
60
61 Viterbi<-viterbi(myhmm, observation=myobs)
62
63
64 #4
65 mystates<-my100sim$states
66
67
68 smoothingMostProb <- sapply(as.data.frame(smoothing), which.max)
69 FilteringMostProb <- sapply(as.data.frame(marginalFilter),which.max)
70 Viterbi
71
72
73 smoothingresult <-table(mystates == smoothingMostProb)
74 Filteringresult <-table(mystates == FilteringMostProb)
75 Viterbiresult <-table(mystates == Viterbi)
76
77 ResultTable <-cbind(smoothingresult, Filteringresult, Viterbiresult)
78
79 ###5
80
81
82 Comparingsimuations<-function(HMM, length){
83   ResultTable<-list()
84   FilterEntropy<-list()
85   library(entropy)
```

```

80 for(i in 1:length(length)){
81   my100sim <-simHMM(myhmm, length[i])
82   myobs <-my100sim$observation # Taking just the real observations (Z)
83   ###Filtering
84   filtering <-exp(forward(myhmm, observation=myobs))#A matrix containing the
      forward probabilities given on a logarithmic scale (natural logarithm).
85   marginalFilter <-apply(as.data.frame(filtering),2, FUN = function(x){prop.table(x)
      })##prop.table already calculated the % on 1.
86   ###Smoothing
87   smoothing <-posterior(myhmm, observation=myobs)
88
89
90   #Getting just the Z states for different models
91   mystates <-my100sim$states
92   smoothingMostProb <- apply(as.data.frame(smoothing), which.max)
93   FilteringMostProb <- apply(as.data.frame(marginalFilter),which.max)
94
95   smoothingresult <-table(mystates == smoothingMostProb)
96   Filteringresult <-table(mystates == FilteringMostProb)
97   Viterbi <-viterbi(myhmm, observation=myobs)##Most probable path
98
99   Viterbiresult <-table(mystates == Viterbi)
100   ResultTable[[i]] <-cbind(smoothingresult, Filteringresult, Viterbiresult)/length[i]
101
102   FilterEntropy[[i]] <- apply(marginalFilter,2,entropy.empirical)
103
104 }
105 }
106 return(list(PercentageSim=ResultTable, Entropy=FilterEntropy))
107 }
108
109
110 length <-rep(100,5)
111 myhmm = initHMM(States = states,
112               Symbols = symbols,
113               startProbs = startProbs,
114               transProbs= transProbs,
115               emissionProbs=emissionProbs)
116
117 Simulations<-Comparingsimulations(myhmm, length = length)
118
119 Simulations$PercentageSim
120
121 #6
122 #install.packages("entropy")
123
124 Entropy<-Simulations$Entropy
125
126 plot(Entropy[[1]], col = "red", type = "l")
127 lines(Entropy[[2]], col = "blue")
128 lines(Entropy[[3]], col = "green")
129 lines(Entropy[[4]], col = "purple")
130 lines(Entropy[[5]], col = "black")
131
132 ##7
133 likelihood<-posterior(myhmm, myobs)
134 prior<-transProbs
135 result<-prior%%likelihood[,100]

```

# 732A96: Lab 3

## Advanced Machine Learning

Carles Sans Fuentes

October 7, 2017

---

### Assignment

#### Question 1

Implementing Gaussian process regression from scratch. This first exercise will have you writing your own code for the Gaussian process regression model:

$$y = f(x) + \epsilon, \epsilon \sim N(0, \sigma_n^2), \text{ with}$$

$$f(x) \sim GP[0, k(x, x')]$$

. When it comes to the posterior distribution for  $f$ , I strongly suggest that you implement Algorithm 2.1 on page 19 of Rasmussen and Williams (RW) book. That algorithm uses the Cholesky decomposition (`chol()` in R) to attain numerical stability. Here is what you need to do:

#### a , b

Question a: Write your own code for simulating from the posterior distribution of  $f(x)$  using the squared exponential kernel. The function (name it `posteriorGP`) should return vectors with the posterior mean and variance of  $f$ , both evaluated at a set of  $x$ -values ( $x^*$ ). You can assume that the prior mean of  $f$  is zero for all  $x$ .

Question b: Now let the prior hyperparameters be  $\sigma_f = 1, l = 0.3$ . Update this prior with a single observation:  $(x, y) = (0.4, 0.719)$ . Assume that the noise standard deviation is known to be  $\sigma_n = 0.1$ . Plot the posterior mean of  $f$  over the interval  $[-1, 1]$ . Plot also 95% probability (pointwise) bands for  $f$ .

Answer:

The function `GaussianKernel()` is the the squared exponential kernel. The `SimGP()` is the function related to the Gaussian Process. The prior has been updated as well as the other parameters. A plot with the mean and the confidence bands is shown below:

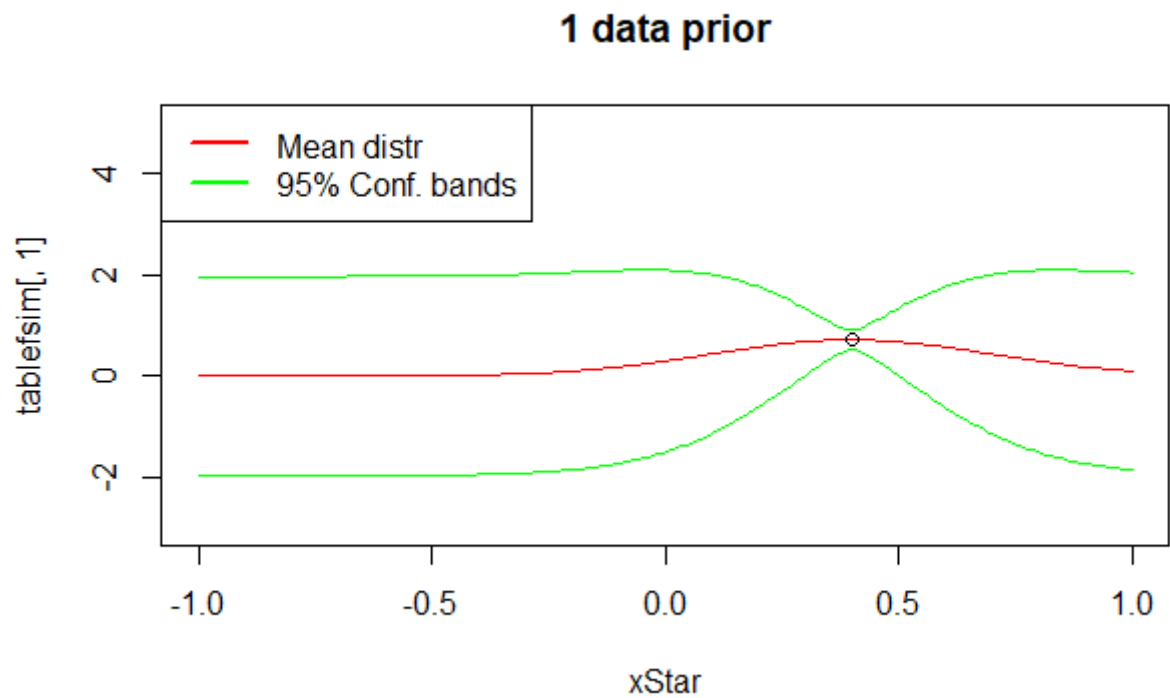


Figure 1: Posterior mean of  $f$  over the interval  $[-1, 1]$  with 95% probability bands for  $f$

**c**

Question: Update your posterior from 1b) with another observation:  $(x, y) = (-0.6, -0.044)$ . Plot the posterior mean of  $f$  over the interval  $[-1, 1]$ . Plot also 95% probability bands for  $f$ . [Hint: updating the posterior after one observation with a new observation gives the same result as updating the prior directly with the two observations. Bayes is beautiful!]

Answer: A plot with the mean and the confidence bands is shown below:

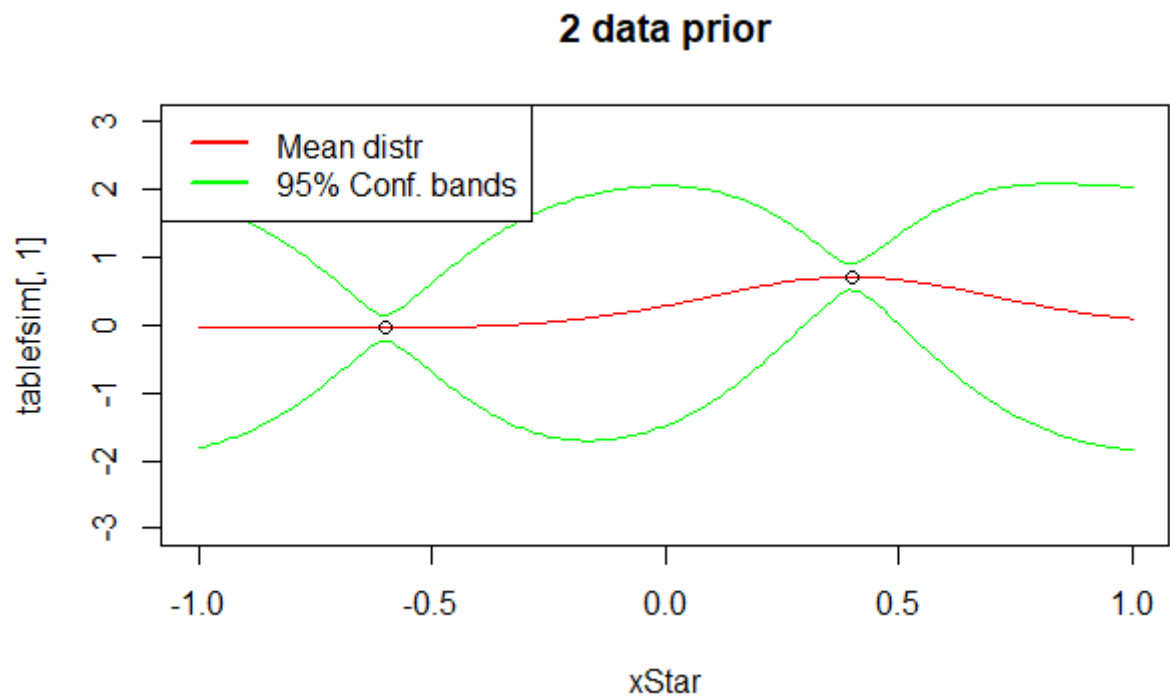


Figure 2: Posterior mean of  $f$  over the interval  $[-1, 1]$  with 95% probability bands for  $f$

**d**

Question: Compute the posterior distribution of  $f$  using all 5 data points in Table 1 below (note that the two previous observations are included in the table). Plot the posterior mean of  $f$  over the interval  $[-1, 1]$ . Plot also 95% probability intervals for  $f$ .

Answer: A plot with the mean and the confidence bands is shown below:

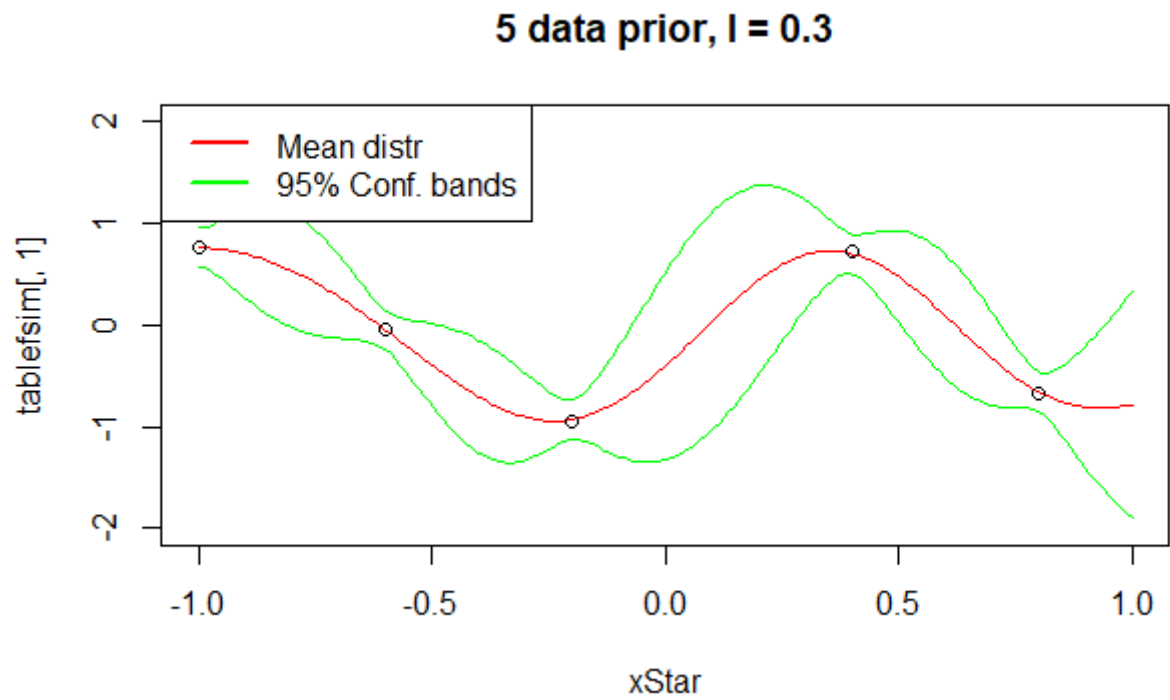


Figure 3: Posterior mean of  $f$  over the interval  $[-1, 1]$  with 95% probability bands for  $f$

e

Question: Repeat 1d), this time with the hyperparameters  $\sigma_f = 1, l = 1$  Compare the results.

Answer: A plot with the mean and the confidence bands is shown below:

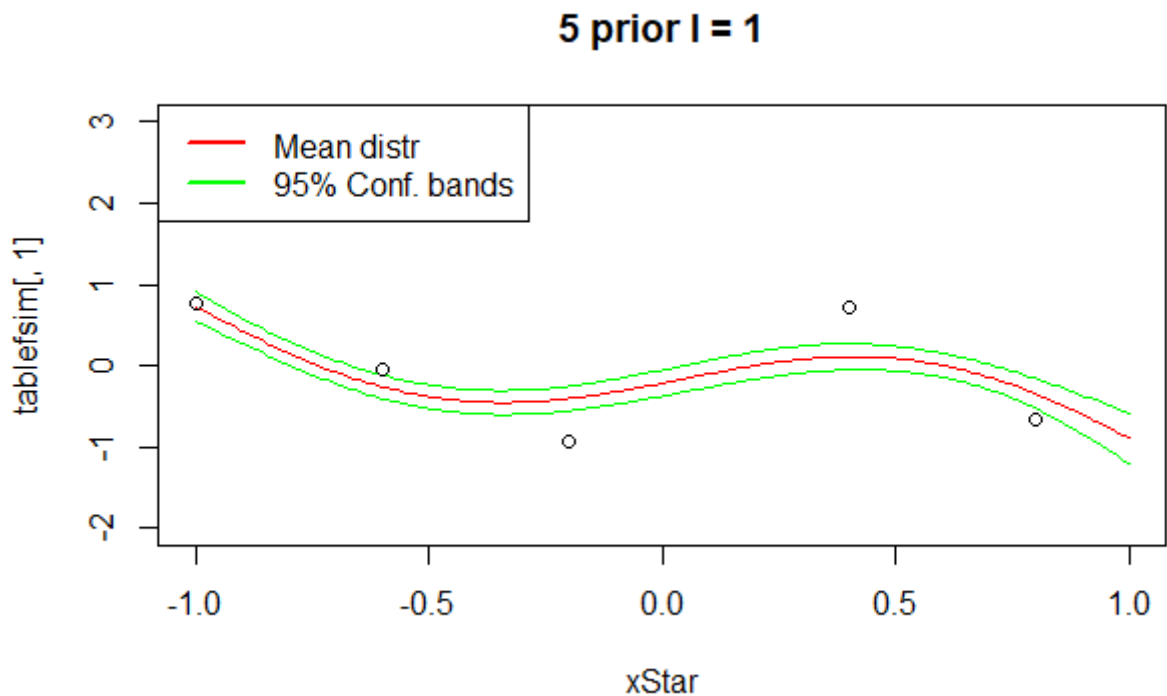


Figure 4: Posterior mean of  $f$  over the interval  $[-1, 1]$  with 95% probability bands for  $f$

It can be seen that the larger the  $l$  the more smoother our prediction will be, and the smaller our confidence bands get. This makes sense since the  $l$  parameter is a smoother hyper-parameter that the large it is the higher the covariance it is for a same  $\sigma_f$ , giving in this way more importance to  $\sigma_f$ . We are assuming by increasing  $l$  that we are more sure about our next distance training point. However, the choice of this  $l$  in our model is worse than having  $l$  equal to 3.

## Question 2

Gaussian process regression on real data using the kernlab package. This exercise lets you explore the kernlab package on a data set of daily mean temperature in Stockholm (Tullinge) during the period January 1, 2010 - December 31, 2015. I have removed the leap year day February 29, 2012 to make your life simpler. A small version of it can be used in order to decrease computation time:

**a**

Question: Familiarize yourself with the following functions in kernlab, in particular the gausspr and kernelMatrix function. Do ?gausspr and read the input arguments and the output. Also, go through myKernLabDemo.R carefully; you will need to understand it. Now, define your own square exponential kernel function (with parameters  $l$  (ell) and  $\sigma_f$  (sigmaf)), evaluate it in the point  $x = 1, x' = 2$ , and use the kernelMatrix function to compute the covariance matrix  $K(x, x_*)$  for the input vectors  $x = (1, 3, 4)^T$  and  $x_* = (2, 3, 4)^T$

Answer: The results of the KernelMatrix and of my own square exponential kernel function called GaussianKernel2 (equal as in the previous activity but in a closure mode for convenience) has been displayed below.

```
1 > cov_matrix <- kernelMatrix(kernel = GaussianKernel2, x=X, y = XStar)
2 > cov_matrix
3 An object of class "kernelMatrix"
4      [,1]      [,2]      [,3]
5 [1,] 2.4261226 0.5413411 0.04443599
```

```

6 [2,] 2.4261226 4.0000000 2.42612264
7 [3,] 0.5413411 2.4261226 4.00000000
8
9 K<-GaussianKernel2(x=x1,y= x2)
10 > K
11      [,1]
12 [1,] 2.426123

```

**b**

Question: Consider first the model:

$$temp = f(time) + \epsilon; \epsilon \sim N(0, \sigma_n^2)$$

$$f \sim GP(0, k(time, time'))$$

. Let  $\sigma_n^2$  be the residual variance from a simple quadratic regression fit (using the `lm()` function in R). Estimate the above Gaussian process regression model using the squared exponential function from 2a) with  $\sigma_f = 20$  and  $l = 0.2$ . Use the `predict` function to compute the posterior mean at every data point in the training datasets. Make a scatter plot of the data and superimpose the posterior mean of  $f$  as a curve (use `type = "l"` in the plot function). Play around with different values on  $\sigma_f$  and  $l$  (no need to write this in the report though).

Answer: The plot is shown below:

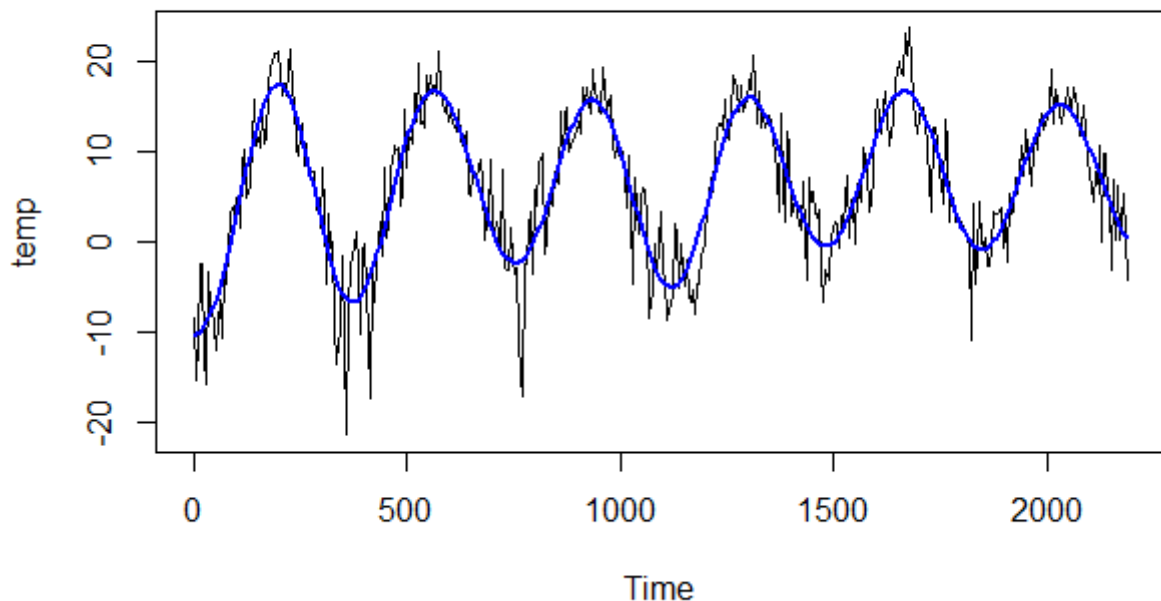


Figure 5: scatter plot of the data superimposed by the posterior mean of  $f$  as a curve

**c**

Question: Kernlab can compute the posterior variance of  $f$ , but I suspect a bug in the code (I get weird results). Do your own computations for the posterior variance of  $f$  (hint: Algorithm 2.1 in RW), and plot 95% (pointwise) posterior probability bands for  $f$ . Use  $\sigma_f = 20$  and  $l = 0.2$ . Superimpose those bands on the figure with the posterior mean in 2b).



Answer: The plot is shown below:

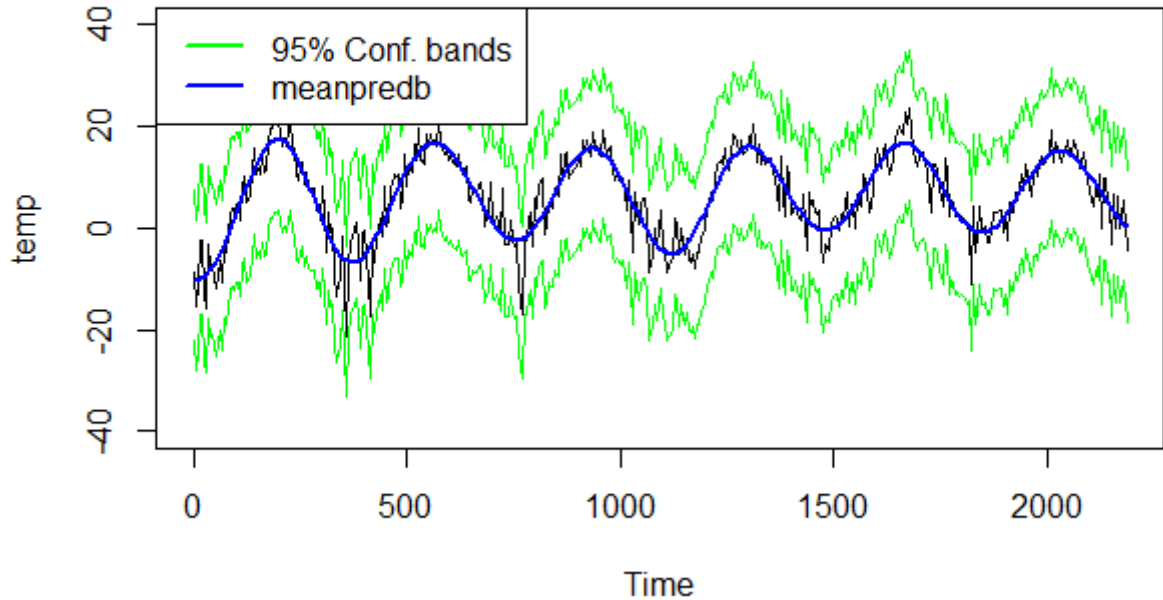


Figure 6: scatter plot of the data superimposed by the posterior mean of  $f$  as a curve with 95% probable confidence bands

**d**

Question:

$$\begin{aligned} temp &= f(day) + \epsilon; \epsilon \sim N(0, \sigma_n^2) \\ f &\sim GP(0, k(day, day')) \end{aligned}$$

. Estimate the model using the squared exponential function from 2a) with  $\sigma_f = 20$  and  $l = 6 * 0.2 = 1.2$ . (I multiplied ' by 6 compared to when you used time as input variable since kernlab automatically standardizes the data which makes the distance between points larger for day compared to time). Superimpose the posterior mean from this model on the fit (posterior mean) from the model with time using  $\sigma_f = 20, l = 0.2$ . Note that this plot should also have the time variable on the horizontal axis.

Answer: The plot is shown below:

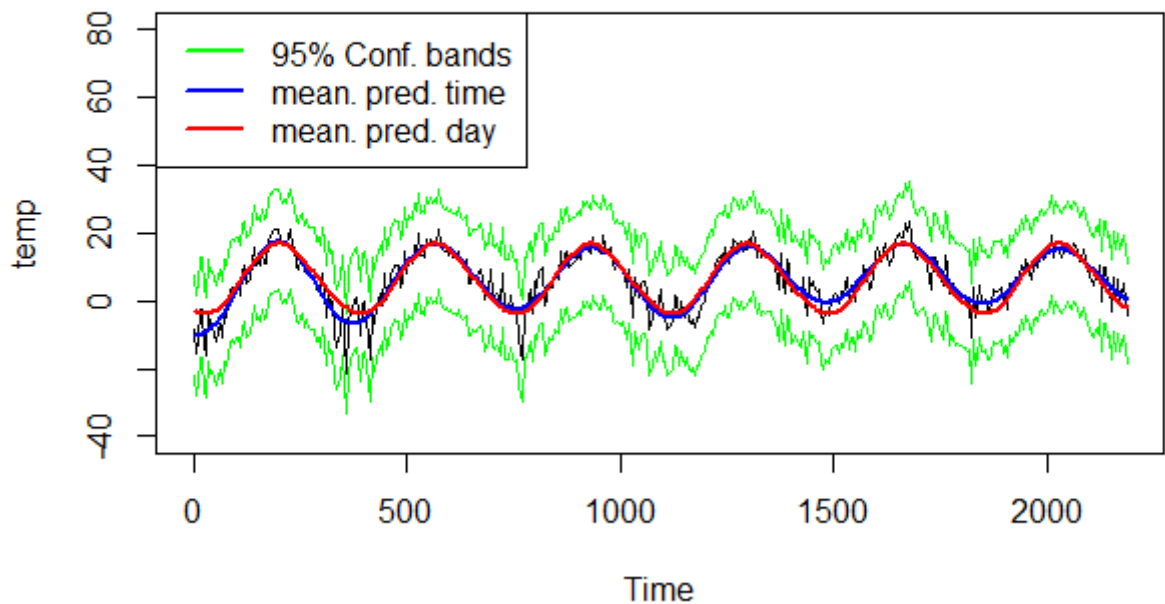


Figure 7: scatter plot of the data superimposed by the posterior mean of  $f$  as a curve with 95% probable confidence bands and new mean estimated by day model

The mean prediction for time (blue line) seems to take into account more larger variance in the data, whereas in the case of the mean of the prediction day (red line), it seems to be more stable even for larger residuals. These makes sense in the day model we are imposing to predict the same result year after year averaging all our data, whereas in the other observations are taken separately as they are linear and not cyclical over time. The pros of the and cons of the model are a possible prediction problem: overfitting vs having a stable good model.

e

Question: Now implement a generalization of the periodic kernel given in my slides from Lecture 2 of the GP topic (Slide 6) Compare the fit to the previous two models (with  $\sigma_f = 20, l = 0.2$ ). Discuss the results.

Answer: The plot is shown below:

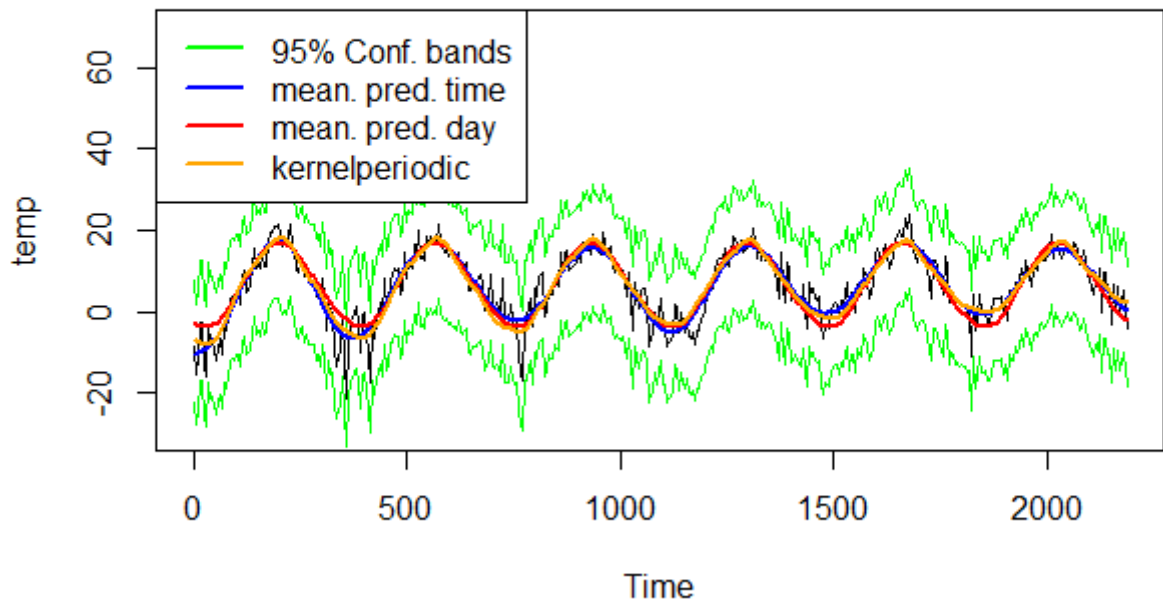


Figure 8: scatter plot of the data superimposed by the posterior mean of  $f$  as a curve with 95% probable confidence bands, new mean estimated by day model and new mean by the last periodic kernel

Having two  $l$  components enables to get a controlled stable mean model since it weights the seasonal and the every year different component. All in all, I think the (orange line) kernel periodic is better model since it combines both of the previous models in order to get a more stable prediction controlling just the parameters.

### Question 3

Download the banknote fraud data:

**a**

Question: Use kernlab to fit a Gaussian process classification model for fraud on the training data, using kernlab. Use kernlab's the default kernel and hyperparameters. Start with using only the first two covariates varWave and skewWave in the model. (1) Plot contours of the prediction probabilities over a suitable grid of values for varWave and skewWave. Overlay the training data for  $fraud = 1$  (as blue points) and  $fraud = 0$  (as red points). You can take a lot of code for this from my KernLabDemo.R. (2) Compute the confusion matrix for the classifier and its accuracy.

Answer: The Confusion Matrix and the accuracy can be seen below

```
1 > predictionfunction(data = data, rows = SelectTraining)
2   true
3 pred  0   1
4    0 512  24
5    1  44 420
6 [1] 0.932
```

**Prob(varWave, Skewwave) is not fraud(red), fraud(blue)**

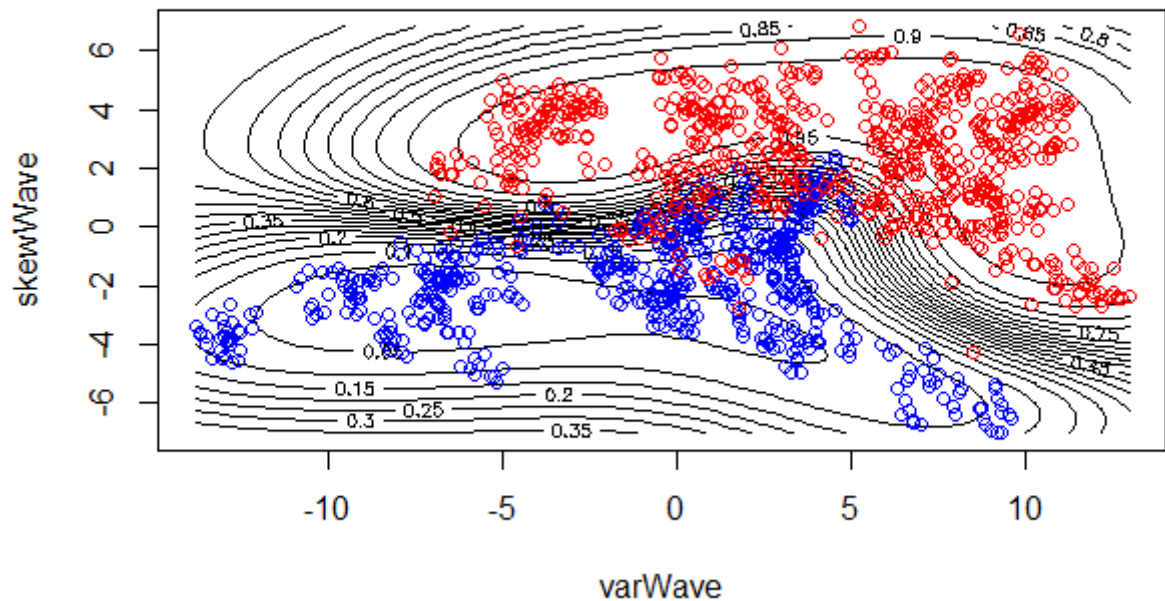


Figure 9: Plot contours of the prediction probabilities over a suitable grid of values for varWave and skewWave

**b**

Question: Using the estimated model from 3a), make predictions for the testset. Compute the accuracy

Answer: The Confusion Matrix and the accuracy can be seen below

```
1 > probPreds<-predictionfunction(data = data, rows = test)
2   true
3 pred  0   1
4     0 191   9
5     1  15 157
6 [1] 0.9354839
```

**c**

Train a model using all four covariates. Make predictions on the test and compare the accuracy to the model with only two covariates.

Answer: The Confusion Matrix and the accuracy can be seen below

```
1 > predictionfunction(data = data, rows = SelectTraining)
2   true
3 pred  0   1
4     0 552   0
5     1   4 444
6 [1] 0.996
7 > predictionfunction(data = data, rows = test)
8   true
9 pred  0   1
10    0 205   0
11    1   1 166
12 [1] 0.9973118
```

With 4 covariates the prediction improves. This makes sense since we have more data to improve the prediction. Also these means that both covariates are important and significant to perform classification analysis.

## Contributions

All results and comments presented have been developed and discussed together by the members of the group.

# Appendix

## Poisson regression-the MCMC way

```
1
2
3 ##Advanced ML lab 3
4 library("mvtnorm")
5
6 ##1a
7 x <- c(0.4)
8 y <- c(0.719)
9 xStar <- seq(-1,1,length=200)
10 sigma_f<-1
11 l<-0.3
12 hyperParam<- c(sigma_f, l)
13 sigmaNoise<- 0.1
14 nSim <- 100
15
16 ###Calculating K from y and x, that are two vector inputs
17 GaussianKernel<- function(x1,x2, sigma_f =sigma_f, l=1){
18   n1<- length(x1)
19   n2<- length(x2)
20   K<- matrix(NA, n1, n2)
21   for(i in 1:n2){
22     K[,i]<- sigma_f^2*exp(-0.5*( (x1-x2[i])/l)^2 )
23   }
24   return(K)
25 }
26
27 GaussianKernel(c(1,4), c(3,1), sigma = 2, l=1)
28 GaussianKernel(c(1,4), c(3,1), sigma = 2, l=2)
29 GaussianKernel(c(1,4), c(3,1), sigma = 2, l=3)
30 GaussianKernel(c(1,4), c(3,1), sigma = 2, l=4)
31
32
33
34 SimGP <- function(K,x,y, xStar, nSim, sigmaNoise, ...){
35   # Simulates nSim realizations (function) form a Gaussian process with mean m(x) and
36   # covariance K(x,x')
37   # over a grid of inputs (x)
38   n <- length(xStar)
39   #if (is.numeric(m)) meanVector <- rep(0,n) else meanVector <- m(xStar)
40
41   covMat <- K(x, x,...)
42   covMatdiff2<- K(x, xStar,...)
43   covMatStar<- K(xStar, xStar,...)
44   L<-t(chol(covMat+ (sigmaNoise^2)*diag(dim(covMat)[2])))
45   alpha<- solve(t(L),solve(L,y))
46   fStar<- t(covMatdiff2)%*%alpha
47   v<-solve(L,covMatdiff2)
48   covfStar <- covMatStar-t(v)%*%v
49   return(list(mean=fStar, covfStar= diag(covfStar)))
50 }
51
52 ##b
53 fSim <- SimGP(y=y, xStar = xStar, x= x, K=GaussianKernel, nSim = nSim, sigma_f =sigma_f, l=1,
54   sigmaNoise = sigmaNoise )
55 plot(fSim$mean)
56 plot(fSim$covfStar)
57 tablefsim<-cbind(mean = fSim$mean, low= fSim$mean-1.96*sqrt(fSim$covfStar),high= fSim$mean+1.96
58   *sqrt(fSim$covfStar))
59 plot(x= xStar,y = tablefsim[,1], col = "red", type = "l", ylim = c(-3,5), main = "1 data prior"
60 )
61 lines(x= xStar, y =tablefsim[,2], col ="green")
62 lines(x= xStar, y = tablefsim[,3], col = "green")
63 points(x = x, y = y)
64 legend("topleft", # places a legend at the appropriate place
65   c("Mean distr ", "95% Conf. bands"), # puts text in the legend
66   lty=c(1,1), # gives the legend appropriate symbols (lines)
67   lwd=c(2.5,2.5),col=c("Red","Green")) # gives the legend lines the correct color and
68   width
69
70 ##c
71
72 x<-c(x,-0.6)
73 y<-c(y, -0.044)
74
75 fSim <- SimGP(y=y, xStar = xStar, x= x, K=GaussianKernel, nSim = nSim, sigma_f =sigma_f, l=1,
76   sigmaNoise = sigmaNoise )
77
78 tablefsim<-cbind(mean = fSim$mean, low= fSim$mean-1.96*sqrt(fSim$covfStar),high= fSim$mean+1.96
79   *sqrt(fSim$covfStar))
```

```

75 plot(x= xStar,y = tablefsim[,1], col = "red", type = "l", ylim = c(-3,3), main = "2 data prior"
76 lines(x= xStar, y =tablefsim[,2], col ="green")
77 lines(x= xStar, y = tablefsim[,3], col = "green")
78 points(x = x, y = y)
79 legend("topleft", # places a legend at the appropriate place
80       c("Mean distr ", "95% Conf. bands"), # puts text in the legend
81       lty=c(1,1), # gives the legend appropriate symbols (lines)
82       lwd=c(2.5,2.5),col=c("Red","Green")) # gives the legend lines the correct color and
           width
83
84 ###d
85 x<- c(-1.0,-0.6,-0.2,0.4,0.8)
86 y<- c(0.768,-0.044,-0.940,0.719, -0.664)
87
88 fSim <- SimGP(y=y, xStar = xStar, x= x, K=GaussianKernel, nSim = nSim, sigma_f =sigma_f, l=1,
           sigmaNoise = sigmaNoise )
89
90 tablefsim<-cbind(mean = fSim$mean, low= fSim$mean-1.96*sqrt(fSim$covfStar),high= fSim$mean+1.96
           *sqrt(fSim$covfStar))
91 plot(x= xStar,y = tablefsim[,1], col = "red", type = "l", main = "5 data prior, l = 0.3", ylim
           = c(-2,2))
92 lines(x= xStar, y =tablefsim[,2], col ="green")
93 points(x = x, y = y)
94 lines(x= xStar, y = tablefsim[,3], col = "green")
95 legend("topleft", # places a legend at the appropriate place
96       c("Mean distr ", "95% Conf. bands"), # puts text in the legend
97       lty=c(1,1), # gives the legend appropriate symbols (lines)
98       lwd=c(2.5,2.5),col=c("Red","Green")) # gives the legend lines the correct color and
           width
99
100 ###e
101
102 sigma_f<-1
103 l<-1
104 hyperParam<- c(sigma_f, l)
105
106 fSim <- SimGP(y=y, xStar = xStar, x= x, K=GaussianKernel, nSim = nSim, sigma_f =sigma_f, l=1,
           sigmaNoise = sigmaNoise )
107
108 tablefsim<-cbind(mean = fSim$mean, low= fSim$mean-1.96*sqrt(fSim$covfStar),high= fSim$mean+1.96
           *sqrt(fSim$covfStar))
109 plot(x= xStar,y = tablefsim[,1], col = "red", type = "l", ylim = c(-2,3), main = "5 prior l = 1
           ")
110 lines(x= xStar, y =tablefsim[,2], col ="green")
111 lines(x= xStar, y = tablefsim[,3], col = "green")
112 points(x = x, y = y)
113 legend("topleft", # places a legend at the appropriate place
114       c("Mean distr ", "95% Conf. bands"), # puts text in the legend
115       lty=c(1,1), # gives the legend appropriate symbols (lines)
116       lwd=c(2.5,2.5),col=c("Red","Green")) # gives the legend lines the correct color and
           width
117
118 #####2
119 #
120 #
121 #
122 # plot(xStar, fSim[1,], type="l", ylim = c(-3,3))
123 # for (i in 2:dim(fSim)[1]) {
124 #   lines(xStar, fSim[i,], type="l")
125 # }
126 # lines(xStar,MeanFunc(xStar), col = "red", lwd = 3)
127 #
128 #
129 # # Plotting using manipulate package
130 # #install.packages("manipulate")
131 # library(manipulate)
132 #
133 # plotGPPrior <- function(sigma_f, l, nSim=20){
134 #   fSim <- SimGP(y=y, xStar = xStar, x= x, K=GaussianKernel, nSim = nSim, sigma_f =sigma_f, l=
           l, sigmaNoise = sigmaNoise )
135 #   plot(xStar, fSim[1,], type="l", ylim = c(-3,3), ylab="f(x)", xlab="x")
136 #   for (i in 2:nSim) {
137 #     lines(xStar, fSim[i,], type="l")
138 #   }
139 #   title(paste('length scale =',l,', sigmaf =',sigma_f))
140 # }
141 #
142 # manipulate(
143 #   plotGPPrior(sigma_f, l, nSim = 20),
144 #   sigma_f = slider(0, 2, step=0.1, initial = 1, label = "SigmaF"),
145 #   l = slider(0, 2, step=0.1, initial = 1, label = "Length scale, l")
146 # )
147 #
148
149 ###2
150

```



```

151 library(kernlab)
152 Data<- read.csv("https://github.com/STIMALiU/AdvMLCourse/raw/master/GaussianProcess/Code/
    TempTullinge.csv", header=TRUE,
153             sep = ";")
154
155 TData<-data.frame(time = 1:2190, date = Data$date, day =rep(1:365,6), temp = Data$temp)
156 GaussKernel<- function(sigmaF , ell){
157   rval <- function(x,y, sigma_f = sigmaF, l = ell){
158     n1 <- length(x)
159     n2 <- length(y)
160     K <- matrix(NA,n1,n2)
161     for (i in 1:n2){
162       K[,i] <- sigma_f^2*exp(-0.5*( (x-y[i])/l)^2 )
163     }
164     return(K)
165   }
166   class(rval) <- "kernel"
167   return(rval)
168 }
169
170
171 SmallData<- TData[seq(1, nrow(TData), 5), ]
172 sigma_f<-2
173 l<-1
174 X<-matrix(c(1,3,4), ncol = 1)
175 XStar<-matrix(c(2,3,4), ncol = 1)
176
177 GaussianKernel2 <- GaussKernel(sigmaF = sigma_f, ell = 1)
178 cov_matrix <- kernelMatrix(kernel = GaussianKernel2, x=X, y = XStar)
179
180 x1<-1
181 x2<-2
182
183 K<-GaussianKernel2(x=x1,y= x2)
184 ##b
185 sigma_f<-20
186 l <-0.2
187
188 x<-SmallData$time
189 x_prime<- seq(1,365,1)
190 GaussianKernel3<-GaussKernel(sigmaF = sigma_f, ell = 1)
191 K<-kernelMatrix(GaussianKernel3, x,x_prime)
192
193 mylm<-lm(temp~time+ I(time^2), SmallData)
194 sigma_2_n<-var(mylm$residuals)
195
196
197 GPfit <- gausspr(temp ~ time, data = SmallData ,
198                 kernel = GaussianKernel3,
199                 kpar = list(sigma = sigma_f, ell = 1), var = sigma_2_n)
200
201 myrange<-range(SmallData$time)
202 Xgrid<- myrange[1]:myrange[2]
203 meanPred<-predict(GPfit, SmallData)
204 length(meanPred)
205 plot(x = SmallData$time, SmallData$temp, type = "l",
206      xlab = "Time", ylab = "temp")
207 lines(SmallData$time, meanPred, col="blue", lwd = 2)
208
209
210 ##c
211
212 fSim <- SimGP(y=SmallData$temp, xStar = SmallData$time, x= SmallData$time,
213             K=GaussianKernel3, nSim = nSim,
214             sigma_f =sigma_f, l=1,
215             sigmaNoise = sqrt(sigma_2_n ))
216
217
218 tablefsim<-cbind(mean = fSim$mean, low= fSim$mean-1.96*sqrt(fSim$covfStar),high= fSim$mean+1.96
    *sqrt(fSim$covfStar))
219 plot(x = SmallData$time, SmallData$temp, type = "l",
220      xlab = "Time", ylab = "temp", ylim= c(-40,40))
221 lines(SmallData$time, meanPred, col="blue", lwd = 2)
222 lines(x= SmallData$time, y =tablefsim[,2], col = "green")
223 lines(x= SmallData$time, y = tablefsim[,3], col = "green")
224 legend("topleft", # places a legend at the appropriate place
225       c( "95% Conf. bands", "meanpredb"), # puts text in the legend
226       lty=c(1,1), # gives the legend appropriate symbols (lines)
227       lwd=c(2.5,2.5),col=c("Green", "blue")) # gives the legend lines the correct color and
    width
228
229 ####d
230
231
232 mylm<-lm(temp~day+ I(day^2), SmallData)
233 sigma_2_n<-var(mylm$residuals)
234 sigma_f<-20

```

```

235 l <- 1.2
236
237
238 GaussianKernel4<-GaussKernel(sigmaF = sigma_f, ell = 1)
239
240
241 GPfit <- gausspr(temp ~ day, data = SmallData ,
242                 kernel = GaussianKernel4,
243                 kpar = list(sigma = sigma_f, ell = 1), var = sigma_2_n)
244
245
246 meanPred2<-predict(GPfit, SmallData)
247
248 plot(x = SmallData$time, SmallData$temp, type = "l",
249      xlab = "Time", ylab = "temp", ylim= c(-40,80))
250 lines(SmallData$time, meanPred, col="blue", lwd = 2)
251 lines(x= SmallData$time, y =tablefsim[,2], col = "green")
252 lines(x= SmallData$time, y = tablefsim[,3], col = "green")
253 lines(SmallData$time, meanPred2, col="red", lwd = 2)
254
255 legend("topleft", # places a legend at the appropriate place
256        c("95% Conf. bands", "mean. pred. time", "mean. pred. day"), # puts text in the legend
257        lty=c(1,1), # gives the legend appropriate symbols (lines)
258        lwd=c(2.5,2.5),col=c("Green", "blue", "red")) # gives the legend lines the correct color
                and width
259
260
261
262 ##e
263
264 periodicKernel<- function(sigmaF, ell1, ell2, di){
265     rval<-function(x1,x2, sigma_f =sigmaF, l_1=ell1, l_2=ell2, d=di){
266
267         K<-  sigma_f^2*exp((-2*sin(pi*abs(x1-x2)/d)**2)/l_1^2)*exp((-1/2*abs(x1-x2)**2)/l_2^2)
268
269     }
270     class(rval) <- "kernel"
271     return(rval)
272 }
273 sigma_f<-20
274 l_1<- 1
275 l_2<-10
276 d <- 365/sd(SmallData$time)
277 periodicKernel5<-periodicKernel(sigmaF=sigma_f, ell1=l_1, ell2=l_2, di=d)
278
279 GPfit <- gausspr(temp ~ time, data = SmallData ,
280                 kernel = periodicKernel5,
281                 kpar = list(sigma = sigma_f, ell = 1), var = sigma_2_n)
282
283 meanPred3<-predict(GPfit, SmallData)
284
285 plot(x = SmallData$time, SmallData$temp, type = "l",
286      xlab = "Time", ylab = "temp", ylim= c(-30,70))
287 lines(SmallData$time, meanPred, col="blue", lwd = 2)
288 lines(x= SmallData$time, y =tablefsim[,2], col = "green")
289 lines(x= SmallData$time, y = tablefsim[,3], col = "green")
290 lines(SmallData$time, meanPred2, col="red", lwd = 2)
291 lines(SmallData$time, meanPred3, col="orange", lwd = 2)
292 legend("topleft", # places a legend at the appropriate place
293        c("95% Conf. bands", "mean. pred. time", "mean. pred. day", "kernelperiodic"), # puts
                text in the legend
294        lty=c(1,1), # gives the legend appropriate symbols (lines)
295        lwd=c(2.5,2.5),col=c("Green", "blue", "red", "orange")) # gives the legend lines the
                correct color and width
296
297
298
299 ##3
300
301
302 data <- read.csv("https://github.com/STIMaLiU/AdvMLCourse/raw/master/GaussianProcess/Code/
                banknoteFraud.csv", header=FALSE, sep=",")
303 names(data) <- c("varWave", "skewWave", "kurtWave", "entropyWave", "fraud")
304
305
306 data[,5] <- as.factor(data[,5])
307 set.seed(111)
308 SelectTraining <- sample(1:dim(data)[1], size = 1000, replace = FALSE)
309 model<-gausspr(fraud~varWave+skewWave, data = data[SelectTraining,])
310 ##Part III
311 data<- data
312 predictionfunction<- function(data, rows){
313     prediction<-predict(model,data[rows,])
314     confusionMat<-table(pred=prediction, true = data[rows,5]) # confusion matrix
315     Accuracy<-sum(diag(confusionMat))/sum(confusionMat)
316     print(confusionMat)
317     print(as.numeric(Accuracy))

```

```

318 }
319 }
320
321 predictionfunction(data = data, rows = SelectTraining)
322
323 ##contour
324 x1 <- seq(min(data[, "varWave"]), max(data[, "varWave"]), length=100)
325 x2 <- seq(min(data[, "skewWave"]), max(data[, "skewWave"]), length=100)
326 gridPoints <- meshgrid(x1, x2)
327 gridPoints <- cbind(c(gridPoints$x), c(gridPoints$y))
328
329 gridPoints <- data.frame(gridPoints)
330 names(gridPoints) <- c("varWave", "skewWave")
331 #####
332 probPreds <- predict(model, gridPoints, type="probabilities")
333 # Plotting for Prob(setosa)
334 contour(x2,x1,matrix(probPreds[,1],100), 20, xlab = "skewWave", ylab = "varWave", main = "Prob(
      varWave, Skewwave) is not fraud(red), fraud(blue)")
335 points(data[data[,5]==1,"skewWave"],data[data[,5]==1,"varWave"],col="blue")
336 points(data[data[,5]==0,"skewWave"],data[data[,5]==0,"varWave"],col="red")
337
338 ###b
339
340 test<-setdiff( 1:dim(data)[1], SelectTraining)
341 probPreds<-predictionfunction(data = data, rows = test)
342
343
344
345
346
347 #c
348
349 model<-gausspr(fraud~., data = data[SelectTraining,])
350 #train
351 predictionfunction(data = data, rows = SelectTraining)
352 #test
353 predictionfunction(data = data, rows = test)

```

# 732A96: Lab 4

## Advanced Machine Learning

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### Assignment

#### Question 1

Question The purpose of the lab is to put in practice some of the concepts covered in the lectures. To do so, you are asked to implement the particle filter for robot localization. For the particle filter algorithm, please check Section 13.3.4 of Bishop's book and/or the slides for the last lecture on SSMs.

Run it for  $T = 100$  time steps to obtain  $z_{1:100}$  (i.e. states) and  $x_{1:100}$  (i.e. observations). Use the observations (i.e. sensor readings) to identify the state (i.e. robot location) via particle filtering. Use 100 particles. For each time step, show the particles, the expected location and the true location. Repeat the exercise after replacing the standard deviation of the emission model with 5 and then with 50. Comment on how this affects the results. Finally, show and explain what happens when the weights in the particle filter are always equal to 1, i.e. there is no correction.

Answer:

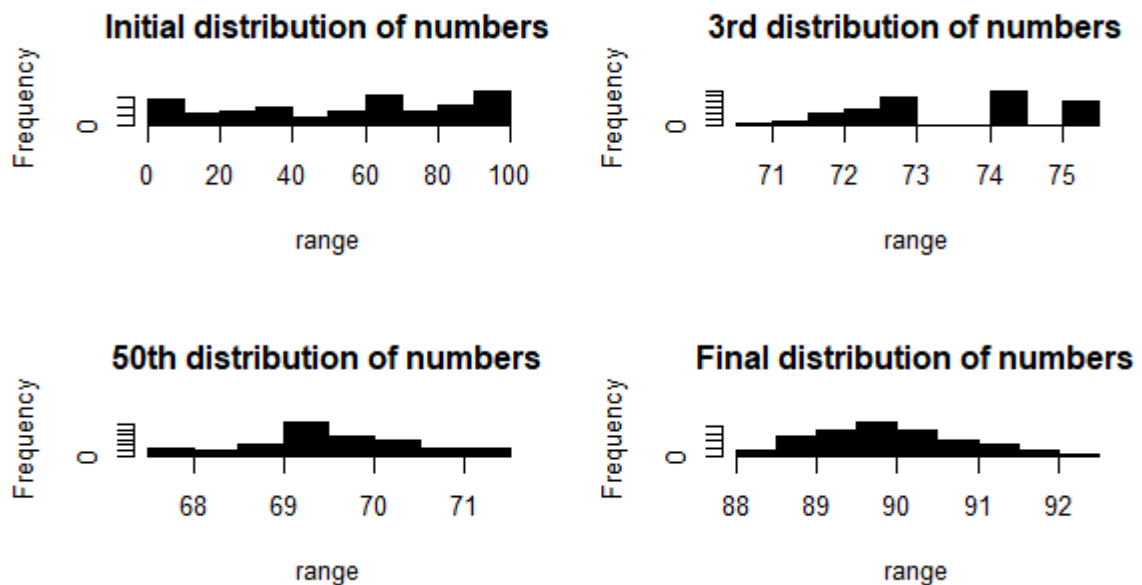


Figure 1: Histogram of the distribution of parameters with sd equal to 1

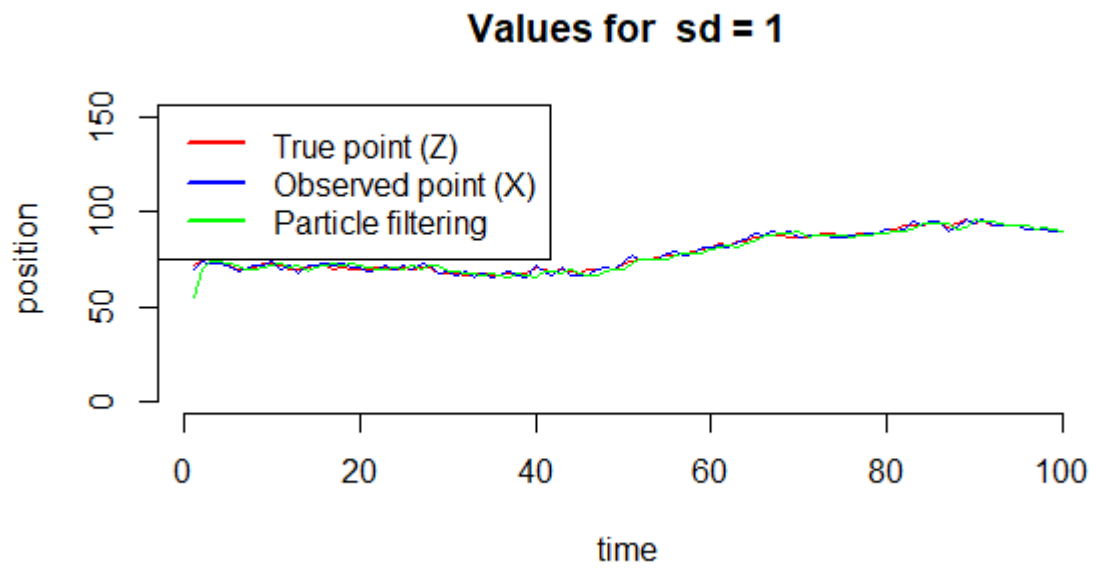


Figure 2: Plot X, Z and the particle filtering for 100 simulations with  $sd$  equal to 1

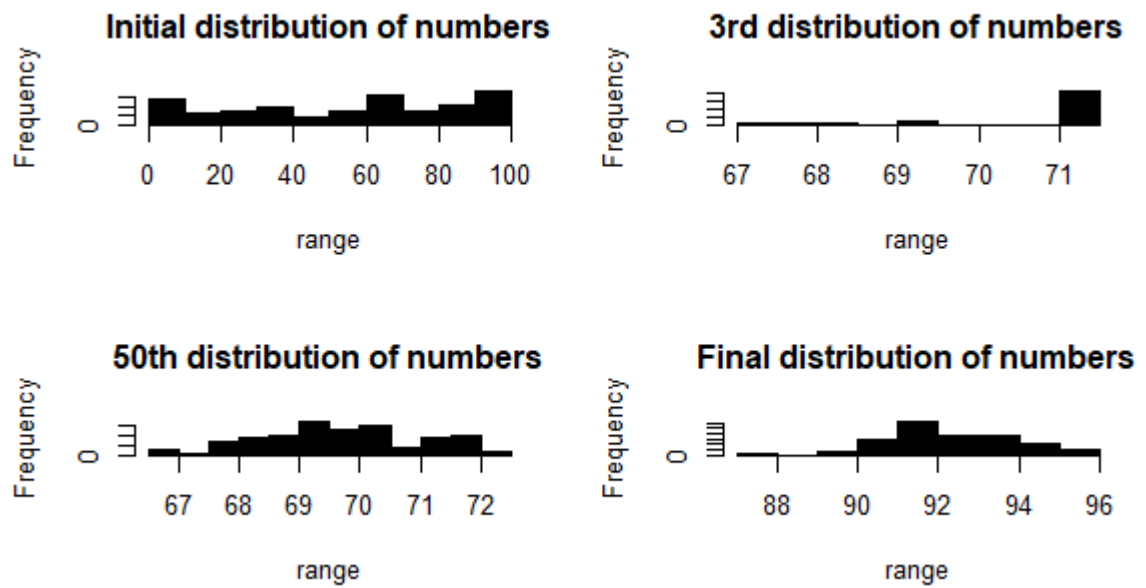


Figure 3: Histogram of the distribution of parameters with  $sd$  equal to 5

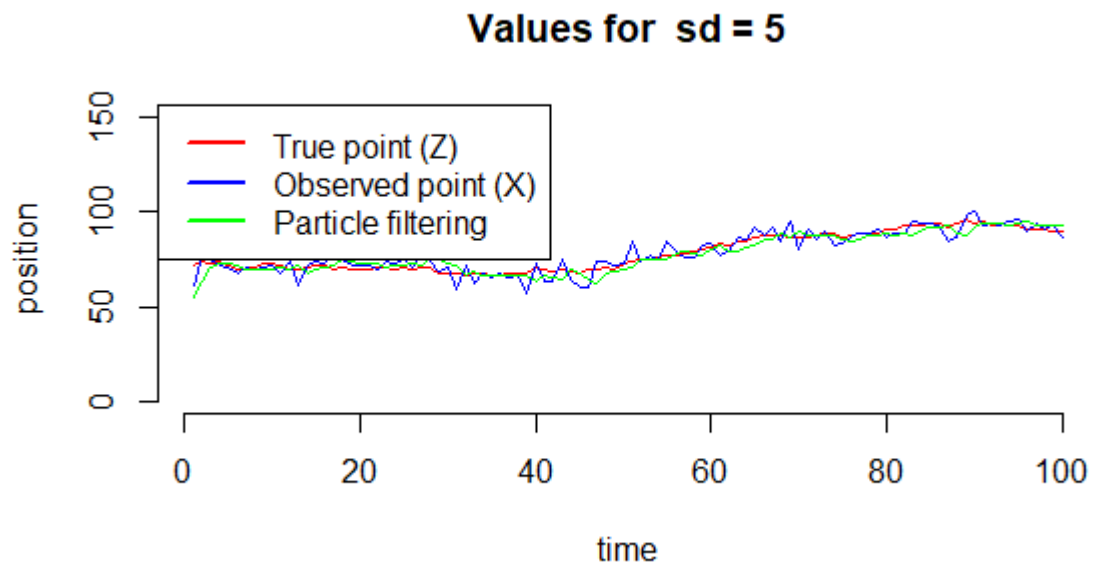


Figure 4: Plot X, Z and the particle filtering for 100 simulations with sd equal to 5

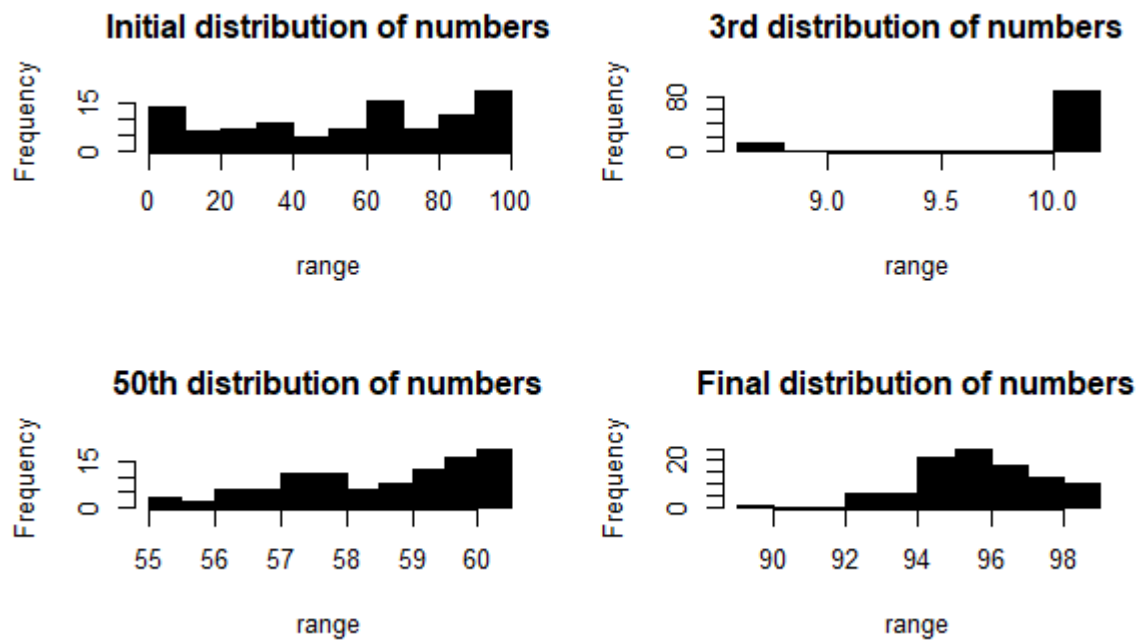


Figure 5: Histogram of the distribution of parameters with sd equal to 50

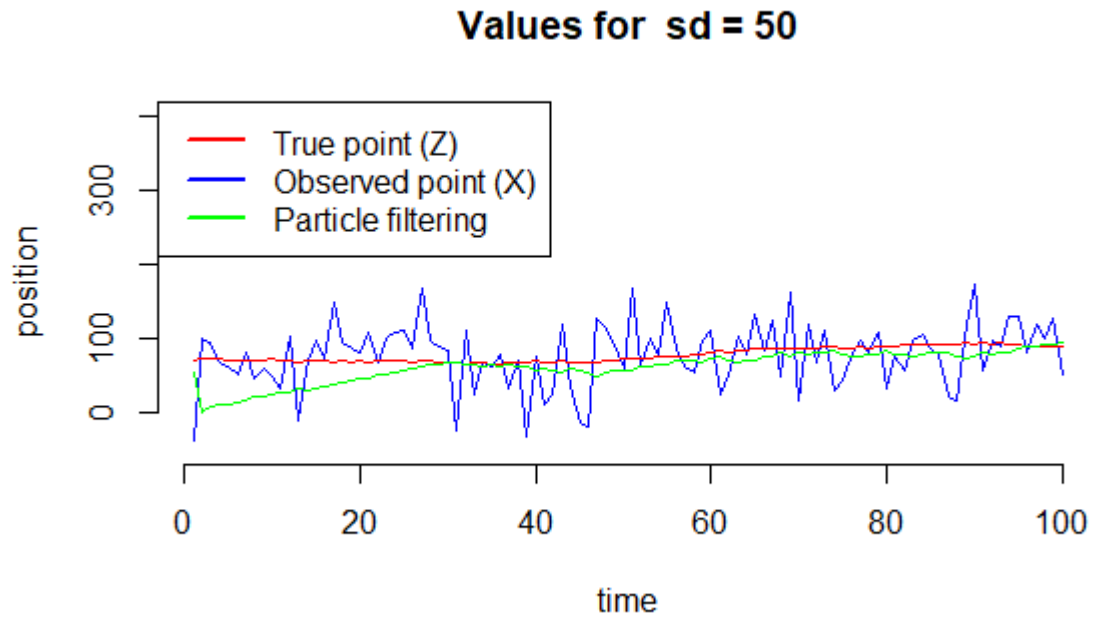


Figure 6: Plot X, Z and the particle filtering for 100 simulations with sd equal to 50

When increasing the standard deviation of the emission function, we are both less certain about the location of the robot given that our sensor is worse. Intuitively, if the sensor is worse (e.g. higher variance), our prediction will also be worse. For that, we can see that the higher the sd, the worse and more bumped our X is (blue line) and the more the particle filtering lasts to converge to the real position. Nevertheless, it ends up giving a good prediction.

If we set every time the weight to be 1 in all cases (e.g. it means all numbers have the same weights), our prediction will be much worse since we are not taking into account the conditional probabilities of the previous points.

## **Contributions**

All results and comments presented have been developed and discussed together by the members of the group.



# Appendix

## Poisson regression-the MCMC way

```
1
2 #####
3 #####1
4 set.seed(12345)
5 #####Generating model
6 #initial model
7 p1<-runif(1,0,100)
8
9 # Transition
10
11 sd_T<-1
12 sd_E<-50
13
14 Transition<- function(n,p0, sd_2){
15   z<- integer(n)
16   z[1]<-p0
17   for(i in 2:n){
18     p<- sample(c(z[i-1]-1,z[i-1],z[i-1]+1),1)
19     z[i]<-rnorm(1,p,sqrt(sd_2))
20   }
21   return(z)
22 }
23 myZ<-Transition(100, p0= p1, sd_2= sd_T**2)
24 ##Emission model
25
26
27 Emission<- function(vectorz, sd_2){
28   n<- length(vectorz)
29   x<- integer(n)
30   for(i in 1:n){
31     p<- sample(c(vectorz[i]-1,vectorz[i],vectorz[i]+1),1)
32     x[i]<-rnorm(1,p,sqrt(sd_2))
33   }
34   return(x)
35 }
36 x_t<- Emission(myZ, sd_2 = sd_E**2)
37
38 #####
39 weights<- rep(0.01,100)
40 X<-runif(100,0,100)
41 calculationprob<- function(simulations, init_weights, grid, sd_E, sd_T, x_t= x_t){
42   n<- length(grid)
43   parameters<- matrix( ncol = n,nrow = simulations)
44   newweights<- matrix(ncol = n,nrow = simulations)
45
46   newweights[1,]<-init_weights/sum(init_weights)
47   parameters[1,]<-sample(x =grid,size = n, replace=TRUE, prob = newweights[1,])
48   ## sample(dnorm(parameters[1,],grid) Left that
49   xt<- integer(100)
50   for(i in 2:simulations){
51
52     for(j in 1:simulations){
53       xt[j]<-Transition(2,parameters[i-1,j],sd_T)[2]
54     }
55     newweights[i,]<-dnorm(x_t[i-1], xt, sqrt(sd_E))/sum(dnorm(x_t[i-1], xt,sqrt(sd_E)))
56     parameters[i,]<-sample(xt, size =n, replace=TRUE, prob = newweights[i,])
57
58   }
59   result<- list(parameters = parameters, weights=newweights)
60   return(result)
61 }
62 sd_2T<-1
63 sd_2E<-1
64
65 trial<-calculationprob(simulations =100, init_weights = weights, grid= X,
66   sd_T = sd_T, sd_E = sd_E,x_t= x_t)
67 parameter_est<-trial$parameters
68 parameter_est
69
70 plotting<- function(Z=myZ, X=x_t , particle= trial$parameters){
71   n<-1:length(Z)
72   for(i in n){
73     plot(0, xlim= c(1,100), ylim = c(0,150 ),bty='n',pch='',ylab='sep representation for
       clearness',xlab='position')
74     points(y = 20, x = Z[i], col ="red")
75     points(y = 30, x = X[i], col ="blue")
76     points(y =rep(40, 100), x = particle[i,], col = "green")
77     legend("topleft", # places a legend at the appropriate place
78       c("True point (Z) ", "Estimated point (X)", "Particle filtering"), # puts text in
       the legend
79       lty=c(1,1), # gives the legend appropriate symbols (lines)
```

```

80         lwd=c(2.5,2.5),col=c("Red"," blue", "Green")) # gives the legend lines the correct
81             color and width
82     Sys.sleep(0.2)
83 }
84 par(mfrow=c(1,1))
85 plotting()
86
87
88 par(mfrow=c(1,1))
89 plot(0, xlim= c(1,100), ylim = c(-50,400),
90      bty='n',pch='',ylab='position',xlab='time', main = paste0("Values for sd = ", sd_E))
91 lines(x = 1:100, y = myZ, col = "red")
92 lines(x = 1:100, y = x_t, col = "blue")
93 lines(x = 1:100, y = apply(trial$parameters,1,mean), col = "green")
94 legend("topleft", # places a legend at the appropriate place
95       c("True point (Z) ", "Observed point (X)", "Particle filtering"), # puts text in the
96         legend
97         lty=c(1,1), # gives the legend appropriate symbols (lines)
98         lwd=c(2.5,2.5),col=c("Red"," blue", "Green")) # gives the legend lines the correct color
99         and width
100
101 par(mfrow=c(2,2))
102 hist(trial$parameters[1,], main= "Initial distribution of numbers", xlab = "range", col = "
103     black")
104 hist(trial$parameters[3,], main= "3rd distribution of numbers", xlab = "range", col = "black")
105 hist(trial$parameters[50,], main= "50th distribution of numbers", xlab = "range", col = "black"
106     )
107 hist(trial$parameters[100,], main= "Final distribution of numbers", xlab = "range", col = "
108     black")

```