

Database Technology

Teaching Session 1: Functional Dependencies and Normalization

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Reasoning About FDs

- Logical implications can be derived by using inference rules called **Armstrong's rules**:
 - *Reflexivity*: If Y is a subset of X , then $X \rightarrow Y$
 - *Augmentation*: If $X \rightarrow Y$, then $XZ \rightarrow YZ$
(we use XY as a short form for $X \cup Y$)
 - *Transitivity*: If $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$
- Additional rules can be derived:
 - *Decomposition*: If $X \rightarrow YZ$, then $X \rightarrow Y$
 - *Union*: If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow YZ$
 - *Pseudo-transitivity*: If $X \rightarrow Y$ and $WY \rightarrow Z$, then $WX \rightarrow Z$

Warmup – Solutions

- Consider the relation **R(A,B,C,D,E,F)** with the following FDs:
FD1: **A → BC**
FD2: **C → AD**
FD3: **DE → F**
- Use the Armstrong rules to derive the following FD: **AC → D**

Warmup – Solutions

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- Use the Armstrong rules to derive the following FD: **AC** → **D**
 - FD4: **AC** → **AD** (Augmentation of FD2 with A)
 - FD5: **AC** → **D** (Decomposition of FD4)

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- Consider the relation **R(A,B,C,D,E,F)** with the following FDs:
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 - FD4: **AC** → **AD** (Augmentation of FD2 with A)
 - FD5: **AC** → **D** (Decomposition of FD4)
- Use the Armstrong rules to derive the following FD: **A** → **D**

Warmup – Solutions

- Consider the relation **R(A,B,C,D,E,F)** with the following FDs:
FD1: **A** → **BC**
FD2: **C** → **AD**
FD3: **DE** → **F**

- Use the Armstrong rules to derive the following FD: **AC** → **D**
 - FD4: **AC** → **AD** (Augmentation of FD2 with A)
 - FD5: **AC** → **D** (Decomposition of FD4)

- Use the Armstrong rules to derive the following FD: **A** → **D**
 - FD6: **A** → **C** (Decomposition of FD1)
 - FD7: **A** → **AD** (Transitivity of FD6 and FD2)
 - FD8: **A** → **D** (Decomposition of FD7)

Warmup (cont'd)

- Consider the relation **R(A,B,C,D,E,F)** with the following FDs:
FD1: **A** → **BC**
FD2: **C** → **AD**
FD3: **DE** → **F**
- Use the Armstrong rules to derive the following FD: **AE** → **ABCDEF**

Warmup (cont'd)

- Consider the relation **R(A,B,C,D,E,F)** with the following FDs:
FD1: **A** → **BC**
FD2: **C** → **AD**
FD3: **DE** → **F**

- Use the Armstrong rules to derive the following FD: **AE** → **ABCDEF**
 - FD9: **AE** → **BCE** (Augmentation of FD1 with E)
 - FD10: **AE** → **C** (Decomposition of FD9)
 - FD11: **AE** → **AD** (Transitivity of FD10 and FD2)
 - FD12: **AE** → **ADE** (Augmentation of FD11 with E)
 - FD13: **AE** → **DE** (Decomposition of FD12)
 - FD14: **AE** → **F** (Transitivity of FD13 and FD3)
 - FD15: **AE** → **ABCDEF** (Union of FD9, FD11, and FD14)

Computing (Super)Keys

```
function ComputeAttrClosure(  $X$ ,  $F$  )  
begin  
     $X^+ := X$ ;  
    while  $F$  contains an FD  $Y \rightarrow Z$  such that  
        (i)  $Y$  is a subset of  $X^+$ , and  
        (ii)  $Z$  is not a subset of  $X^+$  do  
         $X^+ := X^+ \cup Z$ ;  
    end while  
    return  $X^+$ ;  
end
```

Warmup (cont'd)

- Consider the relation **R(A,B,C,D,E,F)** with the following FDs:
FD1: **A** → **BC**
FD2: **C** → **AD**
FD3: **DE** → **F**
- Compute the **attribute closure** of $X = \{ \mathbf{A,E} \}$ w.r.t. $F = \{ \text{FD1, FD2, FD3} \}$ to show that we have: **AE** → **ABCDEF**
 - Initially: $X^+ = \{ \mathbf{A,E} \}$
 - By using FD1: $X^+ = \{ \mathbf{A,E,B,C} \}$
 - By using FD2: $X^+ = \{ \mathbf{A,E,B,C,D} \}$
 - By using FD3: $X^+ = \{ \mathbf{A,E,B,C,D,F} \}$

Your Turn

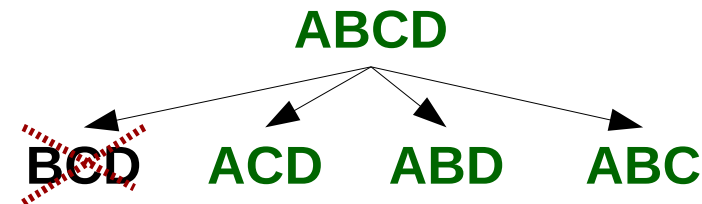
- Consider the relation **R(A,B,C,D)** with the following set F of FDs:
FD1: **AB** \rightarrow **C**
FD2: **BC** \rightarrow **D**
FD3: **D** \rightarrow **B**
- Compute the following attribute closures w.r.t. $F = \{\text{FD1, FD2, FD3}\}$
 - $\{B, C, D\}^+ = ?$
 - $\{A, C, D\}^+ = ?$
 - $\{A, B, D\}^+ = ?$
 - $\{A, B, C\}^+ = ?$

Your Turn

- Consider the relation **R(A,B,C,D)** with the following set *F* of FDs:
FD1: **AB** → **C**
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- Compute the following attribute closures w.r.t. $F = \{FD1, FD2, FD3\}$
 - $\{B, C, D\}^+ = \{B, C, D\}$
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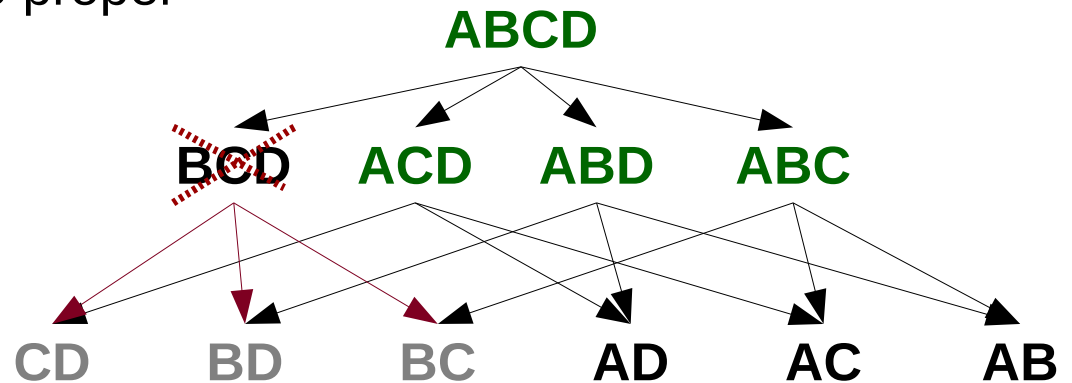
Superkeys and Candidate Keys

- Consider the relation **R(A,B,C,D)** with the following set *F* of FDs:
FD1: **AB** → **C**
FD2: **BC** → **D**
FD3: **D** → **B**
- A set *X* of attributes of *R* is a **superkey** if X^+ contains all the attributes of *R*
 - $\{B,C,D\}^+ = \{B,C,D\}$
 - $\{A,C,D\}^+ = \{A,C,D,B\}$
 - $\{A,B,D\}^+ = \{A,B,D,C\}$
 - $\{A,B,C\}^+ = \{A,B,C,D\}$



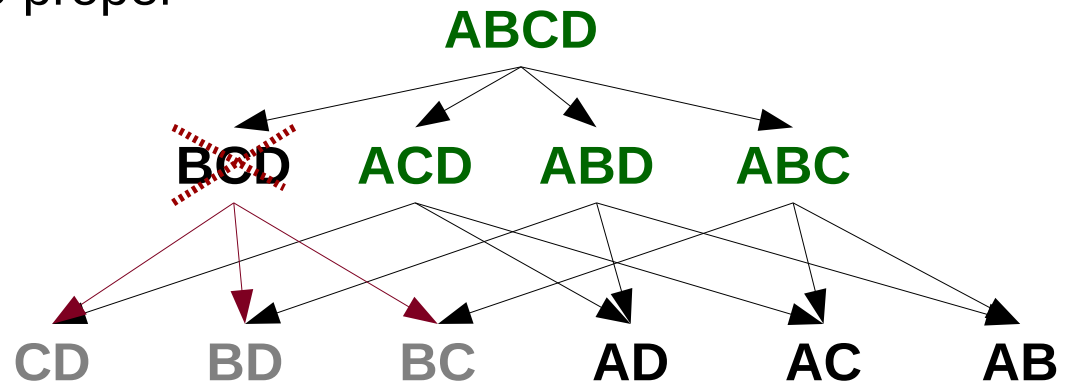
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- A set *X* of attributes of *R* is a **superkey** if X^+ contains all the attributes of *R*
- X* is a **candidate key** (CK) if no proper subset *X'* of *X* is a **superkey**
 - $\{A,D\}^+ = ?$
 - $\{A,C\}^+ = ?$
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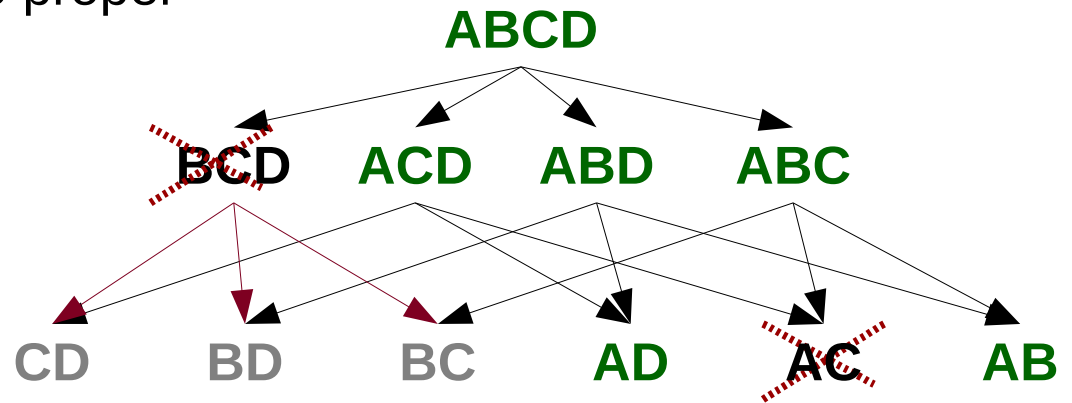
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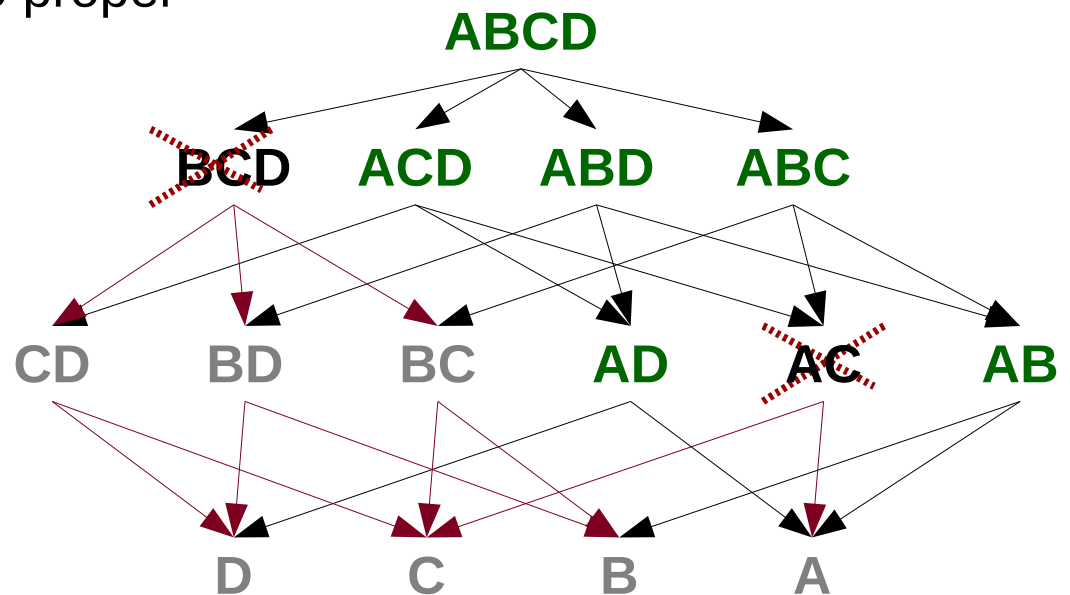
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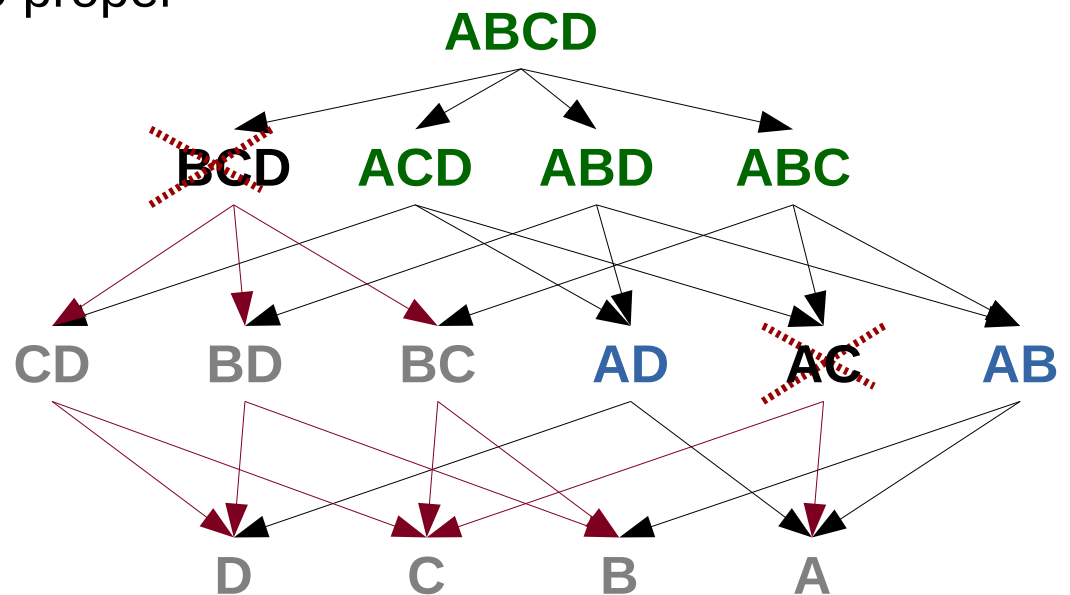
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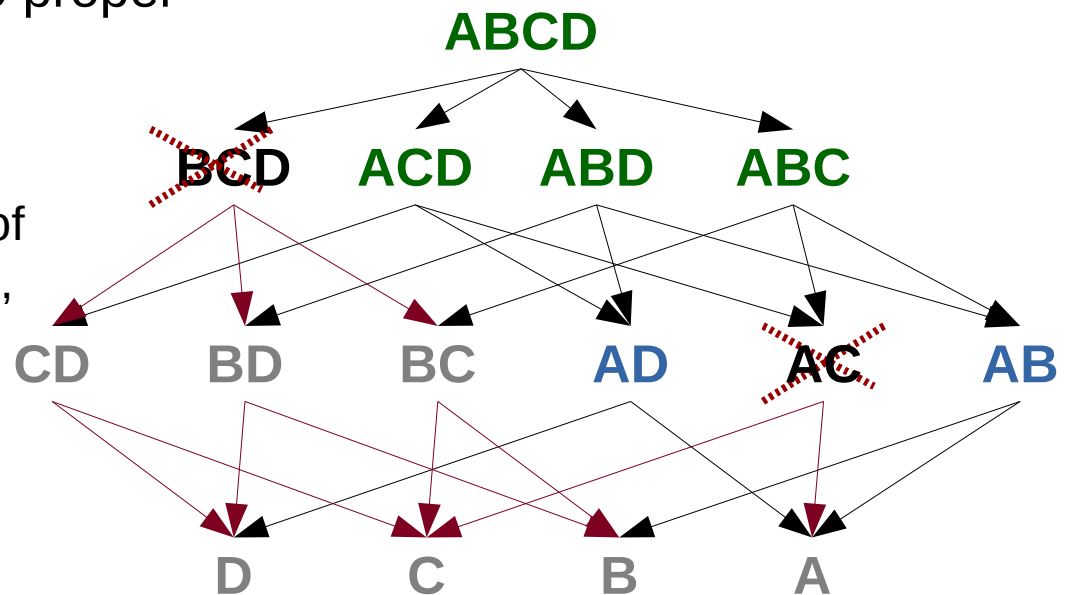
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Superkeys and Candidate Keys

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- A set *X* of attributes of *R* is a **superkey** if X^+ contains all the attributes of *R*
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- Some steps may be skipped
 - If an attribute is in the RHS of FDs but nowhere in the LHS, it cannot be part of any CK
 - If an attribute is nowhere in the RHS, it must be part of any candidate key (such as *A* in the example)



Superkeys and Candidate Keys

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 - ~~FD3: **D** → **B**~~
- A set *X* of attributes of *R* is a **superkey** if X^+ contains all the attributes of *R*
- X* is a **candidate key** (CK) if no proper subset *X'* of *X* is a **superkey**
- Some steps may be skipped
 - If an attribute is in the RHS of FDs but nowhere in the LHS, it cannot be part of any CK
 - If an attribute is nowhere in the RHS, it must be part of every candidate key (such as *A* in the example)
- For instance, if we had only FD1 and FD2 (i.e., not FD3), we could immediately say that:
 - A* and *B* must be in every CK
 - and, since $\{A,B\}^+ = \{A,B,C,D\}$, $\{A,B\}$ is a superkey and thus the only CK

Your Turn

- Consider the relation **R(A,B,C,D,E,F)** with the following FDs:
FD1: **A** → **BC**
FD2: **C** → **AD**
FD3: **DE** → **F**
- What are the candidate keys?

Your Turn

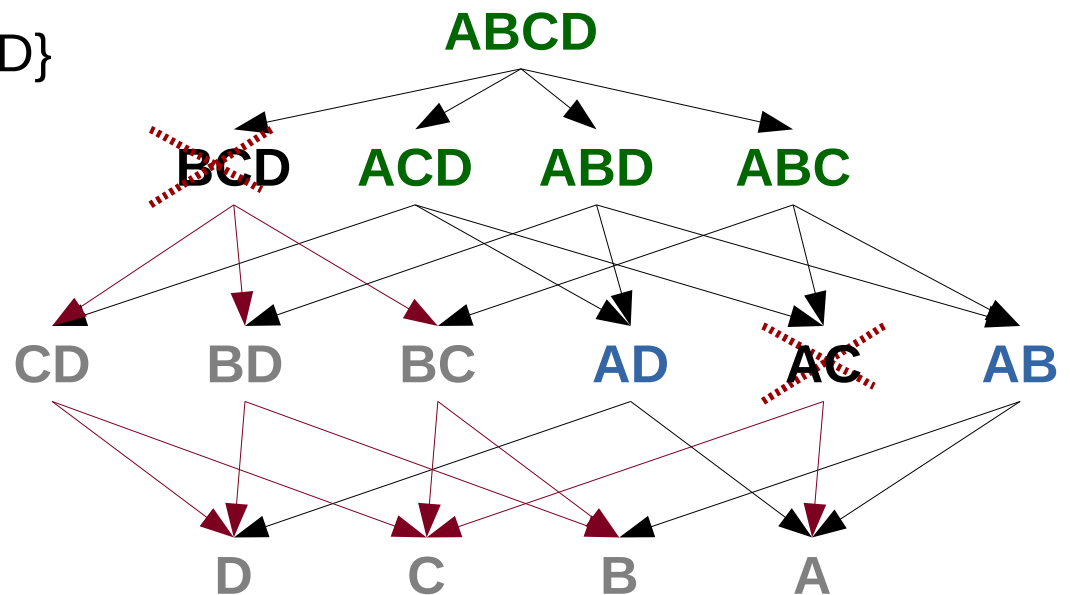
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FD1: **A** → **BC**
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- What are the candidate keys?
 - {A,E} and {C,E}

BCNF

- Relation schema R with a set F of functional dependencies is in BCNF if for **every** non-trivial **FD** $X \rightarrow Y$ in F^+ we have that **X is a superkey**
- Consider the relation **$R(A,B,C,D,E,F)$** with the following FDs:
 - FD1: **$A \rightarrow BC$**
 - FD2: **$C \rightarrow AD$**
 - FD3: **$DE \rightarrow F$**
- What are the candidate keys?
 - $\{A,E\}$ and $\{C,E\}$
- Is the given relation in BCNF?
 - If not, identify the FDs that violate the BCNF condition.

Your Turn

- Relation schema R with a set F of functional dependencies is in BCNF if for **every** non-trivial **FD** $X \rightarrow Y$ in F^+ we have that **X is a superkey**
- Consider the relation $R(A,B,C,D)$ with the following set F of FDs:
 - FD1: $AB \rightarrow C$
 - FD2: $BC \rightarrow D$
 - FD3: $D \rightarrow B$
- Candidate keys: $\{A,B\}$ and $\{A,D\}$
- Is the given relation in BCNF?
 - If not, identify the FDs that violate the BCNF condition.



BCNF Decomposition Step

- By using an FD $X \rightarrow Y$ that violates BCNF, decompose R into
 - $R1$ with all the attributes in X and in Y , and
 - $R2$ with all attributes from R except those that are in Y and not in X
- Consider the relation **$R(A,B,C,D)$** with the following set F of FDs:
 - FD1: **$AB \rightarrow C$**
 - FD2: **$BC \rightarrow D$**
 - FD3: **$D \rightarrow B$**
- Let's decompose based on FD2

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 - FD1: **$AB \rightarrow C$**
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- Let's decompose based on FD2
 - $R1(B,C,D)$
 - $R2(A,B,C)$

BCNF Decomposition Step

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 - FD1: **$AB \rightarrow C$**
 - FD2: **$BC \rightarrow D$**
 - FD3: **$D \rightarrow B$**
- Let's decompose based on FD2 – and don't forget to determine the FDs of the resulting relation schemas
 - $R1(B,C,D)$
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 - FD3: **$D \rightarrow B$**
- Let's decompose based on FD2 – and don't forget to determine the FDs of the resulting relation schemas
 - $R1(B,C,D)$ with FDs: FD2 and FD3
 - $R2(A,B,C)$ with FDs: FD1

BCNF Decomposition Step

- By using an FD $X \rightarrow Y$ that violates BCNF, decompose R into
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 - FD3: **$D \rightarrow B$**
- Let's decompose based on FD2 – and don't forget to determine the FDs of the resulting relation schemas, **and the CKs**
 - $R1(B,C,D)$ with FDs: FD2 and FD3
 - $R2(A,B,C)$ with FDs: FD1

BCNF Decomposition Step

- By using an FD $X \rightarrow Y$ that violates BCNF, decompose R into
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 - FD3: **$D \rightarrow B$**
- Let's decompose based on FD2 – and don't forget to determine the FDs of the resulting relation schemas, and the CKs
 - $R1(B,C,D)$ with FDs: FD2 and FD3, CKs: $\{B,C\}$, $\{C,D\}$
 - $R2(A,B,C)$ with FDs: FD1, CK: $\{A,B\}$

BCNF Decomposition Step

- By using an FD $X \rightarrow Y$ that violates BCNF, decompose R into
 - $R1$ with all the attributes in X and in Y , and
 - $R2$ with all attributes from R except those that are in Y and not in X
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 - FD1: **$AB \rightarrow C$**
 - FD2: **$BC \rightarrow D$**
 - FD3: **$D \rightarrow B$**
- Let's decompose based on FD2 – and don't forget to determine the FDs of the resulting relation schemas, and the CKs
 - $R1(B,C,D)$ with FDs: FD2 and FD3, CKs: $\{B,C\}$, $\{C,D\}$
 - $R2(A,B,C)$ with FDs: FD1, CK: $\{A,B\}$
 - Are they in BCNF?

BCNF Decomposition Step

- By using an FD $X \rightarrow Y$ that violates BCNF, decompose R into
 - $R1$ with all the attributes in X and in Y , and
 - $R2$ with all attributes from R except those that are in Y and not in X
- Consider the relation **$R(A,B,C,D)$** with the following set F of FDs:
 - FD1: **$AB \rightarrow C$**
 - FD2: **$BC \rightarrow D$**
 - FD3: **$D \rightarrow B$**
- Let's decompose based on FD2 – and don't forget to determine the FDs of the resulting relation schemas, and the CKs
 - **$R1(B,C,D)$** with FDs: FD2 and **FD3**, CKs: $\{B,C\}$, $\{C,D\}$
 - $R2(A,B,C)$ with FDs: FD1, CK: $\{A,B\}$
 - Your turn: decompose **$R1$** based on **FD3** (and don't forget ...)

BCNF Decomposition Step

- By using an FD $X \rightarrow Y$ that violates BCNF, decompose R into
 - $R1$ with all the attributes in X and in Y , and
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- Consider the relation **$R(A,B,C,D)$** with the following set F of FDs:
 - FD1: **$AB \rightarrow C$**
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 - FD3: **$D \rightarrow B$**
- Let's decompose based on FD2 – and don't forget to determine the FDs of the resulting relation schemas, and the CKs
 - ~~$R1(B,C,D)$ with FDs: FD2 and FD3, CKs: $\{B,C\}, \{C,D\}$~~
 - $R2(A,B,C)$ with FDs: FD1, CK: $\{A,B\}$
 - $R3(D,B)$ with FD3, CK $\{D\}$ – $R4(C,D)$ only trivial FDs, CK: $\{C,D\}$

Your Turn

- By using an FD $X \rightarrow Y$ that violates BCNF, decompose R into
 - $R1$ with all the attributes in X and in Y , and
 - $R2$ with all attributes from R except those that are in Y and not in X
- Consider the relation **$R(A,B,C,D)$** with the following set F of FDs:
 - FD1: **$AB \rightarrow C$**
 - FD2: **$BC \rightarrow D$**
 - FD3: **$D \rightarrow B$**
- Let's decompose based on **FD3** – and don't forget to determine the FDs of the resulting relation schemas, and the CKs

Your Turn

- By using an FD $X \rightarrow Y$ that violates BCNF, decompose R into
 - $R1$ with all the attributes in X and in Y , and
 - $R2$ with all attributes from R except those that are in Y and not in X
- Consider the relation $R(A,B,C,D)$ with the following set F of FDs:
 - FD1: $AB \rightarrow C$
 - FD2: $BC \rightarrow D$
 - FD3: $D \rightarrow B$
- Let's decompose based on **FD3** – and don't forget to determine the FDs of the resulting relation schemas, and the CKs
 - $R1(B,D)$ with FD3, CK: $\{D\}$
 - $R2(A,C,D)$ with **FD4: $AD \rightarrow C$** , CK: $\{A,D\}$

$R1$ and $R2$ are in BCNF

can be derived from FD3 and FD1 using the augmentation rule and the transitivity rule

Different Example

- By using an FD $X \rightarrow Y$ that violates BCNF, decompose R into
 - $R1$ with all the attributes in X and in Y , and
 - $R2$ with all attributes from R except those that are in Y and not in X
- Consider the relation schema $R(A,B,C,D,E,F)$ with the following FDs:
FD1: $AB \rightarrow CDEF$
FD2: $E \rightarrow F$
- Your turn:
 - Determine candidate key(s)
 - Is R in BCNF?
 - If not, normalize into a set of BCNF relation schemas (and don't forget to determine FDs and CKs along the way)

Different Example

- By using an FD $X \rightarrow Y$ that violates BCNF, decompose R into
 - $R1$ with all the attributes in X and in Y , and
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- Consider the relation schema $R(A,B,C,D,E,F)$ with the following FDs:
FD1: $AB \rightarrow CDEF$
FD2: $E \rightarrow F$
- Your turn:
 - Determine candidate key(s) $\{A,B\}$
 - Is R in BCNF? *No, FD2 violates the BCNF condition.*
 - If not, normalize into a set of BCNF relation schemas
We decompose R based on FD2:
 - $R1(E,F)$ with FD2; candidate key is $\{E\}$
 - $R2(A,B,C,D,E)$ with a new FD: $AB \rightarrow CDE$; candidate key is $\{A,B\}$ *$R1$ and $R2$ are in BCNF*

can be derived from FD1
using the decomposition rule

One More

- By using an FD $X \rightarrow Y$ that violates BCNF, decompose R into
 - $R1$ with all the attributes in X and in Y , and
 - $R2$ with all attributes from R except those that are in Y and not in X
- Consider the relation schema $R(A,B,C,D,E,F)$ with the following FDs:
FD1: **$AB \rightarrow CDEF$**
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FD3: **$A \rightarrow D$**
- Your turn:
 - Determine candidate key(s)
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One More

- By using an FD $X \rightarrow Y$ that violates BCNF, decompose R into
 - $R1$ with all the attributes in X and in Y , and
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- Consider the relation schema $R(A,B,C,D,E,F)$ with the following FDs:
FD1: $AB \rightarrow CDEF$
FD2: $E \rightarrow F$
FD3: $A \rightarrow D$
- Solution: *CK is $\{A,B\}$; R is not in BCNF because of FD2 and FD3.*
We decompose R based on FD2:
 - $R1(E,F)$ with FD2; candidate key is $\{E\}$
 - $R2(A,B,C,D,E)$ with FD3 and a new FD: $AB \rightarrow CDE$; candidate key is $\{A,B\}$ *$R1$ is in BCNF, but $R2$ is not because of FD3. So, we have to decompose $R2$ using FD3:*
 - $R3(A,D)$ with FD3; candidate key is $\{A\}$
 - $R4(A,B,C,E)$ with new FD: $AB \rightarrow CE$; candidate key is $\{AB\}$ *$R3$ and $R4$ are in BCNF. Hence, the result of the decomposition consists of $R1$, $R3$, and $R4$.*

Back to the Earlier Running Example

- By using an FD $X \rightarrow Y$ that violates BCNF, decompose R into
 - $R1$ with all the attributes in X and in Y , and
 - $R2$ with all attributes from R except those that are in Y and not in X
- Consider the relation schema **$R(A,B,C,D,E,F)$** with the following FDs:
FD1: **$A \rightarrow BC$** FD2: **$C \rightarrow AD$** FD3: **$DE \rightarrow F$**
- Recall: CKs are $\{A,E\}$, $\{C,E\}$; all three FDs violate the BCNF condition
- Your turn: Normalize into a set of BCNF relation schemas
(and don't forget to determine FDs and CKs along the way)

Back to the Earlier Running Example

- By using an FD $X \rightarrow Y$ that violates BCNF, decompose R into
 - R_1 with all the attributes in X and in Y , and
 - R_2 with all attributes from R except those that are in Y and not in X
- Consider the relation schema **$R(A,B,C,D,E,F)$** with the following FDs:
FD1: **$A \rightarrow BC$** FD2: **$C \rightarrow AD$** FD3: **$DE \rightarrow F$**
- Recall: CKs are $\{A,E\}$, $\{C,E\}$; all three FDs violate the BCNF condition
- A possible solution: *We decompose R based on FD1:*
 - $R_1(A,B,C)$ with FD1 and a new FD: $C \rightarrow A$; candidate keys are $\{A\}$ and $\{C\}$
 - $R_2(A,D,E,F)$ with FD3 and a new FD: $A \rightarrow D$; candidate key is $\{A,E\}$

R_1 is in BCNF, but R_2 is not because of FD3 and $A \rightarrow D$. Let's decompose R_2 based on FD3:

 - $R_3(D,E,F)$ with FD3; candidate key is $\{D,E\}$
 - $R_4(A,D,E)$ with $A \rightarrow D$; candidate key is $\{A,E\}$

R_3 is in BCNF, but R_4 is not because of $A \rightarrow D$. Let's decompose R_4 based on $A \rightarrow D$

 - $R_5(A,D)$ with $A \rightarrow D$; candidate key is $\{A\}$
 - $R_6(A,E)$ with only trivial FDs; candidate key is $\{A,E\}$

R_5 and R_6 are in BCNF. Hence, the result of the decomposition consists of R_1, R_3, R_4, R_5 .

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