

Mathematical Exercises 2

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Try to solve the problems before class. Don't worry if you fail, the important thing is trying.

You should not hand in any solutions.

This part of the course is not obligatory and is not graded.

1. TAU-CHI.

- (a) Let $x_1, \dots, x_n \stackrel{iid}{\sim} N(\theta, \sigma^2)$. Assume that θ is known, but σ^2 unknown. Derive the posterior distribution for σ^2 . Use the conjugate prior.
- (b) Assume that $\theta = 1$ and that you have observed the data $x_1 = 0.6, x_2 = 3.2, x_3 = 1.2$. Compute the posterior of σ^2 based on these three data points. Use a prior with very little information (it is up to you how to define little information).

2. FEEL THE BERN.

- (a) Let $x_1, \dots, x_n \stackrel{iid}{\sim} \text{Bern}(\theta)$, with a $\text{Beta}(\alpha, \beta)$ prior for θ . Derive the predictive distribution for x_{n+1} .
- (b) You need to decide if you bring your umbrella during your daily walk. It has rained on two days during the last ten days, and you assess those ten days to be representative also for the weather today, the 11th day. Your utility for the action-state combinations are given in the table below. Assume a $\text{Beta}(1, 1)$ prior for θ . Compute the Bayesian decision.
- (c) How sensitive is your decision in (b) to the changes in the prior hyperparameters, α and β ?

	Rainy	Sunny
Bring umbrella	10	20
Leave umbrella	-50	50

3. CAMPAIGN OR NO CAMPAIGN - THAT IS THE QUESTION.

- (a) Let x_i be the sales of product on month i . Let $x_1, \dots, x_n \stackrel{iid}{\sim} N(\theta, \sigma^2)$ be the (approximate) distribution for the sales, and let $\theta \sim N(200, 50^2)$ a priori. Assume that $\sigma^2 = 25^2$ and that we have observed $n = 5$ and $\bar{x} = 320.4$. Compute the predictive distribution for x_6 .
- (b) The company has the choice of performing a marketing campaign for their product. The marketing campaign costs \$400 and is believed to increase sales by 100 items. The company sells the product for $p = 10$ dollar and the cost of producing the product is $q = 5$ dollar. There are no fixed production costs. Should the company perform the marketing campaign?

4. PREDICTIVE DISTRIBUTION FOR A POISSON MODEL

- (a) Do Exercise 13(a) in Chapter 2 of the course book.

Have fun!

- Mattias