LINKÖPING UNIVERSITY
Division of Statistics and Machine Learning
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Bayesian Learning
Advanced level course

Mathematical Exercises 4

Teacher: Mattias Villani
Try to solve the problems before class. Don't worry if you fail, the important thing is trying.
You should not hand in any solutions.
This part of the course is not obligatory and is not graded.

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1. BERNOULLI MEETS LAPLACE
   (a) Let $x_1, ..., x_n \overset{iid}{\sim} Bernoulli(\theta)$ and assume the prior $\theta \sim Beta(\alpha, \beta)$. Derive the marginal likelihood this model.
   (b) Compute the marginal likelihood of the model in a) using the Laplace approximation. Is this approximation accurate if $\alpha = \beta = 1$ and you have observed $s = 6$ success in $n = 10$ trials?

2. FILL IN THE BLANKS - AGAIN
   (a) Derive the marginal likelihood for the Poisson model with Gamma prior at the end of Slide 7 at Lecture 10.

3. PARETO
   (a) Let $x_1, \ldots, x_n \overset{iid}{\sim} \text{Uniform}(0, \theta)$. Show that $\theta \sim \text{Pareto}(\alpha, \beta)$ is a conjugate prior to this Uniform model and derive the posterior for θ [Hint: Don't forget to include an indicator function when you write up the likelihood function. The Uniform$(0, \theta)$ distribution is zero for outcomes larger than θ. Note: Wikipedia parametrizes the Pareto distibution with the two hyperparameters α and x_n. x_n correponds to our β here.
   (b) Derive the predictive distribution of x_{n+1} given $x_1, \ldots, x_n$. [Hint: It is wise to break up the integrals in two parts.]

Have fun!

- Mattias