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EECS 101

Homework 1

Problem 1
Parametric Equations of a line in 3-D space
$x = x_0 + ta$
$y = y_0 + tb$
$z = z_0 + tc$
$-\infty \leq t \leq \infty$

Constants
$x_0 = 0.5$
$y_0 = -1$
$z_0 = 0$
$a = 0$
$b = 1$
$c = -1$
$f' = 1$

Write expressions for the image coordinates *x'* and *y'* in terms of these constants and *f'* and *t* for both the perspective and orthographic projection of the line.

Perspective Projection

$x' = \frac{f'x}{z}$	$y' = \frac{f'y}{z}$
$x' = \frac{f'(x_0 + ta)}{z_0 + tc}$	$y' = \frac{f'(y_0 + tb)}{z_0 + tc}$
$x' = \frac{f'(0.5 + t(0))}{0 + t(-1)}$	$y' = \frac{f'(-1 + t(1))}{0 + t(-1)}$
$x' = \frac{0.5f'}{-t}$	$y' = \frac{-f' + f't}{-t}$
$x' = \frac{0.5(1)}{-t}$	$y' = \frac{-(1) + (1)t}{-t}$
$x' = \frac{0.5}{-t}$	$y' = \frac{1}{t} - 1$

Orthographic Projection

$x' = x$	$y' = y$
$x' = x_0 + ta$	$y' = y_0 + tb$
$x' = 0.5 + t(0)$	$y' = -1 + t(1)$
$x' = 0.5$	$y' = -1 + t$

Is the projection of the line also a line for perspective projection? for orthographic projection?

Both perspective projection and orthographic projection produce a line

Write a C program to generate images of both the perspective and orthographic projections of the line. The sample C program will only display regions in the range $-4 \le x' \le 4$, $-4 \le y' \le 4$. You should compute for which range of t the values of *x'* and *y'* will fall within this range and set *t* accordingly.

(Code in hw1_modified.c)

Perspective Projection

$x' = \frac{0.5}{-t}$

$-4 = \frac{0.5}{-t}$ $\quad -0.01 = \frac{0.5}{-t}$ $\quad 0.01 = \frac{0.5}{-t}$ $\quad 4 = \frac{0.5}{-t}$

$t = 0.125$ $\quad t = 50$ $\quad t = -50$ $\quad t = -0.125$

$y' = \frac{1}{t} - 1$

$-4 = \frac{1}{t} - 1$ $\quad -1.01 = \frac{1}{t} - 1$ $\quad -0.99 = \frac{1}{t} - 1$ $\quad 4 = \frac{1}{t} - 1$

$t = -0.33$ $\quad t = -100$ $\quad t = 100$ $\quad t = 0.2$

Range of t

$0.2 \le t \le 50$

Orthographic Projection

$x' = 0.5$ $\quad y' = -1 + t$

$-4 = -1 + t$ $\quad 4 = -1 + t$

$t = -3$ $\quad t = 5$

Range of t

$0.01 \le t \le 5$

What happens in the perspective projection case as *t* goes to ∞? Is this consistent with the image that your program generates?

As *t* goes to ∞, *x'* goes to 0. As *t* goes to ∞, *y'* goes to -1. This is consistent with the image that the program generates.

Problem 2

Parametric Equations of two parallel lines in a plane that is parallel to the image plane in 3-D space

$x = x_1 + ta$
$x\text{^} = x_2 + ta$
$y = y_1 + tb$
$y\text{^} = y_2 + tb$
$z = z_0$
$z\text{^} = z_0$
$-\infty \leq t \leq \infty$

Constants

$x_1 = 0.5$
$x_2 = -0.5$
$y_1 = -1$
$y_2 = -1$
$z_0 = -1, -2, -3$
$a = 0$
$b = 1$
$f' = 1$

Write expressions for the image coordinates x' and y' in terms of t and the constants above for both the perspective and orthographic projections of the lines.

Perspective Projection

Line A

$x' = \frac{f'x}{z}$ $\quad$ $y' = \frac{f'y}{z}$

$x' = \frac{f'(x_1 + ta)}{z_0}$ $\quad$ $y' = \frac{f'(y_1 + tb)}{z_0}$

$x' = \frac{(1)(0.5 + t(0))}{z_0}$ $\quad$ $y' = \frac{(1)(-1 + t(1))}{z_0}$

$x' = \frac{0.5}{z_0}$ $\quad$ $y' = \frac{-1 + t}{z_0}$

$x' = \frac{0.5}{-1}$ $\quad$ $x' = \frac{0.5}{-2}$ $\quad$ $x' = \frac{0.5}{-3}$ $\quad$ $y' = \frac{-1 + t}{-1}$ $\quad$ $y' = \frac{-1 + t}{-2}$ $\quad$ $y' = \frac{-1 + t}{-3}$

$x' = -0.5$ $\quad$ $x' = -0.25$ $\quad$ $x' = -0.16$

Line B

$x' = \frac{f'x\text{^}}{z\text{^}}$ $\quad$ $y' = \frac{f'y\text{^}}{z\text{^}}$

$x' = \frac{f'(x_2 + ta)}{z_0}$ $\quad$ $y' = \frac{f'(y_2 + tb)}{z_0}$

$x' = \frac{(1)(-0.5 + t(0))}{z_0}$ $\quad$ $y' = \frac{(1)(-1 + t(1))}{z_0}$

$x' = \frac{-0.5}{z_0}$ $\quad$ $y' = \frac{-1 + t}{z_0}$

$x' = \frac{-0.5}{-1}$ $\quad$ $x' = \frac{-0.5}{-2}$ $\quad$ $x' = \frac{-0.5}{-3}$ $\quad$ $y' = \frac{-1 + t}{-1}$ $\quad$ $y' = \frac{-1 + t}{-2}$ $\quad$ $y' = \frac{-1 + t}{-3}$

$x' = 0.5$ $\quad$ $x' = 0.25$ $\quad$ $x' = 0.16$

Orthographic Projection

Line A

$x' = x$ $\quad$ $y' = y$

$x' = x_1 + ta$ $\quad$ $y' = y_1 + tb$

$x' = 0.5 + t(0)$ $\quad$ $y' = -1 + t(1)$

$x' = 0.5$ $\quad$ $y' = -1 + t$

Line B

$x' = x^\wedge$ $\quad$ $y' = y^\wedge$

$x' = x_2 + ta$ $\quad$ $y' = y_2 + tb$

$x' = -0.5 + t(0)$ $\quad$ $y' = -1 + t(1)$

$x' = -0.5$ $\quad$ $y' = -1 + t$

Write a C program to generate images of both the perspective and orthographic projections of the lines by letting *t* range from 0.01 to 10000.

(Code in hw1_modified.c)

Use the magnification equation to show whether or not the projections of the lines will be parallel for both perspective and orthographic projection. Is your answer consistent with the images that your program generates?

$$magnification = \frac{distance\ between\ two\ image\ points}{distance\ between\ corresponding\ scene\ points}$$

$$magnification = \frac{\sqrt{(\frac{f'x}{z} - \frac{f'x^\wedge}{z^\wedge})^2 + (\frac{f'y}{z} - \frac{f'y^\wedge}{z^\wedge})^2}}{\sqrt{(x - x^\wedge)^2 + (y - y^\wedge)^2}}$$

If the two scene points are on the plane z = z0

$$magnification = \frac{\sqrt{(\frac{f'x}{z_0} - \frac{f'x^\wedge}{z_0})^2 + (\frac{f'y}{z_0} - \frac{f'y^\wedge}{z_0})^2}}{\sqrt{(x - x^\wedge)^2 + (y - y^\wedge)^2}}$$

Simplifies to

$$magnification = \frac{f'}{-z_0}$$

Because the parametric equations of the lines are similar and all the points exist at the same z-distance, any two points you choose will have the same magnification, therefore the magnification equation shows that the lines in the perspective projection are parallel.

Orthographic projection does not take into account z-distance when projecting the lines onto the image, so the lines will appear parallel on the image because the lines are parallel in the scene.

The images generated are consistent with my answer.

Is orthographic projection a good approximation to perspective projection for this case? Why?

Orthographic projection is a good approximation to perspective projection in this case because there is no variation in the z-distance in the scene.

What occurs if $z0 = f'$?

If z0 = f’, then magnification = -1 which indicates that the lines are at the same z-distance and produces an orthographic projection of the lines.

Problem 3

Parametric Equations of two parallel lines in a plane that is parallel to the image plane in 3-D space

$x = x_1$
$x\hat{} = x_2$
$y = y_0 + tb$
$y\hat{} = y_0 + tb$
$z = z_0 + tc$
$z\hat{} = z_0 + tc$
$-\infty \leq t \leq \infty$

Constants

$x_1 = -1$
$x_2 = 1$
$y_0 = -1$
$z_0 = 0$
$b = 0, 1, -1$
$c = 1, -1$
$f' = 1$

Write expressions for the image coordinates *x'* and *y'* in terms of *t* and the constants above for both the perspective and orthographic projections of the lines.

Perspective Projection

Line A

$x' = \frac{f'x}{z}$
$x' = \frac{f'x_1}{z_0+tc}$
$x' = \frac{(1)(-1)}{0+tc}$
$x' = \frac{-1}{tc}$
$x' = \frac{-1}{t(1)} \quad x' = \frac{-1}{t(-1)}$

$y' = \frac{f'y}{z}$
$y' = \frac{f'(y_0 + tb)}{z_0+tc}$
$y' = \frac{(1)(-1 + tb)}{0+tc}$
$y' = \frac{-1 + tb}{tc}$
$y' = \frac{-1 + t(0)}{t(1)} \quad y' = \frac{-1 + t(1)}{t(1)} \quad y' = \frac{-1 + t(-1)}{t(1)}$
$y' = \frac{-1 + t(0)}{t(-1)} \quad y' = \frac{-1 + t(1)}{t(-1)} \quad y' = \frac{-1 + t(-1)}{t(-1)}$

Line B

$x' = \frac{f'x\hat{}}{z\hat{}}$
$x' = \frac{f'x_2}{z_0+tc}$
$x' = \frac{(1)(1)}{0+tc}$
$x' = \frac{1}{tc}$
$x' = \frac{1}{t(1)} \quad x' = \frac{1}{t(-1)}$

$y' = \frac{f'y\hat{}}{z\hat{}}$
$y' = \frac{f'(y_0 + tb)}{z_0+tc}$
$y' = \frac{(1)(-1 + tb)}{0+tc}$
$y' = \frac{-1 + tb}{tc}$
$y' = \frac{-1 + t(0)}{t(1)} \quad y' = \frac{-1 + t(1)}{t(1)} \quad y' = \frac{-1 + t(-1)}{t(1)}$
$y' = \frac{-1 + t(0)}{t(-1)} \quad y' = \frac{-1 + t(1)}{t(-1)} \quad y' = \frac{-1 + t(-1)}{t(-1)}$

Orthographic Projection

Line A

$x' = x$ $\qquad y' = y$

$x' = x_1$ $\qquad y' = y_1 + tb$

$x' = -1$ $\qquad y' = -1 + t(0) \qquad y' = -1 + t(1) \qquad y' = -1 + t(-1)$

Line B

$x' = x\^{}$ $\qquad y' = y\^{}$

$x' = x_2$ $\qquad y' = y_2 + tb$

$x' = 1$ $\qquad y' = -1 + t(0) \qquad y' = -1 + t(1) \qquad y' = -1 + t(-1)$

Write a C program to generate images of both the perspective and orthographic projections of the lines by letting *t* range from 0.01 to 10000.

(Code in hw1_modified.c)

Use the magnification equation to show whether or not the projections of the lines will be parallel for both perspective and orthographic projection. Is your answer consistent with the images that your program generates?

$$magnification = \frac{distance\ between\ two\ image\ points}{distance\ between\ corresponding\ scene\ points}$$

$$magnification = \frac{\sqrt{(\frac{f'x}{z} - \frac{f'x\^{}}{z\^{}})^2 + (\frac{f'y}{z} - \frac{f'y\^{}}{z\^{}})^2}}{\sqrt{(x - x\^{})^2 + (y - y\^{})^2}}$$

Because the lines do not exist on the same z-distance, the distance between two image points will vary, therefore the magnification is changing throughout the image and the lines in the perspective projection are not parallel.

Orthographic projection does not take into account z-distance when projecting the lines onto the image, so the lines will appear parallel on the image because the lines are parallel in the scene.

Is orthographic projection a good approximation to perspective projection for this case?

The orthographic projection is not a good approximation to perspective projection in this case because there is a large variation in the z-distance along the lines.

What happens in the perspective projection case as *t* goes to ∞? Is this consistent with the image that your program generates?

As *t* goes to ∞, *x'* goes to 0 for both Line A and Line B. As *t* goes to ∞, *y'* goes to 0, 1, -1 for both Line A and Line B. This is consistent with the images that the program generates.

Bonus Question

Write a description to explain how hw1-bonus.c works. Specifically, explain what each function call (in 1-2 sentences, excluding print statements) in the main function does and how it achieves the effect by examining its arguments, return value and functionality.

The program converts a “.ras” image file into a “.raw” image file. First the 2-D image array is cleared using clear(image) which sets all the values in the image array to 0. Next, fopen(…) is used to open the specified file name stored in ifile and checks to see if it exists. Then read using fread(image[i], 1, COLS, fp), the file is read through and checks if the file has the same number of columns of pixels as specified by the COLS variable. The file is then closed using fclose(fp), and a header is generated for the file to be saved using header(ROWS, COLS, head). Next, fopen(ofile, "wb") is used to create/open the specified file name stored in ofile. Then fwrite(head, 4, 8, fp) is used to write the header of the image file and then fwrite(image[i], 1, COLS, fp) to be save the actual image from the image array. Finally the file is closed using fclose(fp) and the main function returns 0;