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EECS 101

Homework 2

1. Consider a noiseless CCD camera with perfect quantum efficiency (q() = 1) of 501 × 501 exposed potential wells with an integration time of 1 millisecond. Assume that a spot of light of 109 photons per second is focused on the single potential well at the center of the array. All other potential wells receive no light. Assume that the pixel clock operates at 7.5 MHz.

a) Describe in detail the 501 × 501 image (electrons/potential well) that will be generated by the CCD if interline transfer is used.

> The spotlight will make contact with a point between the exposed potential well and the masked well at the center of the array. Therefore, the image generated will be a completely black image.

b) Describe in detail the 501 × 501 image (electrons/potential well) that will be generated by the CCD if frame transfer is used.

> The spotlight will make contact with a single, exposed potential well at the center of the array. Therefore, the image generated will be an image with a single white pixel at the center of the image while the rest of the image is black.

2. Consider an amplified measurement C in electrons for a collection site (potential well) in a cooled CCD camera described by

$$C = (S + N_A + N_P)A$$

where S is the signal in electrons, N_A is a zero-mean amplifier noise source with a variance of 1 electron, N_P is the photon noise source, and A is the constant amplifier gain. (Hint: read problem 4 before working on this problem)

a) Write an expression for the variance of the measurement C.

S = signal of electrons (constant) A = amplifier gain (constant)

$E(N_A) = 0$ $VAR(N_A) = 1$

$E(N_P) = 0$ $VAR(N_P) = \sigma_P^2$

$$VAR(C) = \sigma_C^2 = VAR((S + N_A + N_P)A)$$

$$\sigma_C^2 = A^2 VAR(S) + A^2 VAR(N_A) + A^2 VAR(N_P)$$

$$\sigma_C^2 = A^2(0) + A^2(1) + A^2(\sigma_P^2)$$

$$\sigma_C^2 = A^2 + A^2\sigma_P^2$$

b) Define the signal-to-noise of the quantity C as its mean divided by its standard deviation. What is the signal-to-noise for the measurement C?

$$\text{signal} - \text{to} - \text{noise(C)} = \frac{E(C)}{\sqrt{VAR(C)}}$$

$$\text{signal} - \text{to} - \text{noise(C)} = \frac{SA}{\sqrt{\sigma_C^2}}$$

$$\text{signal} - \text{to} - \text{noise(C)} = \frac{SA}{\sqrt{A^2 + A^2\sigma_P^2}}$$

$$\text{signal} - \text{to} - \text{noise(C)} = \frac{S}{\sqrt{1 + \sigma_P^2}}$$

c) For what minimum value of S will the signal-to-noise exceed 100?

$$100 = \frac{S}{\sqrt{1 + \sigma_P^2}}$$

$$S = 100\sqrt{1 + \sigma_P^2}$$

3. Consider an imaging system using a lens with focal length 4cm having an image plane 6cm behind the lens. Assume the lens diameter is 2cm.

a) How far in front of the lens on the optical axis of the system must we place a point to get an image without blur?

Focal length = f = 4cm Image plane = z' = 6cm

Lens Equation:

$$\frac{1}{z'} + \frac{1}{-z} = \frac{1}{f}$$

$$\frac{1}{6} + \frac{1}{-z} = \frac{1}{4}$$

$$\frac{1}{6} + \frac{-1}{4} = \frac{1}{z}$$

$$\frac{4}{24} + \frac{-6}{24} = \frac{1}{z}$$

$$\frac{-2}{24} = \frac{1}{z}$$

$$z = \frac{24}{-2}$$

$$z = -12\ cm$$

b) Suppose the image plane has an active area of 2cm × 2cm which is partitioned into 500 × 500 square potential wells (collection sites). Suppose the point in part a) images without blur to the center of a potential well. How far can we move the point in focus towards the lens before the image extends to more than one potential well.

Image plane = z' = 6cm Lens diameter = d = 2cm

Blur diameter = $\frac{2\ cm}{500\ squares} = \frac{0.004\ cm}{square} = 0.004\ cm$ Image blur location = $\overline{z'}$

Image Blur Equation:

$$\frac{b}{d} = \frac{|\overline{z'} - z'|}{\overline{z'}}$$

$$\frac{0.004}{2} = \frac{|\overline{z'} - 6|}{\overline{z'}}$$

$$0.002\overline{z'} = |\overline{z'} - 6|$$

$$\pm 0.002\overline{z'} = \overline{z'} - 6$$

$$0.002\overline{z'} = \overline{z'} - 6 \qquad -0.002\overline{z'} = \overline{z'} - 6$$

$$-0.998\overline{z'} = -6 \qquad -1.002\overline{z'} = -6$$

$$\overline{z'} = 6.012\ cm \qquad \overline{z'} = 5.988\ cm$$

You can move the point in focus *z'* up to 0.012cm towards the image extends to more than one potential well. (assuming it is the edge where both CCDs meet)

4. Assume that we have a CCD camera system that is cooled so that noise due to dark current is negligible. Then digitized pixel values will be given by

$$D = (S + N_A + N_P)A + N_Q$$

where S is the signal in electrons, N_A is the zero mean amplifier noise source with variance σ_A^2, N_P is the zero mean photon noise with variance S, A is the gain of the amplifier, and N_Q is the zero mean quantization noise with variance σ_Q^2. If we assume that the three noise sources are independent, show that for a constant signal level S the expected value of D is

$$\mu = SA$$

and the variance of D is given by

$$\sigma_D^2 = A\mu + \sigma_C^2$$

where

$$\sigma_C^2 = A^2\sigma_A^2 + \sigma_Q^2$$

S = signal of electrons (constant) A = amplifier gain (constant)

$E(N_A) = 0$ $VAR(N_A) = \sigma_A^2$

$E(N_P) = 0$ $VAR(N_P) = S$

$E(N_Q) = 0$ $VAR(N_Q) = \sigma_Q^2$

$$E(D) = \mu = E((S + N_A + N_P)A + N_Q)$$

$$\mu = E(S)A + E(N_A)A + E(N_P)A + E(N_Q)$$

$$\mu = (S)A + (0)A + (0)A + (0)$$

$$\mu = SA$$

$$VAR(D) = \sigma_D^2 = VAR((S + N_A + N_P)A + N_Q)$$

$$\sigma_D^2 = A^2VAR(S) + A^2VAR(N_A) + A^2VAR(N_P) + VAR(N_Q)$$

$$\sigma_D^2 = A^2(0) + A^2(\sigma_A^2) + A^2(S) + (\sigma_Q^2)$$

$$\sigma_D^2 = A^2\sigma_A^2 + A^2S + \sigma_Q^2$$

$$\sigma_D^2 = A\mu + \sigma_C^2$$

$$\sigma_C^2 = A^2\sigma_A^2 + \sigma_Q^2$$

Write a program that reads a digital image *I(x, y)* of size $N \times N$ where $N = 100$ and estimates μ by

$$\hat{\mu} = \frac{1}{N^2} \sum_{1 \le x \le N} \sum_{1 \le y \le N} I(x, y)$$

and σ_D^2 by

$$\hat{\sigma}_D^2 = \frac{1}{N^2 - 1} \sum_{1 \le x \le N} \sum_{1 \le y \le N} (I(x, y) - \hat{\mu})^2$$

Run your program to compute $\hat{\mu}$ and $\hat{\sigma}_D^2$ for each of four images provided by your TA. Plot $\hat{\sigma}_D^2$ versus $\hat{\mu}$ for these four points. Estimate A and $\hat{\sigma}_C^2$ using a least squares fit of the line given by (3) to your data. A program that shows how to read an image will be provided in lab. You are required to demonstrate your program to your TA during lab.

(Program in hw2_modified.c)