Jack Melcher

67574625

EECS 101

HW 8

1. Consider a pattern classification system where we want to determine if a coin is head up or tail up by measuring the brightness of the coin. The coin is equally likely to appear head up or tail up. We use a two bit imaging system, so that measured brightness takes on one of the four values in the range 0-3. By supervised learning, we determine that for a head we have the following probabilities of observing each brightness

P(0) = 0.1, P(1) = 0.5, P(2) = 0.3, P(3) = 0.1

and for a tail, we have the following probabilities of observing each brightness

P(0) = 0.0, P(1) = 0.1, P(2) = 0.6, P(3) = 0.3

a) Is perfect classification possible in this system? Explain your answer.

> No, perfect classification is not possible. There is a probability that for a given brightness the system will identify a coin that is Heads as Tails because of the priori probabilities for Heads and Tails and their conditional densities.

b) Suppose our system is presented with an unknown sample and we observe a brightness. For each of the four brightness levels, what is our best guess as to whether we have a head or tail? For each of these guesses, what is the probability of error?

Brightness: 0
Best guess is that we have Heads

$$\mathrm{P(error|0)} = \mathrm{P(Tails|0)} = \frac{p(0|Tails)P(Tails)}{p(0|Heads)P(Heads) + p(0|Tails)P(Tails)} = \frac{0}{(0.1)(0.5) + (0)(0.5)} = 0$$

Brightness: 1
Best guess is that we have Heads

$$\mathrm{P(error|1)} = \mathrm{P(Tails|1)} = \frac{p(1|Tails)P(Tails)}{p(1|Heads)P(Heads) + p(1|Tails)P(Tails)} = \frac{(0.1)(0.5)}{(0.5)(0.5) + (0.1)(0.5)}$$
$$= \frac{1}{6}$$

Brightness: 2
Best guess is that we have Tails

$$\mathrm{P(error|2)} = \mathrm{P(Heads|2)} = \frac{p(2|\mathrm{Heads})P(\mathrm{Heads})}{p(2|Heads)P(Heads) + p(2|Tails)P(Tails)} = \frac{(0.3)(0.5)}{(0.3)(0.5) + (0.6)(0.5)}$$
$$= \frac{1}{3}$$

Brightness: 3
Best guess is that we have Tails

$$\mathrm{P(error|3)} = \mathrm{P(Heads|3)} = \frac{p(3|\mathrm{Heads})P(\mathrm{Heads})}{p(3|Heads)P(Heads) + p(3|Tails)P(Tails)} = \frac{(0.1)(0.5)}{(0.1)(0.5) + (0.3)(0.5)}$$
$$= \frac{1}{4}$$

2. Consider a pattern classification problem where we would like to discriminate between two materials M1 and M2 using a measured color vector (R,G,B) of the unknown material. Assume that the a priori probability of M1 is 3/7 and that the a priori probability of M2 is 4/7. Suppose that the probability density for M1 is uniform over the cube $50 \le R \le 90$, $30 \le G \le 70$, $40 \le B \le 80$. Suppose that the probability density for M2 is uniform over the cube $70 \le R \le 120$, $30 \le G \le 80$, $50 \le B \le 100$.

a) What is the conditional pdf p(R,G,B|M1) as a function of (R,G,B)?

$$\mathrm{p}(\mathrm{R},\mathrm{G},\mathrm{B}|\mathrm{M1}) = \frac{1}{40}$$

b) What is the conditional pdf p(R,G,B|M2) as a function of (R,G,B)?

$$\mathrm{p}(\mathrm{R},\mathrm{G},\mathrm{B}|\mathrm{M2}) = \frac{1}{50}$$

c) What is the a posteriori probability P(M1|R,G,B) as a function of (R,G,B)?

$$\mathrm{P}(\mathrm{M1}|\mathrm{R},\mathrm{G},\mathrm{B}) = \frac{\mathrm{p}(\mathrm{R},\mathrm{G},\mathrm{B}|\mathrm{M1})P(\mathrm{M1})}{\mathrm{p}(\mathrm{R},\mathrm{G},\mathrm{B}|\mathrm{M1})P(\mathrm{M1}) + \mathrm{p}(\mathrm{R},\mathrm{G},\mathrm{B}|\mathrm{M2})P(\mathrm{M2})} = \frac{(1/40)(3/7)}{(1/40)(3/7) + (4/7)(1/50)} = \frac{15}{31}$$

d) What is the a posteriori probability P(M2|R,G,B) as a function of (R,G,B)?

$$\mathrm{P}(\mathrm{M2}|\mathrm{R},\mathrm{G},\mathrm{B}) = \frac{\mathrm{p}(\mathrm{R},\mathrm{G},\mathrm{B}|\mathrm{M2})P(\mathrm{M2})}{\mathrm{p}(\mathrm{R},\mathrm{G},\mathrm{B}|\mathrm{M1})P(\mathrm{M1}) + \mathrm{p}(\mathrm{R},\mathrm{G},\mathrm{B}|\mathrm{M2})P(\mathrm{M2})} = \frac{(1/50)(4/7)}{(1/40)(3/7) + (4/7)(1/50)} = \frac{16}{31}$$

e) What is the best guess for what material we are looking at as a function of (R,G,B)?

Best guess is M2

f) What is the probability of error as a function of (R,G,B) if we take the guess in part e)?

$$\mathrm{P}(\mathrm{error}|\mathrm{R},\mathrm{G},\mathrm{B}) = \mathrm{P}(\mathrm{M1}|\mathrm{R},\mathrm{G},\mathrm{B}) = \frac{15}{31}$$