Iacoviello (2005) Basic Replication

Jack Minchin

11/03/2021

1 Introduction

This is a simple replication of Iacoviello (2005). It explores the impact of changing loan-to-value (LTV) rates in response to house price, inflation, monetary policy and technology shocks.

2 Model

The model includes two types of households, savers and borrowers. Borrowers are relatively more impatient than saves to $\tilde{\beta} < \beta$. There are a continuum of final good firms who bundle intermediate goods, and intermediate goods firms who use labour to create an intermediate good that trades in a monopolistically competitive market. Intermediate firms are subject to a time-dependent pricing model, following Calvo (1983).

2.1 Households

2.1.1 Savers - Unconstrained Households

The unconstrained households are savers. Their maximisation problem is:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t^u + j \ln H_t^u - \frac{(L_t^u)^\eta}{\eta} \right), \tag{1}$$

where $1/(\eta - 1)$ is the labour supply elasticity. $\eta > 0$ and $j > 0$. H_u is the amount of 'housing services' consumers by the savers, and j denotes a weighting term for household. Everything else is standard.

The budget constraint is:

$$C_t^u + q_t\left(H_t^u - H_{t-1}^u\right) + b_t^u \le w_t^u L_t^u + \frac{R_{t-1}b_{t-1}^u}{\pi_t} + F_t \tag{2}$$

The FOC for savers are:

$$\frac{j}{H_t^u} = \frac{1}{C_t^u}q_t - \beta\frac{1}{C_{t+1}^u}q_{t+1} \tag{3}$$

$$w_t^u = L_t^{u\eta-1}C_t^u \tag{4}$$

$$\frac{1}{C_t^u} = \beta E_t\left(\frac{R_t}{\pi_{t+1}C_{t+1}^u}\right) \tag{5}$$

2.1.2 Collateral Constrained Households

We suppose the constrained households are relatively more impatient than the unconstrained households so that $\tilde{\beta} < \beta$. The problem of these households is:

$$\max E_0\sum_{t=0}^{\infty}\widetilde{\beta}^t\left(\ln C_t^c + j\ln H_t^c - \frac{(L_t^c)^\eta}{\eta}\right) \tag{6}$$

subject to the budget constraint:

$$C_t^c + q_t\left(H_t^c - H_{t-1}^c\right) + \frac{R_{t-1}^c b_{t-1}^c}{\pi_t} \le w_t^c L_t^c + b_t^c \tag{7}$$

and the collateral constraint:

$$b_t^c \le \frac{m}{R_t}E_t\pi_{t+1}q_{t+1}H_t^c \tag{8}$$

This household has the FOCs:

$$\frac{1}{C_t^c} = \widetilde{\beta}E_t\left(\frac{R_t}{\pi_{t+1}C_{t+1}^c}\right) + \lambda_t R_t \tag{9}$$

$$w_t^c = (L_t^c)^{\eta-1}C_t^c \tag{10}$$

$$\frac{j}{H_t^c} = \frac{1}{C_t^c}q_t - \widetilde{\beta}E_t\frac{1}{C_{t+1}^c}q_{t+1} - \lambda_t m E_t q_{t+1}\pi_{t+1} \tag{11}$$

Because the collateral constraint is binding, consumption for constrained households can be written as:

$$C_t^c = w_t^c L_t^c + b_t^c + q_t\left(H_{t-1}^c - H_t^c\right) - \frac{R_{t-1}b_{t-1}^c}{\pi_t} \tag{12}$$

and the FOC becomes:

$$\frac{j}{H_t^c} = \frac{1}{C_t^c}\left(q_t - \frac{mE_tq_{t+1}\pi_{t+1}}{R_t^c}\right) - \widetilde{\beta}E_t\frac{1}{C_{t+1}^c}(1-m)q_{t+1} \tag{13}$$

2.2 Firms

2.2.1 Final Goods Firms

There are a continuum of final goods firms bundle together intermediate goods to produce a final good.

$$Y_t = \left[\int_0^1 Y_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{14}$$

subject to the market clearing condition.

$$Y_t = C_t = C_t^u + C_t^c \tag{15}$$

The demand for the intermediate goods can be found to be:

$$Y_{it}^d = \left[\frac{P_t}{P_{it}}\right]^{\frac{1}{1-q}} Y_t \tag{16}$$

2.2.2 Intermediate Goods Firms

Intermediate goods firms combine labour to create an intermediate good that is traded on a monopolistically competitive market. Following Cavlo (1983), only a proportion ω of firms can change their price each period.

The intermediate firms face a simple production function:

$$Y_{it} = A_t L_t^{u\eta} + L_t^{c1-\eta} \tag{17}$$

where A_t follows an AR(1) procces. The profit function for the intermediate goods firms is:

$$\min_{L^u, L^c} \Pi_{it} = w_c L_u + w_c L_c \tag{18}$$

subject to the production function.

From this, we can find the conditions for wages:

$$w_{ut} = \eta(-A_t)\frac{1}{X}L^{\eta-1} \tag{19}$$

and:

$$w_{ct} = (\eta - 1)(-A_t)\frac{1}{X}L^{-\eta} \tag{20}$$

2.3 Monetary Policy

Closing the model with the Taylor rule and the New Keynesian Phillips Curve.

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{\nu}_t \tag{21}$$

κ is $\frac{(1-\omega)(1-\beta\omega)}{\omega}$.

$$r_t = \phi_r r_{t-1} + (1 - \phi_r)\left((1 + \phi_\pi)\,\pi_t + \phi_z z_t\right) + e_t \tag{22}$$

3 Results

3.1 House Price Shocks

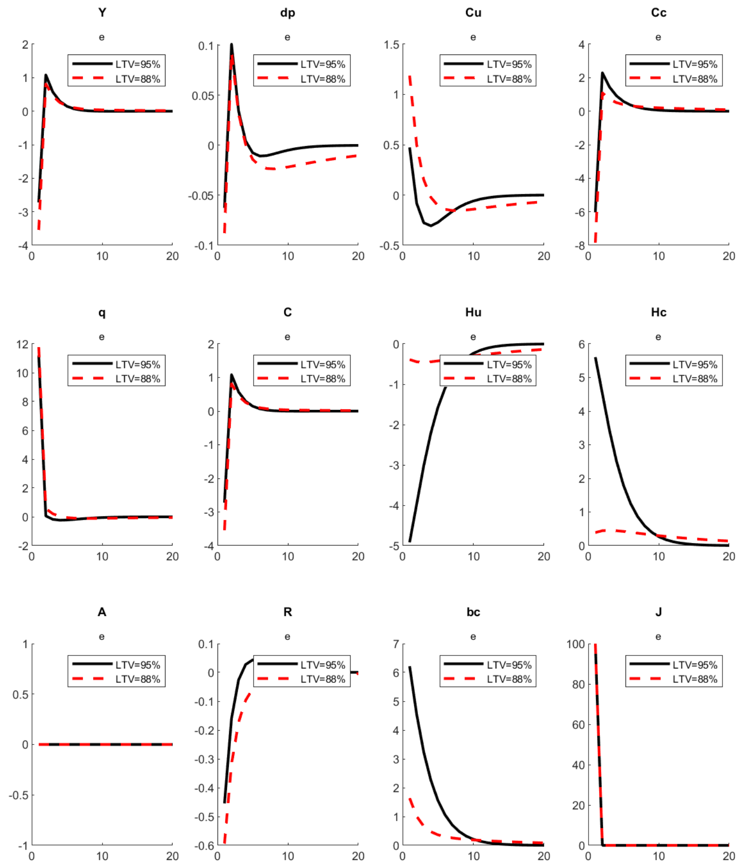

Figure 1

3.1.1 Monetary Policy Shock

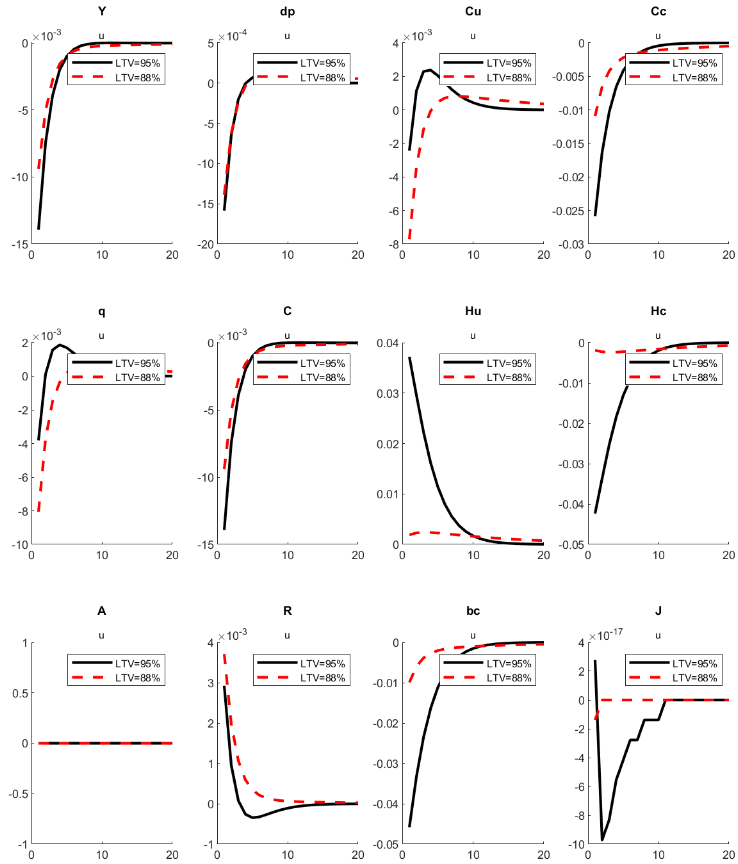

Figure 2

3.1.2 Technology Shock

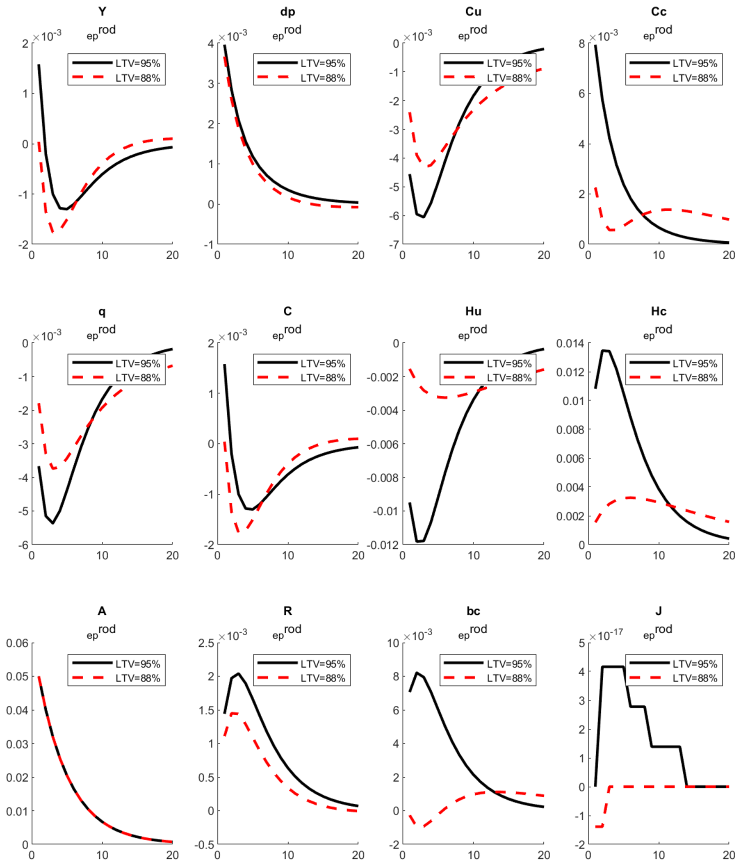

Figure 3

3.1.3 Cost-Push Inflationary Shock

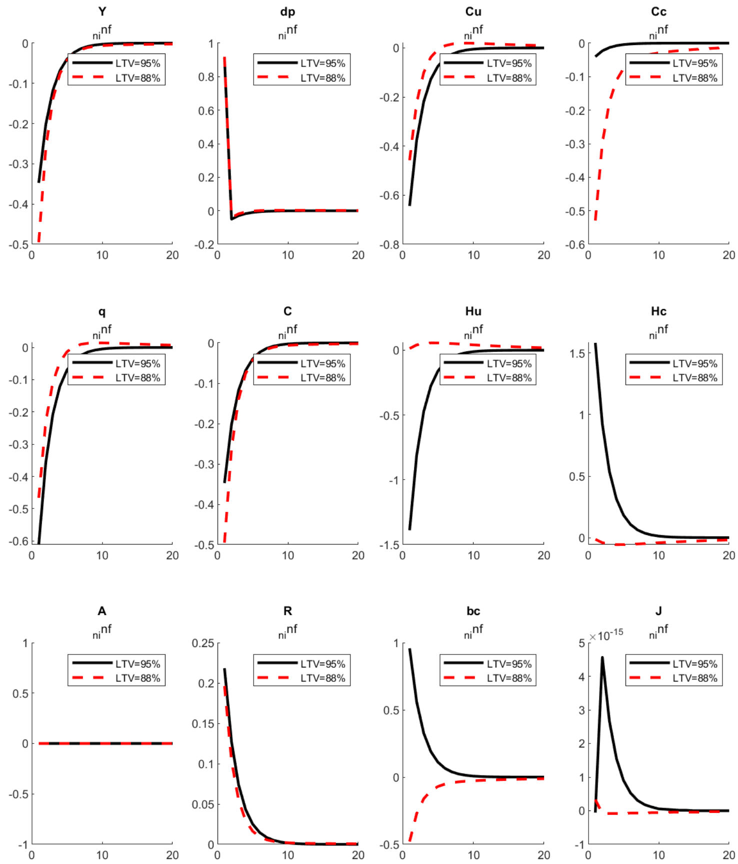

Figure 4

References

Calvo, G. A.

1983. Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 12(3):383–398.

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2005. House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle. *American Economic Review*, 95(3):739–764.

Appendix

Households

The Lagrangian for the unconstrained household problem is:

$$\begin{aligned}
\mathcal{L} = \ln C_t^u + j \ln H^u - \frac{(L_t^u)^\eta}{\eta} + \dots \\
-\lambda_t \Bigg(C_t^u + q_t \left(H_t^u - H_{t-1}^u \right) + b_t^u - w_t^u L_t^u + \frac{R_{t-1} b_{t-1}^u}{\pi_t} + F_t \Bigg) \\
-\beta \lambda_{t+1} \Bigg(C_{t+1}^u + q_{t+1} \left(H_{t+1}^u - H_t^u \right) + b_{t+1}^u - w_{t+1}^u L_{t+1}^u + \frac{R_t b_t^u}{\pi_{t+1}} + F_t + 1 \Bigg)
\end{aligned} \tag{23}$$

To get the FOCs, we maximise w.r.t consumption, labour, housing and the Lagrange multiplier to get:

$$\frac{j}{H_t^u} = \frac{1}{C_t^u} q_t - \beta \frac{1}{C_{t+1}^u} q_{t+1} \tag{24}$$

$$w_t^u = L_t^{u\eta-1} C_t^u \tag{25}$$

$$\frac{1}{C_t^u} = \beta E_t \left(\frac{R_t}{\pi_{t+1} C_{t+1}^u} \right) \tag{26}$$

Intermediate Firms

$$\min_{L^u, L^c} \Pi_{it} = w_u L_u + w_c L_c \tag{27}$$

subject to:

$$Y_{it} = A_t L_t^{u\gamma} L_t^{c1-\gamma} \tag{28}$$

$$\mathcal{L} = w_u L_u + w_c L_c - Z_t \left(Y_{it} - A_t {L_{ut}}^\gamma {L_{ct}}^{1-\gamma} \right) \tag{29}$$

$$\frac{\partial \mathcal{L}}{L_u} = 0 \rightarrow w_{ut} = \gamma A_t \frac{1}{X} L_{ct}^{(1-\gamma)} L_{ut}^{\gamma-1} \tag{30}$$

$$\frac{\partial \mathcal{L}}{L_c} = 0 \rightarrow w_{ct} = (\gamma - 1)(-A_t) \frac{1}{X} L_{ct}^{-\gamma} L_{ut}^{\gamma} \tag{31}$$