1 Introduction

This is a simple replication of Iacoviello (2005). It studies the responses to shocks in the marginal rate of substitution j, generating a rise in housing prices. Additionally, it studies responses monetary policy shocks under a number of different policies.

2 Model

2.1 Households

2.1.1 Unconstrained Households

The unconstrained households are savers. Their maximisation problem is:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t^u + j \ln H_t^u - \frac{(L_t^u)^\eta}{\eta} \right), \tag{1}$$

where $1/(\eta - 1)$ is the labour supply elasticity. $\eta > 0$ and $j > 0$. H_u is the amount of 'housing services' consumers by the savers, and j denotes a weighting term for household. Everything else is standard.

The budget constraint is:

$$C_t^u + q_t \left(H_t^u - H_{t-1}^u \right) + b_t^u \leq w_t^u L_t^u + \frac{R_{t-1} b_{t-1}^u}{\pi_t} + F_t \tag{2}$$

First Order Conditions The household has the FOCs:

$$\frac{j}{H_t^u} = \frac{1}{C_t^u} q_t - \beta \frac{1}{C_{t+1}^u} q_{t+1} \tag{3}$$

$$w_t^u = L_t^{u\eta-1} C_t^u \tag{4}$$

$$\frac{1}{C_t^u} = \beta E_t \left(\frac{R_t}{\pi_{t+1} C_{t+1}^u} \right) \tag{5}$$

2.1.2 Collateral Constrained Households

We suppose the constrained households are relatively more impatient than the unconstrained households so that $\tilde{\beta} > \beta$. The problem of these households is:

$$\max E_0 \sum_{t=0}^{\infty} \widetilde{\beta}^t \left(\ln C_t^c + j \ln H_t^c - \frac{(L_t^c)^\eta}{\eta} \right) \tag{6}$$

subject to the budget constraint:

$$C_t^c + q_t \left(H_t^c - H_{t-1}^c \right) + \frac{R_{t-1}^c b_{t-1}^c}{\pi_t} \leq w_t^c L_t^c + b_t^c \tag{7}$$

and the collateral constraint:

$$b_t^c \leq \frac{m}{R_t} E_t \pi_{t+1} q_{t+1} H_t^c \tag{8}$$

This household has the FOCs:

$$\frac{1}{C_t^c} = \widetilde{\beta} E_t \left(\frac{R_t}{\pi_{t+1} C_{t+1}^c} \right) + \lambda_t R_t \tag{9}$$

$$w_t^c = (L_t^c)^{\eta-1} C_t^c \tag{10}$$

$$\frac{j}{H_t^c} = \frac{1}{C_t^c} q_t - \widetilde{\beta} E_t \frac{1}{C_{t+1}^c} q_{t+1} - \lambda_t m E_t q_{t+1} \pi_{t+1} \tag{11}$$

Because the collateral constraint is binding, consumption for constrained households can be written as:

$$C_t^c = w_t^c L_t^c + b_t^c + q_t \left(H_{t-1}^c - H_t^c \right) - \frac{R_{t-1} b_{t-1}^c}{\pi_t} \tag{12}$$

and the FOC becomes:

$$\frac{j}{H_t^c} = \frac{1}{C_t^c} \left(q_t - \frac{m E_t q_{t+1} \pi_{t+1}}{R_t^c} \right) - \widetilde{\beta} E_t \frac{1}{C_{t+1}^c} (1-m) q_{t+1} \tag{13}$$

2.2 Firms

2.2.1 Final Goods Firms

There are a continuum of final goods firms bundle together intermediate goods to produce a final good.

$$Y_t = \left[\int_0^1 Y_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{14}$$

subject to the market clearing condition.

$$Y_t = C_t = C_t^u + C_t^c \tag{15}$$

The profit function for final goods firms is:

$$\Pi_t = P_t Y_t - \int \Psi_t \Phi_t di = P_t \left[\int \Phi_t^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon-1}{\epsilon}} - \int \Psi_t \Phi_t di \tag{16}$$

Where Φ_t are intermediate goods and Ψ is the price of the intermediate goods. The firms take prices for given so, maximising with respect to Φ, we get :

$$P_t \frac{1}{q} \left[\int \Phi_t^q di \right]^{\frac{\epsilon-1}{\epsilon}} q \Phi_t^{q-1} - \Psi_t = P_t Y_t^{1-q} \Phi_t^{q-1} - \Psi_t = 0 \tag{17}$$

which can be arranged to yield the demand curve for the intermediate good:

$$\Phi_t^D = \left[\frac{P_t}{\Phi_t} \right]^{\frac{\epsilon-1}{\epsilon}} Y_t \tag{18}$$

taking logs we can get:

$$\phi_t^D = [\hat{p}_t - \phi_t] \left(\frac{\epsilon - 1}{\epsilon} \right) + \hat{y}_t \tag{19}$$

2.2.2 Intermediate Goods Firms

Intermediate goods firms combine labour to create an intermediate good that is traded on a monopolistically competitive market. Following Cavlo (1983), only a proportion ω of firms can change their price each period.

The intermediate firms face a simple production function:

$$\Phi_t = L_t^{u\eta} + L_t^{c1-\eta} \tag{20}$$

The profit function for the intermediate goods firms is:

$$\max_{L^u, L^c} \Pi it = w_c L_u + w_c L_c \tag{21}$$

Intermediate firms choose their prices as a discount flow of future profits.

$$E_t \sum_{j=0}^{\infty} \beta^j \pi_{t-j} = E_t \sum_{j=0}^{\infty} \beta^j \left[\Psi_{t+j} - W_{t+j} \right] \left[\frac{P_{t+j}}{\Psi_{t+j}} \right]^{\frac{1}{1-q}} Y_{t+j} \tag{22}$$

To set the optimal price at period t, the producer needs to consider profits in each future period before they can next change their prices. The probability that the firm can change its price after j periods is ω^j, so the part of future profits that depends on today's decision P_t^* is

$$E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \left[P_t^* - W_{t+j}\right] \left[\frac{P_{t+j}}{P_t^*}\right]^{\frac{1}{1-q}} Y_{t+j} \tag{23}$$

this equation can be rearranged to give an expression for the optimal price:

$$\frac{P_t^*}{P_t} = \left(\frac{1}{q}\right) \frac{E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \frac{W_{t+j}}{P_{t+j}} \left[\frac{P_{t+j}}{P_t}\right]^{\frac{1}{1-q}} Y_{t+j}}{E_t \sum_{j=0}^{\infty} (\alpha\beta)^j \left[\frac{P_{t+j}}{P_t}\right]^{\frac{q}{1-q}} Y_{t+j}} \tag{24}$$

We can simplify and log linearise to get:

$$p_{it}^* = \mu + (1 - \alpha\beta) \sum_{j=0}^{\infty} (\alpha\beta)^j E_t mc_{t+j} \tag{25}$$

2.3 Monetary Policy

Closing the model with the Taylor rule and the New Keynesian Phillips Curve.

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{\nu}_t \tag{26}$$

κ is $\frac{(1-\omega)(1-\beta\omega)}{\omega}$.

$$r_t = \phi_r r_{t-1} + (1 - \phi_r)\left((1 + \phi_\pi)\pi_t + \phi_z z_t\right) + e_t \tag{27}$$

3 Results

3.1 House Price Shocks

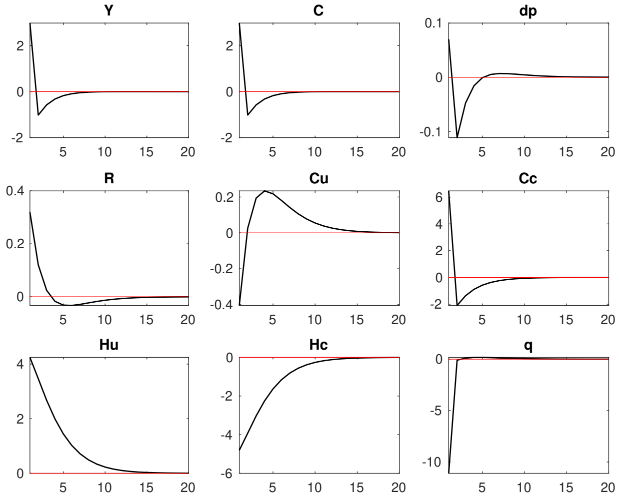

Figure 1

3.1.1 Monetary Policy Shock

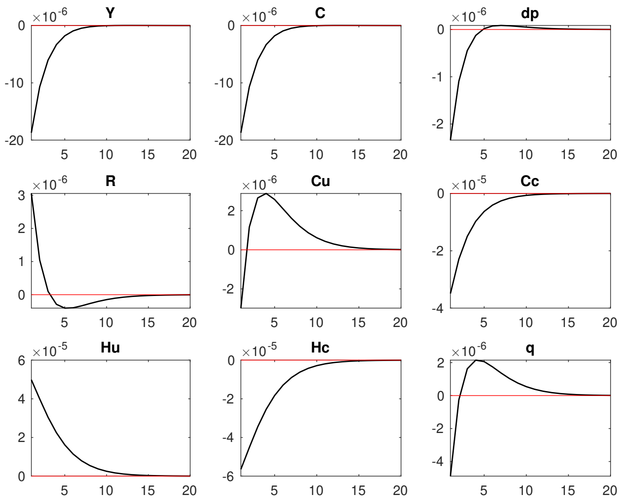

Figure 2

3.1.2 Technology Shock

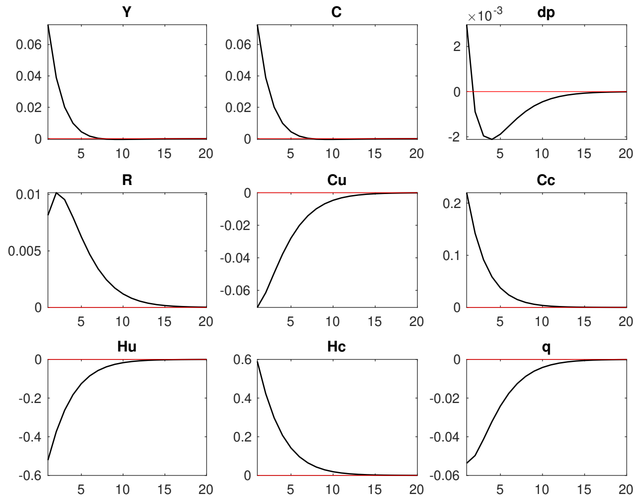

Figure 3

3.1.3 Cost-Push Inflationary Shock

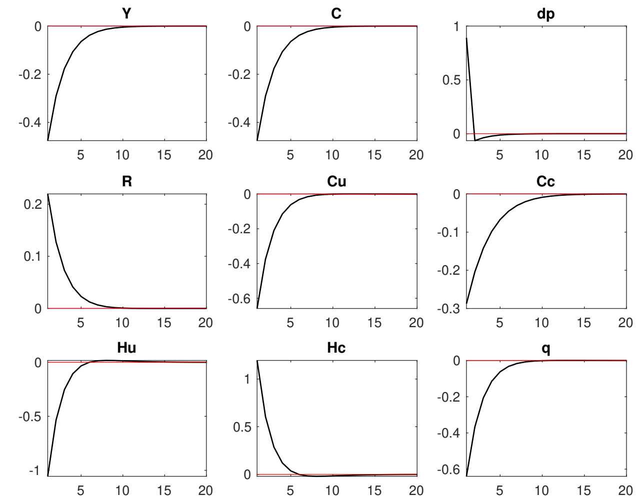

Figure 4

4 Appendix

4.1 Households

The Lagrangian for the unconstrained household problem is:

$$\begin{aligned}\mathcal{L} = \ln C_t^u + j \ln H^u - \frac{(L_t^u)^\eta}{\eta} + \dots \\ -\lambda_t \Bigg(C_t^u + q_t \left(H_t^u - H_{t-1}^u \right) + b_t^u - w_t^u L_t^u + \frac{R_{t-1} b_{t-1}^u}{\pi_t} + F_t \Bigg) \\ -\beta \lambda_{t+1} \Bigg(C_{t+1}^u + q_{t+1} \left(H_{t+1}^u - H_t^u \right) + b_{t+1}^u - w_{t+1}^u L_{t+1}^u + \frac{R_t b_t^u}{\pi_{t+1}} + F_t + 1 \Bigg)\end{aligned} \tag{28}$$

To get the FOCs, we maximise w.r.t consumption, labour, housing and the Lagrange multiplier to get:

$$\frac{j}{H_t^u} = \frac{1}{C_t^u} q_t - \beta \frac{1}{C_{t+1}^u} q_{t+1} \tag{29}$$

$$w_t^u = L_t^{u\eta - 1} C_t^u \tag{30}$$

$$\frac{1}{C_t^u} = \beta E_t \left(\frac{R_t}{\pi_{t+1} C_{t+1}^u} \right) \tag{31}$$