1 Introduction

Balancing for legged robots, which is in fact preventing them from falling over, has always the highest priority in controlling their motions. This problem becomes much more challenging if the supporting surface (i.e. usually the ground) is not "*rigid*". In reality, there is no completely rigid surface but in practice we can assume a surface to be rigid if it is stiff enough (i.e. deflection is negligible). Many of existing legged robots are able to keep their balance on rigid surfaces but still most of them have difficulties in dealing with compliant supporting surfaces. To balance a legged robot on a soft surface (e. g. on a thick carpet), the compliance of the contact has to be considered by the controller, otherwise the robot fails to balance and falls over.

Compliant contacts between robots and their environments have been studied by some researchers in the area of humanoids [5], grasping [17], animated characters [12], etc. However, there have not been much efforts on balancing legged robots on compliant surfaces. Here, we tackle this problem and introduce a momentum-based balancing controller which takes into account the effects of non-rigid contacts between the robot and its environment. A momentum-based controller, controls both linear momentum and angular momentum about the center of mass (CoM) of the robot. This type of controller is first suggested by Goswami and Kallem [9] and then extensively used in recent years by other researchers [1], [3], [10], [14], [15].

Here, we propose a balance control strategy for a legged robot which has multiple rigid and compliant contacts with its environment. This controller regulates both linear momentum and angular momentum about the center of mass of the robot by controlling the contact forces at both rigid and soft contact surfaces. Assuming that contact forces at the compliant surfaces are known (i.e. via force-torque sensors) at the current instant, desired contact forces at the rigid contacts are calculated in order to provide the required rate of change of the robot's momentum. On the other hand, since compliant contact forces are dependent to deformations (and their rates) in the contact areas, the change of compliant forces are functions of robot's states and joint accelerations. Therefore, by converting the balancing problem to an optimization problem, the controller computes desired joint torques in order to provide both desired rigid contact forces (for balance control) and joint acceleration (to regulate the soft contact forces).

Note that, to implement the proposed method in practice, stiffness and damping coefficients of the contact model have to be estimated beforehand by using contact model parameter estimation methods such as [6], [7], [8].

2 Control Strategy

Let $\mathbf{v}$ denote generalized joint velocities of a floating base robot. Therefore, motion equations for this robot will be

$$\mathbf{M}\dot{\mathbf{v}} + \mathbf{C}\mathbf{v} + \mathbf{g} = \mathbf{S}^T\tau + \mathbf{J}_s^T\mathbf{f}_s + \mathbf{J}_r^T\mathbf{f}_r\,, \tag{1}$$

where $\mathbf{M}$ is the joint-space inertia matrix, $\mathbf{C}\mathbf{v}$ is the vector of Coriolis and centrifugal forces, $\mathbf{g}$ is the vector of the gravity forces, $\mathbf{S}$ is the selection matrix for actuated joints, τ is the vector of joint torques, $\mathbf{J}_s$ is the Jacobian of the bodies in contact with soft surfaces, $\mathbf{f}_s$ is the vector of the contact forces at the soft surfaces, $\mathbf{J}_r$ is the Jacobian of the bodies in contact with rigid surfaces and $\mathbf{f}_r$ is the vector of the contact forces at the rigid surfaces. Due to the rigid contacts, we have

$$\mathbf{J}_r\mathbf{v} = \mathbf{0} \implies \dot{\mathbf{J}}_r\mathbf{v} + \mathbf{J}_r\dot{\mathbf{v}} = \mathbf{0}\,. \tag{2}$$

Therefore, from equation (1) we conclude that

$$\tau = (\mathbf{J}_r\mathbf{M}^{-1}\mathbf{S}^T)^{\#}(-\dot{\mathbf{J}}_r\mathbf{v} + \mathbf{J}_r\mathbf{M}^{-1}\mathbf{C}\mathbf{v} + \mathbf{J}_r\mathbf{M}^{-1}\mathbf{g} - \mathbf{J}_r\mathbf{M}^{-1}\mathbf{J}_s^T\mathbf{f}_s - \mathbf{J}_r\mathbf{M}^{-1}\mathbf{J}_r^T\mathbf{f}_r^{des}) + \mathbf{N}\tau_0\,, \tag{3}$$

where superscript $^{\#}$ denotes the generalized weighted pseudo-inverse and $\mathbf{N}$ is the null-space of $\mathbf{J}_r\mathbf{M}^{-1}\mathbf{S}^T$ as

$$\mathbf{N} = \mathbf{I} - (\mathbf{J}_r\mathbf{M}^{-1}\mathbf{S}^T)^{\#}\mathbf{J}_r\mathbf{M}^{-1}\mathbf{S}^T\,. \tag{4}$$

To control the momentum of the robot for its balancing motion, desired force at the rigid contact can be calculated by using

$$\mathbf{f}_r^{des} = \mathbf{A}_r^{\#}(\dot{\mathbf{h}}^{des} - \mathbf{W} - \mathbf{A}_s\mathbf{f}_s) + \mathbf{N}_{A_r}\mathbf{f}_{r_0}\,, \tag{5}$$

where $\mathbf{W} = [0, 0, -9.81, 0, 0, 0]^T$ is the force vector due to the weight of the robot, $\mathbf{h}$ is the vector of linear and angular momentum of the robot about its center of mass (CoM), and $\mathbf{A}_s$ and $\mathbf{A}_r$ are the matrices that transform the soft and rigid contact forces from their local coordinate frames to the forces and moments around the CoM, respectively. Also $\mathbf{N}_{A_r} = \mathbf{I} - \mathbf{A}_r^{\#}\mathbf{A}_r$ is the null-space of $\mathbf{A}_r$. Here, $\mathbf{f}_{r_0}$ can be used to satisfy the constraints (i.e. unilaterality of the normal force and friction cone) on the rigid contact forces.

The null-space term in equation (3) is available to be used for task space control (or impedance control) and also for controlling the soft contact force. For task control we have

$$\tau_t = \mathbf{K}_q(\mathbf{q}_{des} - \mathbf{q}) + \mathbf{K}_{qd}(\dot{\mathbf{q}}_{des} - \dot{\mathbf{q}}) + \ddot{\mathbf{q}}_{des}\,, \tag{6}$$

where $\mathbf{q}$ is the vector of joint angles and $\mathbf{K}_q$ and $\mathbf{K}_{qd}$ are the gains of the controller.

Since compliant contact forces are functions of surface deformations, we do not have any control on them at the current instant and they are usually given via force-torque sensors. However, it is possible to control those forces in the next instant by controlling the acceleration of the contact points. Here, we assume a linear relationship between soft contact forces at the next instant (i.e. $\hat{\mathbf{f}}_s$) and generalized accelerations as

$$\hat{\mathbf{f}}_s = \mathbf{f}_s + \mathbf{A}\dot{\mathbf{v}} + \mathbf{b}\,, \tag{7}$$

where $\mathbf{A}$ and $\mathbf{b}$ are described in the next section. To find a relationship between $\dot{\mathbf{v}}$ and τ, we first solve equation (1) for $\mathbf{f}_r$ as

$$\mathbf{f}_r = (\mathbf{J}_r\mathbf{M}^{-1}\mathbf{J}_r^T)^{-1}(-\dot{\mathbf{J}}_r\mathbf{v} + \mathbf{J}_r\mathbf{M}^{-1}\mathbf{C}\mathbf{v} + \mathbf{J}_r\mathbf{M}^{-1}\mathbf{g} - \mathbf{J}_r\mathbf{M}^{-1}\mathbf{J}_s^T\mathbf{f}_s - \mathbf{J}_r\mathbf{M}^{-1}\mathbf{S}^T\tau)\,. \tag{8}$$

By defining

$$(\mathbf{J}_r\mathbf{M}^{-1})^{\#} = \mathbf{J}_r^T(\mathbf{J}_r\mathbf{M}^{-1}\mathbf{J}_r^T)^{-1}\,, \tag{9}$$

and replacing (8) into (1), we will have

$$\dot{\mathbf{v}} = \mathbf{M}^{-1}\mathbf{N}_{JM}(-\mathbf{C}\mathbf{v} - \mathbf{g} + \mathbf{S}^T\tau + \mathbf{J}_s^T\mathbf{f}_s) - \mathbf{M}^{-1}(\mathbf{J}_r\mathbf{M}^{-1})^{\#}\dot{\mathbf{J}}_r\mathbf{v}\,, \tag{10}$$

where

$$\mathbf{N}_{JM} = \mathbf{I} - (\mathbf{J}_r\mathbf{M}^{-1})^{\#}\mathbf{J}_r\mathbf{M}^{-1}\,. \tag{11}$$

Hence, $\dot{\mathbf{v}}$ is linearly dependent on τ and we can write

$$\dot{\mathbf{v}} = \alpha\tau + \beta\,, \tag{12}$$

where

$$\alpha = \mathbf{M}^{-1}\mathbf{N}_{JM}\mathbf{S}^T\tau\,, \tag{13}$$

and

$$\beta = \mathbf{M}^{-1}\mathbf{N}_{JM}(-\mathbf{C}\mathbf{v} - \mathbf{g} + \mathbf{J}_s^T\mathbf{f}_s) - \mathbf{M}^{-1}(\mathbf{J}_r\mathbf{M}^{-1})^{\#}\dot{\mathbf{J}}_r\mathbf{v}\,. \tag{14}$$

Therefore, by replacing equation (12) into (7) we have

$$\hat{\mathbf{f}}_s = (\mathbf{A}\alpha)\tau + (\mathbf{f}_s + \mathbf{b} + \mathbf{A}\beta)\,, \tag{15}$$

which is a linear relationship between $\hat{\mathbf{f}}_s$ and τ.

Now, we can describe the balancing problem as an optimization problem which its objective is to find τ in order to minimize the following function as

$$\tau = ArgMin(w_1||\tau_0 - \tau_t|| + w_2||\hat{\mathbf{f}}_s - \mathbf{f}_s^{des}||) \tag{16}$$

where $\mathbf{f}_s^{des}$ is the desired forces for the soft contacts and w_1 and w_2 are the values representing the priority of each term. By setting $\mathbf{f}_s^{des}$ we can determine how much we want to rely on the soft contacts during a balancing motion. This optimization problem is subject to the constraints on the saturation limits of the actuators and also the constraints on the soft contact forces (friction cones and unilaterality of the normal forces). Note that contact force constraints for the rigid contacts should be already considered in computing $\mathbf{f}_r$ from (5).

The next section, explains the estimation of the soft contact forces to obtain (7), and also constraints on $\hat{\mathbf{f}}_s$ that should be considered in the above optimization problem.

3 Estimation of the soft contact force

Assume that the body which is in contact with a soft surface is labeled with B_c. We characterize the contact surface of B_c by m fictitious contact points on this body. Let $\mathbf{p}_i$ denote the position of the ith contact point ($i = 1, 2, \ldots, m$) in the world frame. Therefore,

$$\mathbf{p}_i = \mathbf{p} + \mathbf{R}\mathbf{r}_i\,, \tag{17}$$

where $\mathbf{p}$ is the position of the origin of the local frame of B_c with respect to the world frame, $\mathbf{R}$ is the rotation matrix of B_c with respect to the world frame and $\mathbf{r}_i$ is the position of ith contact point in the local frame of B_c. So

$$\dot{\mathbf{p}}_i = \dot{\mathbf{p}} + \dot{\mathbf{R}}\mathbf{r}_i = \dot{\mathbf{p}} - (\mathbf{R}\mathbf{r}_i)^\wedge\omega = [\mathbf{I}_{3\times3} \;\; -(\mathbf{R}\mathbf{r}_i)^\wedge]\mathbf{J}_s\mathbf{v}\,, \tag{18}$$

where ω is the angular velocity of B_c and $()^\wedge$ represents the skew symmetric matrix. We also have

$$\ddot{\mathbf{p}}_i = \ddot{\mathbf{p}} + \ddot{\mathbf{R}}\mathbf{r}_i = \ddot{\mathbf{p}} - (\mathbf{R}\mathbf{r}_i)^\wedge\dot{\omega} + (\omega)^\wedge(\omega)^\wedge\mathbf{R}\mathbf{r}_i = [\mathbf{I}_{3\times3} \;\; -(\mathbf{R}\mathbf{r}_i)^\wedge](\dot{\mathbf{J}}_s\mathbf{v} + \mathbf{J}_s\dot{\mathbf{v}}) + (\omega)^\wedge(\omega)^\wedge\mathbf{R}\mathbf{r}_i\,. \tag{19}$$

According to contact mechanics [13] and its applications in robotics [2, 4, 11, 16], there is a non-linear relationship between compliant contact force and deformation and the rate of the deformation of the surface. However, we can assume a locally linear relationship between the change of the contact force and the change of the deformation and its rate. Therefore, according to this assumption, we can write

$$\delta\mathbf{f}_i = \mathbf{K}\delta\mathbf{p}_i + \mathbf{D}\delta\dot{\mathbf{p}}_i\,, \tag{20}$$

where $\delta\mathbf{f}_i$ and $\delta\mathbf{p}_i$ are the changes of the contact force and the deformation at the ith contact point, respectively, and $\mathbf{K}$ and $\mathbf{D}$ are 3×3 matrices of the coefficients of stiffness and damping, respectively. By using a linear integration method, we can estimate $\delta\mathbf{p}_i$ and $\delta\dot{\mathbf{p}}_i$ as

$$\delta\mathbf{p}_i = \dot{\mathbf{p}}_i\delta t + \frac{1}{2}\ddot{\mathbf{p}}_i\delta t^2\,, \tag{21}$$

and

$$\delta \dot{\mathbf{p}}_i = \ddot{\mathbf{p}}_i \delta t\,, \tag{22}$$

where δt is the sampling time. Hence, by substituting (18) and (19) into (21) and (22), we will have

$$\delta \mathbf{f}_i = \mathbf{A}_i \dot{\mathbf{v}} + \mathbf{b}_i\,, \tag{23}$$

where

$$\mathbf{A}_i = \frac{1}{2}\delta t^2 [\mathbf{K} \quad -\mathbf{K}(\mathbf{R}\mathbf{r}_i)^\wedge]\mathbf{J}_s + \delta t[\mathbf{D} \quad -\mathbf{D}(\mathbf{R}\mathbf{r}_i)^\wedge]\mathbf{J}_s\,, \tag{24}$$

and

$$\begin{aligned}\mathbf{b}_i \;=\;& \delta t[\mathbf{K} \quad -\mathbf{K}(\mathbf{R}\mathbf{r}_i)^\wedge]\mathbf{J}_s\mathbf{v} + \frac{1}{2}\delta t^2[\mathbf{K} \quad -\mathbf{K}(\mathbf{R}\mathbf{r}_i)^\wedge]\dot{\mathbf{J}}_s\mathbf{v} \\ & + \frac{1}{2}\delta t^2 \mathbf{K}(\omega)^\wedge(\omega)^\wedge \mathbf{R}\mathbf{r}_i + \delta t[\mathbf{D} \quad -\mathbf{D}(\mathbf{R}\mathbf{r}_i)^\wedge]\dot{\mathbf{J}}_s\mathbf{v} + \delta t \mathbf{D}(\omega)^\wedge(\omega)^\wedge \mathbf{R}\mathbf{r}_i\,.\end{aligned} \tag{25}$$

Also for the moment of the contact forces we have

$$\delta \mathbf{n}_i = (\mathbf{R}\mathbf{r}_i)^\wedge \delta \mathbf{f}_i\,. \tag{26}$$

Therefore, total change of force vector for B_c will be

$$\delta \mathbf{f}_s = \begin{bmatrix} \sum_{i=1}^m \mathbf{A}_i \dot{\mathbf{v}} + \sum_{i=1}^m \mathbf{b}_i \\ \\ \sum_{i=1}^m (\mathbf{R}\mathbf{r}_i)^\wedge \mathbf{A}_i \dot{\mathbf{v}} + \sum_{i=1}^m \mathbf{b}_i \end{bmatrix} = \begin{bmatrix} \mathbf{A}_f \\ \\ \mathbf{A}_n \end{bmatrix} \dot{\mathbf{v}} + \begin{bmatrix} \mathbf{b}_f \\ \\ \mathbf{b}_n \end{bmatrix} = \mathbf{A}\dot{\mathbf{v}} + \mathbf{b}\,. \tag{27}$$

Hence, the estimated (for the next instant) 6D force-torque vector of the contact force of B_c will be

$$\hat{\mathbf{f}}_s = \mathbf{f}_s + \delta \mathbf{f}_s = \mathbf{A}\dot{\mathbf{v}} + \mathbf{b} + \mathbf{f}_s\,, \tag{28}$$

which is the same as (7) that we already used in the control algorithm. Note that for the constraints on the soft contact force (unilaterality and friction cone), $\hat{\mathbf{f}}_s$ has to be expressed in the local frame of B_c as $\hat{\mathbf{F}}_s = \mathbf{R}^T \hat{\mathbf{f}}_s$.

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