Assignment 2

(1) Evaluate $\int_0^1 \int_{\sqrt{y}}^1 (x^2 + xy^3)\, dx\, dy$. Sketch the region of integration. Change the order of integration and evaluate.

(2) Find the area of the plane $x + y + z = 1$ over the region $x^2 + 2y^2 \leq 1$.

(3) Find the volume enclosed by the surfaces $x^2 + y^2 = z$ and $x^2 + y^2 + z^2 = 2$.

(4) The tetrahedron with vertices $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ is to be sliced into n segments of equal volume by planes parallel to $x + y + z = 1$. Where should the slices be made?

(5) Suppose the density of a solid ball of radius $R > 0$ is given by $\frac{1}{1 + d^3}$, where d is the distance to the center of the ball. Find the total mass of the ball.