Assignment 1

(1) Let $f(x,y) = \sqrt{1-x^2-y^2}$. Show that the tangent plane to the graph of f at the point $\begin{bmatrix} x_0 \\ y_0 \\ f(x_0,y_0) \end{bmatrix}$ is orthogonal to the vector $\begin{bmatrix} x_0 \\ y_0 \\ f(x_0,y_0) \end{bmatrix}$.
Give a geometric interpretation.

(2) Consider a temperature function $T(x,y) = x\sin(y)$
Plot a few level curves. Compute ∇T and explain its meaning.

(3) Does the following limit exist? If so, find it.

$$\lim_{(x,y)\to(0,0)} \frac{\cos(xy)-1}{x}$$

(4) Compute the directional derivative of the function $f(x,y,z) = xy + yz + zx$ at the point $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ in the direction of $\begin{bmatrix} 10 \\ -1 \\ 2 \end{bmatrix}$.

(5) Compute the equation of the tangent plane to the surface $x^3 - 2y^3 + z^3 = 0$ at the point $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.