ECON 21110 - Applied Microeconometrics - Assignment 2

Jack Ogle

Problem 2

(a) We estimate the population model:

$$price_i = \beta_0 + \beta_1 sqrft_i + \beta_2 bdrms_i + \beta_3 lotsize_i + U_i$$

According to our regression results from Table 1 we can interpret the following. All other independent variables constant for every increase in square foot there is an price that is increased by 123 dollars on average.

All other independent variables constant for every unit increase in lot size there is an 2 dollar increase in price on average.

All other independent variables constant for every unit increase in bedrooms there is an 13,853 dollar increase in price on average. This values is not statistically significant in this regression because the p value is greater than 0.05.

There is no logical interpretation for the constant as the price of a house cannot be negative.

(b) The OLS estimate for the parameters in the model we estimated in (a) is biased. This is because it violated MLR.4 (the Zero Conditional Mean Assumption). There are variables in the error term that are correlated with independent variables. For example, variables that measure the location of the house is likely correlated with all three of the independent variables. If the location is a densely populated urban area then it will likely have less square footage, bedrooms, and a smaller lot size. Other variables related to the location of the house like property tax and average income of the zip-code are also in the error term and correlated with the independent variables.

Table 1: Regression Results (a)

	Dependent variable:
	price
sqrft	0.123
	(0.013)***
	t = 9.275***
bdrms	13.853
	(9.010)
	t = 1.537
lotsize	0.002
	(0.001)***
	t = 3.220***
Constant	−21.770
	(29.475)
	t = −0.739
Observations	88
R^2	0.672
Adjusted R^2	0.661
Residual Std. Error	59.833 (df = 84)
F Statistic	57.460*** (df = 3; 84)
Note:	*p<0.1; **p<0.05; ***p<0.01
	Standard errors in parentheses. T-Statistics below standard errors

(c) If we could collect additional data on environmental factors, I would include the variables that measure the location of the house. These variables include proximity to a large urban area, property tax, proximity to a coast, population of the city or town that the house is in. I would want to control for all the variables related to location of the house because these are correlated with the independent size variables and the price. The most important omitted variable that would affect the OLS estimator would be property tax the town that the house is located in. This would reduced our OVB and make MLR.4 more credible. This is because property tax can tell us a lot about the location of house. It can tell us if the house is in a wealthy area or poor area. If it is in a high crime or low crime area. When we add this independent into our regression we can control for crime, relative wealth, quality of schools in the area. And when we control for these variables we make MLR.4 more credible.

(d)

i) We estimate the population model:

$$price_i = \beta_0 + \beta_1 sqrft_i + \beta_2 bdrms_i + \beta_3 lotsize_i + \beta_4 colonial_i + U_i$$

ii) According to our regression results from the table above we can interpret the following. All other independent variables constant for every increase in square foot there is an price that is increased by 124 dollars on average.

All other independent variables constant for every unit increase in lot size there is an 2 dollar increase in price on average.

All other independent variables constant for every unit increase in bedrooms there is an 11,004 dollar increase in price on average. This values is not statistically significant in this regression because the p value is greater than 0.05.

All other independent variables constant if the house is a colonial style there is an 13,716 dollar increase in price compared to non colonial style houses on average. This values is not statistically significant in this regression because the p value is greater than 0.05.

When there are no effects from any independent variables, when they are all equal to zero the average price is -24,127. However, this value is not statistically significant in this regression because the p value is greater than 0.05.

iii) $H_0 : \beta_4 = 0$

We reject this null hypothesis. However, the coefficient for colonial is not statistically significant. Adding the colonial binary variable reduces the effect of bedrooms on price. However, the effect of square feet and lot size remain constant with the addition. Because the

Table 2: Regression Results (d-1)

	Dependent variable:
	price
sqrft	0.124
	(0.013)***
	t = 9.314***
bdrms	11.004
	(9.515)
	t = 1.156
lotsize	0.002
	(0.001)***
	t = 3.230***
colonial	13.716
	(14.637)
	t = 0.937
Constant	−24.127
	(29.603)
	t = −0.815
Observations	88
R^2	0.676
Adjusted R^2	0.660
Residual Std. Error	59.877 (df = 83)
F Statistic	43.252*** (df = 4; 83)
Note:	*p<0.1; **p<0.05; ***p<0.01
	Standard errors in parentheses. T-Statistics below standard errors

(e)

i) We estimate the population model:

$$price_i = \beta_0 + \beta_1 sqrft_i + \beta_2 bdrms_i + \beta_3 lotsize_i + \beta_4 colonial_i + U_i$$

ii) According to our regression results from Table 1 we can interpret the following. All other independent variables constant for every increase in square foot there is an price that is increased by 124 dollars on average.

All other independent variables constant for every unit increase in lot size there is an 2 dollar increase in price on average.

All other independent variables constant for every unit increase in bedrooms there is an 11,004 dollar increase in price on average. This values is not statistically significant in this regression because the p value is greater than 0.05.

All other independent variables constant if the house is a colonial style there is an 13,716 dollar increase in price compared to non colonial style houses on average. This values is not statistically significant in this regression because the p value is greater than 0.05.

When there are no effects from any independent variables, when they are all equal to zero the average price is -24,127. However, this value is not statistically significant in this regression because the p value is greater than 0.05.

To find the OVB we need to subtract all coefficients from the original regression without colonial ($\hat{\beta}_1$) from the coefficients from the new regression with colonial ($\tilde{\beta}_1$).

OVB for each independent variable:

$$\hat{\beta}_1 - \tilde{\beta}_1 = 0.123 - 0.124 = -0.001$$

$$\hat{\beta}_2 - \tilde{\beta}_2 = 13.853 - 11.004 = 2.849$$

$$\hat{\beta}_3 - \tilde{\beta}_3 = 0.002 - 0.002 = 0$$

I did not understand whether this question was referring to OVB in the model or just for each coefficient so I assumed that it was for each coefficient.

(f)

i) We estimate the population model:

$$price_i = \beta_0 + \beta_1 sqrft_i + \beta_2 bdrms_i + \beta_3 lotsize_i + \beta_4 colonial_i + \beta_5 sqrft_i colonial_i + \beta_6 bdrms_i colonial_i + \beta_7 lotsize_i colonial_i + U_i$$

Table 3: Regression Results (f-1)

	Dependent variable:
	price
sqrft	0.090***
	(0.024)
bdrms	11.225
	(20.369)
lotsize	0.007***
	(0.002)
colonial	−30.601
	(62.734)
sqrft:colonial	0.044
	(0.029)
bdrms:colonial	1.644
	(22.833)
lotsize:colonial	−0.006***
	(0.002)
Constant	−2.874
	(50.810)
Observations	88
R^2	0.709
Adjusted R^2	0.684
Residual Std. Error	57.758 (df = 80)
F Statistic	27.877*** (df = 7; 80)
Note:	*p<0.1; **p<0.05; ***p<0.01
	Standard errors in parentheses

ii) According to our regression results from the Table above we can interpret the following.

All other independent variables constant for every increase in square foot a colonial house price increases by 134 dollars on average.

All other independent variables constant for every unit increase in lot size a colonial house price increases 1 dollar in price on average.

All other independent variables constant for every unit increase in bedrooms a colonial house price increases 12,869 dollar in price on average. This values is not statistically significant in this regression because the p value is greater than 0.05.

All other independent variables constant if the house is a colonial style there is an 30,601 dollar decrease in price compared to non colonial style houses on average. This values is not statistically significant in this regression because the p value is greater than 0.05.

When there are no effects from any independent variables, when they are all equal to zero the average price is -2,874. However, this value is not statistically significant in this regression because the p value is greater than 0.05.

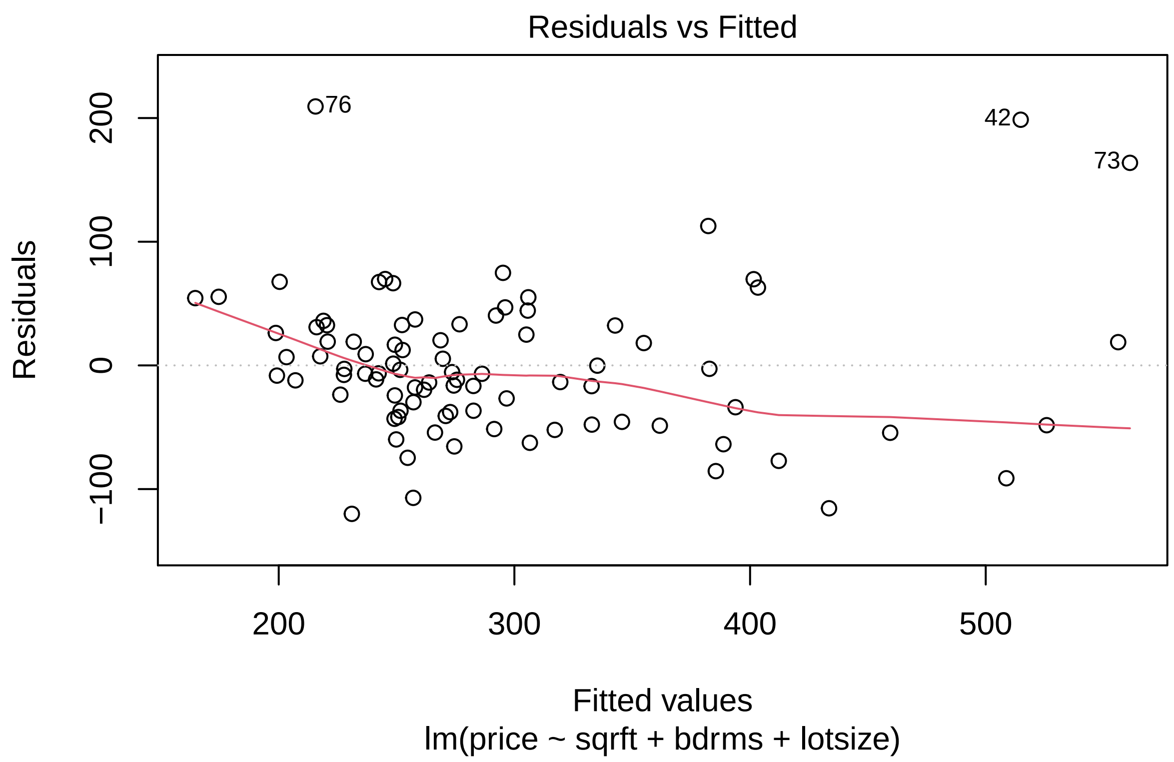
iii) $H_0 : \beta_5 = 0, H_0 : \beta_6 = 0, H_0 : \beta_7 = 0$

Our null hypothesis states that there is no evidence that the effect of house characteristics on price differs by colonial. That in the presence of colonial we see no difference in the effect of house characteristics. However, we can see that we can reject the null hypothesis $H_0 : \beta_6 = 0$ as it equals -0.006 with statistical significance. This is the only one we can reject because the other results are not statistically significant. We reject our null hypothesis and should accept an alternative hypotheses which states that there is a slightly positive correlation between colonial houses and their lot size. If you have a colonial house for every unit increase in lot size there is a 1$ increase in the price of the house on average. This is a result that I would not expect colonial style houses to differ vary much from other styles of houses in terms of prices. I would imagine that the prices of houses colonial or otherwise are affected similarly by house characteristics.

(g)

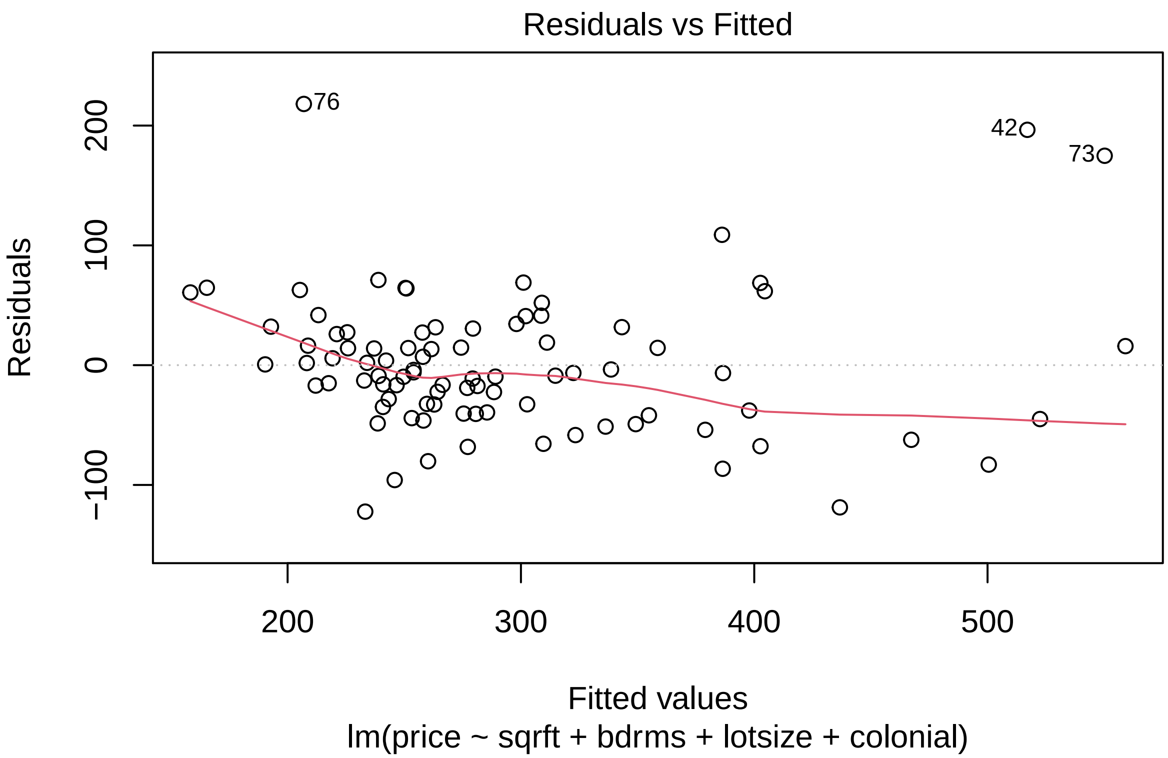
Model (a)

```r
# Graphing the population model
plot(modelA, which=1)
```



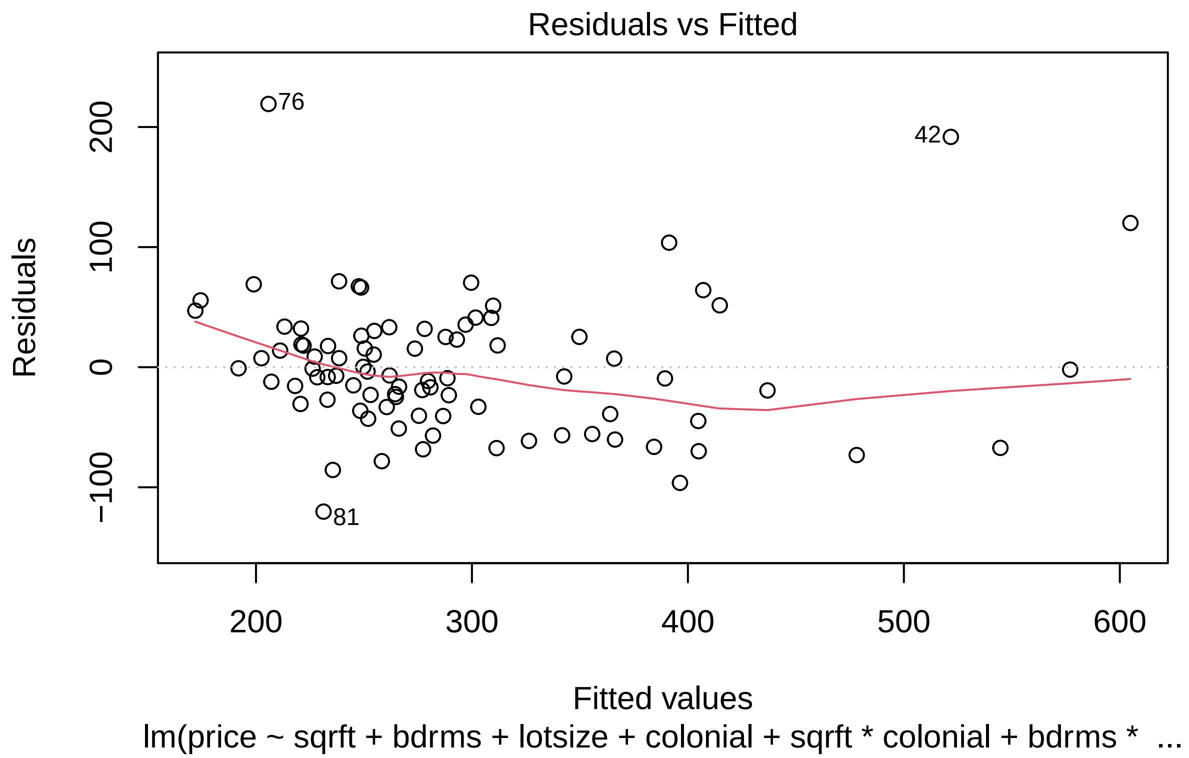
Model (d)

```r
# Graphing the population model
plot(modelD_1, which=1)
```



Model (f)

```
# Graphing the population model
plot(modelF_1, which=1)
```



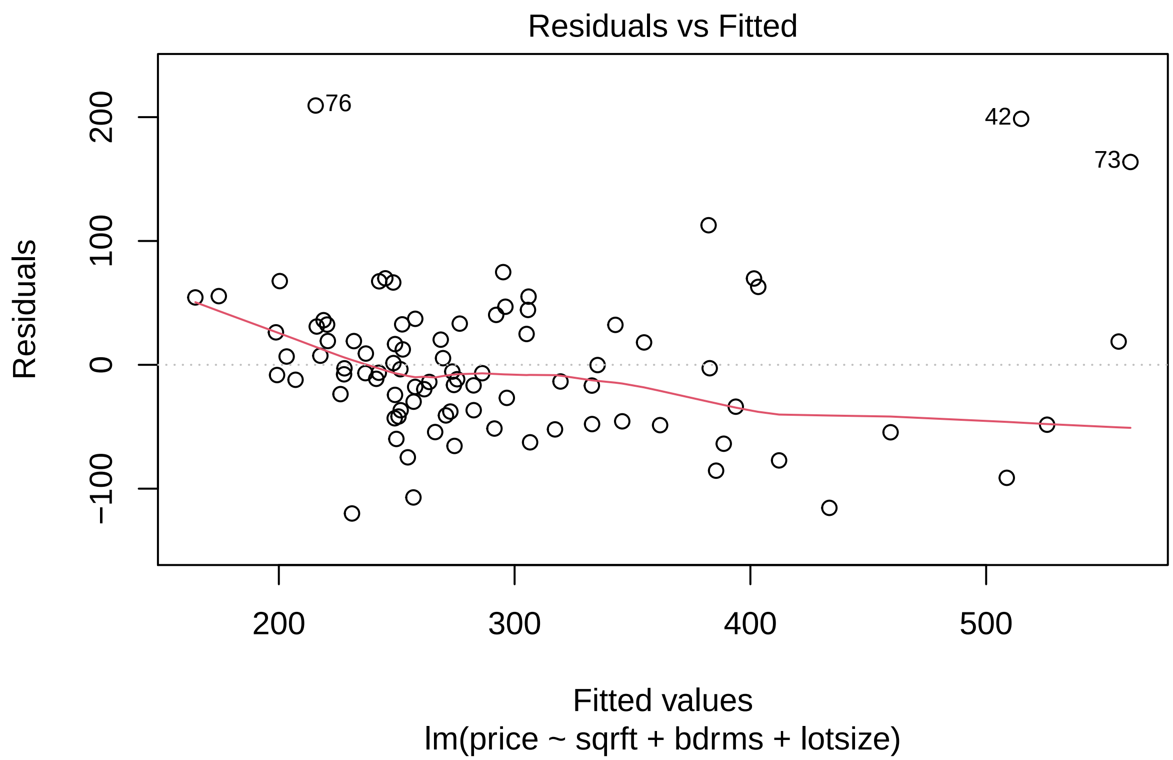
If we compare the figures above and the R-squared values for the various models we learn two important elements about the data and the models that we created to represent them. All the figures show strong evidence that our models have heteroskedasticity. As we increase our fitted values and our independent variables we see a change in the variance of our residuals. If the models were homoskedastic the residuals would be evenly distributed even as we increase our independent variables.

By comparing the R-squared values for (a), (d), and (f): 0.672, 0.676, and 0.709 respectively we know that model (f) has the highest R-squared and thus fits the data the best.

(h) Heteroskedasticity means that Assumption MLR.5 of homoskedasticity is violated. This means that the error variance is not the same across all values of the independent variables. Even though under heteroskedasticity the estimator $\hat{\beta}$ remains unbiased the variance of that estimator is biased and we can no longer do hypothesis testing. Additionally the standard errors and t-statistics we calculated are not correct. Additionally the standard errors are higher in the new model and because standard errors are related to variance we see a higher variance.

We can see this through a graphic representation of the residuals v. the fitted values and a BP test

```
plot(modelA, which = 1)
```



The grpahic supports our argument of heteroskedasticity because as we increase our fitted values and our independent variables we see a change in the variance of our residuals. If the models were homoskedastic the residuals would be evenly distributed even as we increase our independent variables.

Our BP value is 14.092 with three degrees of freedom and a p-value of 0.0028. This is less than our null hypothesis for homoskedasticity which is BP value equal to 0 with statstical significance. Because we have a p-value of less than 0.05 we reject the null and there is strong evidence for heteroskedasticty and a violation of MLR.5.

(i)

Table 4: Regression Results (I-1)

	Dependent variable:	
	price	
	Non-Robust Std. Errors	Robust Std. Errors
	(1)	(2)
sqrft	0.123^{***}	0.123^{***}
	(0.013)	(0.018)
bdrms	13.853	13.853
	(9.010)	(8.479)
lotsize	0.002^{***}	0.002^{***}
	(0.001)	(0.001)
Constant	-21.770	-21.770
	(29.475)	(37.138)
Observations	88	88
R^2	0.672	0.672
Adjusted R^2	0.661	0.661
Residual Std. Error (df = 84)	59.833	59.833
F Statistic (df = 3; 84)	57.460^{***}	23.72^{***}
Note:		*p<0.1; **p<0.05; ***p<0.01
		Standard errors in parentheses

For square footage and lot size coefficients the robust standard errors are larger than the non-robust standard errors. We see a decrease in the robust standard error for the bedroom coefficient, however, this result is not statistically significant. We already know that there is heteroskedasticity present in the model. We use robust standard errors to correct for the heteroskedascity in the model. Therefore our results from a remain the same however, because the standard error increases we have understand that the average distance that the observed values fall from the regression line increases.

(j)

i) We estimate the population model:

$$lprice_i = \beta_0 + \beta_1 sqrft_i + \beta_2 bdrms_i + \beta_3 lotsize_i + U_i$$

Table 5: Regression Results (j-1)

	Dependent variable:
	lprice
sqrft	0.0004***
	(0.00004)
bdrms	0.025
	(0.029)
lotsize	0.00001***
	(0.00000)
Constant	4.759***
	(0.094)
Observations	88
R^2	0.622
Adjusted R^2	0.609
Residual Std. Error	0.190 (df = 84)
F Statistic	46.128*** (df = 3; 84)
Note:	*p<0.1; **p<0.05; ***p<0.01
	Standard errors in parentheses

ii) According to our regression results from the table above we can interpret the following.

All other independent variables constant for every unit increase in square footage the house price in dollars increases 0.04% on average.

All other independent variables constant for every unit increase in bedrooms the house price in dollars increases 2.5% on average.

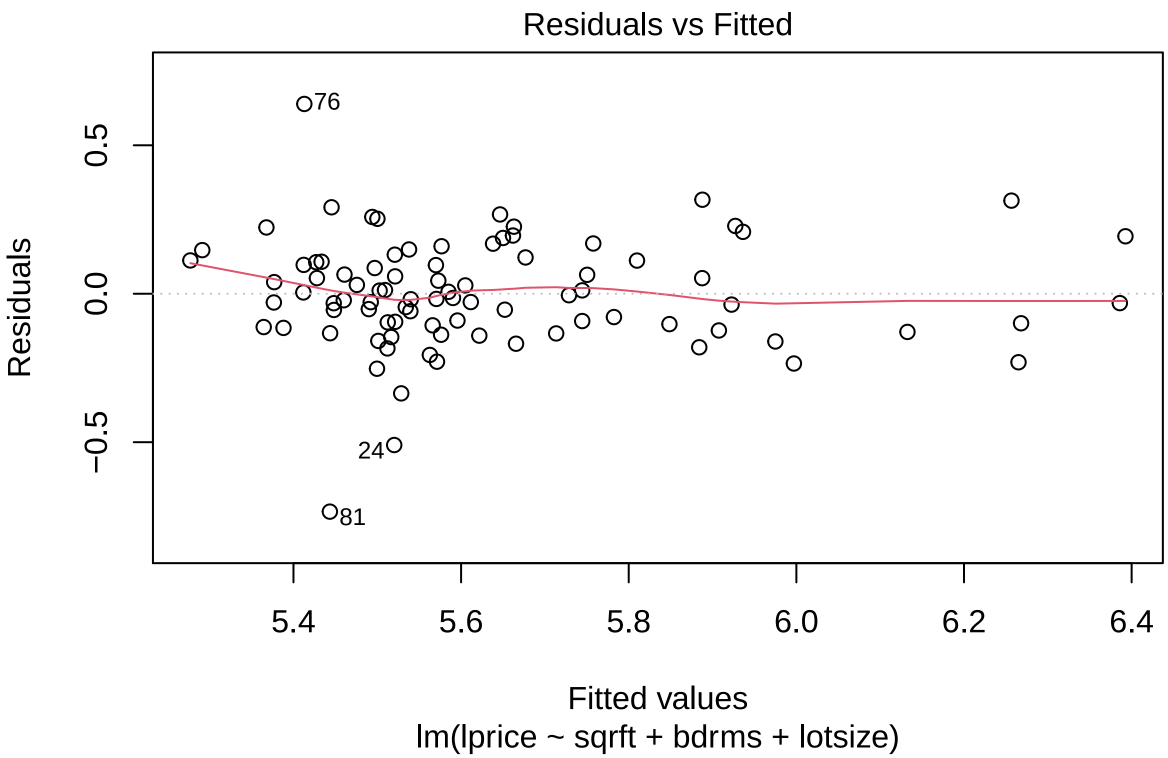
All other independent variables constant for every unit increase in lot size the house price in dollars increases 0.001% on average.

When there are no effects from any independent variables, when they are all equal to zero the average price is 4,759. However, this value is not statistically significant in this regression because the p value is greater than 0.05.

(k) Heteroskedasticity means that Assumption MLR.5 of homoskedasticity is violated. This means that the error variance is not the same across all values of the independent variables. Even though under heteroskedasticity the estimator $\hat{\beta}$ remains unbiased the variance of that estimator is biased and we can no longer do hypothesis testing. Additionally the standard errors and t-statistics we calculated are not correct. Additionally the standard errors are higher in the new model and because standard errors are related to variance we see a higher variance.

We can see this through a graphic representation of the residuals v. the fitted values and a BP test

```
plot(modelJ_1, which = 1)
```



The graphic supports our argument of homooskedasticity because as we increase our fitted values and our independent variables we do not see a change in the variance of our residuals. The model is homoskedastic because the residuals are evenly distributed even as we increase our independent variables.

Our BP value is 3.543 with three degrees of freedom and a p-value of 0.315. Our null hypothesis is that if our BP value is equal to 0 than there is homoskedasticity in the model. Because we have a p-value of greater than 0.05 we fail to reject the null and there is strong evidence for homoskedacitiy and MLR.5 is held. Our boss is right using log price instead of price fixes our model and makes it homoskedastic.

(l) To see if we should include sqrft in the model we need to calculate the OVB for the estimator for bedrooms.

Table 6: Regression Results (l-1)

	Dependent variable:	
	price	
	(1)	(2)
sqrft	0.123^{***}	
	(0.013)	
bdrms	13.853	57.313^{***}
	(9.010)	(10.885)
lotsize	0.002^{***}	0.003^{***}
	(0.001)	(0.001)
Constant	-21.770	63.262
	(29.475)	(39.620)
Observations	88	88
R^2	0.672	0.337
Adjusted R^2	0.661	0.321
Residual Std. Error	59.833 (df = 84)	84.624 (df = 85)
F Statistic	57.460^{***} (df = 3; 84)	21.585^{***} (df = 2; 85)
Note:		*p<0.1; **p<0.05; ***p<0.01
		Standard errors in parentheses

We can see that the coefficient for bedrooms increase from 12.853 to 57.313. Thus the OVB for this coefficient is -44.46. Omitting sqrft increases the bedrooms effect on price. And it increases the variance of that effect on price as we see an increase in the standard error. Standard error is a statistic that is correlated with variance. And variance increases. Yes we should included sqrft because we want to minimize variance and the adjusted r-squared value increases.