ECON 21110 - Applied Microeconometrics - Assignment 2

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Problem 1

(a)

Table 1: Regression Results (a)

	Dependent variable:
	colgpa
female	0.142***
	(0.020)
Constant	2.589***
	(0.014)
Observations	4,137
R^2	0.012
Adjusted R^2	0.011
Residual Std. Error	0.655 (df = 4135)
F Statistic	48.157*** (df = 1; 4135)
Note:	*p<0.1; **p<0.05; ***p<0.01
	Standard errors in parentheses

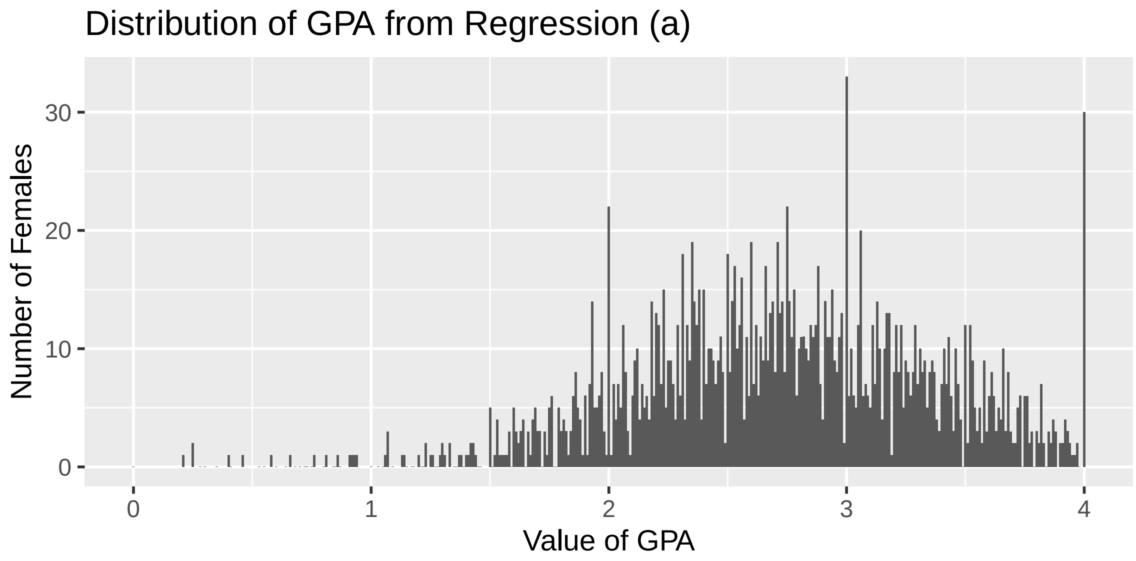
The female coefficient is difference in the average of college gpa between females and males. On average a female's gpa is 0.142 higher than a male's.

(b)

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```


```
##
##  Shapiro-Wilk normality test
##
## data:  modelA$residuals
## W = 0.99245, p-value = 5.352e-14
```

In the figure above we can see a histogram of the residuals. The regression looks normal but we must do a test statistic to know for sure. We will use the shapiro-wilk test for normality. We get the result W = 0.99245, p-value = 5.352e-14. This suggest that the distribution is not normal and that we can reject the null hypothesis of homoskedasticity. The model is heteroskedastic.



(c) We first want to standardize our sat scores by subtracting each observation by the mean and dividing by the standard deviation. We standardize sat scores to reduce multicollinearity among the independent variables. This makes the sat score more meaningful because the interpretation of sat coefficient one standard deviation.

$$colgpa_i = \hat{\beta}_0 + \hat{\beta}_1 female_i + \hat{\beta}_2 standardSat_i + U_i$$

There is an increase in the coefficient from 0.142 to 0.287 when we add in SAT scores as an independent variable.

$$\tilde{\beta}_1 = \hat{\beta}_1 + \hat{\beta}_2\tilde{\delta}_1$$

Where $\tilde{\beta}_1$ is the estimate from SLR and $\hat{\beta}_1$ and $\hat{\beta}_2$ are the estimates from MLR. $\tilde{\delta}_1$ is the coefficient estimate from the following regression.

$$standardSat_i = \tilde{\delta}_0 + \tilde{\delta}_1 female_i + U_i$$

We can see that $\tilde{\delta}_1$ has a negative sign implying that sat and female are negatively correlated. We know this by knowing that $\hat{\beta}_1$, $\tilde{\beta}_1$ and $\hat{\beta}_2$ are positive. If all the estimators are positive using the equation above we know that $\tilde{\delta}_1$ has a negative sign.

Table 2: Regression Results (c)

	Dependent variable:
	colgpa
female	0.231***
	(0.019)
standardSat	0.287***
	(0.009)
Constant	2.549***
	(0.012)
Observations	4,137
R^2	0.197
Adjusted R^2	0.196
Residual Std. Error	0.590 (df = 4134)
F Statistic	506.099*** (df = 2; 4134)
Note:	*p<0.1; **p<0.05; ***p<0.01
	Standard errors in parentheses

Table 3: Regression Results (c)

	Dependent variable:
	standardSat
female	-0.309^{***}
	(0.031)
Constant	0.139^{***}
	(0.021)
Observations	4,137
R^2	0.024
Adjusted R^2	0.023
Residual Std. Error	0.988 (df = 4135)
F Statistic	100.082^{***} (df = 1; 4135)
Note:	*p<0.1; **p<0.05; ***p<0.01
	Standard errors in parentheses

We get that the female coefficient is -0.309 implying that on average being a female results in a SAT score 30.9% lower than being a male.

Because female and sat are negatively correlated, when sat is added into the regression our coefficient increases from 0.142 to 0.231 meaning that there was a negative OVB in the first model. We calculate the negative bias by subtracting the coefficient for female without sat and the coefficient for female with sat. 0.142-0.231 = -0.089.

(d)

We estimate our model with OLS:

$$colgpa_i = \hat{\beta}_0 + \hat{\beta}_1 female_i + \hat{\beta}_2 sat_i + U_i$$

See table 2 for results of that regression. From that regression we obtain the residuals $\hat{U}$.

Then we regress the squared residuals $\hat{U^2}$ on female and sat. From that we obtain an R-squared value $R^2_{\hat{U^2}}$ of 0.00261.

We then use the following formula to compute our F-Statistic:

$$F = \frac{R^2_{\hat{U^2}}/k}{(1 - R^2_{\hat{U^2}})/(n-k-1)}$$

We calculate a chi squared test using the chi value and the degrees of freedom. Which gives us the following values.

F-statistic = 5.409 p-value = 0.00451

Since the p-value is less than 0.05, we can reject the null hypothesis of homoskedasticity. There for heteroskedasticity is present.

For the White test, we must regress our residuals squared over all the independent variables, the squares of the independent variables, and the cross-products between the independent variables. From that regression we see an p-value with a value of 0.0085. Because this p-value is less than 0.05 we reject our null hypothesis of homoskedasticity.

(e)

Table 4: Regression Results (c)

	Dependent variable:	
	colgpa	
	Non-Robust Std. Errors	Robust Std. Errors
	(1)	(2)
female	0.231***	0.231***
	(0.019)	(0.018)
standardSat	0.287***	0.287***
	(0.009)	(0.009)
Constant	2.549***	2.549***
	(0.012)	(0.013)
Observations	4,137	4,137
R^2	0.197	0.197
Adjusted R^2	0.196	0.196
Residual Std. Error (df = 4134)	0.590	0.590
F Statistic (df = 2; 4134)	506.099***	536.1***
Note:		*p<0.1; **p<0.05; ***p<0.01
		Standard errors in parentheses

Observing table 4 we see that for the female coefficient the robust standard error is less than the non-robust standard error. For the sat coefficient we see that the robust and non-robust standard error are the same. And for the constant we get that the robust standard error is slightly larger than the non-robust standard error.

Additionally, our F-statistic increases from 506.099 to 536.19.

These new results support our findings from c. Because the assumption of homoskedasticity the null hypothesis was rejected, there is variance among our residuals as we change our independent variables. The model is heteroskedastic we would employ robust standard errors to correct for the higher variance in among residuals the model as the independent variables change.

(f) We estimate the following

$$colgpa = \beta_0 + \beta_1 sat + \beta_2 female + \beta_3 sat^2$$

Table 5: Regression Results (f)

	Dependent variable:
	colgpa
sat	0.285***
	(0.009)
female	0.239***
	(0.019)
sat^2	0.038***
	(0.006)
Constant	2.507***
	(0.014)
Observations	4,137
R^2	0.204
Adjusted R^2	0.203
Residual Std. Error	0.588 (df = 4133)
F Statistic	352.873*** (df = 3; 4133)
Note:	*p<0.1; **p<0.05; ***p<0.01
	Standard errors in parentheses

With this regression result the functional relationship between colgpa and sat is likely linear. We can reject a non-linear relationship. We fail to reject $H_0 : \beta_3 = 0$. This suggests that there is a linear relationship between colgpa and sat.

The coefficient estimator for sat scores on college gpa is that for every increase in a standard deviation of SAT score there is a 0.287 increase in College GPA. This is pretty low this can be explained by a the fact that we have a non random sample. Colleges choose the students they admit based on various factors including SAT scores. Not all students who took the SAT when to college, so we are missing a part of our random sample. It is likely that those students who did not go to college have lower SAT scores than those who did. Therefore sat scores should have a higher impact on college gpa but because we need to include everyone in the random sample who took the SAT it is smaller.

(g) MLR.4 is the Zero Conditional Mean Assumption which states none of the independent variables are correlated to any variables in the error term. I think that the Zero Conditional Mean assumption does not hold because there are variables in the error term that are correlated with independent variables. For example, household income is correlated with sat scores. If we included metrics like parental income, household income. and IQ it would be more credible. It would be more credible because fewer variables in the error term would be correlated with independent variables.

(h) An ideal randomized experiment would be to clone one man and one women who take the same courses, have the same ability and socio-economic background. We should also attempt to study students from the same college. We want to compare students with the same major and who take the same classes. We should also compare students with similar IQ and ability to study, think, and learn. Essentially, we would like to control for every variable that might lead to a violation of the Zero Conditional Mean assumption. If we can satisfy the Zero Conditional Mean Assumption than we can understand the causal effect of female on college gpa without any confounding variables. This is because if the zero conditional mean is satisfied than the error term does not show any systematic pattern and have a mean of zero regardless of the independent variables that we choose.

(i) If we attempted to implement the ideal randomized experiment we would run into some challenges. It is not possible to control for every confounding variable. It is impossible to find a male clone for every female. However, if we are not able to find perfect clones with the same ability and socio-economic background. We might be able to find matches for ability measures like IQ and KWW. We might also be able to control for the different majors of study. We also would be able to control for socio-economic background by controlling for parental income and household income.

Problem 2

(a) We estimate the population model:

$$price_i = \beta_0 + \beta_1 sqrft_i + \beta_2 bdrms_i + \beta_3 lotsize_i + U_i$$

According to our regression results from Table 1 we can interpret the following. All other independent variables constant for every increase in square foot there is an price that is increased by 123 dollars on average.

All other independent variables constant for every unit increase in lot size there is an 2 dollar increase in price on average.

All other independent variables constant for every unit increase in bedrooms there is an 13,853 dollar increase in price on average. This values is not statistically significant in this regression because the p value is greater than 0.05.

There is no logical interpretation for the constant as the price of a house cannot be negative.

(b) The OLS estimate for the parameters in the model we estimated in (a) is biased. This is because it violated MLR.4 (the Zero Conditional Mean Assumption). There are variables in the error term that are correlated with independent variables. For example, variables that measure the location of the house is likely correlated with all three of the independent variables. If the location is a densely populated urban area then it will likely have less square footage, bedrooms, and a smaller lot size. Other variables related to the location of the house like property tax and average income of the zip-code are also in the error term and correlated with the independent variables.

(c) If we could collect additional data on environmental factors, I would include the variables that measure the location of the house. These variables include proximity to a large urban area, property tax, proximity to a coast, population of the city or town that the house is in. I would want to control for all the variables related to location of the house because these are correlated with the independent size variables and the price. The most important omitted variable that would affect the OLS estimator would be property tax the town that the house is located in. This would reduced our OVB and make MLR.4 more credible. This is because property tax can tell us a lot about the location of house. It can tell us if the house is in a wealthy area or poor area. If it is in a high crime or low crime area. When we add this independent into our regression we can control for crime, relative wealth, quality of schools in the area. And when we control for these variables we make MLR.4 more credible.

(d)

i) We estimate the population model:

$$price_i = \beta_0 + \beta_1 sqrft_i + \beta_2 bdrms_i + \beta_3 lotsize_i + \beta_4 colonial_i + U_i$$

Table 6: Regression Results (a)

	Dependent variable:
	price
sqrft	0.123
	(0.013)***
	t = 9.275***
bdrms	13.853
	(9.010)
	t = 1.537
lotsize	0.002
	(0.001)***
	t = 3.220***
Constant	−21.770
	(29.475)
	t = −0.739
Observations	88
R^2	0.672
Adjusted R^2	0.661
Residual Std. Error	59.833 (df = 84)
F Statistic	57.460*** (df = 3; 84)
Note:	*p<0.1; **p<0.05; ***p<0.01
	Standard errors in parentheses. T-Statistics below standard errors

Table 7: Regression Results (d-1)

	Dependent variable:
	price
sqrft	0.124
	(0.013)***
	t = 9.314***
bdrms	11.004
	(9.515)
	t = 1.156
lotsize	0.002
	(0.001)***
	t = 3.230***
colonial	13.716
	(14.637)
	t = 0.937
Constant	−24.127
	(29.603)
	t = −0.815
Observations	88
R^2	0.676
Adjusted R^2	0.660
Residual Std. Error	59.877 (df = 83)
F Statistic	43.252*** (df = 4; 83)
Note:	*p<0.1; **p<0.05; ***p<0.01
	Standard errors in parentheses. T-Statistics below standard errors

ii) According to our regression results from the table above we can interpret the following. All other independent variables constant for every increase in square foot there is an price that is increased by 124 dollars on average.

All other independent variables constant for every unit increase in lot size there is an 2 dollar increase in price on average.

All other independent variables constant for every unit increase in bedrooms there is an 11,004 dollar increase in price on average. This values is not statistically significant in this regression because the p value is greater than 0.05.

All other independent variables constant if the house is a colonial style there is an 13,716 dollar increase in price compared to non colonial style houses on average. This values is not statistically significant in this regression because the p value is greater than 0.05.

When there are no effects from any independent variables, when they are all equal to zero the average price is -24,127. However, this value is not statistically significant in this regression because the p value is greater than 0.05. Additionally this value has no meaning because the price of the house cannot be negative.

iii) $H_0 : \beta_4 = \beta_1 = \beta_2 = \beta_3$

We reject this null hypothesis. Adding the colonial binary variable reduces the effect of bedrooms on price. However, the effect of square feet and lot size remain constant with the addition. Adding colonial into our regression has very little effect on the price. This makes sense because colonial houses are just a style of house and other factors that are related to the environment or location would have a larger impact on price.

(e)

i) We estimate the population model:

$$price_i = \beta_0 + \beta_1 sqrft_i + \beta_2 bdrms_i + \beta_3 lotsize_i + \beta_4 colonial_i + U_i$$

ii) According to our regression results from Table 1 we can interpret the following. All other independent variables constant for every increase in square foot there is an price that is increased by 124 dollars on average.

All other independent variables constant for every unit increase in lot size there is an 2 dollar increase in price on average.

All other independent variables constant for every unit increase in bedrooms there is an 11,004 dollar increase in price on average. This values is not statistically significant in this regression because the p value is greater than 0.05.

All other independent variables constant if the house is a colonial style there is an 13,716 dollar increase in price compared to non colonial style houses on average. This values is not statistically significant in this regression because the p value is greater than 0.05.

When there are no effects from any independent variables, when they are all equal to zero the average price is -24,127. However, this value is not statistically significant in this regression because the p value is greater than 0.05.

To find the OVB we need to subtract all coefficients from the original regression without colonial ($\hat{\beta}_1$) from the coefficients from the new regression with colonial ($\tilde{\beta}_1$).

OVB for each independent variable:

$$\hat{\beta}_1 - \tilde{\beta}_1 = 0.123 - 0.124 = -0.001$$

$$\hat{\beta}_2 - \tilde{\beta}_2 = 13.853 - 11.004 = 2.849$$

$$\hat{\beta}_3 - \tilde{\beta}_3 = 0.002 - 0.002 = 0$$

I did not understand whether this question was referring to OVB in the model or just for each coefficient so I assumed that it was for each coefficient.

(f)

i) We estimate the population model:

$$price_i = \beta_0 + \beta_1 sqrft_i + \beta_2 bdrms_i + \beta_3 lotsize_i + \beta_4 colonial_i + \beta_5 sqrft_i colonial_i + \beta_6 bdrms_i colonial_i + \beta_7 lotsize_i colonial_i + U_i$$

Table 8: Regression Results (f-1)

	Dependent variable:
	price
sqrft	0.090***
	(0.024)
bdrms	11.225
	(20.369)
lotsize	0.007***
	(0.002)
colonial	−30.601
	(62.734)
sqrft:colonial	0.044
	(0.029)
bdrms:colonial	1.644
	(22.833)
lotsize:colonial	−0.006***
	(0.002)
Constant	−2.874
	(50.810)
Observations	88
R^2	0.709
Adjusted R^2	0.684
Residual Std. Error	57.758 (df = 80)
F Statistic	27.877*** (df = 7; 80)
Note:	*p<0.1; **p<0.05; ***p<0.01
	Standard errors in parentheses

ii) According to our regression results from the Table above we can interpret the following.

All other independent variables constant for every increase in square foot a colonial house price increases by 134 dollars on average.

All other independent variables constant for every unit increase in lot size a colonial house price increases 1 dollar in price on average.

All other independent variables constant for every unit increase in bedrooms a colonial house price increases 12,869 dollar in price on average. This values is not statistically significant in this regression because the p value is greater than 0.05.

All other independent variables constant if the house is a colonial style there is an 30,601 dollar decrease in price compared to non colonial style houses on average. This values is not statistically significant in this regression because the p value is greater than 0.05.

When there are no effects from any independent variables, when they are all equal to zero the average price is -2,874. However, this value is not statistically significant in this regression because the p value is greater than 0.05.

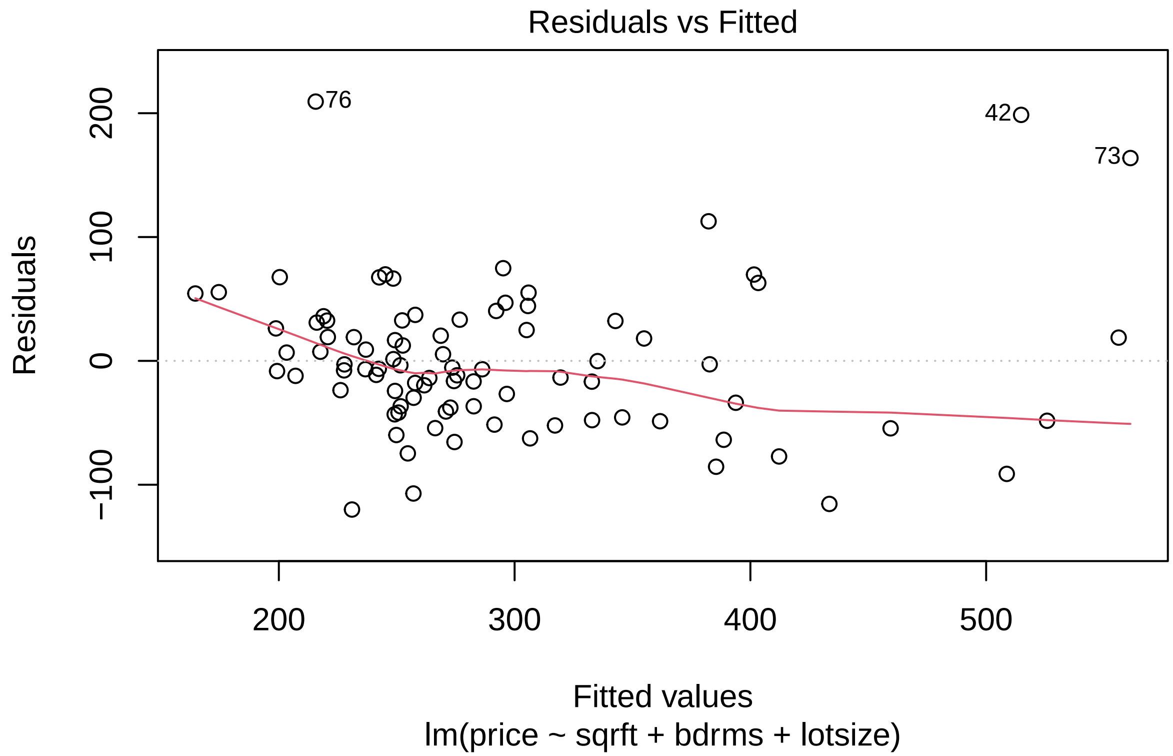
iii) $H_0 : \beta_5 = 0, H_0 : \beta_6 = 0, H_0 : \beta_7 = 0$

Our null hypothesis states that there is no evidence that the effect of house characteristics on price differs by colonial. That in the presence of colonial we see no difference in the effect of house characteristics. However, we can see that we can reject the null hypothesis $H_0 : \beta_6 = 0$ as it equals -0.006 with statistical significance. This is the only one we can reject because the other results are not statistically significant. We reject our null hypothesis and should accept an alternative hypotheses which states that there is a slightly positive correlation between colonial houses and their lot size. If you have a colonial house for every unit increase in lot size there is a 1$ increase in the price of the house on average. This is a result that I would not expect colonial style houses to differ vary much from other styles of houses in terms of prices. I would imagine that the prices of houses colonial or otherwise are affected similarly by house characteristics.

(g)

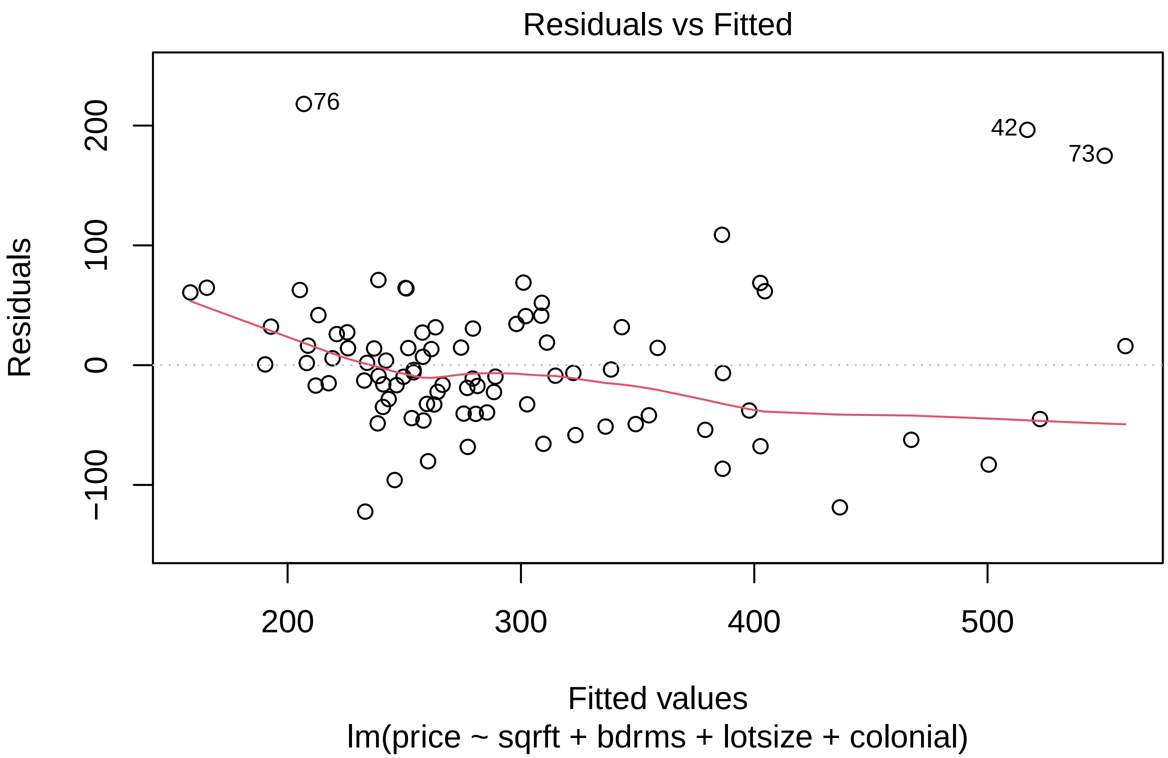
Model (a)

```r
# Graphing the population model
plot(modelA, which=1)
```



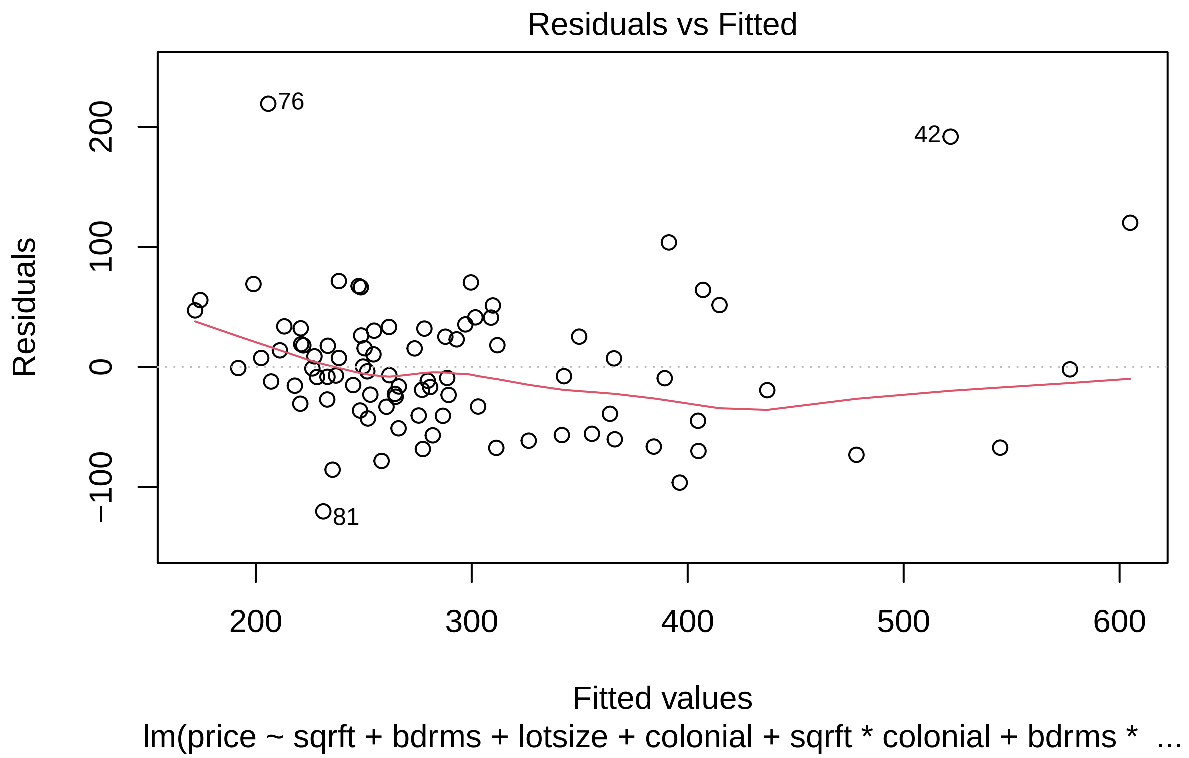
Model (d)

```r
# Graphing the population model
plot(modelD_1, which=1)
```



Model (f)

```
# Graphing the population model
plot(modelF_1, which=1)
```



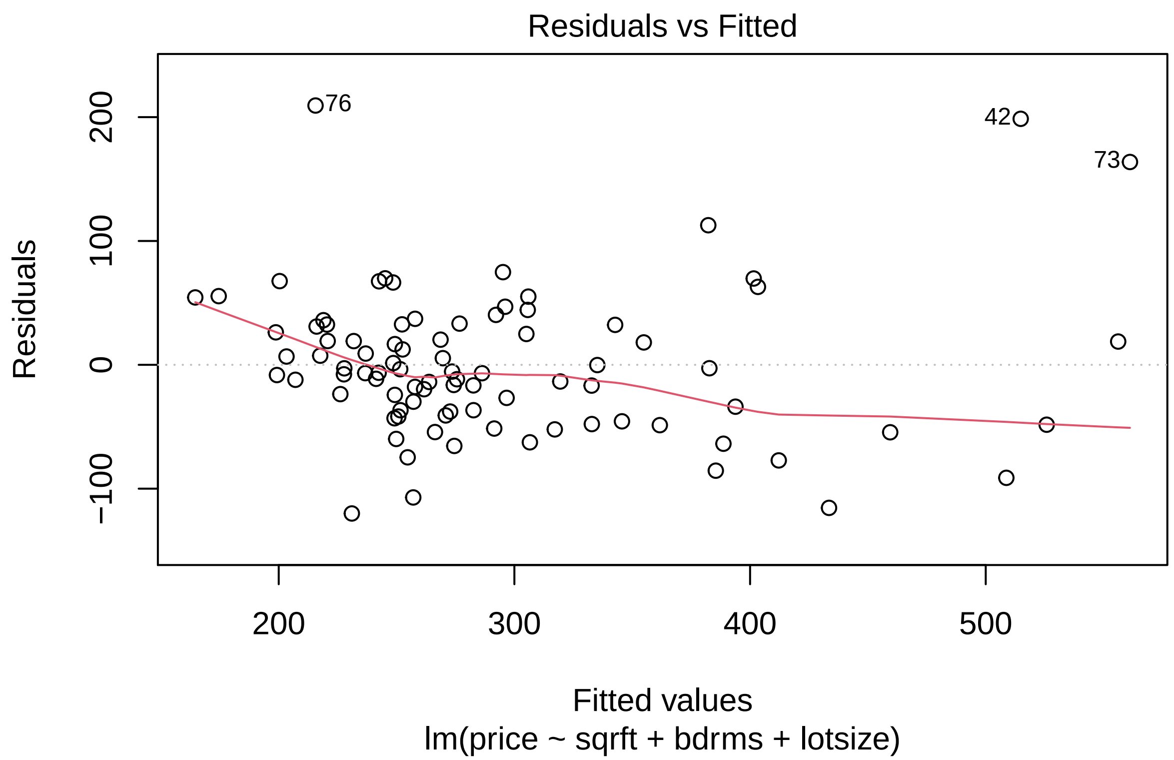
If we compare the figures above and the R-squared values for the various models we learn two important elements about the data and the models that we created to represent them. All the figures show strong evidence that our models have heteroskedasticity. As we increase our fitted values and our independent variables we see a change in the variance of our residuals. If the models were homoskedastic the residuals would be evenly distributed even as we increase our independent variables.

By comparing the R-squared values for (a), (d), and (f): 0.672, 0.676, and 0.709 respectively we know that model (f) has the highest R-squared and thus fits the data the best.

(h) Heteroskedasticity means that Assumption MLR.5 of homoskedasticity is violated. This means that the error variance is not the same across all values of the independent variables. Even though under heteroskedasticity the estimator $\hat{\beta}$ remains unbiased the variance of that estimator is biased and we can no longer do hypothesis testing. Additionally the standard errors and t-statistics we calculated are not correct. Additionally the standard errors are higher in the new model and because standard errors are related to variance we see a higher variance.

We can see this through a graphic representation of the residuals v. the fitted values and a BP test

```
plot(modelA, which = 1)
```



The grpahic supports our argument of heteroskedasticity because as we increase our fitted values and our independent variables we see a change in the variance of our residuals. If the models were homoskedastic the residuals would be evenly distributed even as we increase our independent variables.

Our BP value is 14.092 with three degrees of freedom and a p-value of 0.0028. This is less than our null hypothesis for homoskedasticity which is BP value equal to 0 with statstical significance. Because we have a p-value of less than 0.05 we reject the null and there is strong evidence for heteroskedasticty and a violation of MLR.5.

(i)

Table 9: Regression Results (I-1)

	Dependent variable:	
	price	
	Non-Robust Std. Errors	Robust Std. Errors
	(1)	(2)
sqrft	0.123***	0.123***
	(0.013)	(0.018)
bdrms	13.853	13.853
	(9.010)	(8.479)
lotsize	0.002***	0.002***
	(0.001)	(0.001)
Constant	−21.770	−21.770
	(29.475)	(37.138)
Observations	88	88
R^2	0.672	0.672
Adjusted R^2	0.661	0.661
Residual Std. Error (df = 84)	59.833	59.833
F Statistic (df = 3; 84)	57.460***	23.72***
Note:		*p<0.1; **p<0.05; ***p<0.01
		Standard errors in parentheses

For square footage and lot size coefficients the robust standard errors are larger than the non-robust standard errors. We see a decrease in the robust standard error for the bedroom coefficient, however, this result is not statistically significant. We already know that there is heteroskedasticity present in the model. We use robust standard errors to correct for the heteroskedascity in the model. Therefore our results from a remain the same however, because the standard error increases we have understand that the average distance that the observed values fall from the regression line increases.

(j)

i) We estimate the population model:

$$lprice_i = \beta_0 + \beta_1 sqrft_i + \beta_2 bdrms_i + \beta_3 lotsize_i + U_i$$

Table 10: Regression Results (j-1)

	Dependent variable:
	lprice
sqrft	0.0004***
	(0.00004)
bdrms	0.025
	(0.029)
lotsize	0.00001***
	(0.00000)
Constant	4.759***
	(0.094)
Observations	88
R^2	0.622
Adjusted R^2	0.609
Residual Std. Error	0.190 (df = 84)
F Statistic	46.128*** (df = 3; 84)
Note:	*p<0.1; **p<0.05; ***p<0.01
	Standard errors in parentheses

ii) According to our regression results from the table above we can interpret the following.

All other independent variables constant for every unit increase in square footage the house price in dollars increases 0.04% on average.

All other independent variables constant for every unit increase in bedrooms the house price in dollars increases 2.5% on average.

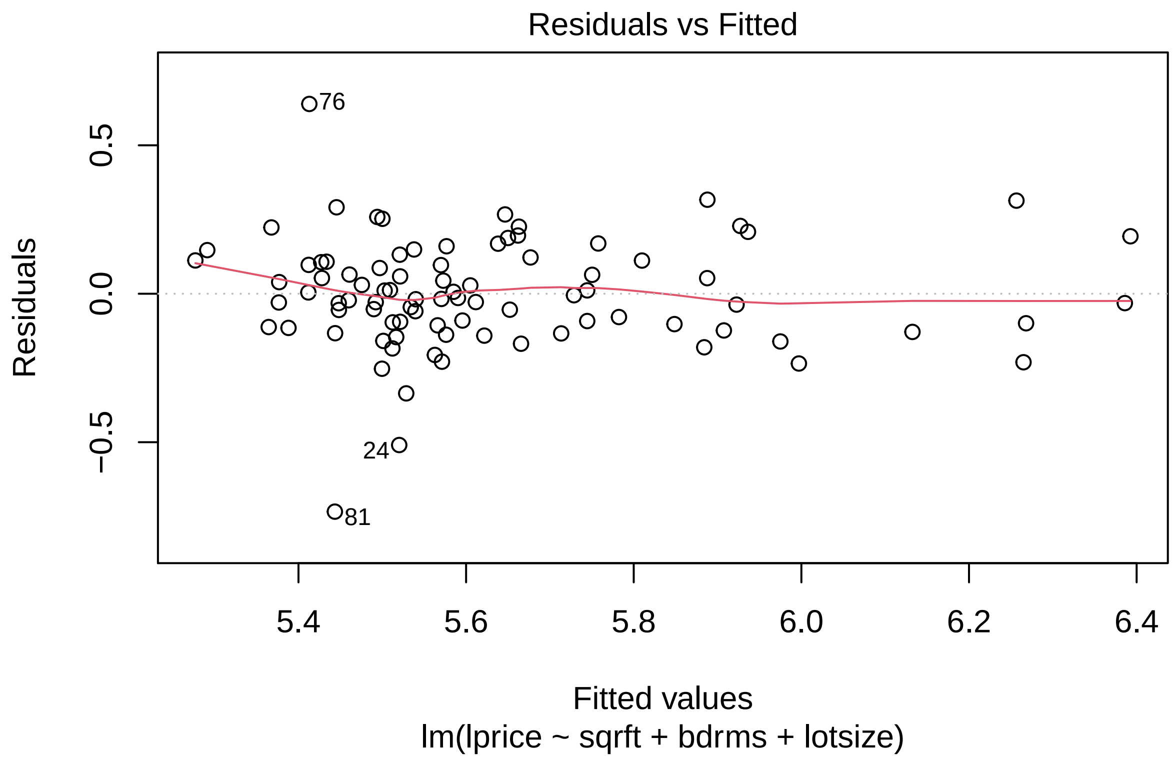
All other independent variables constant for every unit increase in lot size the house price in dollars increases 0.001% on average.

When there are no effects from any independent variables, when they are all equal to zero the average price is 4,759. However, this value is not statistically significant in this regression because the p value is greater than 0.05.

(k) Heteroskedasticity means that Assumption MLR.5 of homoskedasticity is violated. This means that the error variance is not the same across all values of the independent variables. Even though under heteroskedasticity the estimator $\hat{\beta}$ remains unbiased the variance of that estimator is biased and we can no longer do hypothesis testing. Additionally the standard errors and t-statistics we calculated are not correct. Additionally the standard errors are higher in the new model and because standard errors are related to variance we see a higher variance.

We can see this through a graphic representation of the residuals v. the fitted values and a BP test

```r
plot(modelJ_1, which = 1)
```



The graphic supports our argument of homooskedasticity because as we increase our fitted values and our independent variables we do not see a change in the variance of our residuals. The model is homoskedastic because the residuals are evenly distributed even as we increase our independent variables.

Our BP value is 3.543 with three degrees of freedom and a p-value of 0.315. Our null hypothesis is that if our BP value is equal to 0 than there is homoskedasticity in the model. Because we have a p-value of greater than 0.05 we fail to reject the null and there is strong evidence for homoskedacitiy and MLR.5 is held. Our boss is right using log price instead of price fixes our model and makes it homoskedastic.

(l) To see if we should include sqrft in the model we need to calculate the OVB for the estimator for bedrooms.

Table 11: Regression Results (l-1)

	Dependent variable:	
	price	
	(1)	(2)
sqrft	0.123***	
	(0.013)	
bdrms	13.853	57.313***
	(9.010)	(10.885)
lotsize	0.002***	0.003***
	(0.001)	(0.001)
Constant	−21.770	63.262
	(29.475)	(39.620)
Observations	88	88
R^2	0.672	0.337
Adjusted R^2	0.661	0.321
Residual Std. Error	59.833 (df = 84)	84.624 (df = 85)
F Statistic	57.460*** (df = 3; 84)	21.585*** (df = 2; 85)
Note:		*p<0.1; **p<0.05; ***p<0.01
		Standard errors in parentheses

We can see that the coefficient for bedrooms increase from 12.853 to 57.313. Thus the OVB for this coefficient is -44.46. Omitting sqrft increases the bedrooms effect on price. And it increases the variance of that effect on price as we see an increase in the standard error. Standard error is a statistic that is correlated with variance. And variance increases. Yes we should included sqrft because we want to minimize variance and the adjusted r-squared value increases.

Problem 3 (a) The causal question is related to proving the Solow Growth Model through modern data about GDP and GDP growth taken from 1960-2021. Specifically, we want to study if real income is higher in countries with higher savings rates and lower in countries with higher values of $n + g + \delta$. We want to see how log(I/GDP) and $\log(n + g + \delta)$ (X) effect log(GDP) per working age person in 2021 (Y). We assume that g + delta are constant over every country and equal to 0.05.

(b) The ideal experiment would need a large random sample of countries all with the exact same economies and we could measure how Investment / GDP and n the average rate of growth of the working age population (age defined as 14 to 64) on the log gdp of a working age person in 2021.

(c) Some issues with the ideal experiment are that cannot just clone countries economies, population, and size. Countries are huge entities which have complex and intricate differences that cannot be duplicated.

(d) I might add controls like measures for Human capital to control the variability among countries. An example might be the varying levels of education. This would increase the credibility of MLR.4 being true because educational achievement is correlated to investment and GDP an independent variable. Thus if we include it, remove it from our error term it will make the Zero Conditional Mean assumption more credible and make the assumption that our estimator is unbiased.