

termvar, α , β , γ , \mathcal{L} , x , y , z , f , g , h , adv
index, i , j , n , m

Foc	$::=$ A S	Focusing phase Async Sync
t, v	$::=$ $t_1 t_2$ let $\langle p \rangle \leftarrow t_1$ in t_2 case t of $Cases$ $\lambda p.t$ $\lambda p.t$ $[t]$ $\langle t \rangle$ x $C t_0 \dots t_n$ C n let $[p] = t_1$ in t_2 S (t_1, t_2) $-$	Terms Application Effectful Let Binding Case Function Function with graded annotation on its binder Promote Pure Variable Constructor Integer constructors Modal let-binding A pair of terms Hole
C	$::=$ $(,)$ inl inr unit tt ff Just Nothing	Constructors Pair constructor Left injection Right injection Unit
$Cases$	$::=$ $p \rightarrow t; Cases$ $p \rightarrow t$ $\overline{p \mapsto t}$ $p \mapsto t; \dots; p' \mapsto t'$ S	Value-level cases Case cons One case One case overline Many cases (syntactic sugar)
p	$::=$ x $-$ $[p]$ $[[p]]$ $C p_1 \dots p_n$ $C p_1^{Ix_3} \dots p_2^{Ix_3}$ C n (p) y_j^i (p_1, p_2) p^{Ix} S	Patterns Variable Wildcard Unbox Double unboxing Constructor ConstructorIndexed Nullary Constructor Int constructor Hack Pair Pattern at index n+m

Ix	$::=$ $ $ 0 $ $ 1 $ $ 2 $ $ $Ix_1 + Ix_2$ $ $ m $ $ 1 $ $ 2 $ $ 3 $ $ 4 $ $ i $ $ j $ $ n $ $ m	More complex index expressions
Eqn	$::=$ $ $ $x \ p_1 .. p_n = t$	Equations Eq
Def	$::=$ $ $ $x : C; Eqn_1 .. Eqn_n$	Definitions Multi-eq def
A, B, C, E, W, C	$::=$ $ $ \cdot $ $ $A \rightarrow B$ $ $ A_{DEC} $ $ $A^c \rightarrow B$ $ $ $A^c \multimap B$ $ $ $A \multimap B$ $ $ K $ $ $K A \dots B$ $ $ α $ $ $A B$ $ $ $A^{Ix_1}_{Ix_2}$ $ $ $\Box A$ $ $ $\Box_c A$ $ $ $\Box_{c:B} A$ $ $ Int $ $ Char $ $ Unit $ $ \otimes $ $ \mathbb{B} $ $ IO $ $ R $ $ n $ $ $A \text{ op } B$ $ $ $(A \times B)$ $ $ $(A + B)$ $ $ $\{A_1, \dots, A_n\}$ $ $ $\downarrow \kappa$ $ $ Set A	Types Empty Function Dec Graded Function Graded Linear Function S Linear Function Constructor Constructor Variable Application Var2IndexTy BlankBox Box Box with coeffect type Integers Characters unit Products Bool IO Coeffect types Type-level integers InfixOp Tuple Sum

	$B_1^1 \rightarrow \dots \rightarrow B_n^1 \rightarrow A$ S $B_1^{q_1} \rightarrow \dots \rightarrow B_n^{q_n} \rightarrow A$ S	n-Ary Function n-Ary Function w grades
op	$::=$ $+$ $*$ $-$ \leq $<$ $>$ \geq $=$ \neq \sqcup \sqcap	Type operators
R, S	$::=$ Nat Level Ext Interval $A \times B$ $R S$ α β	Coefficient types Nat Level Extending Interval Products Application Variable VariableB
$Cons$	$::=$ $A, Cons$ A $A_1, .., A_n$ \overrightarrow{A} $(Cons)$	Constraints/Predicates
c, r, s, q	$::=$ $c_{Ix_1}^{Ix_1}$ $c_{Ix_2}^{Ix_2}$ c_{Ix_1} $c_1 + c_2$ $c_1 \cdot c_2$ 0 1 $c_1 \sqcup c_2$ $c_1 \sqcap c_2$ $c_1 \sqcap \dots \sqcap c_2$ $c_1 \sqcup \dots \sqcup c_2$ $c_1 = c_2$ $\bigsqcup_1^n c$ $\text{flatten}(c_1, A, c_2, B)$ $c_1 .. c_2$ ∞ (c_1, c_2)	Var2Index Var1Index Addition Multiplication Additive Unit Multiplicative Unit Join Meet MultiMeet MultiJoin CoeffEq BigJoin Flatten

		θc	
		2	S
		3	S
		4	S
		c_{i-1}	
Rel	$::=$	Relations on grades	
		$c_1 \sqsubseteq c_2$	
		$c_1 \sqsubseteq c_2 \sqsubseteq c_3$	
		$c_1 \sqsupseteq c_2$	
		$c_1 \sqsupseteq c_2 \sqsupseteq c_3$	
		$Rel_1 \wedge Rel_2$	
κ	$::=$		
		Type	Type
		$\uparrow A$	Promote a type to a kind
		Effect	Effect grades
		Coeffect	Coeffect grades
		Eff	
		Coeff	
		(Co)eff	
		Predicate	Predicates
		$\kappa_1 \rightarrow \kappa_2$	Kind function
		$\kappa_1 \cup \kappa_2$	
		$\theta \kappa$	Substitutions
		(κ)	
D	$::=$		
		\emptyset	Empty
		D_1, D_2	
		(D)	
As	$::=$		
		$x : C$	Singleton context
		$x :_r C$	Singleton context w/ graded assumption
		$x_{Ix_1}^{Ix_2} :_c C$	Indexed Variable
		$x_{Ix_1}^{Ix_2} :_c C$	Indexed Variable 1
Γ, Δ, Ω	$::=$		
		\emptyset	Empty
		As	Single assumption
$r : ?R$	$::=$		
		c	
		$-$	
		$(r : ?R)$	
		$\theta r : ?R$	
P	$::=$		

		$P_1 \wedge P_2$	
		$P_1 \wedge \dots \wedge P_n$	
		$P_1 \vee P_2$	
		$P_1 \rightarrow P_2$	
		$\forall \alpha. P$	
		$\neg P$	
		$\exists \alpha. P$	
		$\mathbf{t}_1 \equiv \mathbf{t}_2$	
		$\mathbf{t}_1 \sqsubseteq \mathbf{t}_2$	
		\top	
		(P)	
		$\llbracket \theta \rrbracket$	
		$\llbracket Cons \rrbracket$	
\mathbf{t}	$::=$		
		$\llbracket c \rrbracket$	
		$\llbracket A \rrbracket$	
θ, θ_κ	$::=$		
		\emptyset	Empty
		$\theta_1 \uplus \theta_2$	Union
		$x \mapsto B$	SingletonTy
		$x \mapsto \kappa$	SingletonKind
		$x \mapsto c$	Singleton Coeffect
		$\theta\theta'$	Substitution over a substitution
		$\theta_1 \uplus \dots \uplus \theta_2$	
		θ, θ'	Disjoint cat
		$\theta \setminus x$	Remove a substitution for a variable
\mathcal{C}	$::=$		
		$x \mapsto c$	
		$\mathcal{C} + \mathcal{C}'$	
		$\mathcal{C}, \mathcal{C}'$	
		(\mathcal{C})	
		\mathcal{C}	
		\emptyset	
		$c \cdot \mathcal{C}$	

$\Gamma \vdash t : A$ **1 Typing**

$$\begin{array}{c}
\frac{}{0 \cdot \Gamma, x :_1 A \vdash x : A} \quad \text{TYVAR} \\
\\
\frac{\Gamma, x :_r A \vdash t : B}{\Gamma \vdash \lambda x. t : A^r \rightarrow B} \quad \text{TYABS} \\
\\
\frac{\Gamma_1 \vdash t_1 : A^r \rightarrow B \quad \Gamma_2 \vdash t_2 : A}{\Gamma_1 + r \cdot \Gamma_2 \vdash t_1 t_2 : B} \quad \text{TYAPP} \\
\\
\frac{(C : B_1^{q_1} \rightarrow \dots \rightarrow B_n^{q_n} \rightarrow K \vec{A}) \in D}{0 \cdot \Gamma \vdash C : B_1^{q_1} \rightarrow \dots \rightarrow B_n^{q_n} \rightarrow K \vec{A}} \quad \text{TYCON}
\end{array}$$

$$\begin{array}{c}
\frac{\Gamma \vdash t : A}{r \cdot \Gamma \vdash [t] : \Box_r A} \quad \text{TYPR} \\
\\
\frac{\Gamma, x :_r A, \Gamma' \vdash t : B \quad r \sqsubseteq s}{\Gamma, x :_s A, \Gamma' \vdash t : B} \quad \text{TYAPPROX} \\
\\
\frac{\Gamma \vdash t : A \quad r \vdash p_i : A \triangleright \Delta_i \quad \Gamma', \Delta_i \vdash t_i : B}{r \cdot \Gamma + \Gamma' \vdash \mathbf{case} \, t \, \mathbf{of} \, p_1 \mapsto t_1; \dots; p_n \mapsto t_n : B} \quad \text{TYCASE}
\end{array}$$

$c \vdash p : A \triangleright \Gamma$

Declarative pattern checking for Granule Mini (monomorphic)

$$\begin{array}{c}
\frac{0 \sqsubseteq r}{r \vdash _ : A \triangleright \emptyset} \quad \text{PATWILD} \\
\\
\frac{}{r \vdash p : A \triangleright x :_r A} \quad \text{PATVAR} \\
\\
\frac{r \cdot s \vdash p : A \triangleright \Gamma}{r \vdash [p] : \Box_s A \triangleright \Gamma} \quad \text{PATBOX} \\
\\
\frac{(C : B_1^{q_1} \rightarrow \dots \rightarrow B_n^{q_n} \rightarrow K \vec{A}) \in D \quad q_i \cdot r \vdash p_i : B_i \triangleright \Gamma_i \quad |K \vec{A}| > 1 \Rightarrow 1 \sqsubseteq r}{r \vdash C \, p_1 \dots p_n : K \vec{A} \triangleright \vec{\Gamma}_i} \quad \text{PATCON}
\end{array}$$

$\Gamma_1 \vdash A \Rightarrow^- t \mid \Gamma_2$

$\Gamma \vdash A \Rightarrow t \mid \Delta$

$$\begin{array}{c}
\frac{}{\Gamma, x :_r A \vdash A \Rightarrow x \mid 0 \cdot \Gamma, x :_1 A} \quad \text{VAR} \\
\\
\frac{\Gamma, x :_q A \vdash B \Rightarrow t \mid \Delta, x :_r A \quad r \sqsubseteq q}{\Gamma \vdash A^q \rightarrow B \Rightarrow \lambda x. t \mid \Delta} \quad \text{ABS} \\
\\
\frac{\Gamma, x_1 :_{r_1} A^q \rightarrow B, x_2 :_{r_1} B \vdash C \Rightarrow t_1 \mid \Delta_1, x_1 :_{s_1} A^q \rightarrow B, x_2 :_{s_2} B \quad \Gamma, x_1 :_{r_1} A^q \rightarrow B \vdash A \Rightarrow t_2 \mid \Delta_2, x_1 :_{s_3} A^q \rightarrow B}{\Gamma, x_1 :_{r_1} A^q \rightarrow B \vdash C \Rightarrow [(x_1 \, t_2)/x_2] t_1 \mid (\Delta_1 + s_2 \cdot q \cdot \Delta_2), x_1 :_{s_2 + s_1 + (s_2 \cdot q \cdot s_3)} A^q \multimap B} \quad \text{APP} \\
\\
\frac{\Gamma \vdash A \Rightarrow t \mid \Delta}{\Gamma \vdash \Box_r A \Rightarrow [t] \mid r \cdot \Delta} \quad \text{BOX} \\
\\
\frac{(C : B_1^{q_1} \rightarrow \dots \rightarrow B_n^{q_n} \rightarrow K \vec{A}) \in D \quad \Gamma \vdash B_i \Rightarrow t_i \mid \Delta_i}{\Gamma \vdash K \vec{A} \Rightarrow C \, t_1 \dots t_n \mid 0 \cdot \Gamma + (q_1 \cdot \Delta_1) + \dots + (q_n \cdot \Delta_n)} \quad \text{CON} \\
\\
\frac{(C : B_1^{r_1} \rightarrow \dots \rightarrow B_n^{r_n} \rightarrow K A) \in D \quad \Gamma, x :_1 B_1^{q_1} \rightarrow \dots \rightarrow B_n^{q_n} \rightarrow K A \vdash A_1 \Rightarrow t \mid \Delta, x :_1 B_1^{q_1} \rightarrow \dots \rightarrow B_n^{q_n} \rightarrow K A}{\Gamma \vdash A_1 \Rightarrow t \mid \Delta} \quad \text{CONALT} \\
\\
\frac{(C_i : B_1^{q_1^i} \rightarrow \dots \rightarrow B_n^{q_n^i} \rightarrow K \vec{A}) \in D \quad \Gamma, x :_r K A, y_1^i :_{q_1^i} B_1, \dots, y_n^i :_{q_n^i} B_n \vdash B \Rightarrow t_i \mid \Delta_i, x :_{r_i} K A, y_1^i :_{s_1^i} B_1, \dots, y_n^i :_{s_n^i} B_n \quad s_j^i \sqsubseteq q_j^i \quad s_i = s_1^i \sqcup \dots \sqcup s_n^i}{\Gamma, x :_r K A \vdash B \Rightarrow \mathbf{case} \, x \, \mathbf{of} \, C_i \, y_1^i \dots y_n^i \mapsto t_i \mid (\Delta_1 \sqcup \dots \sqcup \Delta_n), x :_{(r_1 \sqcup \dots \sqcup r_n) + (s_1 \sqcup \dots \sqcup s_n)} K A} \quad \text{CASE}
\end{array}$$

$$\begin{array}{c}
(C_i : B_1^{q_1} \rightarrow \dots \rightarrow B_n^{q_n} \rightarrow K \vec{A}) \in D \\
\Gamma, x :_r K \vec{A}, y_1^i :_{r \cdot q_1^i} B_1, \dots, y_n^i :_{r \cdot q_n^i} B_n \vdash B \Rightarrow t_i \mid \Delta_i, x :_{r_i} K \vec{A}, y_1^i :_{s_1^i} B_1, \dots, y_n^i :_{s_n^i} B_n \\
\exists s_j^i. s_j^i \sqsubseteq s_j^i \cdot q_j^i \sqsubseteq r \cdot q_j^i \\
s_i = s_1^i \sqcup \dots \sqcup s_n^i \\
|K \vec{A}| > 1 \Rightarrow 1 \sqsubseteq s_1 \sqcup \dots \sqcup s_m \\
\hline
\Gamma, x :_r K \vec{A} \vdash B \Rightarrow \mathbf{case} \ x \ \mathbf{of} \ \overline{C_i \ y_1^i \dots y_n^i \mapsto t_i \mid (\Delta_1 \sqcup \dots \sqcup \Delta_m)}, x :_{(r_1 \sqcup \dots \sqcup r_m) + (s_1 \sqcup \dots \sqcup s_m)} K \vec{A} \quad \text{CASEALT}
\end{array}$$

$$\begin{array}{c}
(C_i : B_1^{q_1} \rightarrow \dots \rightarrow B_n^{q_n} \rightarrow K A) \in D \\
\Gamma, x :_r K A, y_1^i :_{r \cdot q_1^i} B_1, \dots, y_n^i :_{r \cdot q_n^i} B_n \vdash B \Rightarrow t_i \mid \Delta_i, x :_{r_i} K A, y_1^i :_{s_1^i} B_1, \dots, y_n^i :_{s_n^i} B_n \\
s_i = s_1^i \sqcup \dots \sqcup s_n^i \\
q_i = q_1^i \sqcup \dots \sqcup q_n^i \\
\exists s_i'. s_i \sqsubseteq s_i' \cdot q_i \sqsubseteq r \cdot q_i \\
\hline
\Gamma, x :_r K A \vdash B \Rightarrow \mathbf{case} \ x \ \mathbf{of} \ \overline{C_i \ y_1^i \dots y_n^i \mapsto t_i \mid (\Delta_1 \sqcup \dots \sqcup \Delta_n)}, x :_{(r_1 \sqcup \dots \sqcup r_n) + (s_1' \sqcup \dots \sqcup s_n')} K A \quad \text{CASEALTALT}
\end{array}$$

$$\begin{array}{c}
\Gamma, y :_{r \cdot q} A, x :_r \Box_q A \vdash B \Rightarrow t \mid \Delta, y :_{s_1} A, x :_{s_2} \Box_q A \\
\exists s_3. s_1 \sqsubseteq s_3 \cdot q \sqsubseteq r \cdot q \\
\hline
\Gamma, x :_r \Box_q A \vdash B \Rightarrow \mathbf{case} \ x \ \mathbf{of} \ [y] \rightarrow t \mid \Delta, x :_{s_3 + s_2} \Box_q A \quad \text{UNBOX}
\end{array}$$

$$\begin{array}{c}
(C_i : B_1^{q_1} \rightarrow \dots \rightarrow B_n^{q_n} \rightarrow K A) \in D \\
\Gamma, x :_r K A, y_1^i :_{r \cdot q_1^i} B_1, \dots, y_n^i :_{r \cdot q_n^i} B_n \vdash B \Rightarrow t_i \mid \Delta_i, x :_{r_i^i} K A, y_1^i :_{s_1^i} B_1, \dots, y_n^i :_{s_n^i} B_n \\
\exists r_2^i. r \sqsupseteq r_2^i + r_1^i \cdot (s_1^i \sqcap \dots \sqcap s_n^i) \\
\hline
\Gamma, x :_{r_1} K A \vdash B \Rightarrow \mathbf{case} \ x \ \mathbf{of} \ \overline{C_i \ y_1^i \dots y_n^i \mapsto t_i \mid (\Delta_1 \sqcap \dots \sqcap \Delta_n)}, x :_{r_2^1 \sqcap \dots \sqcap r_2^n} K A \quad \text{CASESUB}
\end{array}$$

$\Gamma; \Omega \vdash A \Uparrow t \mid \Delta$

$$\frac{\Gamma; \Omega \vdash B \Uparrow t \mid \Delta, x :_r A \quad r \sqsubseteq q}{\Gamma; \Omega \vdash A^q \rightarrow B \Uparrow \lambda x. t \mid \Delta} \quad \text{ABSF}$$

$\Gamma; \Omega \Uparrow A \Rightarrow t \mid \Delta$

$$\begin{array}{c}
(C_i : B_1^{q_1} \rightarrow \dots \rightarrow B_n^{q_n} \rightarrow K A) \in D \\
\Gamma; \Omega, x :_r K A_1, y_1^i :_{q_1^i} B_1, \dots, y_n^i :_{q_n^i} B_n \Uparrow B \Rightarrow t_i \mid \Delta_i, x :_{r_i} K A, y_1^i :_{s_1^i} B_1, \dots, y_n^i :_{s_n^i} B_n \\
s_j^i \sqsubseteq q_j^i \\
s_i = s_1^i \sqcup \dots \sqcup s_n^i \\
\hline
\Gamma; \Omega, x :_r K A \Uparrow B \Rightarrow \mathbf{case} \ x \ \mathbf{of} \ \overline{C_i \ y_1^i \dots y_n^i \mapsto t_i \mid (\Delta_1 \sqcup \dots \sqcup \Delta_n)}, x :_{(r_1 \sqcup \dots \sqcup r_n) + (s_1 \sqcup \dots \sqcup s_n)} K A \quad \text{CASEF}
\end{array}$$

$$\begin{array}{c}
\Gamma; \Omega, y :_{r \cdot q} A, x :_r \Box_q A \Uparrow B \Rightarrow t \mid \Delta, y :_{s_1} A, x :_{s_2} \Box_q A \\
\exists s_3. s_3 \cdot q \sqsubseteq s_1 \\
\hline
\Gamma; \Omega, x :_r \Box_q A \Uparrow B \Rightarrow \mathbf{case} \ x \ \mathbf{of} \ [y] \rightarrow t \mid \Delta, x :_{s_3 + s_2} \Box_q A \quad \text{UNBOXF}
\end{array}$$

$\Gamma \vdash A \Downarrow t \mid \Delta$

$$\frac{\Gamma \vdash A \Downarrow t \mid \Delta}{\Gamma \vdash \Box_r A \Downarrow [t] \mid r \cdot \Delta} \quad \text{BoxF}$$

$$\frac{(C : B_1^1 \rightarrow \dots \rightarrow B_n^1 \rightarrow K A) \in D \quad \Gamma \vdash B_i \Downarrow t_i \mid \Delta_i}{\Gamma \vdash K A \Downarrow C \ t_1 \dots t_n \mid \Delta_1 + \dots + \Delta_n} \quad \text{CONF}$$

$\Gamma; \Omega \Downarrow A \Rightarrow t \mid \Delta$

$$\frac{}{\Gamma; x :_r A \Downarrow A \Rightarrow x \mid 0 \cdot \Gamma, x :_1 A} \quad \text{VARF}$$

$$\frac{\begin{array}{l} \Gamma; x_1 :_{r_1} A^q \rightarrow B, x_2 :_{r_1} B \Downarrow \vdash C \Rightarrow t_1 \mid \Delta_1, x_1 :_{s_1} A^q \rightarrow B, x_2 :_{s_2} B \\ \Gamma; x_1 :_{r_1} A^q \rightarrow B \Downarrow \vdash A \Rightarrow t_2 \mid \Delta_2, x_1 :_{s_3} A^q \rightarrow B \end{array}}{\Gamma; x_1 :_{r_1} A^q \rightarrow B \Downarrow \vdash C \Rightarrow [(x_1 t_2)/x_2]t_1 \mid (\Delta_1 + s_2 \cdot q \cdot \Delta_2), x_1 :_{s_2+s_1+(s_2 \cdot q \cdot s_3)} A^q \multimap B} \text{ APPF}$$

Definition rules: 29 good 0 bad

Definition rule clauses: 78 good 0 bad