$termvar$, α, β, γ, $\mathcal{L}$, x, y, z, f, g, h, adv
$index$, i, j, n, m

Foc	::=			Focusing phase
	|	A		Async
	|	S		Sync
$t,\ v$	::=			Terms
	|	$t_1\ t_2$		Application
	|	$\textbf{let}\ \langle p \rangle \leftarrow t_1\ \textbf{in}\ t_2$		Effectful Let Binding
	|	$\textbf{case}\ t\ \textbf{of}\ \mathit{Cases}$		Case
	|	$\lambda p.t$		Function
	|	$\lambda p.t$		Function with graded annotation on its binder
	|	$[t]$		Promote
	|	$\langle t \rangle$		Pure
	|	x		Variable
	|	$C\ t_0 \ldots t_n$		Constructor
	|	C		
	|	n		Integer constructors
	|	$\textbf{let}\ [p] = t_1\ \textbf{in}\ t_2$	S	Modal let-binding
	|	$\textbf{inl}\ t$		Inl
	|	$\textbf{inr}\ t$		Inr
	|	$\textbf{let}\ (x_1, x_2) = t_1\ \textbf{in}\ t_2$	S	Pair let-binding
	|	(t_1, t_2)		A pair of terms
	|	$-$		Hole
	|	$\textbf{let}\ () = t_1\ \textbf{in}\ t_2$		UnitElim
	|	$()$		Unit
C	::=			Constructors
	|	$(,)$		Pair constructor
	|	**inl**		Left injection
	|	**inr**		Right injection
	|	unit		Unit
	|	tt		
	|	ff		
	|	Just		
	|	Nothing		
Cases	::=			Value-level cases
	|	$p \rightarrow t; \mathit{Cases}$		Case cons
	|	$p \rightarrow t$		One case
	|	$\overline{p \mapsto t}$		One case overline
	|	$p \mapsto t; \ldots; p' \mapsto t'$	S	Many cases (syntactic sugar)
p	::=			Patterns
	|	x		Variable
	|	$\_$		Wildcard
	|	$[p]$		Unbox
	|	$[[p]]$		Double unboxing
	|	$C\ p_1 \ldots p_n$		Constructor
	|	$C\ \ p_1{}^{Ix_3}_{Ix_1} \ldots p_2{}^{Ix_3}_{Ix_2}$		ConstructorIndexed
	|	C		Nullary Constructor
	|	n		Int constructor

	|	(p)		
	|	y^{i}_{j}		Hack
	|	(p_1, p_2)		Pair
	|	p^{Ix}	S	Pattern at index n+m
Ix	::=			More complex index expressions
	|	0		
	|	1		
	|	2		
	|	$Ix_1 + Ix_2$		
	|	m		
	|	1		
	|	2		
	|	3		
	|	4		
	|	i		
	|	j		
	|	n		
	|	m		
Eqn	::=			Equations
	|	$x\, p_1 \,..\, p_n = t$		Eq
Def	::=			Definitions
	|	$x : C; Eqn_1 \,..\, Eqn_n$		Multi-eq def
$A,\ B,\ C,\ E,\ W,\ C$	::=			Types
	|	$\cdot$		Empty
	|	$A \rightarrow B$		Function
	|	A_{DEC}		Dec
	|	$A^{c} \rightarrow B$		Graded Function
	|	$A^{c} \multimap B$		Graded Linear Function
	|	$A \multimap B$	S	Linear Function
	|	K		Constructor
	|	$K\, A \ldots B$		Constructor
	|	α		Variable
	|	$A\, B$		Application
	|	$A^{Ix_1}_{Ix_2}$		Var2IndexTy
	|	$\Box A$		BlankBox
	|	$\Box_{c} A$		Box
	|	$\Box_{c:B} A$		Box with coeffect type
	|	Int		Integers
	|	Char		Characters
	|	$\mathsf{1}$		unit
	|	$\otimes$		Products
	|	$\mathbb{B}$		Bool
	|	IO		IO
	|	R		Coeffect types
	|	n		Type-level integers
	|	$A\, \mathsf{op}\, B$		InfixOp

	$\mid$	$A \otimes B$		Tuple
	$\mid$	$A \oplus B$		Sum
	$\mid$	$\{A_1, ..., A_n\}$		
	$\mid$	$\downarrow\kappa$		
	$\mid$	$\mathsf{Set}\ A$		
	$\mid$	${B_1}^1 \to ... \to {B_n}^1 \to A$	S	n-Ary Function
	$\mid$	${B_1}^{q_1} \to ... \to {B_n}^{q_n} \to A$	S	n-Ary Function w grades
op	$::=$			Type operators
	$\mid$	$+$		
	$\mid$	$*$		
	$\mid$	$-$		
	$\mid$	$\leq$		
	$\mid$	$<$		
	$\mid$	$>$		
	$\mid$	$\geq$		
	$\mid$	$=$		
	$\mid$	$\neq$		
	$\mid$	$\sqcup$		
	$\mid$	$\sqcap$		
$R,\ S$	$::=$			Coeffect types
	$\mid$	Nat		Nat
	$\mid$	Level		Level
	$\mid$	Ext		Extending
	$\mid$	$\mathsf{Interval}$		Interval
	$\mid$	$A \times B$		Products
	$\mid$	$R\ S$		Application
	$\mid$	α		Variable
	$\mid$	β		VariableB
$Cons$	$::=$			Constraints/Predicates
	$\mid$	$A, Cons$		
	$\mid$	A		
	$\mid$	$A_1,\ ..,\ A_n$		
	$\mid$	$\overrightarrow{A}$		
	$\mid$	$(Cons)$		
$c,\ r,\ s,\ q$	$::=$			
	$\mid$	$c^{Ix_1}_{Ix_2}$		Var2Index
	$\mid$	c_{Ix_1}		Var1Index
	$\mid$	$c_1 + c_2$		Addition
	$\mid$	$c_1 \cdot c_2$		Multiplication
	$\mid$	0		Additive Unit
	$\mid$	1		Multiplicative Unit
	$\mid$	$c_1 \sqcup c_2$		Join
	$\mid$	$c_1 \sqcap c_2$		Meet
	$\mid$	$c_1 \sqcap ... \sqcap c_2$		MultiMeet
	$\mid$	$c_1 \sqcup ... \sqcup c_2$		MultiJoin
	$\mid$	$c_1 = c_2$		CoeffEq

	$\mid$	$\bigsqcup_1^n c$		BigJoin
	$\mid$	$\mathsf{flatten}(c_1, A, c_2, B)$		Flatten
	$\mid$	$c_1..c_2$		
	$\mid$	∞		
	$\mid$	(c_1, c_2)		
	$\mid$	θc		
	$\mid$	2	S	
	$\mid$	3	S	
	$\mid$	4	S	
	$\mid$	5	S	
	$\mid$	6	S	
	$\mid$	10	S	
	$\mid$	15	S	
	$\mid$	20	S	
	$\mid$	c_{i-1}		
	$\mid$	Unused		
	$\mid$	$[r...s]$		
Rel	$::=$			Relations on grades
	$\mid$	$c_1 \sqsubseteq c_2$		
	$\mid$	$c_1 \sqsubseteq c_2 \sqsubseteq c_3$		
	$\mid$	$c_1 \sqsupseteq c_2$		
	$\mid$	$c_1 \sqsupseteq c_2 \sqsupseteq c_3$		
	$\mid$	$\mathit{Rel}_1 \wedge \mathit{Rel}_2$		
κ	$::=$			
	$\mid$	Type		Type
	$\mid$	$\uparrow A$		Promote a type to a kind
	$\mid$	Effect		Effect grades
	$\mid$	$\mathsf{Coeffect}$		Coeffect grades
	$\mid$	Eff		
	$\mid$	Coeff		
	$\mid$	$\mathsf{(Co)eff}$		
	$\mid$	$\mathsf{Predicate}$		Predicates
	$\mid$	$\kappa_1 \to \kappa_2$		Kind function
	$\mid$	$\kappa_1 \cup \kappa_2$		
	$\mid$	$\theta\kappa$		Substitutions
	$\mid$	(κ)		
D	$::=$			
	$\mid$	$\emptyset$		Empty
	$\mid$	D_1, D_2		
	$\mid$	(D)		
As	$::=$			
	$\mid$	$x : C$		Singleton context
	$\mid$	$x :_r C$		Singleton context w/ graded assumption
	$\mid$	$x^{\mathit{Ix}_1}_{\mathit{Ix}_2} :_c C$		Indexed Variable
	$\mid$	$x^{\mathit{Ix}_1} :_c C$		Indexed Variable 1

$\Gamma,\ \Delta,\ \Omega$	$::=$		
	$\vert$	$\emptyset$	Empty
	$\vert$	As	Single assumption
$r :?R$	$::=$		
	$\vert$	c	
	$\vert$	$-$	
	$\vert$	$(r :?R)$	
	$\vert$	$\theta r :?R$	
P	$::=$		
	$\vert$	$P_1 \wedge P_2$	
	$\vert$	$P_1 \wedge ... \wedge P_n$	
	$\vert$	$P_1 \vee P_2$	
	$\vert$	$P_1 \rightarrow P_2$	
	$\vert$	$\forall \alpha . P$	
	$\vert$	$\neg P$	
	$\vert$	$\exists \alpha . P$	
	$\vert$	$\mathbf{t}_1 \equiv \mathbf{t}_2$	
	$\vert$	$\mathbf{t}_1 \sqsubseteq \mathbf{t}_2$	
	$\vert$	$\top$	
	$\vert$	(P)	
	$\vert$	$[\![\theta]\!]$	
	$\vert$	$[\![Cons]\!]$	
$\mathbf{t}$	$::=$		
	$\vert$	$[\![c]\!]$	
	$\vert$	$[\![A]\!]$	
$\theta,\ \theta_\kappa$	$::=$		
	$\vert$	$\emptyset$	Empty
	$\vert$	$\theta_1 \uplus \theta_2$	Union
	$\vert$	$x \mapsto B$	SingletonTy
	$\vert$	$x \mapsto \kappa$	SingletonKind
	$\vert$	$x \mapsto c$	Singleton Coeffect
	$\vert$	$\theta\theta'$	Substitution over a substitution
	$\vert$	$\theta_1 \uplus \ldots \uplus \theta_2$	
	$\vert$	θ, θ'	Disjoint cat
	$\vert$	$\theta \setminus x$	Remove a substition for a variable
$\mathcal{C}$	$::=$		
	$\vert$	$x \mapsto c$	
	$\vert$	$\mathcal{C} + \mathcal{C}'$	
	$\vert$	$\mathcal{C}, \mathcal{C}'$	
	$\vert$	$(\mathcal{C})$	
	$\vert$	$\mathcal{C}$	
	$\vert$	$\emptyset$	
	$\vert$	$c \cdot \mathcal{C}$	

$\boxed{\Gamma \vdash t : A}$

1 Typing

$$\frac{}{0 \cdot \Gamma, x :_1 A \vdash x : A}\ \textsc{TyVar}$$

$$\frac{\Gamma, x :_r A \vdash t : B}{\Gamma \vdash \lambda x.t : A^r \to B}\ \textsc{TyAbs}$$

$$\frac{\Gamma_1 \vdash t_1 : A^r \to B \qquad \Gamma_2 \vdash t_2 : A}{\Gamma_1 + r \cdot \Gamma_2 \vdash t_1\, t_2 : B}\ \textsc{TyApp}$$

$$\frac{(C : {B_1}^{q_1} \to ... \to {B_n}^{q_n} \to K\, \vec{A}) \in D}{0 \cdot \Gamma \vdash C : {B_1}^{q_1} \to ... \to {B_n}^{q_n} \to K\, \vec{A}}\ \textsc{TyCon}$$

$$\frac{\Gamma \vdash t : A}{r \cdot \Gamma \vdash [t] : \Box_r A}\ \textsc{TyPr}$$

$$\frac{\Gamma, x :_r A, \Gamma' \vdash t : B \qquad r \sqsubseteq s}{\Gamma, x :_s A, \Gamma' \vdash t : B}\ \textsc{TyApprox}$$

$$\frac{\Gamma \vdash t : A \qquad r \vdash\ p_i : A \rhd \Delta_i \qquad \Gamma', \Delta_i \vdash t_i : B}{r \cdot \Gamma + \Gamma' \vdash \textbf{case}\ t\ \textbf{of}\ p_1 \mapsto t_1; ...; p_n \mapsto t_n : B}\ \textsc{TyCase}$$

$\boxed{c \vdash\ p : A \rhd \Gamma}$ Declarative pattern checking for Granule Mini (monomorphic)

$$\frac{0 \sqsubseteq r}{r \vdash\ \_ : A \rhd \emptyset}\ \textsc{PatWild}$$

$$\frac{}{r \vdash\ p : A \rhd x :_r A}\ \textsc{PatVar}$$

$$\frac{r \cdot s \vdash\ p : A \rhd \Gamma}{r \vdash\ [p] : \Box_s A \rhd \Gamma}\ \textsc{PatBox}$$

$$\frac{\begin{array}{l}(C : {B_1}^{q_1} \to ... \to {B_n}^{q_n} \to K\, \vec{A}) \in D \\ q_i \cdot r \vdash\ p_i : B_i \rhd \Gamma_i \qquad |K\, \vec{A}| > 1 \Rightarrow 1 \sqsubseteq r\end{array}}{r \vdash\ C\, p_1 ... p_n : K\, \vec{A} \rhd \vec{\Gamma_i}}\ \textsc{PatCon}$$

$\boxed{\Gamma_1 \vdash A \Rightarrow^- t \mid \Gamma_2}$

$\boxed{\Gamma \vdash A \Rightarrow t \mid \Delta}$

$$\frac{}{\Gamma, x :_r A \vdash A \Rightarrow x \mid 0 \cdot \Gamma, x :_1 A}\ \textsc{Var}$$

$$\frac{\Gamma, x :_q A \vdash B \Rightarrow t \mid \Delta, x :_r A \qquad r \sqsubseteq q}{\Gamma \vdash A^q \to B \Rightarrow \lambda x.t \mid \Delta}\ \textsc{Abs}$$

$$\frac{\begin{array}{l}\Gamma, x_1 :_{r_1} A^q \to B, x_2 :_{r_1} B \vdash C \Rightarrow t_1 \mid \Delta_1, x_1 :_{s_1} A^q \to B, x_2 :_{s_2} B \\ \Gamma, x_1 :_{r_1} A^q \to B \vdash A \Rightarrow t_2 \mid \Delta_2, x_1 :_{s_3} A^q \to B\end{array}}{\Gamma, x_1 :_{r_1} A^q \to B \vdash C \Rightarrow [(x_1\, t_2)/x_2]t_1 \mid (\Delta_1 + s_2 \cdot q \cdot \Delta_2), x_1 :_{s_2 + s_1 + (s_2 \cdot q \cdot s_3)} A^q \multimap B}\ \textsc{App}$$

$$\frac{\Gamma \vdash A \Rightarrow t \mid \Delta}{\Gamma \vdash \Box_r A \Rightarrow [t] \mid r \cdot \Delta}\ \textsc{Box}$$

$$\frac{\begin{array}{l}(C : {B_1}^{q_1} \to ... \to {B_n}^{q_n} \to K\, \vec{A}) \in D \\ \Gamma \vdash B_i \Rightarrow t_i \mid \Delta_i\end{array}}{\Gamma \vdash K\, \vec{A} \Rightarrow C\, t_1 ... t_n \mid 0 \cdot \Gamma + (q_1 \cdot \Delta_1) + \ ...\ + (q_n \cdot \Delta_n)}\ \textsc{Con}$$

$$\frac{\begin{array}{l}(C : {B_1}^{r_1} \to ... \to {B_n}^{r_n} \to K\, A) \in D \\ \Gamma, x :_1 {B_1}^{q_1} \to ... \to {B_n}^{q_n} \to K\, A \vdash A_1 \Rightarrow t \mid \Delta, x :_1 {B_1}^{q_1} \to ... \to {B_n}^{q_n} \to K\, A\end{array}}{\Gamma \vdash A_1 \Rightarrow t \mid \Delta}\ \textsc{ConAlt}$$

$$\dfrac{\begin{array}{l}(C_i : {B_1}^{q^i_1} \to \ldots \to {B_n}^{q^i_n} \to K\,\vec{A}) \in D \\ \Gamma, x :_r K\,A, y^i_1 :_{q^i_1} B_1, \ldots, y^i_n :_{q^i_n} B_n \vdash B \Rightarrow^{+} t_i;\ \Delta_i, x :_{r_i} K\,A, y^i_1 :_{s^i_1} B_1, \ldots, y^i_n :_{s^i_n} B_n \\ s^i_j \sqsubseteq q^i_j \\ s_i = s^i_1 \sqcup \ldots \sqcup s^i_n \end{array}}{\Gamma, x :_r K\,A \vdash B \Rightarrow \textbf{case}\ x\ \textbf{of}\ \overline{C_i\ y^i_1 \ldots y^i_n \mapsto t_i} \mid (\Delta_1 \sqcup \ldots \sqcup \Delta_n), x :_{(r_1 \sqcup \ldots \sqcup r_n) + (s_1 \sqcup \ldots \sqcup s_n)} K\,A}\ \textsc{Case}$$

$$\dfrac{\begin{array}{l}(C_i : {B_1}^{q^i_1} \to \ldots \to {B_n}^{q^i_n} \to K\,\vec{A}) \in D \\ \Gamma, x :_r K\,\vec{A}, y^i_1 :_{r \cdot q^i_1} B_1, \ldots, y^i_n :_{r \cdot q^i_1} B_n \vdash B \Rightarrow t_i \mid \Delta_i, x :_{r_i} K\,\vec{A}, y^i_1 :_{s^i_1} B_1, \ldots, y^i_n :_{s^i_n} B_n \\ \exists {s'}^i_j.\ s^i_j \sqsubseteq {s'}^i_j \cdot q^i_j \sqsubseteq r \cdot q^i_j \\ s_i = {s'}^i_1 \sqcup \ldots \sqcup {s'}^i_n \\ |K\,\vec{A}| > 1 \Rightarrow 1 \sqsubseteq s_1 \sqcup \ldots \sqcup s_m \end{array}}{\Gamma, x :_r K\,\vec{A} \vdash B \Rightarrow \textbf{case}\ x\ \textbf{of}\ \overline{C_i\ y^i_1 \ldots y^i_n \mapsto t_i} \mid (\Delta_1 \sqcup \ldots \sqcup \Delta_m), x :_{(r_1 \sqcup \ldots \sqcup r_m) + (s_1 \sqcup \ldots \sqcup s_m)} K\,\vec{A}}\ \textsc{CaseAlt}$$

$$\dfrac{\begin{array}{l}(C_i : {B_1}^{q^i_1} \to \ldots \to {B_n}^{q^i_n} \to K\,A) \in D \\ \Gamma, x :_r K\,A, y^i_1 :_{r \cdot q^i_1} B_1, \ldots, y^i_n :_{r \cdot q^i_1} B_n \vdash B \Rightarrow t_i \mid \Delta_i, x :_{r_i} K\,A, y^i_1 :_{s^i_1} B_1, \ldots, y^i_n :_{s^i_n} B_n \\ s_i = s^i_1 \sqcup \ldots \sqcup s^i_n \\ q_i = q^i_1 \sqcup \ldots \sqcup q^i_n \\ \exists {s'}_i.\ s_i \sqsubseteq {s'}_i \cdot q_i \sqsubseteq r \cdot q_i \end{array}}{\Gamma, x :_r K\,A \vdash B \Rightarrow \textbf{case}\ x\ \textbf{of}\ \overline{C_i\ y^i_1 \ldots y^i_n \mapsto t_i} \mid (\Delta_1 \sqcup \ldots \sqcup \Delta_n), x :_{(r_1 \sqcup \ldots \sqcup r_n) + ({s'}_1 \sqcup \ldots \sqcup {s'}_n)} K\,A}\ \textsc{CaseAltAlt}$$

$$\dfrac{\begin{array}{l}\Gamma, y :_{r \cdot q} A, x :_r \Box_q A \vdash B \Rightarrow t \mid \Delta, y :_{s_1} A, x :_{s_2} \Box_q A \\ \exists s_3.\ s_1 \sqsubseteq s_3 \cdot q \sqsubseteq r \cdot q \end{array}}{\Gamma, x :_r \Box_q A \vdash B \Rightarrow \textbf{case}\ x\ \textbf{of}\ [y] \to t \mid \Delta, x :_{s_3 + s_2} \Box_q A}\ \textsc{Unbox}$$

$$\dfrac{\begin{array}{l}(C_i : {B_1}^{q_1} \to \ldots \to {B_n}^{q_n} \to K\,A) \in D \\ \Gamma, x :_r K\,A, y^i_1 :_{r \cdot q_1} B_1, \ldots, y^i_n :_{r \cdot q_n} B_n \vdash B \Rightarrow t_i \mid \Delta_i, x :_{r^i_1} K\,A, y^i_1 :_{s^i_1} B_1, \ldots, y^i_n :_{s^i_n} B_n \\ \exists r^i_2.\ r \sqsupseteq r^i_2 + r^i_1 \cdot (s^i_1 \sqcap \ldots \sqcap s^n_1) \end{array}}{\Gamma, x :_{r_1} K\,A \vdash B \Rightarrow \textbf{case}\ x\ \textbf{of}\ \overline{C_i\ y^i_1 \ldots y^i_n \mapsto t_i} \mid (\Delta_1 \sqcap \ldots \sqcap \Delta_n), x :_{r^1_2 \sqcap \ldots \sqcap r^n_2} K\,A}\ \textsc{CaseSub}$$

$\boxed{\Gamma; \Omega \vdash A \Uparrow\Rightarrow t \mid \Delta}$

$$\dfrac{\Gamma; \Omega \vdash B \Uparrow\Rightarrow t \mid \Delta, x :_r A \qquad r \sqsubseteq q}{\Gamma; \Omega \vdash A^q \to B \Uparrow\Rightarrow \lambda x.t \mid \Delta}\ \textsc{AbsF}$$

$\boxed{\Gamma; \Omega \Uparrow\vdash A \Rightarrow t \mid \Delta}$

$$\dfrac{\begin{array}{l}(C_i : {B_1}^{q_1} \to \ldots \to {B_n}^{q_n} \to K\,A) \in D \\ \Gamma; \Omega, x :_r K\,A_1, y^i_1 :_{q_1} B_1, \ldots, y^i_n :_{q_n} B_n \Uparrow\vdash B \Rightarrow t_i \mid \Delta_i, x :_{r_i} K\,A, y^i_1 :_{s^i_1} B_1, \ldots, y^i_n :_{s^i_n} B_n \\ s^i_j \sqsubseteq q_j \\ s_i = s^i_1 \sqcup \ldots \sqcup s^i_n \end{array}}{\Gamma; \Omega, x :_r K\,A \Uparrow\vdash B \Rightarrow \textbf{case}\ x\ \textbf{of}\ \overline{C_i\ y^i_1 \ldots y^i_n \mapsto t_i} \mid (\Delta_1 \sqcup \ldots \sqcup \Delta_n), x :_{(r_1 \sqcup \ldots \sqcup r_n) + (s_1 \sqcup \ldots \sqcup s_n)} K\,A}\ \textsc{CaseF}$$

$$\dfrac{\begin{array}{l}\Gamma; \Omega, y :_{r \cdot q} A, x :_r \Box_q A \Uparrow\vdash B \Rightarrow t \mid \Delta, y :_{s_1} A, x :_{s_2} \Box_q A \\ \exists s_3.\ s_3 \cdot q \sqsubseteq s_1 \end{array}}{\Gamma; \Omega, x :_r \Box_q A \Uparrow\vdash B \Rightarrow \textbf{case}\ x\ \textbf{of}\ [y] \to t \mid \Delta, x :_{s_3 + s_2} \Box_q A}\ \textsc{UnboxF}$$

$\boxed{\Gamma \vdash A \Downarrow t \mid \Delta}$

$$\dfrac{\Gamma \vdash A \Downarrow t \mid \Delta}{\Gamma \vdash \Box_r A \Downarrow [t] \mid r \cdot \Delta}\ \textsc{BoxF}$$

$$\frac{\begin{array}{l}(C : {B_1}^1 \to \ldots \to {B_n}^1 \to K\ A) \in D \\ \Gamma \vdash B_i \Downarrow t_i \mid \Delta_i\end{array}}{\Gamma \vdash K\ A \Downarrow C\ t_1 \ldots t_n \mid \Delta_1 + \ldots + \Delta_n} \quad \textsc{ConF}$$

$\boxed{\Gamma; \Omega \Downarrow\!\vdash A \Rightarrow t \mid \Delta}$

$$\frac{}{\Gamma; x :_r A \Downarrow\!\vdash A \Rightarrow x \mid 0 \cdot \Gamma, x :_1 A} \quad \textsc{VarF}$$

$$\frac{\begin{array}{l}\Gamma; x_1 :_{r_1} A^q \to B, x_2 :_{r_1} B \Downarrow\!\vdash C \Rightarrow t_1 \mid \Delta_1, x_1 :_{s_1} A^q \to B, x_2 :_{s_2} B \\ \Gamma; x_1 :_{r_1} A^q \to B \Downarrow\!\vdash A \Rightarrow t_2 \mid \Delta_2, x_1 :_{s_3} A^q \to B\end{array}}{\Gamma; x_1 :_{r_1} A^q \to B \Downarrow\!\vdash C \Rightarrow [(x_1\ t_2)/x_2]t_1 \mid (\Delta_1 + s_2 \cdot q \cdot \Delta_2), x_1 :_{s_2 + s_1 + (s_2 \cdot q \cdot s_3)} A^q \multimap B} \quad \textsc{AppF}$$

$\boxed{\Gamma \vdash A \Rightarrow^- t \mid \Delta}$

$$\frac{}{\Gamma, x : A \vdash A \Rightarrow^- x \mid \Gamma} \quad \textsc{Syn\_sub\_lin\_var}$$

$$\frac{\texttt{<<no parses (char 8): exists e***x >>}}{\Gamma, x :_r A \vdash A \Rightarrow^- x \mid \Gamma, x :_s A} \quad \textsc{Syn\_sub\_gr\_var}$$

$$\frac{\begin{array}{l}\Gamma, x :_r A, y : A \vdash B \Rightarrow^- t \mid \Delta, x :_s A \\ y \notin |\Delta| \\ \texttt{<<no parses (char 22): exists s' . s = s' + 1*** >>}\end{array}}{\Gamma, x :_r A \vdash B \Rightarrow^- [y/x]t \mid \Delta, x :_{s'} A} \quad \textsc{Syn\_sub\_der}$$

$$\frac{\begin{array}{l}x \notin |\Delta| \\ \Gamma, x : A \vdash B \Rightarrow^- t \mid \Delta\end{array}}{\Gamma \vdash A \to B \Rightarrow^- \lambda x.t \mid \Delta} \quad \textsc{Syn\_sub\_abs}$$

$$\frac{\begin{array}{l}x_2 \notin |\Delta_1| \\ \Gamma, x_2 : B \vdash C \Rightarrow^- t_1 \mid \Delta_1 \\ \Delta_1 \vdash A \Rightarrow^- t_2 \mid \Delta_2\end{array}}{\Gamma, x_1 : A \to B \vdash C \Rightarrow^- [(x_1\ t_2)/x_2]t_1 \mid \Delta_2} \quad \textsc{Syn\_sub\_app}$$

$$\frac{\Gamma \vdash A \Rightarrow^- t \mid \Delta}{\Gamma \vdash \Box_r A \Rightarrow^- t \mid \Gamma - r \cdot (\Gamma - \Delta)} \quad \textsc{Syn\_sub\_box}$$

$$\frac{\Gamma, x_2 :_r A \vdash B \Rightarrow^- t \mid \Delta, x_2 :_s A \qquad 0 \leq s}{\Gamma, x_1 : \Box_r A \vdash B \Rightarrow^- \mathbf{let}\ [x_2] = x_1\ \mathbf{in}\ t \mid \Delta} \quad \textsc{Syn\_sub\_unbox}$$

$$\frac{\begin{array}{l}\Gamma \vdash A \Rightarrow^- t_1 \mid \Delta_1 \\ \Delta_1 \vdash B \Rightarrow^- t_2 \mid \Delta_2\end{array}}{\texttt{<<no parses (char 8): G |- A **** B =>- t1, t2 ; D2 >>}} \quad \textsc{Syn\_sub\_pair\_intro}$$

$$\frac{\begin{array}{l}x_1 \notin |\Delta| \\ x_2 \notin |\Delta| \\ \Gamma, x_1 : A, x_2 : B \vdash C \Rightarrow^- t_2 \mid \Delta\end{array}}{\texttt{<<no parses (char 11): G, x3 : A **** B |- C =>- let x1, x2 = x3 in t2 ; D >>}} \quad \textsc{Syn\_sub\_pair}$$

$$\frac{\Gamma \vdash A \Rightarrow^- t \mid \Delta}{\texttt{<<no parses (char 8): G |- A +*** B =>- inl t ; D >>}} \quad \textsc{Syn\_sub\_sum\_intro\_left}$$

$$\frac{\Gamma \vdash B \Rightarrow^- t \mid \Delta}{\texttt{<<no parses (char 8): G |- A +*** B =>- inr t ; D >>}} \quad \textsc{Syn\_sub\_sum\_intro\_right}$$

$$x_2 \notin |\Delta_1|$$
$$x_3 \notin |\Delta_2|$$
$$\Gamma, x_2 : A \vdash C \Rightarrow^- t_1 \mid \Delta_1$$
$$\Gamma, x_3 : B \vdash C \Rightarrow^- t_2 \mid \Delta_2$$

<<no parses (char 14): G, x1 : A + B ***|- C =>- case x1 of inl x2 -> t1 | inr x3 -> t2 ; D1

$$\frac{}{\Gamma \vdash 1 \Rightarrow^- () \mid \Gamma} \quad \textsc{Syn\_sub\_unit\_intro}$$

$$\frac{\Gamma \vdash C \Rightarrow^- t \mid \Delta}{\Gamma, x : 1 \vdash C \Rightarrow^- \textbf{let}\,() = x\,\textbf{in}\,t \mid \Delta} \quad \textsc{Syn\_sub\_unit\_elim}$$

$\boxed{\Gamma \vdash A \Rightarrow^+ t;\, \Delta}$

$$\frac{}{\Gamma, x : A \vdash A \Rightarrow^+ x;\, x : A} \quad \textsc{Syn\_add\_lin\_var}$$

$$\frac{}{\Gamma, x :_r A \vdash A \Rightarrow^+ x;\, x :_1 A} \quad \textsc{Syn\_add\_gr\_var}$$

$$\frac{\Gamma, x :_s A, y : A \vdash B \Rightarrow^+ t;\, \Delta, y : A}{\Gamma, x :_s A \vdash B \Rightarrow^+ [y/x]t;\, \Delta + x :_1 A} \quad \textsc{Syn\_add\_der}$$

$$\frac{\Gamma, x : A \vdash B \Rightarrow^+ t;\, \Delta, x : A}{\Gamma \vdash A \to B \Rightarrow^+ \lambda x.t;\, \Delta} \quad \textsc{Syn\_add\_abs}$$

$$\frac{\begin{array}{c}\Gamma, x_2 : B \vdash C \Rightarrow^+ t_1;\, \Delta_1, x_2 : B \\ \Gamma - (\Delta_1, x_2 : B) \vdash A \Rightarrow^+ t_2;\, \Delta_2\end{array}}{\Gamma, x_1 : A \to B \vdash C \Rightarrow^+ [(x_1\ t_2)/x_2]t_1;\, (\Delta_1 + \Delta_2), x_1 : A \to B} \quad \textsc{Syn\_add\_app}$$

$$\frac{\Gamma \vdash A \Rightarrow^+ t;\, \Delta}{\Gamma \vdash \Box_r A \Rightarrow^+ [t];\, r \cdot \Delta} \quad \textsc{Syn\_add\_box}$$

$$\frac{\Gamma, x_2 :_r A \vdash B \Rightarrow^+ t;\, \Delta, x_2 :_s A \qquad s \leq r}{\Gamma, x_1 : \Box_r A \vdash B \Rightarrow^+ \textbf{let}\,[x_2] = x_1\,\textbf{in}\,t;\, \Delta, x_1 : \Box_r A} \quad \textsc{Syn\_add\_unbox}$$

$$\Gamma \vdash A \Rightarrow^+ t_1;\, \Delta_1$$
$$\Gamma - \Delta_1 \vdash B \Rightarrow^+ t_2;\, \Delta_2$$

<<no parses (char 8): G |- A **** B =>+ t1, t2 ; D1 + D2 >>   SYN_ADD_PAIR_INTRO

$$\Gamma, x_1 : A, x_2 : B \vdash C \Rightarrow^+ t_2;\, \Delta, x_1 : A, x_2 : B$$

<<no parses (char 11): G, x3 : A **** B |- C =>+ let x1, x2 = x3 in t2 ; D, x3 : A * B >>

$$\Gamma \vdash A \Rightarrow^+ t;\, \Delta$$

<<no parses (char 8): G |- A +*** B =>+ inl t ; D >>   SYN_ADD_SUM_INTRO_LEFT

$$\Gamma \vdash B \Rightarrow^+ t;\, \Delta$$

<<no parses (char 8): G |- A +*** B =>+ inr t ; D >>   SYN_ADD_SUM_INTRO_RIGHT

$$\frac{}{\Gamma \vdash 1 \Rightarrow^+ ();\, \emptyset} \quad \textsc{Syn\_add\_unit\_intro}$$

$$\Gamma \vdash C \Rightarrow t \mid \Delta$$

<<no parses (char 49): G, x : Unit |- C => let () = x in t ; D, x : Unit*** >>   SYN_ADD_U

$$\Gamma, x_2 : A \vdash C \Rightarrow^+ t_1;\, \Delta_1, x_2 : A$$
$$\Gamma, x_3 : B \vdash C \Rightarrow^+ t_2;\, \Delta_2, x_3 : B$$

<<no parses (char 14): G, x1 : A + B ***|- C =>+ case x1 of inl x2 -> t1 | inr x3 -> t2 ; (D

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Definition rules:         44 good    13 bad
Definition rule clauses: 130 good    13 bad
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