

*termvar*,  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\mathcal{L}$ ,  $x$ ,  $y$ ,  $z$ ,  $f$ ,  $g$ ,  $h$ , *adv*  
*index*,  $i$ ,  $j$ ,  $n$ ,  $m$

$Foc$	$::=$		Focusing phase
		A	Async
		S	Sync
$t, v$	$::=$		Terms
		$t_1 t_2$	Application
		<b>let</b> $\langle p \rangle \leftarrow t_1$ <b>in</b> $t_2$	Effectful Let Binding
		<b>case</b> $t$ <b>of</b> $Cases$	Case
		$\lambda p.t$	Function
		$\lambda p.t$	Function with graded annotation on its binder
		$[t]$	Promote
		$\langle t \rangle$	Pure
		$x$	Variable
		$C t_0 \dots t_n$	Constructor
		$C$	
		$n$	Integer constructors
		<b>let</b> $[p] = t_1$ <b>in</b> $t_2$	S Modal let-binding
		<b>inl</b> $t$	Inl
		<b>inr</b> $t$	Inr
		<b>let</b> $(x_1, x_2) = t_1$ <b>in</b> $t_2$	S Pair let-binding
		$(t_1, t_2)$	A pair of terms
		$-$	Hole
		<b>let</b> $() = t_1$ <b>in</b> $t_2$	UnitElim
		$()$	Unit
		$\llbracket A \rrbracket_{\text{pull}}^{\Gamma} t$	Pull
		$\llbracket A \rrbracket_{\text{push}}^{\Gamma} t$	Push
		$\llbracket A \rrbracket_{\text{pull}}^{\Gamma}$	PullPartial
		$\llbracket A \rrbracket_{\text{push}}^{\Gamma}$	PushPartial
		$\llbracket A \rrbracket_{\text{fmap}}^{\Gamma} t$	Fmap
		$\llbracket A \rrbracket_{\text{fmap}}^{\Gamma}$	FmapPartial
			Nothing
		<b>letrec</b> $t_1 = t_2$ <b>in</b> $t_3$	LetRec
		$\llbracket A \rrbracket_{\text{copyShape}}^{\Gamma} t$	CopyShape
		$\llbracket A \rrbracket_{\text{drop}}^{\Gamma} t$	Drop
$C$	$::=$		Constructors
		$(,)$	Pair constructor
		<b>inl</b>	Left injection
		<b>inr</b>	Right injection
		<b>unit</b>	Unit
		<b>tt</b>	
		<b>ff</b>	
		<b>Just</b>	
		<b>Nothing</b>	
$Cases$	$::=$		Value-level cases
		$p \rightarrow t; Cases$	Case cons
		$p \rightarrow t$	One case
		$\overline{p \mapsto t}$	One case overline
		$p \mapsto t; \dots; p' \mapsto t'$	S Many cases (syntactic sugar)

$p$	$::=$ $ $ $x$ $ $ $-$ $ $ $[p]$ $ $ $[[p]]$ $ $ $C\ p_1 \dots p_n$ $ $ $C\ p_1^{Ix_3} \dots p_2^{Ix_2}$ $ $ $C$ $ $ $n$ $ $ $(p)$ $ $ $y_j^i$ $ $ $(p_1, p_2)$ $ $ $p^{Ix}$	Patterns Variable Wildcard Unbox Double unboxing Constructor ConstructorIndexed Nullary Constructor Int constructor  Hack Pair S Pattern at index n+m
$Ix$	$::=$ $ $ $0$ $ $ $1$ $ $ $2$ $ $ $Ix_1 + Ix_2$ $ $ $m$ $ $ $1$ $ $ $2$ $ $ $3$ $ $ $4$ $ $ $i$ $ $ $j$ $ $ $n$ $ $ $m$	More complex index expressions
$Eqn$	$::=$ $ $ $x\ p_1 \dots p_n = t$	Equations Eq
$Def$	$::=$ $ $ $x : C; Eqn_1 \dots Eqn_n$	Definitions Multi-eq def
$A, B, C, E, W, C$	$::=$ $ $ $\cdot$ $ $ $A \rightarrow B$ $ $ $A_{DEC}$ $ $ $A^c \rightarrow B$ $ $ $A^c \multimap B$ $ $ $A \multimap B$ $ $ $K$ $ $ $KA \dots B$ $ $ $\alpha$ $ $ $AB$ $ $ $A^{Ix_1}_{Ix_2}$ $ $ $\Box A$ $ $ $\Box_c A$ $ $ $\Box_{c:B} A$	Types Empty Function Dec Graded Function Graded Linear Function S Linear Function Constructor Constructor Variable Application Var2IndexTy BlankBox Box Box with coeffect type

		Int		Integers
		Char		Characters
		1		unit
		$\otimes$		Products
		$\mathbb{B}$		Bool
		IO		IO
		$R$		Coeffect types
		$n$		Type-level integers
		$A \text{ op } B$		InfixOp
		$A \otimes B$		Tuple
		$A \oplus B$		Sum
		$\{A_1, \dots, A_n\}$		
		$\downarrow \kappa$		
		Set $A$		
		$B_1^1 \rightarrow \dots \rightarrow B_n^1 \rightarrow A$	S	n-Ary Function
		$B_1^{q_1} \rightarrow \dots \rightarrow B_n^{q_n} \rightarrow A$	S	n-Ary Function w grades
		$A \uparrow$		AsyncTy
		$A \downarrow$		SyncTy
		$X$		Recursion Variables
		$\mu X. A$		Recursive Type
op	::=			Type operators
		+		
		*		
		−		
		$\leq$		
		$<$		
		$>$		
		$\geq$		
		$=$		
		$\neq$		
		$\sqcup$		
		$\sqcap$		
$R, S$	::=			Coeffect types
		Nat		Nat
		Level		Level
		Ext		Extending
		Interval		Interval
		$A \times B$		Products
		$R S$		Application
		$\alpha$		Variable
		$\beta$		VariableB
$Cons$	::=			Constraints/Predicates
		$A, Cons$		
		$A$		
		$A_1, \dots, A_n$		
		$\overrightarrow{A}$		
		$(Cons)$		

$c, r, s, q$	$::=$	
		$c_{Ix_1}$ Var2Index
		$c_{Ix_2}$ Var1Index
		$c_1 + c_2$ Addition
		$c_1 \cdot c_2$ Multiplication
		0 Additive Unit
		1 Multiplicative Unit
		$c_1 \sqcup c_2$ Join
		$c_1 \sqcap c_2$ Meet
		$c_1 \sqcap \dots \sqcap c_2$ MultiMeet
		$c_1 \sqcup \dots \sqcup c_2$ MultiJoin
		$c_1 = c_2$ CoeffEq
		$\bigsqcup_1^n c$ BigJoin
		$\text{flatten}(c_1, A, c_2, B)$ Flatten
		Hi
		Lo
		$c_1..c_2$
		$\infty$
		$(c_1, c_2)$
		$\theta c$
		2 S
		3 S
		4 S
		5 S
		6 S
		10 S
		15 S
		20 S
		$c_{i-1}$
		Unused
		$[r...s]$
$Rel$	$::=$	Relations on grades
		$c_1 \sqsubseteq c_2$
		$c_1 \sqsubseteq c_2 \sqsubseteq c_3$
		$c_1 \sqsupseteq c_2$
		$c_1 \sqsupseteq c_2 \sqsupseteq c_3$
		$Rel_1 \wedge Rel_2$
$\kappa$	$::=$	
		Type Type
		$\uparrow A$ Promote a type to a kind
		Effect Effect grades
		Coeffect Coeffect grades
		Eff
		Coeff
		(Co)eff
		Predicate Predicates
		$\kappa_1 \rightarrow \kappa_2$ Kind function
		$\kappa_1 \cup \kappa_2$

	$\theta\kappa$	Substitutions
	$(\kappa)$	
$D$	$::=$	
	$\emptyset$	Empty
	$D_1, D_2$	
	$(D)$	
$As$	$::=$	
	$x : C$	Singleton context
	$x :_r C$	Singleton context w/ graded assumption
	$x^{Ix_1} :_c C$	Indexed Variable
	$x^{Ix_1} :_c C$	Indexed Variable 1
$\Gamma, \Delta, \Omega$	$::=$	
	$\emptyset$	Empty
	$As$	Single assumption
$r : ?R$	$::=$	
	$c$	
	$-$	
	$(r : ?R)$	
	$\theta r : ?R$	
$P$	$::=$	
	$P_1 \wedge P_2$	
	$P_1 \wedge \dots \wedge P_n$	
	$P_1 \vee P_2$	
	$P_1 \rightarrow P_2$	
	$\forall \alpha. P$	
	$\neg P$	
	$\exists \alpha. P$	
	$\mathbf{t}_1 \equiv \mathbf{t}_2$	
	$\mathbf{t}_1 \sqsubseteq \mathbf{t}_2$	
	$\top$	
	$(P)$	
	$\llbracket \theta \rrbracket$	
	$\llbracket Cons \rrbracket$	
$\mathbf{t}$	$::=$	
	$\llbracket c \rrbracket$	
	$\llbracket A \rrbracket$	
$\theta, \theta_\kappa$	$::=$	
	$\emptyset$	Empty
	$\theta_1 \uplus \theta_2$	Union
	$x \mapsto B$	SingletonTy
	$x \mapsto \kappa$	SingletonKind
	$x \mapsto c$	Singleton Coeffect

	$\theta\theta'$	Substitution over a substitution
	$\theta_1 \uplus \dots \uplus \theta_2$	
	$\theta, \theta'$	Disjoint cat
	$\theta \setminus x$	Remove a substitution for a variable

$\mathcal{C}$	$::=$
	$x \mapsto c$
	$\mathcal{C} + \mathcal{C}'$
	$\mathcal{C}, \mathcal{C}'$
	$(\mathcal{C})$
	$\mathcal{C}$
	$\emptyset$
	$c \cdot \mathcal{C}$

## $\Gamma \vdash t : A$ 1 Typing

$\frac{}{0 \cdot \Gamma, x :_1 A \vdash x : A}$	TYVAR
$\frac{\Gamma, x :_r A \vdash t : B}{\Gamma \vdash \lambda x. t : A^r \rightarrow B}$	TYABS
$\frac{\Gamma_1 \vdash t_1 : A^r \rightarrow B \quad \Gamma_2 \vdash t_2 : A}{\Gamma_1 + r \cdot \Gamma_2 \vdash t_1 t_2 : B}$	TYAPP
$\frac{(C : B_1^{q_1} \rightarrow \dots \rightarrow B_n^{q_n} \rightarrow K \vec{A}) \in D}{0 \cdot \Gamma \vdash C : B_1^{q_1} \rightarrow \dots \rightarrow B_n^{q_n} \rightarrow K \vec{A}}$	TYCON
$\frac{\Gamma \vdash t : A}{r \cdot \Gamma \vdash [t] : \Box_r A}$	TYPR
$\frac{\Gamma, x :_r A, \Gamma' \vdash t : B \quad r \sqsubseteq s}{\Gamma, x :_s A, \Gamma' \vdash t : B}$	TYAPPROX
$\frac{\Gamma \vdash t : A \quad r \vdash p_i : A \triangleright \Delta_i \quad \Gamma', \Delta_i \vdash t_i : B}{r \cdot \Gamma + \Gamma' \vdash \mathbf{case} \ t \ \mathbf{of} \ p_1 \mapsto t_1; \dots; p_n \mapsto t_n : B}$	TYCASE

$c \vdash p : A \triangleright \Gamma$  Declarative pattern checking for Granule Mini (monomorphic)

$\frac{0 \sqsubseteq r}{r \vdash \_ : A \triangleright \emptyset}$	PATWILD
$\frac{}{r \vdash p : A \triangleright x :_r A}$	PATVAR
$\frac{r \cdot s \vdash p : A \triangleright \Gamma}{r \vdash [p] : \Box_s A \triangleright \Gamma}$	PATBOX
$\frac{(C : B_1^{q_1} \rightarrow \dots \rightarrow B_n^{q_n} \rightarrow K \vec{A}) \in D \quad q_i \cdot r \vdash p_i : B_i \triangleright \Gamma_i \quad  K \vec{A}  > 1 \Rightarrow 1 \sqsubseteq r}{r \vdash C p_1 \dots p_n : K \vec{A} \triangleright \vec{\Gamma}_i}$	PATCON

$\Gamma_1 \vdash A \Rightarrow^- t \mid \Gamma_2$

$\Gamma \vdash A \Rightarrow t \mid \Delta$

$\frac{}{\Gamma, x :_r A \vdash A \Rightarrow x \mid 0 \cdot \Gamma, x :_1 A}$  VAR

$$\begin{array}{c}
\frac{\Gamma, x :_q A \vdash B \Rightarrow t \mid \Delta, x :_r A \quad r \sqsubseteq q}{\Gamma \vdash A^q \rightarrow B \Rightarrow \lambda x. t \mid \Delta} \text{ ABS} \\
\\
\frac{\Gamma, x_1 :_{r_1} A^q \rightarrow B, x_2 :_{r_1} B \vdash C \Rightarrow t_1 \mid \Delta_1, x_1 :_{s_1} A^q \rightarrow B, x_2 :_{s_2} B \quad \Gamma, x_1 :_{r_1} A^q \rightarrow B \vdash A \Rightarrow t_2 \mid \Delta_2, x_1 :_{s_3} A^q \rightarrow B}{\Gamma, x_1 :_{r_1} A^q \rightarrow B \vdash C \Rightarrow [(x_1 t_2)/x_2] t_1 \mid (\Delta_1 + s_2 \cdot q \cdot \Delta_2), x_1 :_{s_2+s_1+(s_2 \cdot q \cdot s_3)} A^q \rightarrow B} \text{ APP} \\
\\
\frac{\Gamma \vdash A \Rightarrow t \mid \Delta}{\Gamma \vdash \Box_r A \Rightarrow [t] \mid r \cdot \Delta} \text{ BOX} \\
\\
\frac{(C : B_1^{q_1} \rightarrow \dots \rightarrow B_n^{q_n} \rightarrow K \vec{A}) \in D \quad \Gamma \vdash B_i \Rightarrow t_i \mid \Delta_i}{\Gamma \vdash K \vec{A} \Rightarrow C t_1 \dots t_n \mid 0 \cdot \Gamma + (q_1 \cdot \Delta_1) + \dots + (q_n \cdot \Delta_n)} \text{ CON} \\
\\
\frac{(C : B_1^{r_1} \rightarrow \dots \rightarrow B_n^{r_n} \rightarrow K A) \in D \quad \Gamma, x :_1 B_1^{q_1} \rightarrow \dots \rightarrow B_n^{q_n} \rightarrow K A \vdash A_1 \Rightarrow t \mid \Delta, x :_1 B_1^{q_1} \rightarrow \dots \rightarrow B_n^{q_n} \rightarrow K A}{\Gamma \vdash A_1 \Rightarrow t \mid \Delta} \text{ CONALT} \\
\\
\frac{(C_i : B_1^{q_1^i} \rightarrow \dots \rightarrow B_n^{q_n^i} \rightarrow K \vec{A}) \in D \quad \Gamma, x :_r K \vec{A}, y_1^i :_{r \cdot q_1^i} B_1, \dots, y_n^i :_{r \cdot q_1^i} B_n \vdash B \Rightarrow t_i \mid \Delta_i, x :_{r_i} K \vec{A}, y_1^i :_{s_1^i} B_1, \dots, y_n^i :_{s_n^i} B_n \quad \exists s_j'^i. s_j^i \sqsubseteq s_j'^i \cdot q_j^i \sqsubseteq r \cdot q_j^i \quad s_i = s_1'^i \sqcup \dots \sqcup s_n'^i \quad |K \vec{A}| > 1 \Rightarrow 1 \sqsubseteq s_1 \sqcup \dots \sqcup s_m}{\Gamma, x :_r K \vec{A} \vdash B \Rightarrow \text{case } x \text{ of } \overline{C_i} y_1^i \dots y_n^i \mapsto t_i \mid (\Delta_1 \sqcup \dots \sqcup \Delta_m), x :_{(r_1 \sqcup \dots \sqcup r_m) + (s_1 \sqcup \dots \sqcup s_m)} K \vec{A}} \text{ CASE} \\
\\
\frac{(C_i : B_1^{q_1^i} \rightarrow \dots \rightarrow B_n^{q_n^i} \rightarrow K A) \in D \quad \Gamma, x :_r K A, y_1^i :_{r \cdot q_1^i} B_1, \dots, y_n^i :_{r \cdot q_1^i} B_n \vdash B \Rightarrow t_i \mid \Delta_i, x :_{r_i} K A, y_1^i :_{s_1^i} B_1, \dots, y_n^i :_{s_n^i} B_n \quad s_i = s_1^i \sqcup \dots \sqcup s_n^i \quad q_i = q_1^i \sqcup \dots \sqcup q_n^i \quad \exists s'_i. s_i \sqsubseteq s'_i \cdot q_i \sqsubseteq r \cdot q_i}{\Gamma, x :_r K A \vdash B \Rightarrow \text{case } x \text{ of } \overline{C_i} y_1^i \dots y_n^i \mapsto t_i \mid (\Delta_1 \sqcup \dots \sqcup \Delta_n), x :_{(r_1 \sqcup \dots \sqcup r_n) + (s'_1 \sqcup \dots \sqcup s'_n)} K A} \text{ CASEALTALT} \\
\\
\frac{\Gamma, y :_{r \cdot q} A, x :_r \Box_q A \vdash B \Rightarrow t \mid \Delta, y :_{s_1} A, x :_{s_2} \Box_q A \quad \exists s_3. s_1 \sqsubseteq s_3 \cdot q \sqsubseteq r \cdot q}{\Gamma, x :_r \Box_q A \vdash B \Rightarrow \text{case } x \text{ of } [y] \rightarrow t \mid \Delta, x :_{s_3+s_2} \Box_q A} \text{ UNBOX} \\
\\
\frac{(C_i : B_1^{q_1} \rightarrow \dots \rightarrow B_n^{q_n} \rightarrow K A) \in D \quad \Gamma, x :_r K A, y_1^i :_{r \cdot q_1} B_1, \dots, y_n^i :_{r \cdot q_n} B_n \vdash B \Rightarrow t_i \mid \Delta_i, x :_{r_1^i} K A, y_1^i :_{s_1^i} B_1, \dots, y_n^i :_{s_n^i} B_n \quad \exists r_2^i. r \sqsupseteq r_2^i + r_1^i \cdot (s_1^i \sqcap \dots \sqcap s_n^i)}{\Gamma, x :_{r_1} K A \vdash B \Rightarrow \text{case } x \text{ of } \overline{C_i} y_1^i \dots y_n^i \mapsto t_i \mid (\Delta_1 \sqcap \dots \sqcap \Delta_n), x :_{r_1^1 \sqcap \dots \sqcap r_2^n} K A} \text{ CASESUB} \\
\\
\boxed{\Gamma; \Omega \vdash A \Uparrow \Rightarrow t \mid \Delta} \\
\\
\frac{\Gamma; \Omega, x :_q A \vdash B \Uparrow \Rightarrow t \mid \Delta, x :_r A \quad r \sqsubseteq q}{\Gamma; \Omega \vdash A^q \rightarrow B \Uparrow \Rightarrow \lambda x. t \mid \Delta} \text{ ABSF} \\
\\
\boxed{\Gamma; \Omega \Uparrow \vdash A \Rightarrow t \mid \Delta} \\
\\
\frac{(C_i : B_1^{q_1^i} \rightarrow \dots \rightarrow B_n^{q_n^i} \rightarrow K \vec{A}) \in D \quad \Gamma; \Omega, x :_r K \vec{A}, y_1^i :_{r \cdot q_1^i} B_1, \dots, y_n^i :_{r \cdot q_1^i} B_n \Uparrow \vdash B \Rightarrow t_i \mid \Delta_i, x :_{r_i} K \vec{A}, y_1^i :_{s_1^i} B_1, \dots, y_n^i :_{s_n^i} B_n \quad \exists s_j'^i. s_j^i \sqsubseteq s_j'^i \cdot q_j^i \sqsubseteq r \cdot q_j^i \quad s_i = s_1'^i \sqcup \dots \sqcup s_n'^i \quad |K \vec{A}| > 1 \Rightarrow 1 \sqsubseteq s_1 \sqcup \dots \sqcup s_m}{\Gamma; \Omega, x :_r K \vec{A} \Uparrow \vdash B \Rightarrow \text{case } x \text{ of } \overline{C_i} y_1^i \dots y_n^i \mapsto t_i \mid (\Delta_1 \sqcup \dots \sqcup \Delta_m), x :_{(r_1 \sqcup \dots \sqcup r_m) + (s_1 \sqcup \dots \sqcup s_m)} K \vec{A}} \text{ CASEF}
\end{array}$$



$$\frac{\begin{array}{c} \Gamma; \Omega, y :_{r \cdot q} A, x :_r \Box_q A \uparrow\vdash B \Rightarrow t \mid \Delta, y :_{s_1} A, x :_{s_2} \Box_q A \\ \exists s_3. s_3 \cdot q \sqsubseteq s_1 \end{array}}{\Gamma; \Omega, x :_r \Box_q A \uparrow\vdash B \Rightarrow \mathbf{case} \ x \ \mathbf{of} \ [y] \rightarrow t \mid \Delta, x :_{s_3+s_2} \Box_q A} \text{ UNBOXF}$$

$$\boxed{\Gamma \vdash A \Downarrow t \mid \Delta}$$

$$\frac{\begin{array}{c} (C : B_1^1 \rightarrow \dots \rightarrow B_n^1 \rightarrow K \vec{A}) \in D \\ \Gamma \vdash B_i \Downarrow \Rightarrow t_i \mid \Delta_i \end{array}}{\Gamma \vdash K A \Downarrow C t_1 \dots t_n \mid \Delta_1 + \dots + \Delta_n} \text{ CONF}$$

$$\boxed{\Gamma; \Omega \Downarrow\vdash A \Rightarrow t \mid \Delta}$$

$$\overline{\Gamma; x :_r A \Downarrow\vdash A \Rightarrow x \mid 0 \cdot \Gamma, x :_1 A} \text{ VARF}$$

$$\frac{\begin{array}{c} \Gamma; x_1 :_{r_1} A^q \rightarrow B, x_2 :_{r_1} B \Downarrow\vdash C \Rightarrow t_1 \mid \Delta_1, x_1 :_{s_1} A^q \rightarrow B, x_2 :_{s_2} B \\ \Gamma; x_1 :_{r_1} A^q \rightarrow B \Downarrow\vdash A \Rightarrow t_2 \mid \Delta_2, x_1 :_{s_3} A^q \rightarrow B \end{array}}{\Gamma; x_1 :_{r_1} A^q \rightarrow B \Downarrow\vdash C \Rightarrow [(x_1 t_2)/x_2] t_1 \mid (\Delta_1 + s_2 \cdot q \cdot \Delta_2), x_1 :_{s_2+s_1+(s_2 \cdot q \cdot s_3)} A^q \multimap B} \text{ APPF}$$

$$\boxed{\Gamma \vdash A \Rightarrow^- t \mid \Delta}$$

$$\overline{\Gamma, x : A \vdash A \Rightarrow^- x \mid \Gamma} \text{ SYN\_SUB\_LIN\_VAR}$$

Definition rules: 28 good 0 bad  
Definition rule clauses: 73 good 0 bad