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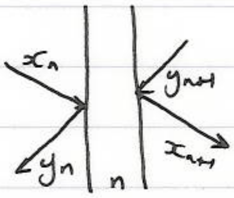
T-Matrix

The T-matrix relates waves on either side of a dielectric layer



$$\begin{pmatrix} x_n \\ y_n \end{pmatrix} = \tilde{M}_n \begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix}$$

$$\tilde{M}_n = \frac{1}{t_{n-1,n}} \begin{pmatrix} 1 & r_{n-1,n} \\ r_{n-1,n} & 1 \end{pmatrix} \begin{pmatrix} e^{-i\delta_n} & 0 \\ 0 & e^{i\delta_n} \end{pmatrix} = \frac{1}{t_{n-1,n}} \begin{pmatrix} e^{-i\delta_n} & r_{n-1,n} e^{i\delta_n} \\ r_{n-1,n} e^{-i\delta_n} & e^{i\delta_n} \end{pmatrix}$$

with N layers we have N+1 interfaces each w/ a matrix $\tilde{M}_n$

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \tilde{M}_1 \cdot \tilde{M}_2 \cdot \tilde{M}_3 \dots \tilde{M}_N \cdot \frac{1}{t_{N,N+1}} \begin{pmatrix} 1 & r_{N,N+1} \\ r_{N,N+1} & 1 \end{pmatrix} \begin{pmatrix} x_{N+1} \\ y_{N+1} \end{pmatrix}$$

$$t_N = \frac{x_{N+1}}{x_1}, \quad r = \frac{y_1}{x_1} \quad ; \quad \begin{aligned} x_1 &= M_{11} x_{N+1} + M_{12} y_{N+1} \\ y_1 &= M_{21} x_{N+1} + M_{22} y_{N+1} \end{aligned}$$

if $y_{N+1} = 0$

$$x_1 = M_{11} x_{N+1}, \quad y_1 = M_{21} x_{N+1}$$

$$\boxed{\therefore \quad r = \frac{M_{21}}{M_{11}}, \quad t = \frac{1}{M_{11}}}$$

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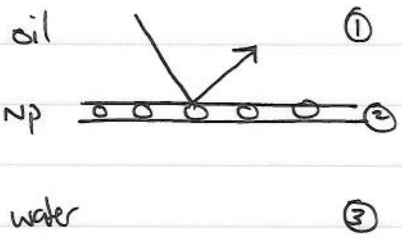

$$M_2 = \frac{1}{t_{1,2}} \begin{pmatrix} e^{-i\delta_2} & r_{1,2} e^{i\delta_2} \\ r_{1,2} e^{-i\delta_2} & e^{i\delta_2} \end{pmatrix}$$

$$M_3 = \frac{1}{t_{2,3}} \begin{pmatrix} 1 & r_{2,3} \\ r_{2,3} & 1 \end{pmatrix}$$

$$M = \frac{1}{t_{1,2} t_{2,3}} \begin{pmatrix} e^{-i\delta_2} & r_{1,2} e^{i\delta_2} \\ r_{12} e^{-i\delta_2} & e^{i\delta_2} \end{pmatrix} \begin{pmatrix} 1 & r_{23} \\ r_{23} & 1 \end{pmatrix}$$

$$M = \frac{1}{t_{12} t_{23}} \begin{pmatrix} e^{-i\delta_2} + r_{1,2} e^{i\delta_2} r_{2,3} & r_{23} e^{-i\delta_2} + r_{1,2} r_{2,3} e^{i\delta_2} \\ r_{12} e^{-i\delta_2} + e^{i\delta_2} r_{23} & r_{12} e^{-i\delta_2} r_{23} + e^{i\delta_2} \end{pmatrix}$$

$$r = \frac{r_{1,2} e^{-\delta_2} + e^{i\delta_2} r_{2,3}}{e^{-i\delta_2} + r_{1,2} r_{2,3} e^{i\delta_2}} = \frac{r_{1,2} + r_{2,3} e^{2i\delta_2}}{1 + r_{1,2} r_{2,3} e^{2i\delta_2}}$$

$$R = \left| \frac{r_{1,2} + r_{2,3} e^{2i\delta_2}}{1 + r_{1,2} r_{2,3} e^{2i\delta_2}} \right|^2 , \quad \delta_2 = k^{\ell}_{\parallel,\perp} d$$

$$r_{1,2}^{(s)} = \frac{k_1 - k_{2,\parallel}}{k_1 + k_{2,\parallel}} \qquad r_{2,3}^{(s)} = \frac{k_{2,\parallel} - k_3}{k_{2,\parallel} + k_3}$$

$$R = \left| \frac{(k_1 - k_{2,\parallel})(k_{2,\parallel} + k_3) + (k_{2,\parallel} - k_3)(k_1 + k_{2,\parallel}) e^{2i\delta_2}}{(k_1 + k_{2,\parallel})(k_{2,\parallel} + k_3) + (k_1 - k_{2,\parallel})(k_{2,\parallel} - k_3) e^{2i\delta_2}} \right|^2$$

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$$R = \left| \frac{k_1 k_{2,\parallel} + k_1 k_3 - k_{2,\parallel}^2 - k_{2,\parallel} k_3 \; + (k_{2,\parallel} - k_3)(k_1 + k_{2,\parallel}) e^{2i\delta_2}}{(k_1 + k_{2,\parallel})(k_{2,\parallel} + k_3) \; + (k_1 - k_{2,\parallel})(k_{2,\parallel} - k_3)\, e^{2i\delta_2}} \right|^2$$

$$= \left| \frac{2k_\parallel (k_1 - k_3) + (k_\parallel - k_3)(k_1 + k_\parallel)(e^{2i\delta_2} - 1)}{(k_1 + k_\parallel)(k_\parallel + k_3) \mp (k_1 - k_\parallel)(k_\parallel - k_3)\, e^{2i\delta_2}} \right|^2$$

$$\cancel{= \left(\frac{2k_\parallel (k_1 - k_3) + (k_\parallel - k_3)(k_1 + k_\parallel)(e^{2i\delta_2} - 1)}{(k_1 + k_\parallel)(k_\parallel + k_3) +} \right)}$$

$$* \quad \boxed{R_s = \left| \frac{2k_\parallel (k_1 - k_3) + (k_\parallel - k_3)(k_1 + k_\parallel)(e^{2i\delta_2} - 1)}{2k_\parallel (k_1 + k_3) \mp (k_1 - k_\parallel)(k_\parallel - k_3)(e^{2i\delta_2} - 1)} \right|^2}$$

$$\boxed{R_s = \left| \frac{2k_\parallel (k_1 - k_3) + (k_\parallel - k_3)(k_1 + k_\parallel)(e^{2i\delta_2} - 1)}{2k_\parallel (k_1 + k_3) - (k_\parallel - k_1)(k_\parallel - k_3)(e^{2i\delta_2} - 1)} \right|^2}$$

✓✓ for s.

~~p-polarised light.~~

$$\cancel{r_{1,2} = \frac{\varepsilon_{2,\parallel} k_1 - \varepsilon_1 k_{2,\perp}}{\varepsilon_{2,\parallel} k_1 + \varepsilon_1 k_{2,\perp}}} \qquad \cancel{r_{2,3} = \frac{\varepsilon_3 k_{2,\perp} - \varepsilon_{2,\parallel} k_3}{\varepsilon_{2,\parallel} k_3 + \varepsilon_3 k_{2,\perp}}}$$

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p-polarised

$$r_{1,2} = \frac{\varepsilon_1 k_\perp - k_1 \varepsilon_\parallel}{\varepsilon_1 k_\perp + k_1 \varepsilon_\parallel} \qquad r_{2,3} = \frac{\varepsilon_\parallel k_3 - k_\perp \varepsilon_3}{\varepsilon_\parallel k_3 + k_\perp \varepsilon_3}$$

$$R_p = \left| \frac{(\varepsilon_1 k_\perp - k_1 \varepsilon_\parallel)(\varepsilon_\parallel k_3 + k_\perp \varepsilon_3) + (\varepsilon_\parallel k_3 - k_\perp \varepsilon_3)(\varepsilon_1 k_\perp + k_1 \varepsilon_\parallel) e^{2i\delta_2}}{(\varepsilon_1 k_\perp + k_1 \varepsilon_\parallel)(\varepsilon_\parallel k_3 + k_\perp \varepsilon_3) + (\varepsilon_\parallel k_3 - k_\perp \varepsilon_3)(\varepsilon_1 k_\perp - k_1 \varepsilon_\parallel) e^{2i\delta_2}} \right|^2$$

$$= \left| \frac{2(\varepsilon_\parallel \varepsilon_1 k_3 - k_1 \varepsilon_3 \varepsilon_\parallel) k_\perp + (\varepsilon_\parallel k_3 - k_\perp \varepsilon_3)(\varepsilon_1 k_\perp + k_1 \varepsilon_\parallel)(e^{2i\delta_2} - 1)}{(\varepsilon_1 k_\perp + k_1 \varepsilon_\parallel)(\varepsilon_\parallel k_3 + k_\perp \varepsilon_3) + (\varepsilon_\parallel k_3 - k_\perp \varepsilon_3)(\varepsilon_1 k_\perp - k_1 \varepsilon_\parallel) e^{2i\delta_2}} \right|^2$$

$$= \left| \frac{2(\varepsilon_\parallel \varepsilon_1 k_3 - k_1 \varepsilon_3 \varepsilon_\parallel) k_\perp + (\varepsilon_\parallel k_3 - k_\perp \varepsilon_3)(\varepsilon_1 k_\perp + k_1 \varepsilon_\parallel)(e^{2i\delta_2} - 1)}{2k_\perp(\varepsilon_\parallel \varepsilon_1 k_3 + \varepsilon_\parallel \varepsilon_3 k_1) + (\varepsilon_\parallel k_3 - k_\perp \varepsilon_3)(\varepsilon_1 k_\perp - k_1 \varepsilon_\parallel)(e^{2i\delta_2} - 1)} \right|^2$$

$$\cancel{R_p = \left| \frac{2(\varepsilon_1 k_3 - k_1 \varepsilon_3) k_\perp + \left(k_3 - k_\perp \frac{\varepsilon_3}{\varepsilon_\parallel}\right)(\varepsilon_1 k_\perp + \varepsilon_\parallel)(e^{2i\delta_2} - 1)}{2(\varepsilon_1 k_3 + \varepsilon_3 k_1) + \left(k_3 - k_\perp \frac{\varepsilon_3}{\varepsilon_\parallel}\right)(\varepsilon_1 k_\perp - k_1 \varepsilon_\parallel)(e^{2i\delta_2} - 1)} \right|^2}$$

$$\cancel{R_p = \left| \frac{2(\varepsilon_1 k_3 - k_1 \varepsilon_3) + \left(\varepsilon_\parallel k_3 - k_\perp \frac{\varepsilon_3}{\varepsilon_\parallel}\right)(\varepsilon_1 k_\perp + k_1)(e^{2i\delta_2} - 1)}{2(\varepsilon_1 k_3 + \varepsilon_3 k_1) - \left(k_\perp \frac{\varepsilon_3}{\varepsilon_\parallel}\right.}\right.}$$

compare to feature

$$R_p = \left| \frac{2(\varepsilon_1 k_3 - k_1 \varepsilon_3)k_\perp + (k_3 - k_\perp \varepsilon_3)\left(\frac{\varepsilon_1}{\varepsilon_\parallel} k_\perp + k_1 \varepsilon_\parallel\right)\left(e^{2i\delta_2} - 1\right)}{2(\varepsilon_1 k_3 + \varepsilon_3 k_1)k_\perp + (k_3 - k_\perp \varepsilon_3)\left(\frac{\varepsilon_1}{\varepsilon_\parallel} k_\perp - k_1 \varepsilon_\parallel\right)\left(e^{2i\delta_2} - 1\right)} \right|^2$$

$$\boxed{R_p = \left| \frac{2(\varepsilon_1 k_3 - k_1 \varepsilon_3)k_\perp + (k_3 - k_\perp \varepsilon_3)\left(\frac{\varepsilon_1}{\varepsilon_\parallel} k_\perp + k_1 \varepsilon_\parallel\right)\left(e^{2i\delta_2} - 1\right)}{2(\varepsilon_1 k_3 + \varepsilon_3 k_1)k_\perp + (k_3 - k_\perp \varepsilon_3)\left(k_1 \varepsilon_\parallel - \frac{\varepsilon_1}{\varepsilon_\parallel} k_\perp\right)\left(e^{2i\delta_2} - 1\right)} \right|^2}$$

$\checkmark\checkmark$

So T-Matrix is harder for anisotropy.