Analog Circuit Design Notes

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As you know from elementary school...

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Chapter 1

MOS Overview

1.1 Current with modulation effect

$$I_{DS} = \frac{1}{2}\mu C'_{ox}\frac{W}{L}(V_{GS} - V_t)^2(1 + \frac{V_D - V_S}{V_A}) \tag{1.1}$$

1.2 Weak inversion regime

Taxonomy	IC values	Bias range
Weak inversion	IC≤0.1	$V_{GS} \leq V_T$-0.1V
Moderate inversion	0.1≤IC≤10	V_T-0.1V<V_{GS}<V_T-0.2V
Strong inversion	IC≥10	$V_{GS} \geq V_T$+0.2V

Inversion coefficient

$$IC = \frac{I}{2n\mu C'_{ox}V_{th}^2 W/L} \tag{1.2}$$

With n=1.5 subthreshold coefficient.
Transconductance

$$g_m = \frac{I}{nV_{th}}\frac{2}{1+\sqrt{1+4IC}} \tag{1.3}$$

V_{ov} can't be defined.

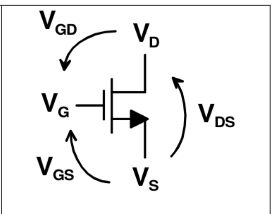
1.3 Saturation conditions

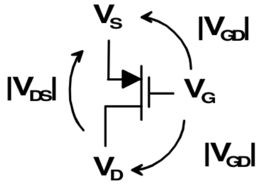
Saturation condition for a NMOS:

$$\begin{cases} V_{GS} > V_T \\ V_{DS} > V_{OV} = V_{GS} - V_T \end{cases}.$$

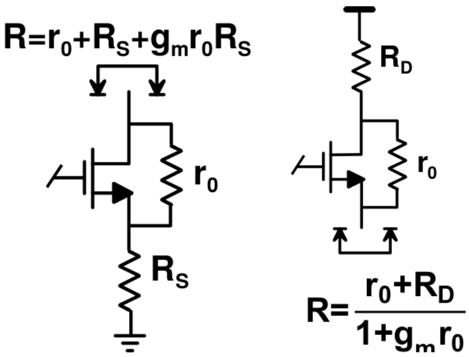
The first condition assures that the transistor is on.

The second condition corresponds to $V_{GD} < V_T$, and assures to be in the saturation region.



Saturation condition for a PMOS: $\begin{cases} V_{SG} > \lvert V_T \rvert \\ V_{SD} > V_{OV} = V_{GS} - \lvert V_T \rvert \end{cases}$	

1.4 Input and output resistance in MOS circuits



1.5 Noise

Reistor voltage noise

$$S_V(f) = 4kTR \tag{1.4}$$

Resistor current noise

$$S_I(f) = \frac{4kT}{R} \tag{1.5}$$

Transistor current noise

$$S_I(f) = 4kT\frac{2}{3}g_m \tag{1.6}$$

1.6 Pelgrom costants

1.6.1 For resistors

$$\frac{\Delta R}{R} = \frac{K_{\Delta R/R}}{\sqrt{WL}} \tag{1.7}$$

1.6.2 For transistors

Threshold voltage

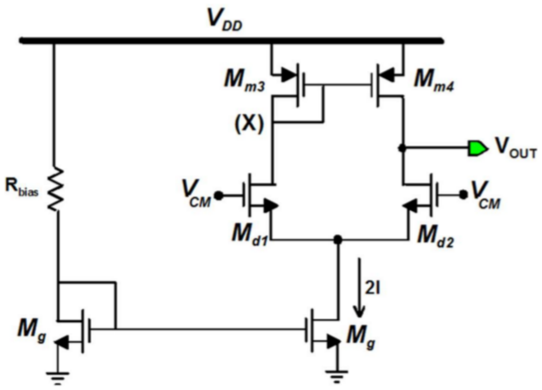
$$\sigma(\Delta V_t) = \frac{K_{\Delta V_t/V_t}}{\sqrt{WL}} \tag{1.8}$$

Parameter k

$$\sigma(\frac{\Delta k}{k}) = \frac{K_{\Delta V_t/V_t}}{\sqrt{WL}} \tag{1.9}$$

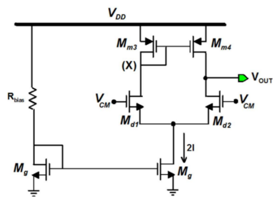
Chapter 2

One stage OTA



2.1 Dynamics

2.1.1 n-mos stage



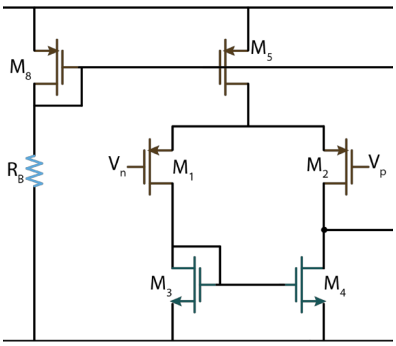
Common mode dynamic

$$V_{CM}^{min} = V_{ov}^{mg} + V_{gs}^{in} \qquad\qquad V_{CM}^{max} = V_{dd} - V_{gs}^{mM} + V_t^{in} \tag{2.1}$$

Output dynamic

$$V_{out}^{min} = V_{CM} - V_t^{in} \qquad\qquad V_{out}^{max} = V_{dd} - V_{ov}^{mM} \tag{2.2}$$

2.1.2 p-mos stage



Common mode dynamic

$$V_{CM}^{min} = V_{gs}^{m3} - V_t \tag{2.3}$$

$$V_{CM}^{max} = V_{dd} - V_{OV}^{m5} - V_{sg}^{m1} \tag{2.4}$$

2.2 Differential gain

$$G_d = g_{m1} r_{0,2} // r_{0,4} \tag{2.5}$$

2.3 Common mode gain

$$G_{CM} = \frac{\varepsilon(r_{0,2}//r_{0,4})}{r_{0,g}} \tag{2.6}$$

The variable ε has 2 dependance on deterministic and non deterministic effects.

2.3.1 Deterministic contributions

Mirror error
Current read from M3 is a partition ,caused by $r_0 3$, of the total current.

$$\varepsilon_{det,mirror} = \frac{1/g_{mM}}{1/g_{mM} + r_{03}} = \frac{1}{1 + g_{mM} \cdot r_{03}} \simeq \frac{1}{g_{mM} \cdot r_{03}} \tag{2.7}$$

Unballace of the stage
The impedence seen from the source of the input generators if the two branches is not the same.

$$\varepsilon_{det,unballace} \simeq \frac{1}{r_{01} \cdot g_{mM}} \tag{2.8}$$

2.3.2 Statistical contributions

Statistical mismatch of the mirror
Taking $\Delta g_m = g_{m3} - g_{m4}$

$$\varepsilon_{stat,mirror} = \frac{\Delta g_m}{g_m} \tag{2.9}$$

Statistical mismatch of the input mos

Taking $\Delta g_m = g_{m1} - g_{m2}$ and $g_m = (g_{m1} + g_{m2})/2$

$$\varepsilon_{stat,input} = -\frac{\Delta g_m}{g_m}(1 + \frac{2r_{0G}}{r_{01}}) \tag{2.10}$$

2.4 CMRR

$$CMRR = G_d/G_{CM} = \frac{2g_{m1}r_{0g}}{\varepsilon} \tag{2.11}$$

2.4.1 Deterministic CMRR

Common mode rejection ratio due to deterministic effect

$$CMRR_{det} \simeq \frac{2g_{m1}r_{0g}}{\frac{1}{g_{mM} \cdot r_{03}} + \frac{1}{r_{01} \cdot g_{mM}}} \tag{2.12}$$

2.4.2 Statistical CMRR

Common mode rejection ratio due to statistical effect with very bit r_g approaches to

$$CMRR = G_d/G_{CM} = \frac{2g_{m1}r_{0g}}{\varepsilon_{stat}} \rightarrow \frac{g_{mM} \cdot r_0 1}{\frac{\Delta g_{m1}}{g_{m1}}} \tag{2.13}$$

2.5 Input voltage offset

Input voltage offset caused due to statistical unbalace of the transistors.

2.5.1 Input mismatch

Offset caused by k variations

$$V_{OS}^{in,k} = \frac{V_{ov1}}{2} \cdot \frac{\Delta k}{k} \tag{2.14}$$

Offset caused by V_t variatons

$$V_{OS}^{in,V_t} = \Delta V_t \tag{2.15}$$

So the total offset due to input transistors is

$$V_{OS}^{in} = \frac{V_{ov1}}{2} \cdot \frac{\Delta k}{k} + \Delta V_t \tag{2.16}$$

2.5.2 Mirror mismatch

Offset caused by k variations

$$V_{OS}^{mirror,k} = \frac{V_{ov,g}}{2} \cdot \frac{\Delta k_M}{k_M} \tag{2.17}$$

Offset caused by V_t variations

$$V_{OS}^{mirror,V_t} = \frac{V_{ov,g}}{V_{ov,m}} \Delta V_{t,M} \tag{2.18}$$

2.5.3 Total offset

Defining with the Pelgrom constants the factors

$$\sigma^2(\frac{\Delta k}{k}) = \frac{K^2_{\Delta k/k}}{WL} \qquad \sigma^2(\Delta V_t) = \frac{K^2_{\Delta V_t}}{WL} \tag{2.19}$$

we can define the statistic offset as

$$\sigma(V_{OS}) = \sqrt{\sigma^2(\Delta V_{t,in}) + \sigma^2(\Delta V_{t,M}) \cdot \left(\frac{V_{ov,in}}{V_{ov,M}}\right)^2 + \left(\frac{V_{ov,in}}{2}\right)^2 [\sigma^2(\frac{\Delta k}{k}_{in}) + \sigma^2(\frac{\Delta k}{k}_{M})]} \tag{2.20}$$

2.6 Input equivalent noise

2.6.1 White noise

Voltage noise:

$$S_v^{in} = \frac{8kT\gamma}{g_{m1}}(1 + \frac{g_{mM}}{g_{m1}}) = \frac{8kT\gamma}{g_{m1}}(1 + \frac{V_{ov,in}}{V_{ov,m}}) \tag{2.21}$$

Current noise:

$$S_i^{in} = kT\gamma(4g_{m,in} + 2g_{m,M} + g_{m,G})(\frac{\omega}{\omega_T})^2 \tag{2.22}$$

With $\omega_T = \frac{1}{C_{gs}1/g_m}$ cut off radial frequency. This noise has a quadratic dependence with frequency.
The voltage noise is dominant over the current one until the crossover frequency

$$f \simeq \frac{f_t}{g_m R_s} \tag{2.23}$$

with R_s input resistance of the circuit. In practice we will current noise is considered only in RF applications.

2.6.2 Nigger 1/f noise

Same transfer of the white noise but the current generator is

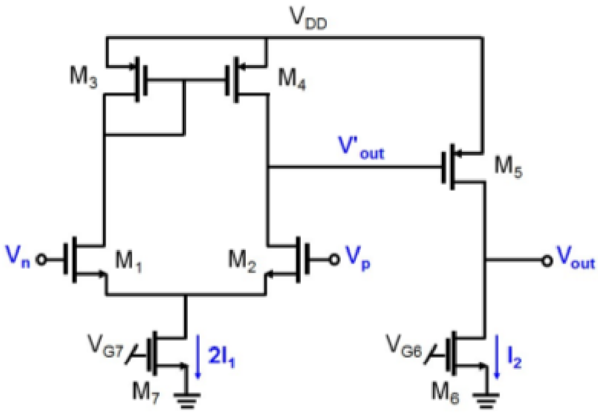
$$S_I(f) = \frac{K^{1/f}}{C'_{ox}WL} \tag{2.24}$$

So the overall tranfer function is

$$E_{1/f} = [\frac{K^{1/f}}{C'_{ox}W_{in}L_{in}} + \frac{K^{1/f}}{C'_{ox}W_{mM}L_{mM}}\left(\frac{g_m^{mM}}{g_m^{m,in}}\right)]\frac{1}{f} \tag{2.25}$$

Chapter 3

Two stage OTA

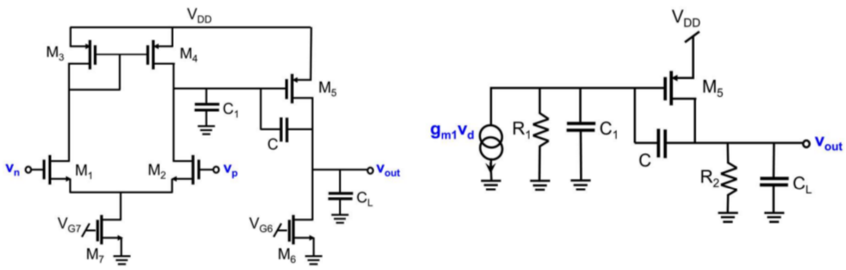


3.1 Differential gain

$$G_d = G_1 G_2 = g_{m,1}(r_{0,2}//r_{0,4})g_{m,5}(r_{0,5}//r_{0,6}) \tag{3.1}$$

$$G_d = \frac{4V_a^2}{L_{min}^2 V_{ov}^{m5} V_{ov}^{m1}}(\frac{L_2 L_4}{L_2 + L_4})(\frac{L_5 L_6}{L_5 + L_6}) \tag{3.2}$$

3.2 Compensation capacitance



GBWP:

$$GBWP = \frac{g_{m1}}{2\pi C} \tag{3.3}$$

Poles:

$$f_L = \frac{1}{2\pi(R_1C_1 + R_1C_1(1 + g_{m5}R_2) + (C + C_L)R_2)} \simeq \frac{1}{2\pi(R_1C(1 + g_{m5}R_2))} \tag{3.4}$$

$$f_H = \frac{CR_1g_{m5}R_2}{2\pi(R_1C_1R_2(C_L + C) + R_2C_LCR_1)} \simeq \frac{Cg_{m5}}{2\pi(C(C_1 + C_L) + C_1C_L)} \tag{3.5}$$

Increasing the compensation capacitance C the low frequency pole decrease while the high frequency pole approaches his asymptotic value

$$f_H \rightarrow \frac{g_{m5}}{2\pi(C_1 + C_L)} \tag{3.6}$$

To find this poles it's useful to use the time constant method.
To find the first pole we have to compute what resistance is seen by a capacitor while the others are open in order to compute the τ^0:

$$\tau^0_{C_1} = R_1C_1 \tag{3.7}$$

$$\tau^0_{C_L} = R_2C_L \tag{3.8}$$

$$\tau^0_C = C(R_2 + R_1(1 + g_{m5}R_2)) \tag{3.9}$$

Doing so we get the first pole

$$f_L = \frac{1}{\tau^0_{C_1} + \tau^0_{C_L} + \tau^0_C} \tag{3.10}$$

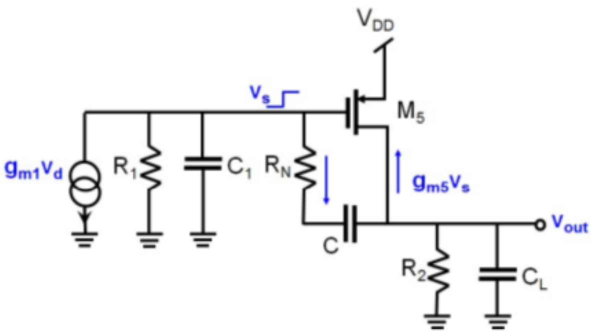
To estimate the second pole ,since the 3 capacitors are not indipendent, we may consider the C as a short and doing so we find that C_1 and C_2 are in parallel and they see an impedence of $1/g_{m5}$ that is the asymptotic value.
Zero:

$$f_z = \frac{g_{m5}}{2\pi C} \tag{3.11}$$

It's a positive zero that add a -90 shift on the phase.

3.3 Frequency compensation

3.3.1 Nulling resistor



Zero:
The circuit has still a zero in

$$f_z = \frac{1}{2\pi C(R_n - 1/g_{m5})} \tag{3.12}$$

so depending on the value of R_n the zero can be negative or at infinite frequency.
Poles:
We will consider $R_n \simeq 2/g_{m5} << R_1, R_2$
Low frequency pole estimated with time constants

$$f_L = \frac{1}{2\pi(R_1C_1 + R_1C_1(1+g_{m5}R_2) + (C+C_L)R_2) + R_nC} \simeq \frac{1}{2\pi(R_1C(1+g_{m5}R_2))} \tag{3.13}$$

Middle frequency pole estimated considering C a short

$$f_M \simeq \frac{g_{m5}}{2\pi(C_1+C_L)} \tag{3.14}$$

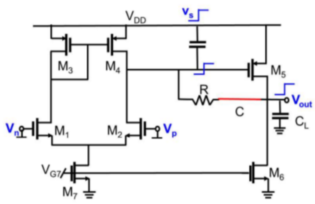
High frequency pole estimated with time constants

$$f_H = \frac{1}{2\pi(C_1//C_L//C)R_n} \tag{3.15}$$

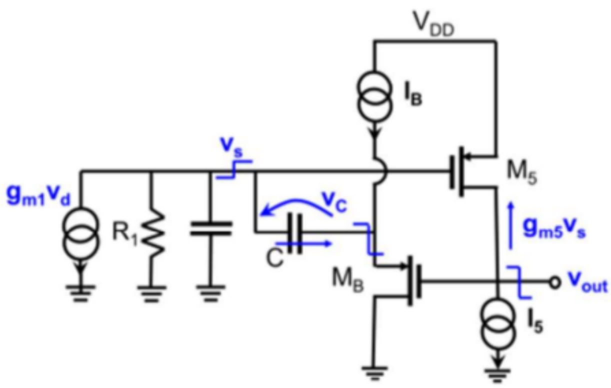
GBWP:

$$GBWP = \frac{g_{m1}}{2\pi C} \tag{3.16}$$

Remember: this circut has a bad power supply rejection ration (PSRR) at middle fequency



3.3.2 Voltage buffer



Zero:

$$f_z = \frac{1}{2\pi(C/g_{mB})} \qquad [LHP] \tag{3.17}$$

Poles:

$$f_L \simeq \frac{1}{2\pi C R_1(1+g_{m5}R_2)} \tag{3.18}$$

The high frequency poles are complex conjugate with

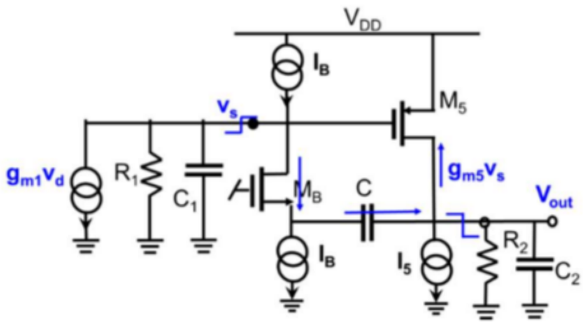
$$\omega_0 = \sqrt{\frac{g_{m5}g_{mB}}{C_1 C_L}} \tag{3.19}$$

$$Q = \sqrt{\frac{g_{m5}C_1}{g_{mB}C_L}} \tag{3.20}$$

GBWP:

$$GBWP = \frac{g_{m1}}{2\pi C} \tag{3.21}$$

3.3.3 Ahuja compensation



Zero:

$$f_z = \frac{1}{2\pi(C/g_{mB})} \qquad [LHP] \tag{3.22}$$

Poles:

$$f_L \simeq \frac{1}{2\pi C R_1(g_{m5}R_2)} \tag{3.23}$$

The high frequency poles are complex conjugate so we get

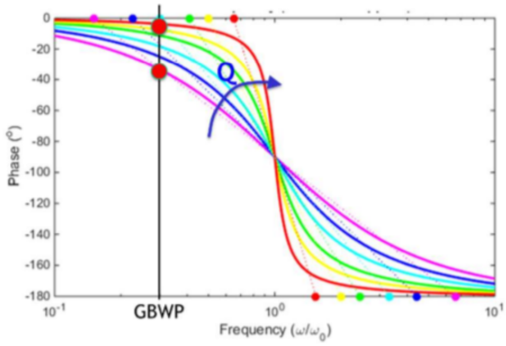
$$\omega_0 = \sqrt{\frac{g_{m5}g_{mB}}{C_1 C_L}} \tag{3.24}$$

$$Q = \sqrt{\frac{g_{m5}C_L}{g_{mB}C_1}} \tag{3.25}$$

GBWP:

$$GBWP = \frac{g_{m1}}{2\pi C} \tag{3.26}$$

Remember the lower the Q factor of the pole pair, the higher the phase shift at the GBWP



3.3.4 Simple cascode

No additional current needed and better PSRR.
Poles:
Same pole of the Ahuja structure so

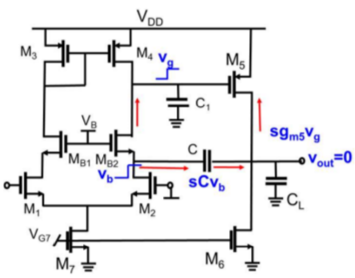
$$f_L \simeq \frac{1}{2\pi C R_1 (g_{m5} R_2)} \tag{3.27}$$

The high frequency poles are complex conjugate so we get

$$\omega_0 = \sqrt{\frac{g_{m5} g_{mB}}{C_1 C_L}} \tag{3.28}$$

$$Q = \sqrt{\frac{g_{m5} C_L}{g_{mB} C_1}} \tag{3.29}$$

Zeros:



To find the zeros we have to derive the following equation

$$i_5 = \frac{g_{m5} g_{mB}}{s C_1} v_b = s C v_b \tag{3.30}$$

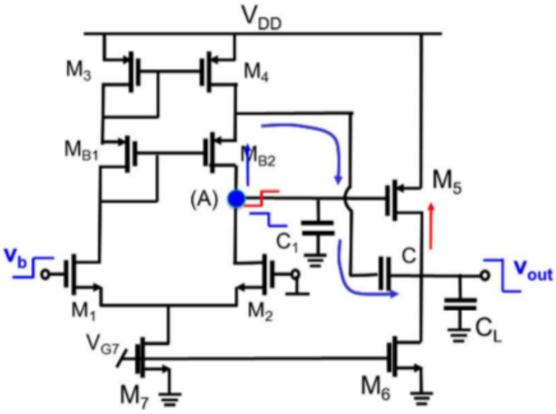
that gives us the 2 zeros one positive and one negative

$$s = \pm \sqrt{\frac{g_{m5} g_{mB}}{C C_L}} \tag{3.31}$$

Taking into account all parassitic terms the 2 zeros are not equal but the positive one is a little bit at lower frequency but provided that $g_{mB}/C_1 >> g_{m5}/C$ this has a limited impact on the phase margin.

3.3.5 Cascoded mirror

To avoid the RHP zero we can use this configuration



The miller effect is retained and we remove the RHP zero.

3.4 Slew Rate SR

3.4.1 Internal SR

For this we don't have to take into account the load capacitance so

$$SR_{int} = \frac{2I_1}{g_{m1}}(2\pi GBWP) = \frac{2I_1}{C} \tag{3.32}$$

3.4.2 Power Bandwith

The maximum frequncy that can be reached by full swing armonic signals without suffering from slew-rate distortion

$$f_{PBW} = \frac{SR}{2\pi A_{max}} \tag{3.33}$$

where A_{max} is the maximum value allowed to the voltage dynamics at the output node.

3.4.3 External SR

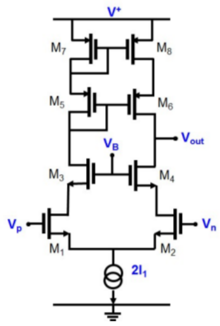
$$SR_{ext} = \frac{I_5}{C + C_L} \tag{3.34}$$

Chapter 4

High gain one stage OTA

4.1 Telescopic cascode

4.1.1 Gain



Considering all transistors with the same r_0

$$G_d = \frac{(g_{m1} r_0)^2}{2} \tag{4.1}$$

If the r_0 are different we get

$$r_{down} = \frac{r_{04} + 2r_{02} + 2g_{m4} r_{04} r_{02}}{2} \simeq \frac{2g_{m4} r_{04} r_{02}}{2} = g_{m4} r_{04} r_{02} \tag{4.2}$$

$$r_{up} = r_{06} + r_{08} + g_{m6} r_{06} r_{08} \simeq g_{m6} r_{06} r_{08} \tag{4.3}$$

so for Norton theorem

$$G_d = g_{m1} \cdot r_{down} // r_{up} \tag{4.4}$$

4.1.2 Dynamics

Output voltage swing
Upper voltage limited by M6 saturation

$$V_{out}^{max} = V_{dd} - V_{gs8} + V_{ov6} \tag{4.5}$$

Lower voltage limited by M4 saturation

$$V_{out}^{min} = V_B - V_t \tag{4.6}$$

Input common mode range
Upper voltage limited by saturation of M1-M2

$$V_{CM}^{max} = V_B - V_{gs4} + V_t \tag{4.7}$$

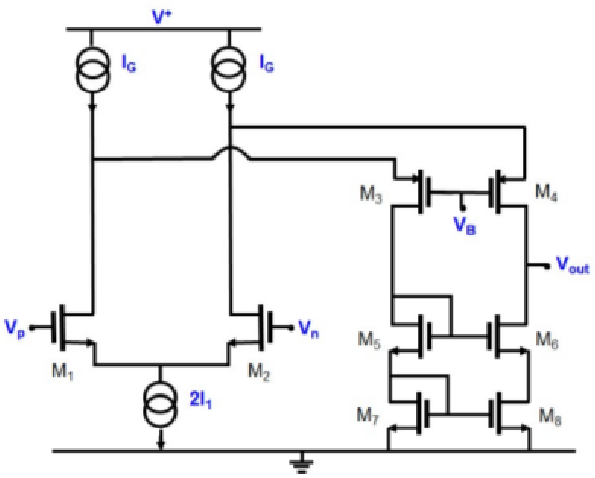
Lower voltage limited by saturation of the tail current generator

$$V_{CM}^{min} = V_{ovG} + V_{gs1} \tag{4.8}$$

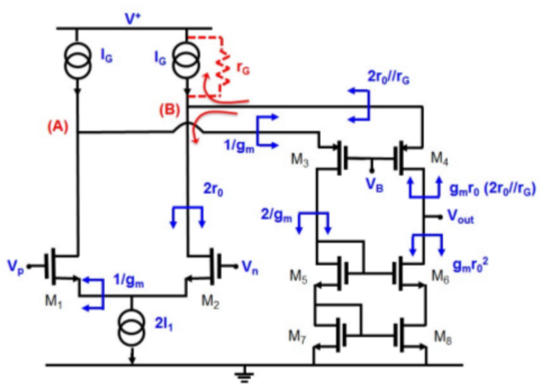
The setting of V_B is quite critical. By decreasing V_B , the output swing increases, but reducing the common-mode swing.

$$V_B^{max} = V_{dd} - V_{gs7} - V_{ov6} \qquad V_B^{min} = V_{ovG} + V_{ov2} + V_{gs3} \tag{4.9}$$

4.2 Folded cascode



4.2.1 Gain



Using Norton theorem

$$r_{down} = r_{06} + r_{08} + g_{m6} r_{06} r_{08} \simeq g_{m6} r_{06} r_{08} \tag{4.10}$$

The upper resistance
without taking into account the loop is

$$r_{up}^{no-loop} = r_{04} + r_{0g} // 2r_{02} + g_{m4} r_{04} (r_{0g} // 2r_{02} \simeq g_{m4} r_{04} (r_{0g} // 2r_{02}) \tag{4.11}$$

The G_{loop} is now

$$G_{loop} = -\frac{r_{0g}}{r_{0g} + 2r_{02}} \tag{4.12}$$

Therefore we have

$$r_{up} \simeq g_{m4} r_{04} (r_{04} // r_{0g}) \tag{4.13}$$

and a differential gain of

$$G_d = g_{m1} g_{m4} \cdot r_{up} // r_{down} \simeq \frac{(g_m r_0)^2}{3} \tag{4.14}$$

4.2.2 n-mos dynamics

Output voltage swing
Upper voltage limited by the saturation of M4

$$V_{out}^{max} = V_B + V_t \tag{4.15}$$

Lower voltage limited by the voltage drop across the mirror

$$V_{out}^{min} = V_{gs7} + V_{ov6} \tag{4.16}$$

Input common mode range
Upper limit is the saturation of M1 and M2

$$V_{CM}^{max} = V_B + V_{gs4} + V_t \tag{4.17}$$

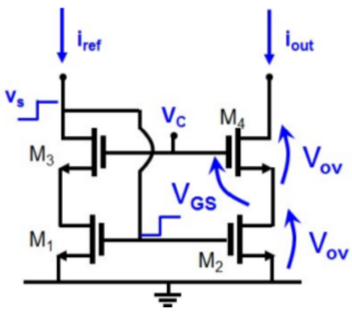
Lower limit is the current surce saturation

$$V_{CM}^{min} = V_{ovG} + V_{gs1} \tag{4.18}$$

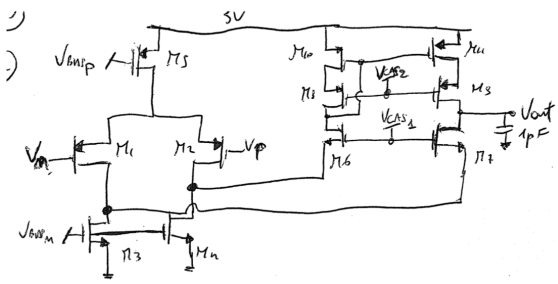
By increasing the V_B value both the output swing and the common-mode range increases. The upper limit for V_B is setted by the saturation of the 2 generators

$$V_B^{max} = V^+ - V_{ovG} - |V_{gs4}| \tag{4.19}$$

To further increase the voltage dynamic is possible to use this topology instead of the mirror-load.



4.2.3 p-mos dynamics



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4.2.4 Noise

Cascode doesn't ifluence noise so we get

$$E_{wn} = \frac{8kT\gamma}{g_m^{in}} \left(1 + \frac{g_m^{mirror\ high}}{g_m^{in}} + \frac{g_m^{g,current}}{g_m^{in}}\right) \tag{4.20}$$

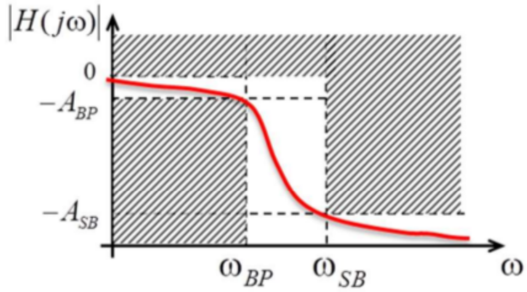
Remember to prevent the degradetion of the amplifier time response I_G *must be* $> 2I_1$

Chapter 5

Filters

$$2\pi f = \omega \tag{5.1}$$

5.1 Parameters



Selectivity index

$$k = \frac{\omega_{BP}}{\omega_{SB}} \tag{5.2}$$

Attenuation index

$$\varepsilon_{BP} = \sqrt{10^{A_{BP}/10} - 1} \qquad \varepsilon_{BP} = \sqrt{10^{A_{SB}/10} - 1} \tag{5.3}$$

Discriminator factor

$$k_\varepsilon = \frac{\varepsilon_{BP}}{\varepsilon_{SB}} < 1 \tag{5.4}$$

Higher the attenuation in stop-band lower k_ε is.

5.2 Butterworth Filters

Maximum flatness in passband.
General formula for the module

$$|H(j\omega)| = \frac{1}{\sqrt{1 + (\frac{\omega}{\Omega_0})^{2n}}} \tag{5.5}$$

where n is the filter order.

To mach the in-band attenuation spec. we have to respect

$$\frac{\Omega_{BP}}{\Omega_0} \leq \varepsilon_{BP}^{1/n} \tag{5.6}$$

To mach the stop-band attenuation spec. we have to respect

$$\frac{\Omega_{SB}}{\Omega_0} \geq \varepsilon_{SB}^{1/n} \tag{5.7}$$

So we get an interval of Ω_0 and a formula for the filter order

$$\frac{\Omega_{BP}}{\varepsilon_{BP}^{1/n}} \geq \Omega_0 \leq \frac{\Omega_{SB}}{\varepsilon_{SB}^{1/n}} \qquad\qquad n \leq \frac{\ln(k_\varepsilon)}{\ln(k)} \tag{5.8}$$

Poles in the Gauss plane are disposed as in figure

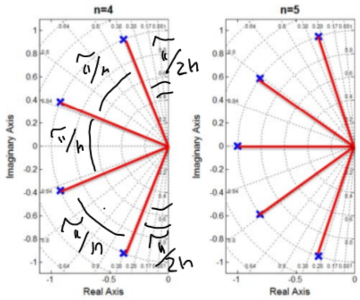


Table of Butterworth polynomials

Table 1: Butterworth polynomials normalized with respect the -3dB cut-off frequency.

N	$B_n(s)$
1	s +1
2	s^2 +1.414s + 1
3	(s +1)(s^2 + s + 1)
4	(s^2 +0.765s + 1)(s^2 + 1.848s + 1)
5	(s +1)(s^2 + 0.618s +1)(s^2 + 1.618s +1)
6	(s^2 +0.518s + 1)(s^2 + 1.414s + 1)(s^2 + 1.932s +1)

5.3 Bessel Filters

Maximum phase linearity.
General formula for the module

$$|H(s)| = J_n(0)/J_n(s) \tag{5.9}$$

where n is the filter order.
Table of poles for a n order Bessel

Table 2: Poles of the Bessel filters				
Order	Re Part	Im Part	Ω	Q
1	1,0000	0,000	1,000	0,500
2	1,1030	0,637	1,274	0,577
3	1,0509	1,003	1,452	0,691
	1,3270	0,000	1,327	0,500
4	1,3596	0,407	1,419	0,522
	0,9877	1,248	1,591	0,806
5	1,3851	0,720	1,561	0,564
	0,9606	1,476	1,761	0,916
	1,5069	0,000	1,507	0,500

5.4 Chebyshev Type-I

Low number of poles good selectivity but bad phase.
General formula is

$$H(s) = \frac{K_n}{D_n(s)} \tag{5.10}$$

where $D_n(s)$ are the Cheby. polynomials and K_n is a coefficient to have $|H(0)| = 1$ for filters of odd order and $|H(0)| = 1/\sqrt{1+\varepsilon_{BP}^2}$ for even filters order (count the ripple).

Order of the filter form the specs can be derived as

$$n \geq \frac{arCosh(k_\varepsilon^{-1})}{arCosh(k^{-1})} \tag{5.11}$$

Poles derived from the following formulas to mach exactly the BP attenuation

$$\Gamma = \left(\frac{1+\sqrt{1+\varepsilon_{BP}^2}}{\varepsilon_{BP}} \right)^{1/n}$$

$$s_k = -\sin\left[(2k-1)\frac{\pi}{2n}\right]\cdot\frac{\Gamma^2-1}{2\Gamma} + j\cos\left[(2k-1)\frac{\pi}{2n}\right]\cdot\frac{\Gamma^2+1}{2\Gamma}$$

where we remember that

$$\omega = \sqrt{r^2+i^2} \qquad Q = \frac{|omega|}{|2r|} \tag{5.12}$$

Where k ranges form 1,2,...2n and only poles with negative real part have to be considered.
To mach SB attenuation exactly we have to use in Γ

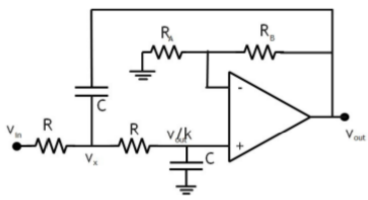
$$\varepsilon'_{BP} = \frac{\varepsilon_{SB}}{Ch(n * arCh(1/k))} \tag{5.13}$$

Table 3: Low-pass Chebyshev-I polynomials with 3dB ripple and normalized reference radial frequency.	
N	$D_n(s)$
1	$s+1.002$
2	$s^2+0.645s+0.708$
3	$(s+0.299)(s^2+0.299s+0.839)$
4	$(s^2+0.170s+0.903)(s^2+0.441s+0.196)$
5	$(s+0.178)(s^2+0.110s+0.936)(s^2+0.287s+0.377)$
6	$(s^2+0.077s+0.955)(s^2+0.209s+0.522)(s^2+0.285s+0.089)$

Poles in Gauss plane placed in an ellipse.

5.5 Sallen-Key Cell

5.5.1 With K>1



With all R equal ,all capacitors equal C and K the gain of the amplifier

$$\omega_0 = \frac{1}{RC} \qquad\qquad Q = \frac{RC}{RC(k-1) + 2RC} \tag{5.14}$$

Form the Q equation we get the dependaces with k

$$k = 3 - \frac{1}{Q} \qquad Q = \frac{1}{(3-k)} \tag{5.15}$$

A variation of $\Delta k/k = \%$ is mapped in a Q variation of

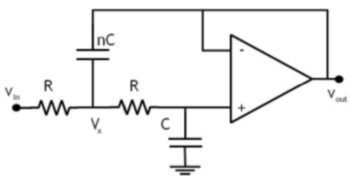
$$\Delta Q \simeq \Delta k \cdot Q^2 \tag{5.16}$$

The most significant change in the transfer function due to a change of Q is at the cut off frequency Ω_{bp}; there we can define the proportion

$$10\% \;\; of \;\; \frac{\Delta Q}{Q} \propto 1dB \tag{5.17}$$

Highes the Q bigger the variation for a given percentage bigger attenuation.

5.5.2 With K=1



With all R equal and K=1 the capacitor has to be different to add a degree of freedom to the system so

$$\omega_0 = \frac{1}{\sqrt{n}RC} \qquad\qquad Q = \frac{\sqrt{Q}}{2} \tag{5.18}$$

5.6 Non idealities

When we get a non ideal op-amp with finite GBWP and DC gain and olso others poles or positive zeros this imperfections leads us to a change of Q like in the relations

$$\frac{\Delta Q}{Q} \simeq 2Q \left(\frac{f_0}{GBWP} + \frac{f_0}{f_z} + \frac{f_z}{f_p} - \frac{1}{A_0} \right) \tag{5.19}$$