Electron Devices Notes

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...it's negligible...

Contents

Chapter 1

Semiconductors basics

1.1 Material distinction and energy gap

Distinction betweeen materials
In silicon-cristal electrons are subjected to periodical potential as a consequence to periodic disposition of atoms. We have bands of permitted energy level divided by gaps.
Importants bands are valence band VB ,that is the last band fully filled with electrons at 0 K, and conduction band CB ,that is the first band totally empty of electrons. Between this 2 bands there is the so called bandgap.
We can classificate all materials in 3 categories:
Metals - $E_{gap} = 0$ or "negative" and $\rho < 10^{-2}\Omega$cm .
Semiconductors - $E_{gap} = 1eV$ and $10^{-2} < \rho < 10^{5}\Omega$cm .
Isolators - $E_{gap} > 7eV$ and $\rho > 10^{5}\Omega$cm.

Energy gap function of T
In Si $E_{gap} = 1.12eV$ at room temperature (RT) but this value is a function of T; at high temperatures the silicon stretches and so does the periodical potential that influence the electrons.
The relation of E_{gap} with temperature is

$$E_{gap}(T) = E_{gap}(0) - \frac{\alpha T^2}{\beta + T} \tag{1.1}$$

where α and β change from material to material.
We can consider a sensitivity parameter of the temperature as

$$\frac{dE_{gap}(T)}{dT} = \frac{-2\alpha T(\beta + T) + \alpha T^2}{(\alpha + T)^2} \tag{1.2}$$

At RT for Si we have a change of 25meV over 100 degrees.

1.2 Silicon concentration

Conduction Band
Using the effective band approximation we want to know the densitiy of states in CB or VB and

after this the number of e or h in this bands.
Refering to the space of momentum we can say that

$$E - E_c = \frac{\hbar k_x^2}{2m_x} + \frac{\hbar k_y^2}{2m_y} + \frac{\hbar k_z^2}{2m_z} \tag{1.3}$$

The iso-energetic surface in the space of momentum is an ellipsoid that is longer in the direction with effective mass higer. In the case 2 effective mass are equal we obtain a rotiation ellipsoide just as the case of Si in witch we have $m_x = m_z = 0.19m_0 = m_t$ and $m_y = 0.92m_0 = m_l$. Observing that energy and the ellipsoide dimention are directly proportional we can say that all the points inside the ellipsoide have less energy than border ones.
The volume of the ellipsoide is

$$\mathcal{V} = \frac{4}{3}\pi\sqrt{\frac{2m_x(E-E_c)}{\hbar^2}}\sqrt{\frac{2m_y(E-E_c)}{\hbar^2}}\sqrt{\frac{2m_z(E-E_c)}{\hbar^2}} = \frac{4}{3}\pi\frac{\sqrt{8m_xm_ym_z}}{\hbar^3}\sqrt{(E-E_c)^3} \tag{1.4}$$

So the density of states will be

$$N = \frac{\mathcal{V}}{(\frac{2\pi}{L})^3}\frac{1}{L^3} \tag{1.5}$$

Making thefirst derivative of the equation by dE we can obtain finally

$$g_c(E) = \frac{4\pi}{h^3}\sqrt{2m_xm_ym_z}\sqrt{E-E_c}\cdot 2\cdot deg \tag{1.6}$$

where the last 2 terms are a corrective coefficients for the spin and the degeneration of the material we consider.
We can say how many states are occupied only in a very specific case of thermodinamical equilibrium. If this condition is satisfacted we can use the Fermi-Dirac statistic

$$f(E) = \frac{1}{1+e^{(E-E_f)/kT}} \tag{1.7}$$

where E_f is the Fermi level defined as the level at witch the prbability of occupation of an energy state by an electron is 1/2. $f(E)$ makes a smooth transition from 1 to 0 as the energy increases; the width of the transition is governed by kT. That can be approximated with the Maxwell-Bolzmann statistic if $E - E_f >> kT$

$$f(E) \simeq e^{-\frac{E-E_f}{kT}} \tag{1.8}$$

This is a good approximation if we are close to E_c or at least well above E_f at least a few kT. Concentretion of electrons in CB is (under the condition of thermodinamic equilibrium)

$$n = \int_{E_c}^{+\infty} g_c(E)f(E)dE \tag{1.9}$$

Making a change of variable as $x = (E-E_c)/kT$ and $\eta = (E_f - E_c)/kT$ we can write

$$n = \left[\frac{4\cdot 2\cdot deg\cdot\pi}{h^3}(kT)^{3/2}\frac{\sqrt{\pi}}{2}\right]\frac{2}{\sqrt{\pi}}\int_0^{+\infty}\frac{\sqrt{x}}{1+e^{x-\eta}}dx = N_c\cdot F_{1/2}(\eta) \tag{1.10}$$

where N_c is the density of states $F_{1/2}(\eta)$ is the Fermi-Dirac integral of order 1/2.
Assuming $x - \eta >> 1$ (that is the M-B approximation) we arrive at

$$n = N_c\frac{2}{\sqrt{\pi}}e^{\eta}\int_0^{+\infty}\sqrt{x}e^{-x}dx = N_ce^{\eta} = N_ce^{-\frac{E_c-E_f}{kT}} \tag{1.11}$$

Valence band

Valence band of silicon has 3 sub-bands: heavy hole band and light hole that stay at VB level and the split-off band that stays 44meV under the VB. This 3 band mass are isotopic so the iso-energetic surface in the k space are spheres; bigger the sphere bigger the effective mass. We take into account for calculations only heavy hole band:

$$E_v - E = \frac{\hbar^2 k^2}{2m_{hh}} \rightarrow \hat{k}_{hh} = \frac{\sqrt{2m_{hh}(E_v - E}}{\hbar} \tag{1.12}$$

as done before we calculate the volume of the sphere and the number of state per unit volume

$$\mathcal{V} = 4/3\frac{\pi}{\hbar^3}\sqrt{8m_{hh}^3}\sqrt{(E_v - E)^3} \tag{1.13}$$

$$N = \frac{\mathcal{V}}{\frac{2\pi}{L}}\frac{1}{L^3} \tag{1.14}$$

we can calculate the density of states as

$$g_V(E) = \frac{4\pi}{h^3}\sqrt{2m_{hh}}\sqrt{E_v - E} \cdot 2 \cdot deg \tag{1.15}$$

we proceed as for the CB obtaining the total concentration per unit volume of holes that is

$$p = p_{hh} + p_{lh} + p_{so} \tag{1.16}$$

but the concentration of holes in the split-off band is negligible due to the distance from the valence band (the transition of 1-f(E) that stays in a few kT).

$$p \simeq N_v e^{-\frac{E_f - E_v}{kT}} \tag{1.17}$$

Intrinsic and estrinsic Silicon

For intrinsic Si p=n and so we can calcuate E_f from this eq as

$$E_f = \frac{E_c - E_v}{2} - \frac{kT}{2}\ln(N_c/N_v) = E_i \tag{1.18}$$

E_i is the intrinsic level that stays in the middle of the gap (the correction is in the order of 0.3kT so negligible).

Putting 1.18 into p expression we obtain

$$p = n = \sqrt{N_c N_v}e^{-\frac{E_{gap}}{2kT}} = n_i \tag{1.19}$$

n_i is the intrinsic carrier concentration that has a strong dependance with T as it is involved in $N_c N_v$ and in E_{gap}.

From 1.11 by adding and subtracting E_i and dividing the exponential in 2 parts we can write

$$n = n_i e^{(E_f - E_i)/kT} \qquad p = n_i e^{(E_i - E_f)/kT} \tag{1.20}$$

this are 2 expressions valid in general not only for intrinsic semiconductors.

We can introduce the law of mass action as

$$pn = n_i^2 \tag{1.21}$$

For estrinsic semiconductors ,in the band diagram ,there is another band near to CB (E_d) (or near to VB (E_a) depending on the type of dope).
We want to extimate the position of E_f in n-doped Si. We can write that $n = p + N_d^+$ where N_d^+ are the ionized donor concentration so

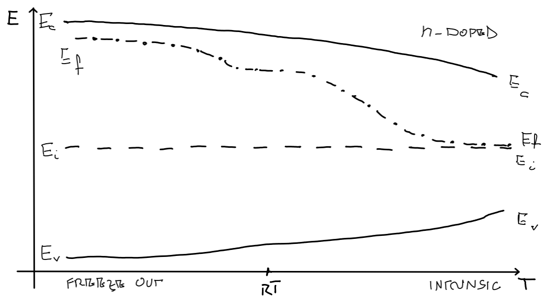
$$n = p + N_d(1 - \frac{1}{1 + 1/2e^{(E_d - E_f)/kT}}) = p + \frac{N_d}{1 + 2e^{-(E_d - E_f)/kT}} \tag{1.22}$$

the factor 1/2 it's a spin correction coefficient. At RT $E_d - E_f >> kT$ and p concentration is negligible in comparison with N_d so $n \simeq N_d$ and from the law of mass action $p = n_i^2/N_d$. From this we can compute E_f as

$$E_c - E_f = kT \ln(N_c/N_d) \tag{1.23}$$

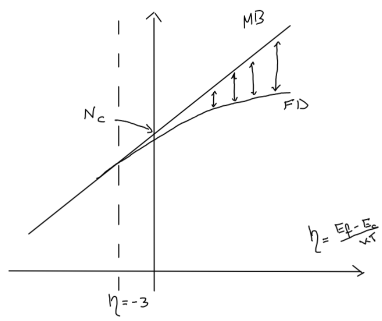
Moving E_f upwards some hypothesis falls: when E_f reachs E_d the complete ionization theory is not true anymore and from there further the M-B approximation is no longer valid.

Fermi level dependence on temperature in doped semiconductors



With high T E_g decreases and E_f will move distant to E_c following 1.18 until the hypothesis of negligible holes concentration decades and so at very high temperatures E_f will tend asimptotically to E_i.
At very low temperature E_f will increase until we reach the 0K where there is no conduction so E_f has to stay a little bit higher than E_d.

Low-temperature approssimation



In the case of a material that has its Fermi level higher than the conduction band level we cannot use the M-B approximaton. The approximation we can do is a low-temperature one that trasforms the Fermi Dirac distribution into a step so

$$n = \int_{E_c}^{E_f} \frac{48\pi}{h^3} \sqrt{2m_t^2 m_l} \sqrt{E - E_c} \tag{1.24}$$

that multiplying and dividing the risult by $(kT)^{3/2}\sqrt{\pi}/2$ we obtain

$$n = N_c 2/3 \frac{2}{\sqrt{\pi}} \left(\frac{E_f - E_c}{kT} \right)^{3/2} = N_c 2/3 \frac{2}{\sqrt{\pi}} \eta^{3/2} \tag{1.25}$$

In the graph below we can notice the difference between F-D and M-B distribution over η.

1.3 Current transport

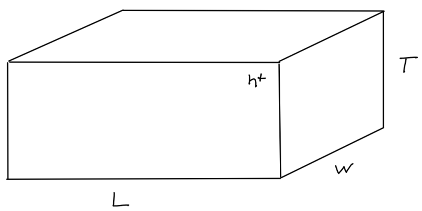
There are 2 most important mechanisms that generates current: drift and diffusion process.
Drift current is caused by the application of a electric field F; electrons are not only influenced by F but olso from scattering events so we can define a drift velocity (that is an average velocity) as

$$v_d = \mu_n F \tag{1.26}$$

So drift velocity is proportional to F with a constant μ_n called mobility that depends on doping concentration, temperature and dimensionality of the system taking in account all scattering events.It's important to note that in a bulk of doped Si if we are near the surface the mobility is much lower than the inside (dimensionality).
Mobility has a drop for Si around doping concentration of $\simeq 10^6 cm^{-3}$ beacuse at that level scattering with impurities becomes dominant in respect of thermal scattering.
Increasing too much F we arrive at the phenomena of velocity saturation ($\simeq 10^7 cm/s$) caused by scattering with high energetic optical phonons.
The density current caused by drift process is

$$J = (qn\mu_n + qp\mu_p)F = \sigma F = F/\rho \tag{1.27}$$

where σ is the conductivity of the material and rho is the resistivity.



When we calculate the resistivity of a block of doped silicon like in figure we define $\rho_{sh} = \rho/T$ as the sheet resistivity

$$R = \rho \frac{L}{WT} = \rho_{sh} \frac{W}{L} \tag{1.28}$$

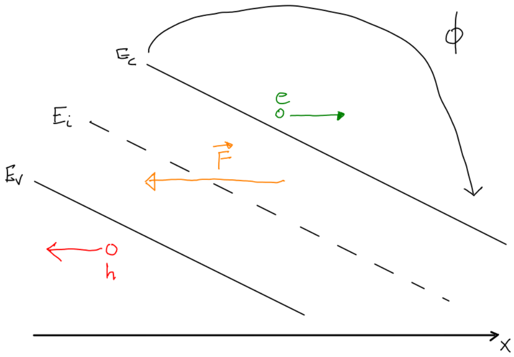
Diffusion current is driven by a gradient of concentration so the density of current is

$$J_n = qD_n \frac{dn}{dx} \qquad J_p = qD_p \frac{dp}{dx} \tag{1.29}$$

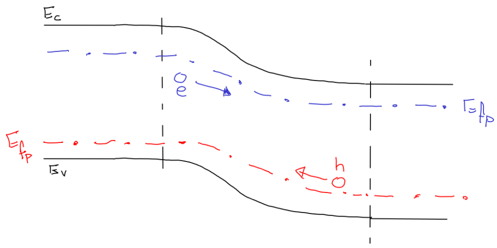
where D_n , D_p are the diffusion coefficients defined by Einstein's relations as

$$D_n = \mu_n \frac{kT}{q} \quad D_p = \mu_p \frac{kT}{q} \tag{1.30}$$

1.4 Bands "geography"



We can obtain external potential ϕ as $\phi = \frac{-E_i(x)}{q}$.
The external potential increases in the direction the bands band downwards. The electric field is in the opposite direction of the growth of potential $\vec{F} = -\frac{d\phi}{dx}$.
Electrons move by drift in the opposite direction of F and holes in the same direction of F.



Without condition of thermal equilibrium electrons move from regions of high quasi fermi level to regions of low quasi fermi level and holes vice-versa.

1.5 Poisson equation

Poisson equation is important to study the electrostatic of a system.
We can derive it from Maxwell's first law

$$F = -\vec{\nabla}\phi \rightarrow F = -\frac{d\phi}{dx} \tag{1.31}$$

and Gauss equation

$$\vec{\nabla}\vec{F} = \rho/\varepsilon_{Si} \tag{1.32}$$

from witch we obtain considering that all charges we have in a semiconductor are holes, electrons and ionized donors

$$\frac{d^2\phi}{dx^2} = -\rho/\varepsilon_{Si} = -\frac{q}{\varepsilon_{Si}}(p - n + N_d^+ - N_a^-) \tag{1.33}$$

In order to study the electrostatic of a system we have olso to know the coninutity equations of the electric field that are

$$F_{t1} = F_{t2} \Longleftrightarrow \epsilon_1 F_{n1} - \epsilon_2 F_{n2} = Q'_{int} \tag{1.34}$$

Debye length

Band banding is the phenomena caused by applying an electric field on a material at termodinamic equilibrium. This band banding causes different concentrations of e an h over the space so we can re-write the electron and holes concentrations taking in account the change of $\phi(x)$ over the space as

$$n = n_i e^{q\frac{\phi(x)-\phi_f}{kT}} \qquad p = n_i e^{q\frac{\phi_f-\phi(x)}{kT}} \tag{1.35}$$

This band banding is tipically caused by the change of doping concentration over the space.With a smooth change of N_d in a material we can assume that $n = N_d$.
Let's assume a step-like function in doping concentration in this case we cannot consider $n = N_d$ over the space beacuse this assumption will lead us to an infinite electric field.
Using Poisson equation and neglecting holes and ionized acceptors we can say that $N_d(x) = \hat{N}_d + \delta N_d(x)$ corrisponding to $\phi(x) = \hat{\phi} + \delta\phi(x)$ and assuming $\delta N_d << N_d$ we can write

$$\frac{d^2\phi}{dx^2} = -\rho/\varepsilon_{Si} = -\frac{q}{\varepsilon_{Si}}(\hat{N}_d + \delta N_d(x) - n_i e^{q\frac{\hat{\phi}+\delta\phi(x)-\phi_f}{kT}}) = -\frac{q}{\varepsilon_{Si}}(\hat{N}_d + \delta N_d(x) - \hat{N}_d - n_i e^{q\frac{\delta\phi(x)}{kT}}) \tag{1.36}$$

we can write the first order expansion of the exponential term beacuse the exponent is small obtaining the following differential equation

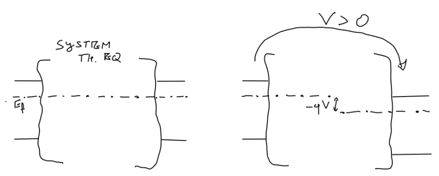
$$\frac{d^2\phi}{dx^2} = \frac{q^2\hat{N}_d}{\varepsilon_{Si}kT}\phi(x) - \frac{q}{\varepsilon_{Si}}\delta N_d(x) \tag{1.37}$$

the solution of this equation give us an exponential decrease of the potential with a length called Debye length

$$L_D = \sqrt{\frac{\varepsilon_{Si}kT}{q^2 N_d}} \tag{1.38}$$

this will cause a net charge (both negative and positive near the discontinuity.

1.6 Quasi-Fermi levels



For having a net current transport (we're leaving the thermodinamic equilibrium hypotesis) we have to perturbe our system by for example applying at the contact (:region of dispositive from witch we can externally apply a F, a good contact stay at th. eq.) a $V \neq 0$. As the figure shows we can't introduce a E_f level and so we cannot use the FD distribution. If the perturbation is weak we can recover all expressions of th.eq by introducing different Fermi levels both for e and h called quasi Fermi levels

$$n = n_i e^{\frac{E_{fn}-E_i}{kT}} \qquad p = n_i e^{\frac{E_i-E_f}{kT}} \tag{1.39}$$

so we can write the law of mass action generalized as

$$pn = n_i^2 e^{\frac{E_{fn}-E_{fp}}{kT}} \tag{1.40}$$

Now let's try to analise the total current density of electrons. The formula has a strong dependance from the electrostatic potential (from F and n) so

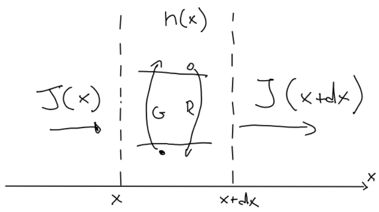
$$J_n = -qn\mu_n F + qD_n\frac{dn}{dx} = -qn\mu_n\frac{d\phi}{dx} + qD_n\frac{d}{dx}\left(n_i e^{q\frac{\phi-\phi_f}{kT}}\right) = -qn\mu_n\frac{d\phi_f}{dx} \tag{1.41}$$

But since the Fermi potential is costant the result is 0 and it's obvius beacuse we assume th.eq. with formula (1.20). If we substitute the (1.39) in n we obtain

$$J_n = -qn\mu_n \frac{d\phi_{fn}}{dx} \tag{1.42}$$

that isn't 0 beacuse ϕ_{fn} can change.
Electrons moves from the region of high quasi Fermi level to region of low quasi Fermi level, holes the opposite.

1.7 Equation of for e and h



Let's consider the figure above if we make a ballace of charges between x and x+dx we have

$$\frac{\partial n}{\partial t}dx = -J_n(x)/q + J_n(x+dx)/q - (G-R)dx \tag{1.43}$$

where G and R are generation and ricombination processes per unit time per unit volume. If we expand at the first order $J(x+dx)$ and simplify the expression we have

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} + G - R \qquad \frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} + G - R \tag{1.44}$$

that are the continuity equation for electrons and hole current.
In a stationary system nothing change with time so subtractring the 2 equation we have

$$\frac{1}{q}\frac{\partial (J_n + J_p)}{\partial x} = 0 \tag{1.45}$$

so the sum of the 2 contributions of current from e and h stay costant. If we neglect G and R the 2 contrbutes are costant separatly.
G and R process restabilize equilibrium of a system if we perturb the minority carrier concentration. If we disturb majority carrier the system returns in equilibrium with a very short time costant. Let's analyse the 2 case.
Majority carrier perturbation
Assume a n-doped material and let us add Δn concentration of electrons. Using poisson equation we have

$$\frac{\partial \phi^2}{\partial x^2} = -\frac{q}{2\varepsilon}\left(-n - \delta n + N_d\right) = \frac{q\Delta n}{\varepsilon} = -\frac{dF}{dx} \tag{1.46}$$

from this result we obtain a field that has a linear dependence over the space that tends to move away the excess of charge with a drift current (no diffusion the concentration is costant) so using 1.44

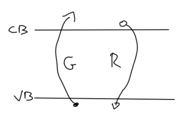
$$\frac{\partial J_n}{\partial x} = qn\mu_n \frac{\partial F}{\partial x} \tag{1.47}$$

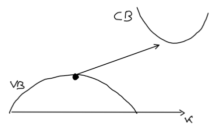
$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} = \frac{-1}{q} q n \mu_n \frac{q \Delta n}{\partial \varepsilon} = -\frac{\Delta n}{\varepsilon \rho} \tag{1.48}$$

solving this differential equation we have a solution that has an exponential dependence with $\varepsilon\rho$ that is called the dialectric relaxation time of the material; it's the time costant that the system use to return to its equilibrium ($\simeq ps$)

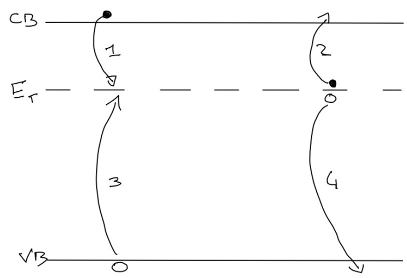
Minority carrier perturbation

The phenomena described for majority carrier concentration isn't valid for minority carrier. Considering a n-doped material like before the ρ in equation 1.48 is very high (we are considering holes now) so the real result of the field is that attracts a concentration Δn over Δp creating a quasi-neutral region where G and R slowly decrease the 2 extra concentration. In the processes of G and R in Si electrons cannot move thought the bandgap from CB to VB they need another particle to respect the conservation of momentum.





We need a difect of the cristal that creates another "band" in the bandgap this process called difect assisted and is described by Shockley-Reed-Hall theory. The difect can only be empty or filled with one electron so there can be only 4 process: 1)Difect empty captures e from CB 2) Difect filled realeses an e to CB 3) Difect filled captures hole from VB 4) Difect empty realeses an hole from VB.



For R we need 1+3 for G 2+4.
The rate of 1 can be described as

$$r_1 = N_T(1 - f(E_T))n(v_{th}\delta_n) \tag{1.49}$$

where: N_T is the total difect concentration ,f probability of the difect to be filled and the last term is a costant where δ_n is the capture crossection.
The rate of process 2 can be described as

$$r_2 = N_T f(E_T) e_n \tag{1.50}$$

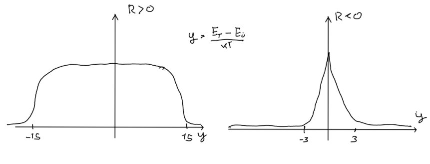
where e_n is a proportionality costant emission rate for e. Under th.eq. $r_1 = r_2$ and we can use the FD distribution for f. Under this condition we can calculate e_n (and using the same consideration olso e_p)

$$e_n = n_i v_{th} \delta_n e^{\frac{E_T - E_i}{kT}} \qquad e_p = n_i v_{th} \delta_p e^{\frac{E_i - E_T}{kT}} \tag{1.51}$$

So assuming $\delta_p = \delta_n = \delta$ and $\tau = 1/(v_{th} N_T \delta)$ under stationary condition $R = r_1 - r_2 = r_3 - r_4$ from this we obtain $f(E_T)$ and

$$R = \frac{pn - n_i}{\tau \left[p + n + 2n_i Cosh(\frac{E_T - E_i}{kT})\right]} \tag{1.52}$$

re-writing the numerator we can obtain that if $E_{fn} > Efp \rightarrow R > 0$ and if $E_{fn} < Efp \rightarrow R < 0$.
Looking at the graph of G and R



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1.8 Quasi-neutral condition

Under quasi-neutral condition $\Delta n + n_0 = n$ and $\Delta p + p_0 = p$ assuming low injection level ($\Delta n << p_0 + n_0$) we can neglect in the eq of R all the terms with "Δ" so we obtain

$$R = \frac{\Delta n}{\tau_n} = \frac{(p_0 + n_0)\Delta n}{\tau\left[p_0 + n_0 + 2n_i Cosh(\frac{E_T - E_i}{kT})\right]} \tag{1.53}$$

if we suppose $E_T - E_i 0 = 0$
So the continuity equation for electrons becomes

$$-\frac{d\Delta n}{dt} = R \rightarrow \Delta n \propto e^{-t/\tau_n} \tag{1.54}$$

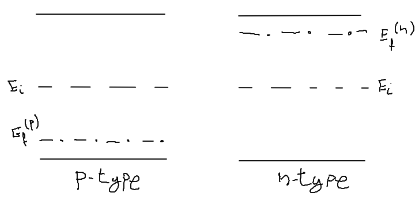
where τ_n is dipendent on the quality of the material

Chapter 2

Diodes

Diodes or pn junction are basically 2 zones of opposite doping polarity attached toghether. To study pn junctions we have to make some simplyfing assumptions : we consider 1D materials with constant doping concentration and a step-like change of doping.

2.1 Built-in potential

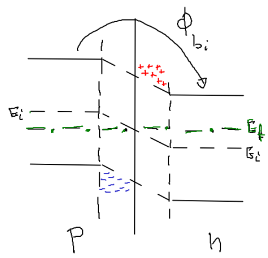


Let's consider the 2 regions isolated from each other and under th.eq. We don't have a single Fermi level so depending on the zone we have

$$E_i - E_f^{(n)} = kT\ln(\frac{N_a}{n_i}) \qquad E_f^{(n)} - E_i = kT\ln(\frac{N_d}{n_i}) \tag{2.1}$$

where $E_f^{(n/p)}$ is the Fermi level in that zone.
When we put the 2 materials toghether there will be first a diffusion current and than olso a drift current due to the band banding and the exposed charge; we will reach th.eq when the total current will be 0.
Under th.eq we will have a costant Fermi level all over the space and a band banding at the interface. Far from the intarface we will reach zones where the bands are like the isolated case. The drop of electrostatic potential in the junction under th.eq is called built in potential or ϕ_{bi} and is

$$q\phi_{bi} = (E_f^{(n)} - E_f^{(p)})|_{n.e} = kT\ln(\frac{N_aN_d}{n_i^2}) \tag{2.2}$$

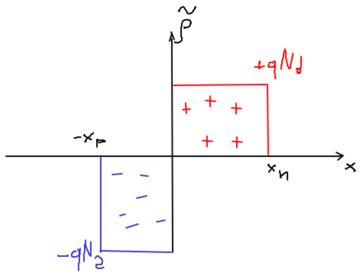


that is close to the E_g of the material.

2.2 Depletion approximation

Beacuse the distance $E_f - E_c$ increase near the junction we have a exponential deacrease of n so in the transition region we can say that $n << N_d$ and so $p << N_a$. From this we can say that the transition region si complitely depleted of free carriers beacuse their concentration is negligible so this region is called depletion region.

2.3 Electrostatics of a pn junction

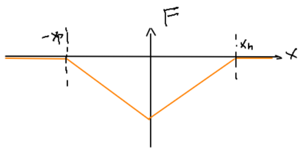


We have to solve
poisson equation in the depletion region so using depletion approximation and considering complete ionization

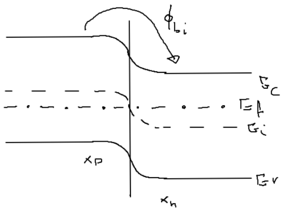
$$\frac{d\phi^2}{dx^2} = -\frac{q}{\varepsilon}(N_d - N_a) \tag{2.3}$$

and considering
the concentration of fixed charges like the graph
we can split the Poisson eq in 2 parts for $-x_p < x < 0$ we have $\frac{d\phi^2}{dx^2} = \frac{q}{\varepsilon}N_a$ and for $0 < x < x_n$ we have $\frac{d\phi^2}{dx^2} = -\frac{q}{\varepsilon}N_d$.
If we integrate both side of this equation and remembering that $F(x_n) = 0$ we can deduce the electric field in the junction as

$$\int_x^{x_n} d\frac{d\phi}{dx} = \int_x^{x_n} -\frac{q}{\varepsilon}N_d dx \tag{2.4}$$



$$F(-x_p < x < 0) = -\frac{qN_d}{\varepsilon}(x_p + x) \qquad F(0 < x < x_n) = -\frac{qN_d}{\varepsilon}(x_n - x) \tag{2.5}$$



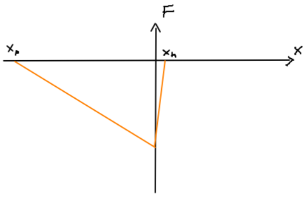
for continuity
equations of the electric field the F must be continuos in 0 so from this condition we get that $N_a x_p = N_d x_n$ that means that the total charge in the depletion area is 0 (from Gauss law olso).
Integrating
again both parts of the equation of the field we finally obtain the potential over the space that has a parabolic dependance

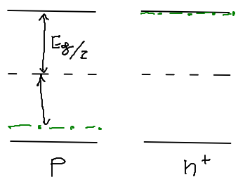
$$\phi(-x_p < x < 0) = \phi(-x_p) + \frac{qN_a}{2\varepsilon}(x + x_p)^2 \qquad \phi(0 < x < x_n) = \phi(x_n) - \frac{qN_d}{2\varepsilon}(x_n - x)^2 \tag{2.6}$$

To know x_n and x_p we have to set some boundary condition: ϕ must be continuos in 0, and $N_a x_p = N_d x_n$ from this 2 condition we get

$$x_n = \sqrt{\frac{2\varepsilon}{q}\phi_{bi}(\frac{1}{N_a} + \frac{1}{N_d})} \times \frac{N_a}{N_a + N_d} \qquad x_p = \sqrt{\frac{2\varepsilon}{q}\phi_{bi}(\frac{1}{N_a} + \frac{1}{N_d})} \times \frac{N_d}{N_a + N_d} \qquad W = \sqrt{\frac{2\varepsilon}{q}\phi_{bi}(\frac{1}{N_a} + \frac{1}{N_d})} \tag{2.7}$$

2.4 Unilateral Junction



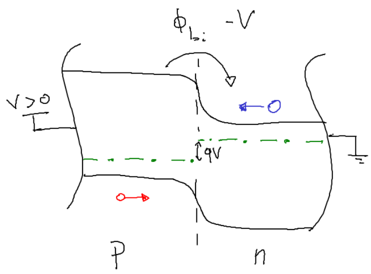


Let's consider a junction n^+p so the depletion area is almost all in the less doped material and so the electric field. We can make some approximation that the depletion area is almost in the p zone and that the Fermi level of the n^+ zone is almost at E_c so this lead us to

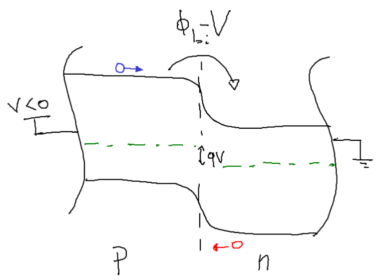
$$W \simeq x_p \qquad \phi_{bi} \simeq E_g/2 + kT \ln(\frac{N_a}{n_i}) \tag{2.8}$$

2.5 Bias

Forward Bias



Applying a external voltage like in figure we are reducing the total voltage drop over the junction. There will be a net flow of electrons from left to right that is a current by diffusion that will be large beacuse we have a lot of charges.
Invers Bias

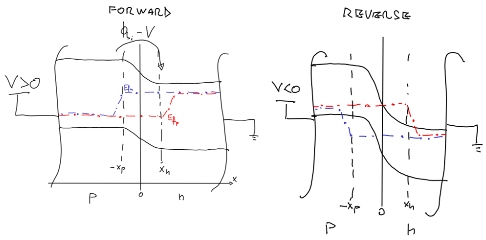


In this case $V < 0$ electrons and holes move for drift the total voltae drop is will be greater than the built in. The flow of current is given by minority carrier so small I.

2.6 The V-I characteristic of pn

First we want to investigate on foward bias electrostatic but we have to make some other simplyfing assumption: the contacts of our junction are at th.eq. and looking to the n region moving closer to the contact we will enter in a region where electrons are majority carrier. Beacuse we are injecting holes in that region we will have $p = p_n 0 + \Delta p$ so a region of quasi-equilibrium.
If we assume stationarity and that G and R process are negligible and so the $J_n = n\mu_n \frac{dE_{fn}}{dx} = cost$ so beacuse n isn't costant all over the space $\frac{dE_{fn}}{dx}$ will change over the

space. The final graph is this



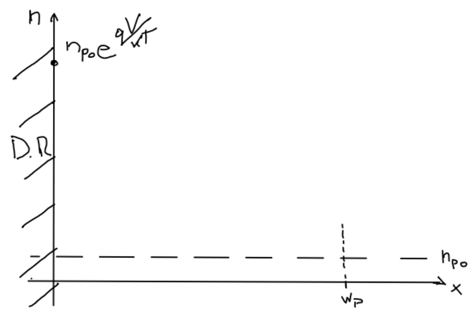
we have 2 region of quasi-equilibrium and a transition region that is depleted from free charges. This is the same result of electrostatic without bias so the depletion region will be

$$W = \sqrt{\frac{2\varepsilon}{q}(\phi_{bi} - V)(\frac{1}{N_a} + \frac{1}{N_d})} \tag{2.9}$$

E_{fn} remains costant until $-x_p$ but n changes beacuse E_c moves upwards.
For reverse bias we have to follow the same path of forward bias arriving at the graph above. Bands are flat near the contact beacuse we suppose quasi-neutral region under low level of injection with costant doping and a negligible gradient of quasi-Fermi level if one of this supposition falls the bands will no more be flat.
Now we want to know the quasi-Fermi level in the minority region. We know that at $x = x_n$ $n \simeq N_d$ and for the law of mass action generalized $p = \frac{n_i^2}{n} e^{\frac{E_{fn} - E_{fp}}{kT}} \simeq p_{n0} e^{\frac{qV}{kT}}$ and at $x = -x_p$ with the same passages we obtain $n \simeq n_{p0} e^{\frac{qV}{kT}}$. Now we can solve the continuity equation only in the quasi-neutral region.
Beacus the bands are flat we have only diffusion so $J_n = qD_n \frac{dn}{dx}$ and $G - R = \frac{-\Delta n}{\tau_n}$ from this two in the coninuity equations under stationary condition we get

$$\frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{L_n^2} = 0 \tag{2.10}$$

where $L_n = \sqrt{D_n \tau_n}$ = diffusion length of electrons in the quasi neutral p-region. Introducing the system of coordinate like in figure we can solve this differential equation.



Solving that as a polynomial we get $\lambda^2 - \frac{1}{L_n^2} = 0 \leftarrow \lambda = \pm\frac{1}{\lambda}$ so we get two exponential terms that are $\Delta n(x) = Ae^{\frac{x}{L_n}} + Be^{-\frac{x}{L_n}}$ using the boundary condition $\Delta n(0) = n_{p0}(e^{\frac{qV}{kT}} - 1)$ and $\Delta n(W_p) = 0$ we arrive at

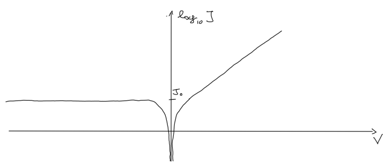
$$\Delta x(n) = n_{p0}(e^{\frac{qV}{kT}} - 1)\frac{\sinh(\frac{W_p - x}{L_n})}{\sinh(\frac{W_p}{L_n})} \tag{2.11}$$

Having only diffusion processes we can know the current but we are intrested only in $J_n(0)$ at the edge of the quasi-neutral region so

$$J_n(0) = (qD_n \frac{d\Delta n}{dx})|_{x=0} = -\frac{qD_n n_{p0}}{L_n \tanh(W_p/L_n)}(e^{\frac{qV}{kT}} - 1) \tag{2.12}$$

so in general the total current in the pn junction is

$$J = [\frac{qD_n n_{p0}}{L_n \tanh(W_p/L_n)} + \frac{qD_p p_{n0}}{L_p \tanh(W_n/L_p)}](e^{\frac{qV}{kT}} - 1) = J_0(e^{\frac{qV}{kT}} - 1) \tag{2.13}$$

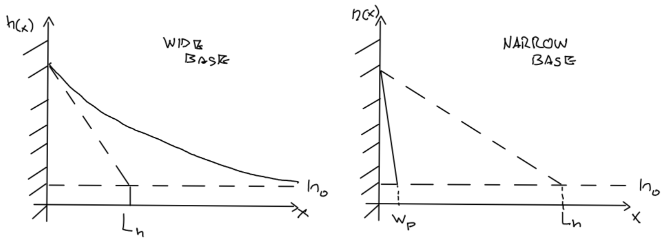


If $V >> kT/q$ we can neglect 1 and plot a log-log scale of this characteristic that is

$$\log(J) = log(J_0) + \frac{qV}{kT\ln(10)} \tag{2.14}$$

One important parameter of the characteristic is the slope in foward bias that is $kT/q\ln(10) = 60mV/dec@RT$. The dominant contribute to the current is from the zone less doped like in the electrostatic.

2.7 Wide and narrow base diode

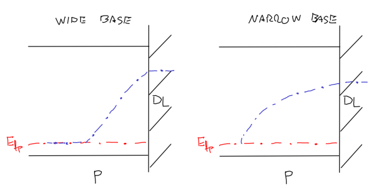


If we have a wide-base diode if $W_p >> L_n$ so we can approximate as follow the expression of charge and of current density

$$\Delta n(x) = n_{p0}(e^{\frac{qV}{kT}} - 1)e^{\frac{-x}{L_n}} \qquad J_n = \frac{qD_n n_{p0}(e^{\frac{qV}{kT}} - 1)}{L_n} \tag{2.15}$$

If we have a wide-base diode if $W_p << L_n$ so we can approximate as follow the expression of charge and of current density

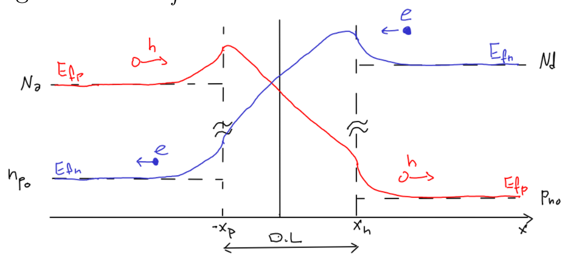
$$\Delta n(x) = n_{p0}(e^{\frac{qV}{kT}} - 1)\frac{W_p - x}{W_p} \qquad J_n = \frac{qD_n n_{p0}(e^{\frac{qV}{kT}} - 1)}{W_n} \tag{2.16}$$



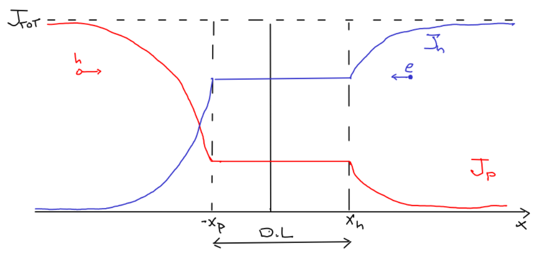
In a wide base diode n(x) decrease exponentially in the p region therefore E_fn will decrease linearly with a slope of kT/L_n and after the intersection with E_fn it will be costant as this last one. For a narrow base the decrease of E_fn will be logaritmic beacuse n(x) decreases linearly. In the quasi neutral p (n) region we have less electrons (holes) than in all other zone so the highest resistivity. The limit for conduction is given by the quasi-neutral region where we have less concentration of minority carrier; this is why the pn junction is called a minority carrier device , the limit for current transport is given by minority carriers.

2.7.1 Current profile

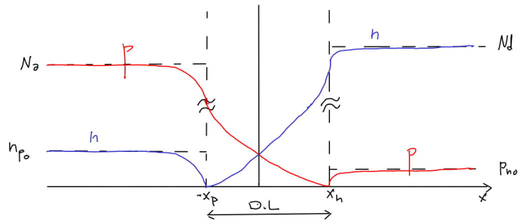
Wide base forward bias
The exponential behaviour of the minority will be followed by a consequent increase of majority charges in order to mantain the region quasi-neutral. In the depletion layer a connection between the 2 region. Minority carriers moves for diffusion but majority carrier moves for drift beacuse there is a small gradient of $E_f n$ that let electrons move from right to left.



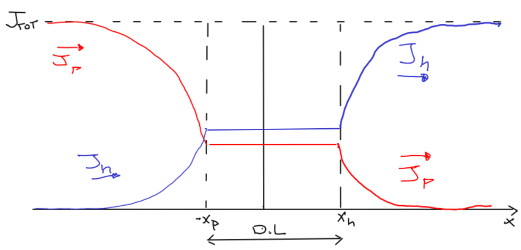
To know J_n in n region we have to remember that under stationary condition (so in quasi neutral region) $J_n + J_p = const$. Olso we remember that under stationary conditions and neglecting G-R both current contributions are constant. From this considerations we can draw the graph of J_n, J_p over the space.



Wide base reverse bias
Same path of before : minority carrier in quasi neutral regions decrease before the depletion layer so this phenomena will be copied by the majority carrier in order to mantain quasi-neutrality.



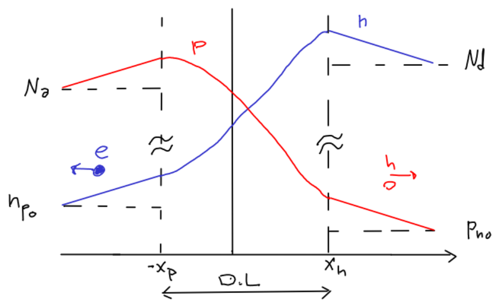
Knowing $J_n + J_p = const$ in stationary conditions and that if we neglect olso G-R process we have both currents constat we obtain the following graph.



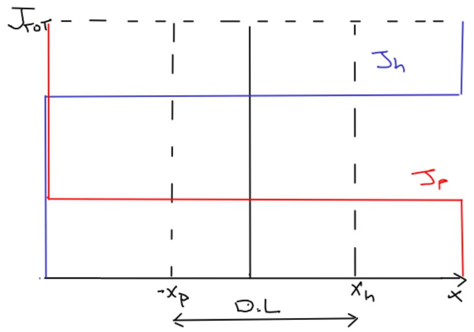
The flow of holes becomes zero at n contact but than in grows for generation processes.
We have the same behaviour of the current both in reverse and in forward bias but J_{tot} changes of orders of magnitude. As before minority carriers move by diffusion, majority by drift and in the depletion layer we have movement by drift.

Narrow base forward bias

Linear increase of minority carriers so olso majority in order to mantain the region quasi-neutral.

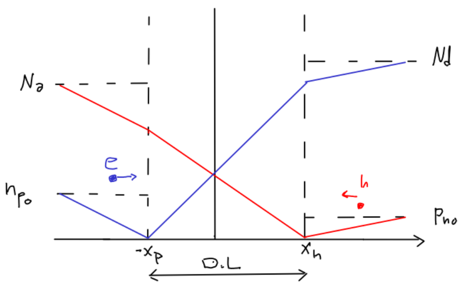


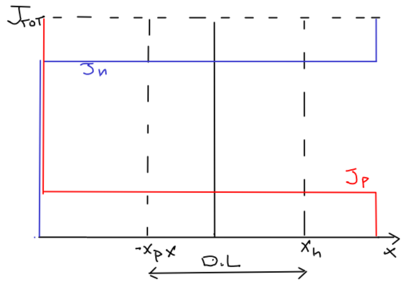
The derivate of a linear dependence is constat so J_n and J_p remain constat. There is no space to G-R process beacuse the device is much shorter than L_n so all generation and ricombination process are at the contacts.



Narrow base reverse bias

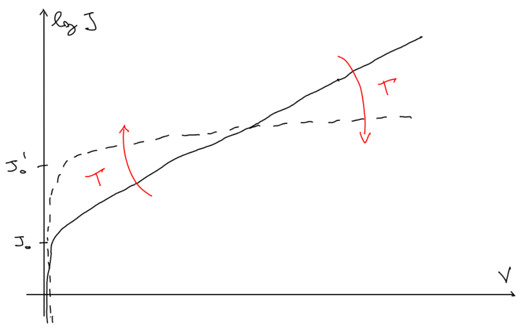
With the same path of before we arrive at





2.8 T change

Let's consider a pn junction in foward bias $J = J_0(e^{\frac{qV}{kT}} - 1) \simeq J_0 e^{\frac{qV}{kT}}$. If we increase the T the slope of the straight part will increase for the exponential term (be careful to units in the graph the line becomes flatter) but olso $J_0 = \frac{qD_n n_i^2}{N_a L_n}$ will increase for his strong dependance with temperature caused by $n_i \propto T^3 e^{-\alpha/T}$.



So with the increase of T we can have both an increase or a decrease of voltage corrisponding a fixed J. We have to find the typical regime of our device so from $J = J_0 e^{\frac{qV}{kT}}$ we extract the voltage as $V = \frac{kT}{q}\ln(J/J_0)$ with J a fixed current. Now we cand derivate obtaining

$$\frac{dV}{dT} = \frac{k}{q}\ln(J/J_0) + \frac{kT}{q} J_0/J \frac{-J\frac{dJ_0}{dT}}{J_0^2} = V/T - \frac{kT}{q}\frac{dJ_0}{dT} \tag{2.17}$$

for semplicity we can hilight J_0 dependences on temperature writing $J_0 = aT^{\gamma}e^{-E_g/kT}$ so we obtain that $\frac{dJ_0}{dT} = J_0\gamma/T + J_0[-\frac{dE_g}{dT}\frac{1}{kT}] + J_0\frac{E_g}{kT^2}$ so coming back at voltage we obtain

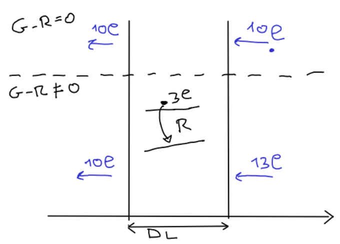
$$\frac{dV}{dT} = (V - \frac{E_g}{q})\frac{1}{T} - \frac{k\gamma}{q} + \frac{1}{q}\frac{dE_g}{dT} \tag{2.18}$$

That are all negative terms so we are in the firs zone where we find a decrease of voltage w.r.t an increase of the temperature (at RT $dV/dT \simeq -1.9mV/K$)

2.9 Second order effects on current

There are some phenomena that we have to consider in order to make more real the JV graph of a pn junction.

2.9.1 Low current regime



We have always neglect the G-R in the depletion layer but now we have to make some considerations. Under reverse bias $E_{fn} < E_{fp}$ so there will be some generation processes, knowing the quasi-Fermi level and using the law of mass action generalized we can write the SRH R coefficient as

$$R = \frac{n_i^2(e^{(qV)/(kT) - 1})}{\tau_0(p + n + 2n_i Ch((E_t - E_i)/(kT))} \tag{2.19}$$

so the sign of the voltage applied V change the sign of R. Refering to forward bias the impact of R is that from majority region there will be more electron moving to the depletion region beacuse of minority constrain of the pn junction So we aspect a larger current under foward bias where

$R \simeq \frac{n_i^2(e^{\frac{qV}{kT}}-1)}{\tau_0[p+n]}$ to get the worst case we take p=n so R will be maximum and we get $R = \frac{n_i(e^{\frac{qV}{2kT}})}{2\tau_0}$.
Under forward bias so we can define

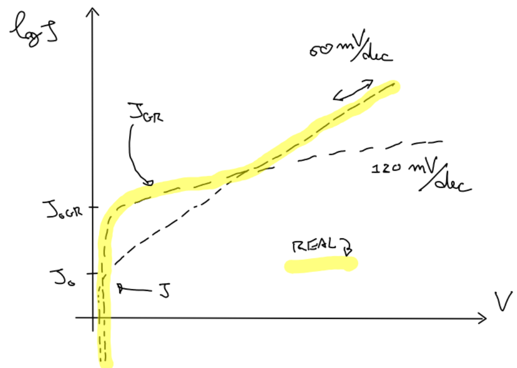
$$J_R = R_{max} W_d q = q W_d \frac{n_i(e^{\frac{qV}{2kT}}}{2\tau_0} \tag{2.20}$$

Under reverse bias $R \simeq -\frac{n_i^2}{2n_i\tau_0} = -\frac{n_i}{2\tau_0}$ and so as for forward bias we can define

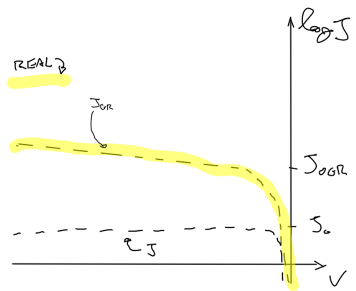
$$J_G = -\frac{n_i}{2\tau_0} W_d q \tag{2.21}$$

In general for both cases we can define

$$J_{GR} = \frac{qn_i}{2\tau_0} W_d(e^{\frac{qV}{2kT}} - 1) = J_{0GR}(e^{\frac{qV}{2kT}} - 1) \tag{2.22}$$



Similar to ideal but J_{0GR} is bigger and we have a factor 2 at the exp so there is a slight difference at very low current. Note that $J_{0GR} \propto 1/\tau_0$ so is very process dependent and olso that the ideal J_0 has a quadratic dependance on n_i (and this term has a linear dependance) so at high temperature we recover the ideal diode characteristic.
With reverse bias the current is dominated by J_{RG} and have a slight increase due to the dependence of W_d with V.



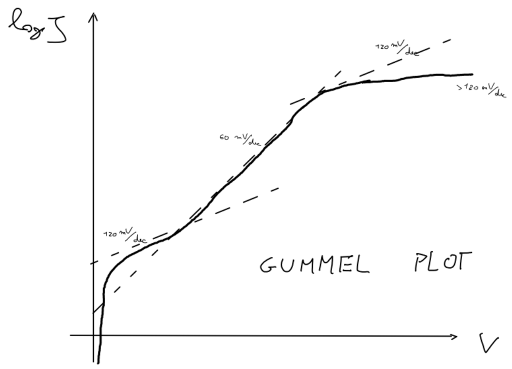
2.9.2 High current regime

At high current regime there are 2 problems that create distortion in the ideal characteristic.

High injection regime
At high current the hypotesis of low injection regime decades so taking p quasi-neutral region we have $n = n_0 + \Delta n \simeq \Delta n$ and $p = p_0 + \Delta n \simeq \Delta n$ so with the law of mass action generalized we obtain $pn = n_i^2 e^{\frac{qV}{kT}} = \Delta n^2$ and so $\Delta n = n_i e^{\frac{qV}{2kT}}$ from this through continuity equation we get that

$$J \propto n_i e^{\frac{qV}{2kT}} \tag{2.23}$$

so an additional solpe of 120mv/dec.
Resistive drops
E_f gradient is no more negligible so we have parasitic resistance due to the law $J_n = n\mu_n \frac{dE_{fn}}{dx}$. To introduce this non ideality we can say that $J = J_0 e^{\frac{qv}{mkT}}$ where m is a factor of non ideality. What it follows is the Gummel plot of the pn junction that include all non idealities.



2.10 Small signal model

The pn junction is, of course, a non linear device if we assume that is polarized at $\overline{V} > 0$ and we change that voltage of σV we are interested in its response. We have 3 small signal parameters: conductance, depletion capacitance and diffusion capacitance.

2.10.1 Conductance

Changing the voltage of σV we have a variation of the current so a condctance that we can calculate as

$$g_m = \frac{\partial I}{\partial V} = \frac{I}{kt/q} \tag{2.24}$$

2.10.2 Deplition capacitance

Changing δV we remove a part of the depletion region as a consequence of majority injected in the junction that neutralize the fixed charges in the DL. We have a variation of the charge stored in the device as a consequence of a variation of the voltage so a capacitance.
We can introduce

$$C_{dep} = \frac{\partial Q_{DL}}{\partial V} = -\frac{\partial}{\partial V}(qN_d x_n) = \frac{\varepsilon_{si}}{W_d} \tag{2.25}$$

The minus sign is place in order to have a positive C and this formula is valid olso for non constant doping concentration.

2.10.3 Diffusion capacitance

Olso in the quasi-neutral region we have charges stored that change if σV change due to the excess of minority. This capacitance is called diffusion capacitance and is defined as

$$C_{diff} = \frac{\partial Q_{diff}}{\partial V} \tag{2.26}$$

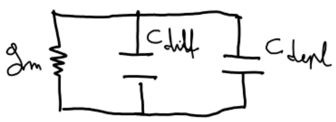
we have therefore define $Q_d iff$. Assuming forward bias and so neglecting n_{p0} we can write $Q_d iff = \int_0^{W_p} q\Delta n(x)dx$.
For a wide base diode we have

$$Q_{diff} = q\Delta n(0)\frac{L_n^2}{L_n}\frac{D_n}{D_n} = J_n\tau_n \tag{2.27}$$

For a narrow base diode

$$Q_{diff} = q\Delta n(0)\frac{W_p^2}{2W_p}\frac{D_n}{D_n} = J_n t_p \tag{2.28}$$

where t_p is defined as electron transit time through the quasi-neutral p region.



It's called like
this beacuse from the current equation we can derive that $J_n = qD_n\Delta n(0)/W_p = q\Delta n(x)v_{diff} = q\Delta n(0)\frac{W_p - x}{W_p}v_{diff}$ from this we obtain $v_d iff = \frac{D_n}{W_p - x}$ and from this integrating from 0 to W_p in dx we obtain exactly $t_p = \frac{W_p^2}{2D_n}$. Going back
to the diffusion capacitance we have for a wide base diode

$$C_{diff}^{wide} = g_m\tau_n \tag{2.29}$$

and for a narrow base

$$C_{diff}^{narrow} = g_m t_p \tag{2.30}$$

In the end the small signal model is showed in figure.
In reverse bias the modulation of the quasi-neutral change is negligible and so is the diffusion capacitance.

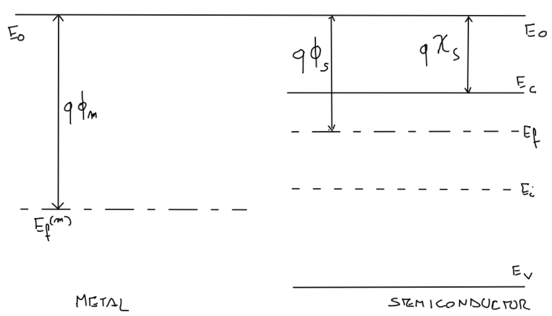
Chapter 3

Metal-semiconductor junction

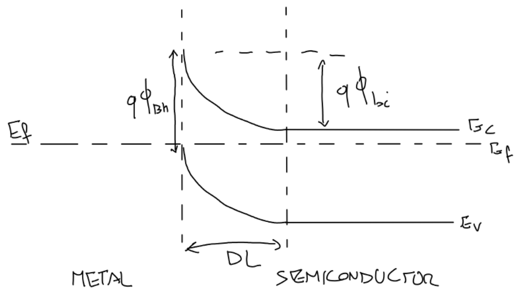
Metal-semiconductor junctions can be divided in 2 categories: ohmic contacts ,that are what we've called "a good contact", and rectifing devices ,that work like diodes. A good contact is a low parasitic resistance device with 2 pin through wich can flow a lot of J with a small V. Rectifing devices let current flow only in one direction, for this type of device is needed a low doped semiconductor and a metal with a proper work function; this type of devices are called Schottky diodes.

3.1 Schottky diode

We will study the electrostatic of this device as we've done with diodes. We initially suppose the 2 zones isolated and under thermodinamic equilibrium. Let's olso take the semiconductor n-doped.
For the metal we're not interested in the conduction or valence band but only in the position of the Fermi level. All type of material's bands are refered to the vacuum level. The distance between the Fermi-level of the metal and the vacuum level E_0 is calle work-function ϕ_m. For silicon we can define the electron affinity that is the distance between the conduction band and the vacuum level χ_s. In this way we can allign the two materials like in figure.

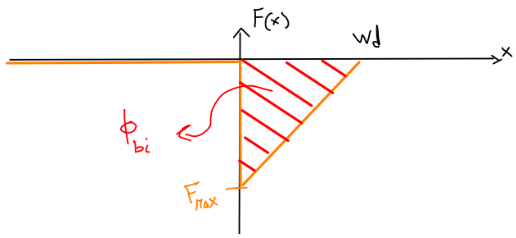


Requirement for a Schottky diode are $E_f^{(m)} < E_f^{(s)}$ and a low doped semicondctor. If
we put the 2 materials toghether the distance between the difference $q\phi_m - q\chi_s = q\phi_{bn}$ is preserved in the semiconductor the bands goes up to align E_f and we therefore have a built in potential ϕ_{bi}. There is a diffusion process from semiconductor to metal and not in the opposite direction beacuse electrons of metal don't have enough energy.



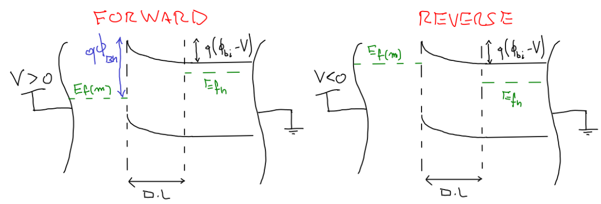
We can olso draw a graph of the electric field where we can view the deplition layer and the built in potential as area.
This device is an ideal unilateral $p^+ - n$ junction; we have the depliton layer only in the n side.
We can recove the expression from the diode

$$W_d = \sqrt{\frac{2\varepsilon_{si}}{q}\frac{1}{N_d}\phi_{bi}} = x \tag{3.1}$$



3.1.1 Bias

We will always consider the semiconductor part of the device n-doped and grounded; a voltage will be applied to the metal. As for the pn junction we consider foward bias if the voltage applied is grater than 0 and reverse bias if the voltage applied is less than zero.



Reverse bias
With reverse bias $E_{f(m)}$ becomes higher the depletion layer increases as the total voltage drop over the device.
Foward bias
With foward bias $E_{f(m)}$ becomes lower, far from the contact we will have in the semiconductor a quasi-neutral region ,near the contact a transition zone with a depletion region. The total voltage drop decreases as the width of the depletion region.
We have two major difference with the pn junction: the flow of electrons from the semiconductor to the metal don't have a minority constrain (the metal is full of electrons) so we will expect a high current but the transport of holes from metal to semiconductor has this

limitation (the semiconductor is n-doped). J_p will be small, negligible with respect to electrons flow.
Metal-semiconductor junction is a majority carrier device we won't deal with minority carrier in current transport phenomena. We will always neglect G-R processes beacuse are relevant only at low current regime.

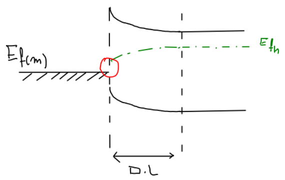
3.1.2 Schottcky model

We don't know the behaviour of E_{fn} in the depletion layer but we know that in that zone we have the lowe concetration of electrons so it can't be flat. Under stationary condition we can say that $J_n = qn\mu_n F + qD_n \frac{dn}{dx}$ = constant and, as boundary condition, that $n(W_d) = N_d$ $n(0) = N_c e^{\frac{q\phi_{bn}}{kT}}$. F is given the only parameter we don't know is n and $\frac{dn}{dx}$ so solving this differential equation we can get that

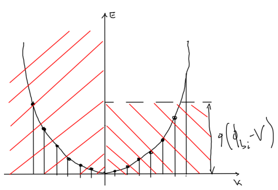
$$J = J_0(e^{\frac{qV}{kT}} - 1) \tag{3.2}$$

that is a relation identical to the pn junction but with a different J_0.
However this relation does not mach the experimental results. This is due to an implicit assumption that we've made using $n(0) = N_c e^{\frac{q\phi_{bn}}{kT}}$ as boundary condition; this happens only if the interface is always at thermodynamic equilibrium.
This model is valid for low mobility semicondutors (not for Si, Ge and GaAs).

3.1.3 Bethe's model



E_{fn} can arrive at the interfae higher than $E_{f(m)}$. We have a very thin interface that is of the order of fractions of nm we can't describe current transport with drift and diffusion theory beacuse they're dominated by scattering events that occurs in tens of nanometers. We have to use a thermioinc transport model. The electrons that pass from semiconductor to metal are that with an energy greater than $q(\phi_{bi} - V)$ from the potential material [?]. From energy dispersion relation $E = E_c + \frac{\hbar^2 k^2}{2m_x} + \frac{\hbar^2 k^2}{2m_y} + \frac{\hbar^2 k^2}{2m_z}$ we define $E_x = \frac{\hbar^2 k^2}{2m_x}$ the energy related to x transport and let's take into account only one ellipsoide.



Refering to the interface between the depletion layer and the quasi neutral region, half of the E_x parabola can be neglected beacuse we want electrons that move from right to left (semiconductor to metal). We want olso to consider only electrons with energy higher than $q(\phi_{bi} - V)$ so with a $k_x > \overline{k}$ so we can neglect other states. The equivalent current of this electrons will be

$$J^{(1)}_{s-m} = \sum_{k_x > \overline{k}} 2\frac{q}{L^3} v_x(k_x) f(k_x, k_y, k_z) \tag{3.3}$$

the 2 factor is a spin correction. Adding a corrective term we can transform the summation into an integral

$$J^{(1)}_{s-m} = \frac{1}{(\frac{2\pi}{L})^3} \int_{\overline{k_x}}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{q}{L^3} v_x(k_x) f(k_x, k_y, k_z) dk_x dk_y dk_z \tag{3.4}$$

remembering that $\hbar k = mv$ therefore $dk = \frac{m}{\hbar}dv$, we can change integration variable as

$$J^{(1)}_{s-m} = \frac{2q}{(2\pi)^3}\int_{+\infty}^{v_x}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} v_x(k_x)f(k_x,k_y,k_z)\frac{m_x m_y m_z}{\hbar^3}dv_x dv_y dv_z \tag{3.5}$$

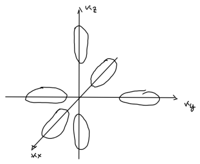
for f we can use M-B approximation and using $\hbar k = mv$ in the energy dispertion relation we get that

$$f \simeq e^{-\frac{E_c-E_{fn}}{kT}} \cdot e^{-1/2\frac{m_x v_x^2}{kT}} \cdot e^{-1/2\frac{m_y v_y^2}{kT}} \cdot e^{-1/2\frac{m_z v_z^2}{kT}} \tag{3.6}$$

so solving the 3 integral we get

$$J^{(1)}_{s-m} = \frac{4\pi q}{h^3}\sqrt{m_x m_y}(kT)^2 e^{\frac{-\phi_{bn}}{kT}} e^{\frac{qV}{kT}} = A\cdot\sqrt{m_x m_y}T^2 e^{\frac{-\phi_{bn}}{kT}} e^{\frac{qV}{kT}} \tag{3.7}$$

where A=120$\frac{A^2}{cm^2K^2}$ it's called the Richardson constant.



Taking into account all the ellipsoide we get

$$J_{s-m} = A\frac{2m_t + 4\sqrt{m_t m_l}}{m_0}T^2 e^{\frac{-\phi_{bn}}{kT}} e^{\frac{qV}{kT}} = A^*T^2 e^{\frac{-\phi_{bn}}{kT}} e^{\frac{qV}{kT}} \tag{3.8}$$

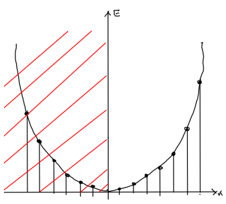
with $A^* = A\cdot 2.05$.
From m-s we can say that under th.eq the 2 current must be equal and that the barrier is costant equal to $q\phi_{bn}$ so $J_{m-s} = A^*T^2 e^{\frac{-\phi_{bn}}{kT}}$.
So finally the total current throught the device will be

$$J = A^*T^2 e^{\frac{-q\phi_{bn}}{kT}}(e^{\frac{qV}{kT}} - 1) = J_{0,th}(e^{\frac{qV}{kT}} - 1) \tag{3.9}$$

ϕ_{bn} it's a crucial parameter for current flow in metal semiconductor junction. We have a $J_{0,th} \simeq 10^5 A/cm^2$ orders of magnitude higher with respect to the pn junction and a turn on voltage of 0.3-0.4 V.
This model is valid for high mobility semiconductors.

3.1.4 Universal model

We can connect the two models in an universal one.
Starting from Schottcky model we can use his assumption of $J_n = qn\mu_n F + qD_n\frac{dn}{dx} =$ and $n(W_d) = N_d$ but we have to change n(0). We can say something about $J_n(0)$ as the current density without scattering.



We can calculate that term using energy dispersion relation but beacuse we are at the interface we will take all the left state of the parabola (we don't have a barrier) so we get

$$J_n(0) = A^*T^2 e^{-\frac{E_c-E_{fn}(0)}{kT}} \tag{3.10}$$

that is the correct boundary condition. If we muliply and divide this expression by N_c we obtain

$$J_n(0) = A^*T^2\frac{n(0)}{N_c} \tag{3.11}$$

this for the electron flow from semiconductor to the metal. For the opposite direction we use the th.eq. condition and we get

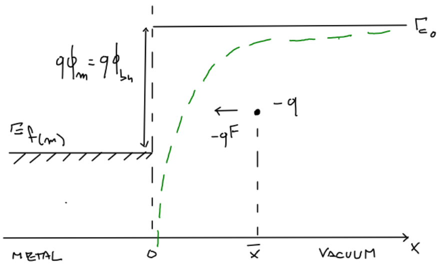
$$J_n(0) = A^*T^2\frac{n_0}{N_c} \tag{3.12}$$

with n_0 electron concentration at the interface under th.eq.
So the final boundary condition is

$$J_n(0) = \frac{A^* T^2}{N_c}[n(0) - n_0] \tag{3.13}$$

We are using a drift-diffusion approach with thermionic boundary condition.
Solving this system we get an expression valid for all semiconductors that can be simplified in the 2 models depending on what type of semiconductor we have.
If mobility is large than it's easy to move electrons but difficult to remove them form the interface that becomes the bottom neck of the system. With low mobility it's difficult to move electrons but easy to remove them form the interfece so they don't pile up there.

3.1.5 Schottky effect



The Schottky effect is a pure electrostatic effect that take place in the Bethe's model.
To study this
effect we start from a metal-vacuum "junction" placing in the vacuum a single electron at a certain distance $\overline{x}$.
We have an
electrostatic induction at the surface of the metal that attracts the electron in the vacuum near the surface.
In order to calculate the electric field that attracts the single electron we can use the image method placing a positive charge +q at $-\overline{x}$ and removing the metal.
From this assumption we get

$$-qF = \frac{1}{4\pi\varepsilon_0}\frac{-q^2}{(2\overline{x})^2} \tag{3.14}$$

and so the electric field

$$F = \frac{q}{16\pi\varepsilon_0 x^2} \tag{3.15}$$

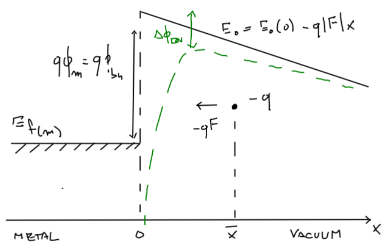
from witch we can calculate ,integrating both part of the equation from x to $+\infty$, the potential as

$$\phi(x) = \phi(+\infty) + \frac{q}{16\pi\varepsilon_0 x} \tag{3.16}$$

Multipling for -q we can obtain the energy

$$E_0(x) = E_0(+\infty) - \frac{q^2}{16\pi\varepsilon_0 x} \tag{3.17}$$

and so the profile in green in the graph.



If we consider that in the vacuum there is a costant electric field (and so $E_0(x) = E_0(0) - q|F|x$ like in figure beacuse the sistem is linear we obtain that

$$E_0(x) = E_0(0) - q|F|x - \frac{q^2}{16\pi\varepsilon_0 x} \tag{3.18}$$

and the behaviour in red in figure. We have change of the peak and so of the barrier that block our electrons. The Δ of the barrier is called Schottky barrier lowering.
We can calculate the new peak deriving by dx the $E_0(x)$ function finding the x_max obtaining the energy at that point

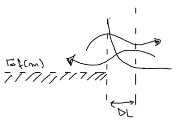
$$E_0(x_{max}) = E_0(0) - \sqrt{\frac{q^3|F|}{4\pi\varepsilon_0}} \tag{3.19}$$

So the difference in the barrier is $\Delta\phi_{bn} = \sqrt{\frac{q^3|F|}{4\pi\varepsilon_0}}$.
Now let's apply this effect on our metal semiconductor junction: when an electron travels throught the depletion layer from the semiconductor to the metal we have a barrier lowering of $\Delta\phi_{bn} = \sqrt{\frac{q^3|F_{max}|}{4\pi\varepsilon_{Si}}}$ (we place F_{max} beacuse the maximum effect we have is with the max F and the narrower distance).
In our device we have a change of the J-V characteristic

$$J = A^* T^2 e^{\frac{-q(\phi_{bn} - \Delta\phi_{bn})}{kT}} (e^{\frac{qV}{kT}} - 1) \tag{3.20}$$

that creates an increment of J_0 and 2 opposit behaviour for reverse and forward bias: with $V < 0$ the current increases increasing $|V|$, with $V > 0$ J decrease this due to the dependance of $\Delta\phi_{bn} \propto \sqrt{F_{max}} \propto V_{rv-bias}$.

3.2 Ohmic contact



In order to have an ohmic contact we need a high doped semiconductor. With N_d very high the depletion layer becomes narrower and increasing the doping concentration we arrive at a condition when there is a strong factor of J due to quantum-mechanical tunneling. That process does not depend on the type of bias so we don't have anymore a rectifing behaviour. Taking into account the figure as reference system from the studied pn junction we know the behaviour of $E_c(x) = \frac{q^2 N_d}{2\varepsilon_{Si}} x^2$ so we can calculate T ,transparancy coefficient as

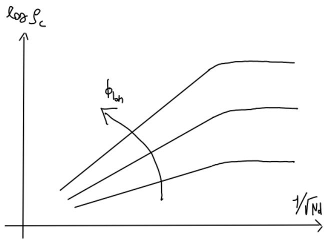
$$T = e^{-2\int Im\{E_c(x)\}dx} = e^{-2\int \sqrt{\frac{2m^*(E_c - E)}{\hbar^2}} dx} \tag{3.21}$$

we have to remember that in this case m^* is the effective mass for tunneling process. So if we consider $E = E_f$

$$T = e^{-2\int \sqrt{\frac{2m^* E_c}{\hbar^2}} dx} = e^{-2\sqrt{\frac{2m^*}{\hbar^2}}\sqrt{\frac{q^2 N_d}{2\varepsilon_{Si}}} W_d/2} \tag{3.22}$$

That substituiting W_d with its expession we get

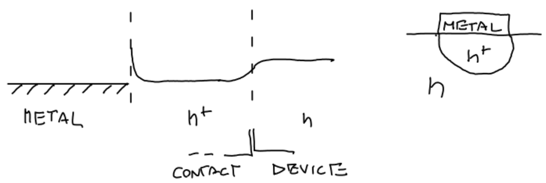
$$T = e^{-2\sqrt{\frac{2m^*}{\hbar^2}}\sqrt{\frac{q^2 N_d}{2\varepsilon_{Si}}\frac{2\varepsilon_{Si}}{qN_d}(\phi_{bi} - V)}} = e^{-q\frac{(\phi_{bi} - V)}{E_{00}}} \tag{3.23}$$



One important parameter of this device is the contact resistivity defined as

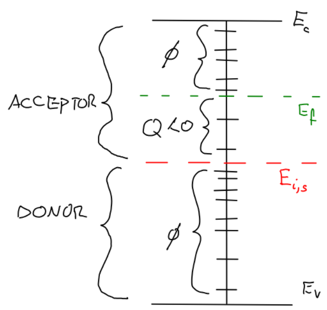
$$\rho_c = (\frac{\partial J}{\partial V})^{-1}|_{V=0} = \frac{E_{00}}{q} e^{q\phi_{bi}/E_{00}} \tag{3.24}$$

This parameter depends on the barrier hight and on $\sqrt{N_d}$. In the graph shows the $\log(\rho_c) - 1/\sqrt{N_d}$ dependance. It's a straight line until the concentration becomes too low and so tunneling effect is no more relevant.
To achive large current flow we need to perturbe only al little bit the device so it stays always near thermodynamic equilibrium.
The two figures below show how a contact is done in a device.

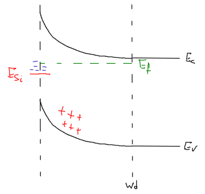


3.3 Interface states

We've always considered Si as a periodic infinite cristal but in metal-semiconductor junction we have te interface that interrupts the sequence of atoms. Some atoms of Si can't share all theyr 4 valence electrons creating some dangling bonds.



Electrons remains localize close to theyr silicon atoms this creates spurius energy level at the interface. In the bulk region electrons and holes are only in the conduction band or in the valenve band but at the interface can stay in a lot of states between this two bands. The spurious states in the upper part of the bandgap have an acceptor behaviour the others in the bottom part have a donor behaviour. The energy that divide this two type of states is the E_{is}. The states with a donor behaviour are neutral when filled and positivly charged when empty. Vice-versa states with acceptor behaviour are negativly charged when empty and neutral when filled. All states above E_f will be filled all the other states empty. So the previous band-diagram is incoherent with the Gauss'law since we have some exposed charge and no band banding.



We have a negative charge so $\phi < 0$ the bands band upward and the distance $E_f - E_{is}$ becomes narrower. The bands banding create a depletion region that expose a positive charge equal to the interface states' charge. The distance between $E_c - E_f$ is not set only by doping concentration but olso from interface states. We define interface state density $N_{is} = [cm^{-2}eV^{-1}$ in order to find the total charge introsuced by the interface

$$|Q_{is}| = N_{is}(E_f - E_{is})q \tag{3.25}$$

The distance $E_c(0) - E_{is} \simeq E_{gap}/2$ so writing $\Delta E_{is} = E_c(0) - E_{is}$ we can say that

$$|Q_{is}| = N_{is}[\Delta E_{is} - (E_c(0) - E_{is})]q \tag{3.26}$$

We know from the analysis of the pn junction that the charge in the depletion leyer is

$$Q_{dep} = qN_dW_d = \sqrt{2\varepsilon_{si}qN_d}\frac{E_c(0) - E_c(W_d)}{q} \tag{3.27}$$

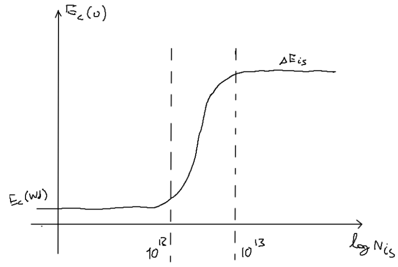
where the last term is the voltage drop in the depletion layer.
For Gauss law this 2 terms have to be equal so solving the equation we get the level of the conduction band at the interface

$$E_c(0) = \Delta E_{is} + \frac{\varepsilon_{si}N_d}{q^2N_{is}^2} - \sqrt{(\frac{\varepsilon_{si}N_d}{q^2N_{is}^2})^2 + \frac{2\varepsilon_{si}N_d}{q^2N_{is}^2}(\Delta E_{is} - E_c(W_d))} \tag{3.28}$$

This effect changes the barrerier hight and so theflow of current throught the device. ϕ_{bn} becomes strongly dependent on N_{is}

3.3.1 Limit cases

Starting from the equation $|Q_{is}| = Q_{dep}$ we have 2 limit cases.
$N_{is} = 0$ in this case $E_c(0) = E_c(W_d)$ so we don't have a band banding we restore the ideal case.
$N_{is} \to +\infty$ in this case we notice that the first term has to be finite and so $\Delta E_{is} - (E_c(0) - E_{is}$ has to be zero that means that $E_c(0) = E_f + \Delta E_{is}$.
This case means we are moving E_{is} up to E_f that is totally indipendent from doping concentration but depends only from the interface states. This condition is called "Fermi level pinning at the surface".
The plot of $E_c(0) - \log(N_{is})$ make a transition between $10^12 - 10^13$ as order of magnitude.



Chapter 4

MOS capacitor