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CRYPTO HW1 P2:

Q1) for our simplified box we have:

$$S_0 = \begin{bmatrix} 1, 0, 3, 2 \\ 3, 2, 1, 0 \\ 0, 2, 1, 3 \\ 3, 1, 3, 2 \end{bmatrix}$$

Now there are 16 pairs
for (x, x^*) in this box.

$$x' = x \oplus x^*$$

each spot varies to values up to 4

$$y = S(x), \quad y^* = S(x^*)$$

$$y' = y \oplus y^*$$

we can now make differential Distribution table
from all of these values.

input x'	0	1	2	3
00	16	0	0	0
01	0	10	6	0
02	0	2	10	4
03	2	4	0	10
04	2	4	8	2
05	4	2	2	8
06	8	2	2	4
07	2	8	4	2
08	2	4	8	2
09	0	2	2	12
A	10	0	4	2
B	4	10	2	0
C	8	2	2	4
D	2	8	4	2
E	2	4	8	2
F	4	2	2	8

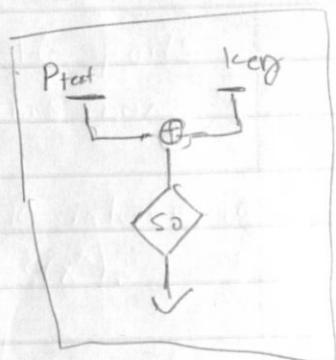
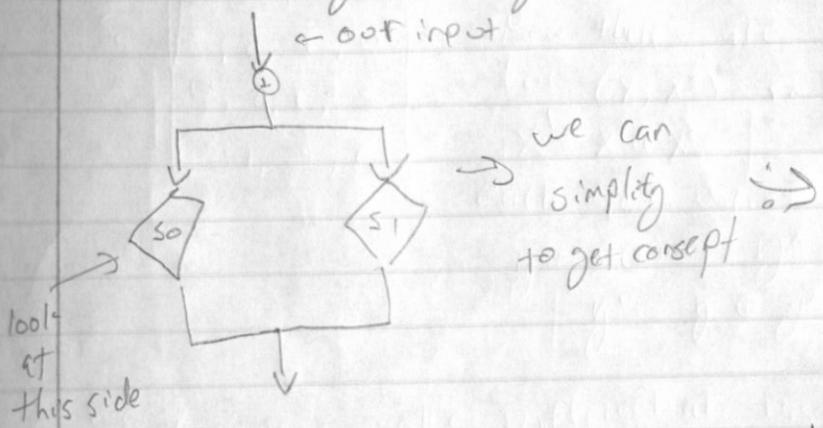
I used Python to make
the DPTable (0, left)
using each x as 4 bit
value. Picking all combos
of $(x,)$

Now let's start to
get the actual key :

On next page we
continue

Now that we have all the above lets pick 4 bit key = 0xa lets say.

We are gonna look at:



Lets Pick the input of:

1: 6, 2 to encrypt

our results will be:

$$- b \times A = 13 \text{ look up } \rightarrow 3$$

$$- 2 \times A = 9 \text{ look up } \rightarrow 0$$

xor 4

$$\begin{array}{l} \text{Output: } D(1) 2 \ 3 \\ \text{occurs: } 2(4) 0 \ 2 \end{array}$$

Now values that (X, X^4)

$$4 \rightarrow 1: 0, 4, 9, 13$$

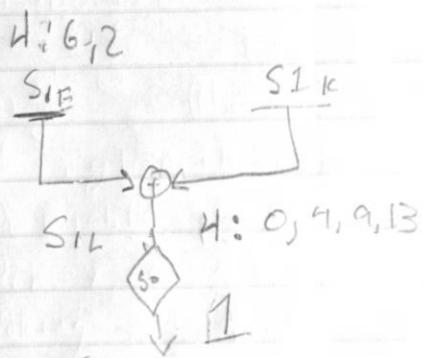
Now to find key:

SIE

SIL

SIK

SIO



Key:

$$SIK = SIE \oplus SIL:$$

$$0 \oplus 6 = 6 \quad | \quad 0 \oplus 2 = 2$$

$$4 \oplus 6 = 2 \quad | \quad 4 \oplus 2 = 6$$

$$9 \oplus 6 = 15 \quad | \quad 9 \oplus 2 = 11$$

$$13 \oplus 6 = 11 \quad | \quad 6 \oplus 2 = 4$$

$$\text{Possible vals: } \{2, 4, 6, 11, 15\}$$

we can continue doing this till we end up getting that the $11: 15$ is the key.

Basically the we could continue get another set and intersection until we got it as key. with more complex system. This all can be expanded to work. Just more boxes and rows make generating harder.

$$Q2) H(k|c) = H(k) + H(p) - H(c)$$

$$\begin{aligned} H(p) &= - \sum_{i=1}^n p_i \log_2 p_i \\ &= - \left(\frac{1}{3} \log_2 \frac{1}{3} + \frac{1}{6} \log_2 \frac{1}{6} + \frac{1}{2} \log_2 \frac{1}{2} \right) \\ &= 1.45 \end{aligned}$$

$P = \{a, b, c\}$ with
 $p_a(a) = 1/3$
 $p_p(b) = 1/6$
 $p_p(c) = 1/2$

$$\begin{aligned} H(k) &= - \left(\frac{1}{2} \log_2 \frac{1}{2} + \frac{1}{4} \log_2 \frac{1}{4} + \frac{1}{4} \log_2 \frac{1}{4} \right) \\ &= 1.5 \end{aligned}$$

$e_{k1}(a) = 1$	$e_{k1}(b) = 2$	$e_{k1}(c) = 3$
$\text{Prob dist } P_c$	$e_{k2}(a) = 2$	$e_{k2}(b) = 3$
$\overline{P(c)} = \left(\frac{1}{3}, \frac{1}{2}\right) + \left(\frac{1}{2}, \frac{1}{4}\right)$	$e_{k2}(c) = 3$	$e_{k3}(b) = 4$
$= \frac{1}{6} + \frac{1}{8} = \frac{14}{48} = 7/24$	$e_{k3}(a) = 4$	$e_{k3}(c) = 4$

$$\begin{aligned} P_c(2) &= \left(\frac{1}{3} \cdot \frac{1}{4}\right) + \left(\frac{1}{6} \cdot \frac{1}{2}\right) + \left(\frac{1}{2} \cdot \frac{1}{2}\right) \\ &= \frac{1}{12} + \frac{1}{12} + \frac{1}{4} = \frac{5}{12} \end{aligned}$$

$$\begin{aligned} P(3) &= \left(\frac{1}{4} \cdot \frac{1}{3}\right) + \left(\frac{1}{4} \cdot \frac{1}{6}\right) \\ &= \frac{1}{12} + \frac{1}{24} = \frac{3}{24} = \frac{1}{8} \end{aligned}$$

$$\begin{aligned} P(4) &= \left(\frac{1}{4} \cdot \frac{1}{6}\right) + \left(\frac{1}{4} \cdot \frac{1}{2}\right) \\ &= \frac{1}{24} + \frac{1}{8} = \frac{9}{24} \end{aligned}$$

$$\begin{aligned} H(c) &= - \left(\frac{7}{24} \log_2 \left(\frac{7}{24}\right) + \frac{5}{12} \log_2 \left(\frac{5}{12}\right) + \frac{1}{8} \log_2 \left(\frac{1}{8}\right) \right. \\ &\quad \left. + \frac{9}{24} \log_2 \left(\frac{9}{24}\right) \right) \\ &= 1.85 \end{aligned}$$

$$H(\text{KIO}) = 1.5 + 1.45 - 1.797 = \boxed{1.85}$$