

---

## WHAT IS A KERNEL?

```
k <- function(x) return(dnorm(x)) # gaussian kernel

fhat <- function(x, xs, h) {
  # estimate the density at point x
  n <- length(xs)
  return(sum(k((x - xs) / h))) / (n * h))
}
```

## CHOOSING OPTIMAL BANDWIDTH

```
loocv <- function(xs, hs) {
  # calculate the leave-one-out cross-validation score for various bandwidths
  n <- length(xs)
  scores <- vector()
  for (h in hs) {
    score <- 0
    for (xi in xs) {
      for (xj in xs) {
        score <- score + dnorm((xi-xj)/h, 0, sqrt(2)) - 2 * k((xi-xj)/h)
      }
    }
    score <- score / (h * n^2)
    score <- score + 2 * k(0) / (n * h)
    scores <- c(scores, score)
  }
  return(scores)
}
```

```
optimal_h <- function(loocv_scores, hs) {
  # determine the bandwidth which corresponds to minimum loocv score
  min_cv <- min(loocv_scores)
  indexes <- which(loocv_scores %in% min_cv)
  index <- min(indexes)
  return(hs[index])
}
```

## PLOTTING THE DENSITY ESTIMATE

```
kde <- function(xs, h) {
  # produce estimates of the density at all values of x
  vals <- vector()
  for (x in xs) {
    vals <- c(vals, fhat(x, xs, h))
  }
  return(vals)
}
```