Homework 2
Course : S520 Intro to Statistics
Author : Jack McShane
Date : February 8, 2022

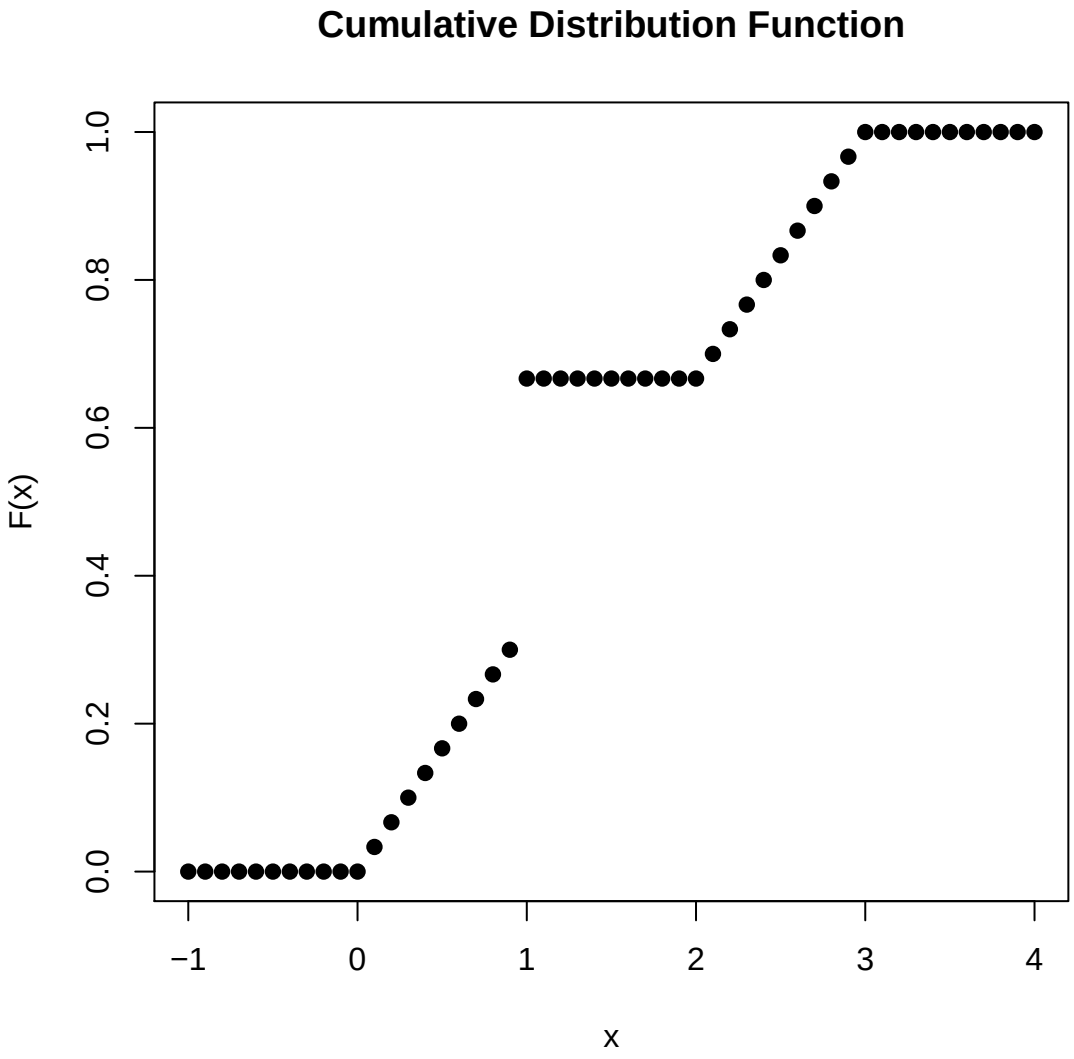
Question 1: Suppose 5 cards are dealt from a standard deck of playing cards. How many hands are possible?

In this instance, the order in which the cards are dealt does not matter. We are therefore looking for the number of combinations, not the number of permutations.

$$\begin{aligned} C(n,r) &= \frac{P(n,r)}{P(r,r)} \\ &= \frac{P(52,5)}{P(5,5)} \\ &= \frac{52*51*50*49*48}{5!} \\ &= \underline{\underline{2598960}} \end{aligned}$$

Question 2: Suppose that X is a random variable with cdf:

$$F(y) = \begin{cases} 0 & y \leq 0 \\ \frac{y}{3} & y \in [0,1) \\ \frac{2}{3} & y \in [1,2] \\ \frac{y}{3} & y \in [2,3] \\ 1 & y \geq 3 \end{cases}$$


Cumulative Distribution Function
F(x)
0.0
0.2
0.4
0.6
0.8
1.0
−1
0
1
2
3
4
x

(a) What is P(X > .5) ?

$$\begin{aligned} P(X > .5) &= 1 - P(X \leq .5) \\ &= 1 - F(.5) \\ &= 1 - \frac{.5}{3} \\ &= 1 - \frac{1}{6} \\ &= \underline{\underline{\frac{5}{6}}} \end{aligned}$$

(b) What is P(2 < X ≤ 3) ?

$$\begin{aligned} P(2 < X \leq 3) &= P(X \leq 3) - P(X \leq 2) \\ &= F(3) - F(2) \\ &= 1 - \frac{2}{3} \\ &= \underline{\underline{\frac{1}{3}}} \end{aligned}$$

(c) What is P(.5 < X ≤ 2.5) ?

$$\begin{aligned} P(.5 < X \leq 2.5) &= P(X \leq 2.5) - P(X \leq .5) \\ &= F(2.5) - F(.5) \\ &= \frac{5}{6} - \frac{1}{6} \\ &= \underline{\underline{\frac{2}{3}}} \end{aligned}$$

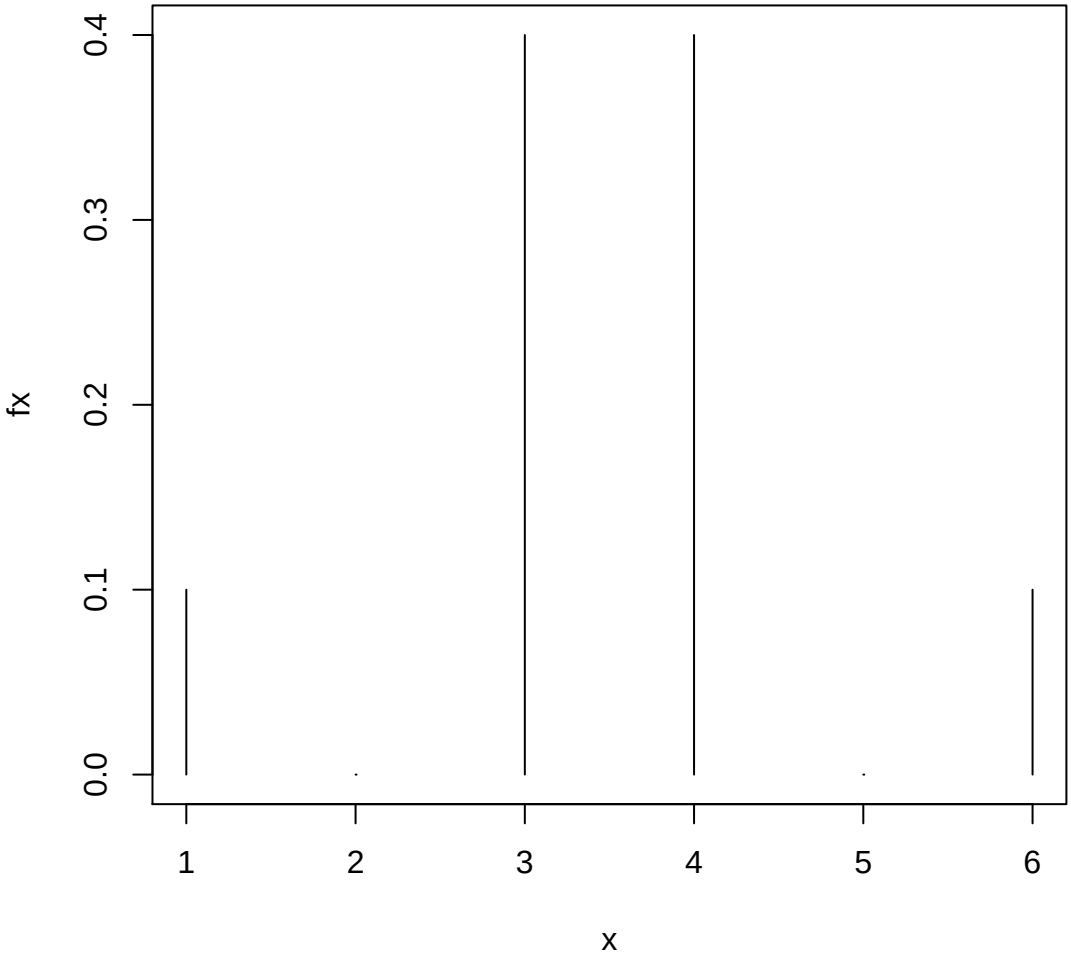
(d) What is P(X = 1) ?

$$\begin{aligned} P(X=1) &= P(X \leq 1) - P(X < 1) \\ &= F(1) - F(1^-) \\ &= \frac{2}{3} - \frac{1}{3} \\ &= \underline{\underline{\frac{1}{3}}} \end{aligned}$$

Question 3: Suppose that X(S) = 1, 3, 4, 6 and P(X = 1) = P(X = 6) = .1, P(X = 3) = P(X = 4) = .4.

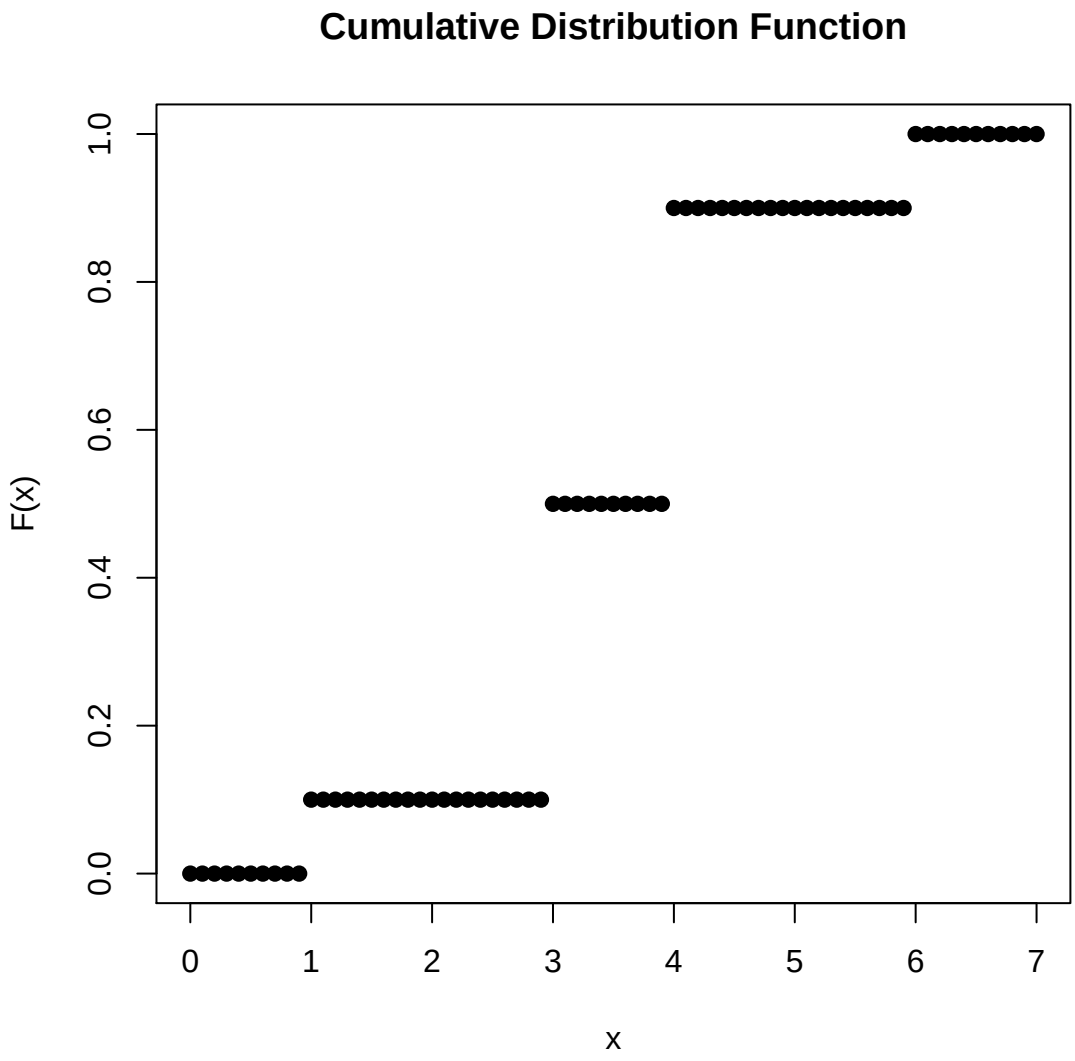
(a) Determine the pmf.

$$pmf(X) = \begin{cases} .1 & X = 1 \\ .4 & X = 3 \\ .4 & X = 4 \\ .1 & X = 6 \\ 0 & otherwise \end{cases}$$



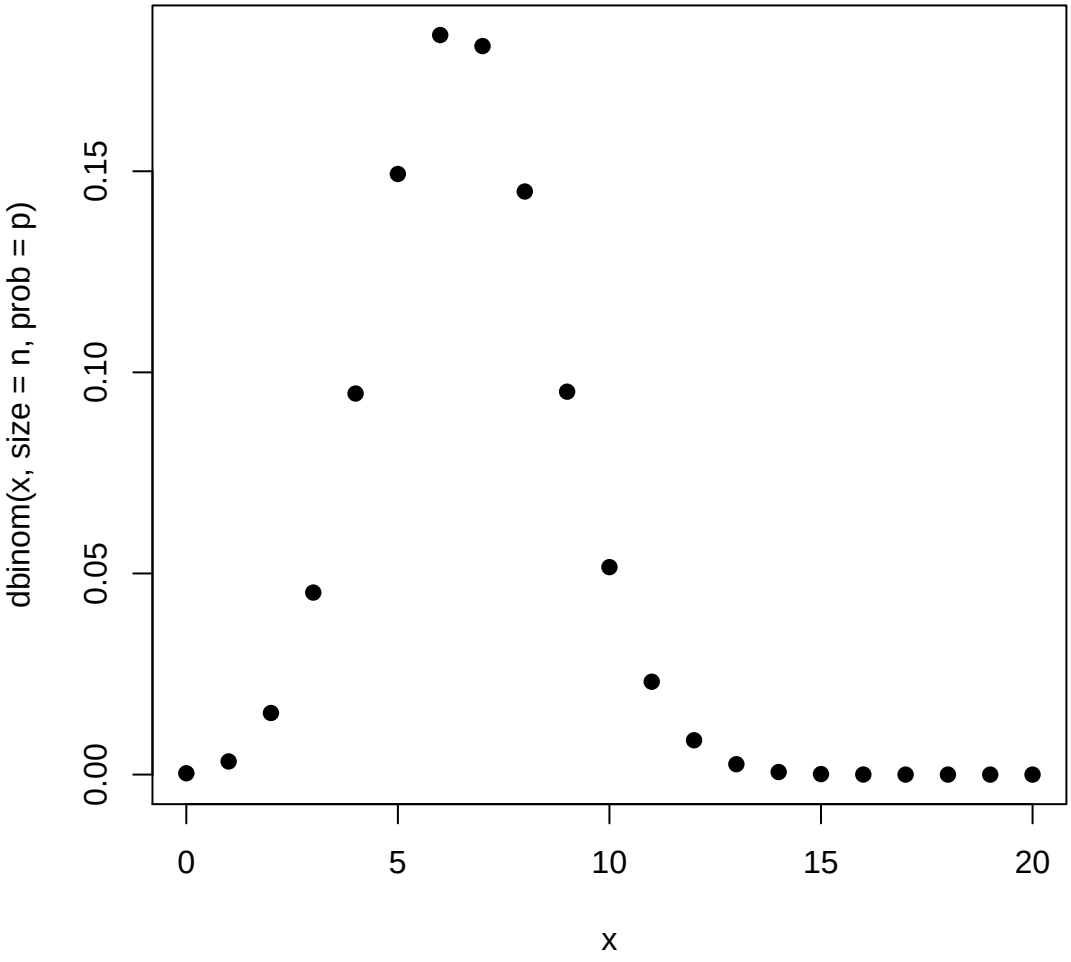
(b) Determine the cdf.

$$F(X = x) = \begin{cases} 0 & x < 1 \\ .1 & x \in [1, 3) \\ .5 & x \in [3, 4) \\ .9 & x \in [4, 6) \\ 1 & x \geq 6 \end{cases}$$



Question 4: In a multiple choice exam, there are 20 questions and each question has three choices. If someone is totally unprepared and randomly chooses answers. Let Y be the number of questions (s)he guesses correctly.

(a) What is the distribution of Y?




```
x <- 0:20
n <- 20
p <- .33
plot(x, dbinom(x, size=n, prob=p), pch=19)
```

(b) What is P(Y=6)?

$$P(Y = 6) = \underline{\underline{.184}}$$

```
dbinom(6, size=n, prob=p)
```

(c) What is P(Y $\leq$ 6)?

$$P(Y \leq 6) = \underline{\underline{.492}}$$

```
sum(dbinom(0:6, size=n, prob=p))
```

(d) What is P(Y > 6)?

$$\begin{aligned} P(Y > 6) &= 1 - P(Y \leq 6) \\ &= \underline{\underline{.508}} \end{aligned}$$

```
1 - dbinom(0:6, size=20, prob=p)
```