

hw9-report

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1. Let $Y_1, Y_2, \dots, Y_n \sim_{iid} N(\theta, \sigma^2)$ where σ^2 is known. Consider $H_0 : \theta \leq \theta_0$ vs. $H_1 : \theta > \theta_0$.

(a) What would be Type I Error? What would be Type II Error?

(b) In HW8, we show that the likelihood ratio test procedure is to reject H_0 if

$$z = \frac{\bar{y} - \theta_0}{\sigma/\sqrt{n}} \geq k_1 = \sqrt{-2\log(k)}$$

The power function of this test is:

$$\gamma(\theta) = P(N(0,1) \geq k_1 + \frac{\theta_0 - \theta}{\sigma/\sqrt{n}})$$

Let $\theta_0 = 105$, $\sigma = 10$, $n = 100$ and $k_1 = 1.8$, plot this function, and comment on your plot.

(c) For this test, what is the probability of Type I Error when $\theta = 105$?

(d) For this test, what is the probability of Type II Error when $\theta = 110$? What is the power of rejecting H_0 when $\theta = 110$?

(e) If we set the significance level $\alpha = 0.05$, what is k_1 ?

(f) For this test procedure with $\alpha = 0.05$, what sample size n is necessary to ensure that the power of rejecting H_0 at $\theta = 108$ is at least 80%?

2. Consider the carprice example from ‘Notes 7’.

(a) What is the $\hat{\beta}_1$? How do you interpret this number?

(b) To test $H_0 : \beta_1 = 0$ vs $H_1 : \beta_1 \neq 0$, what is the P-value? What is your conclusion?