

Homework 1

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January 20, 2022

1 Question 3

Given :

$$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$A = \{1, 3, 5, 7, 9\}, B = \{1, 2, 3, 5, 7\}$$

(a) What is \bar{A} ?

$$\begin{aligned}\bar{A} &= S - A \\ &= \{1 \dots 10\} - \{1, 3, 5, 7, 9\} \\ &= \underline{\underline{\{2, 4, 6, 8\}}}\end{aligned}$$

(b) What is \bar{B} ?

$$\begin{aligned}\bar{B} &= S - B \\ &= \{1 \dots 10\} - \{1, 2, 3, 5, 7\} \\ &= \underline{\underline{\{4, 6, 8, 9, 10\}}}\end{aligned}$$

(c) What is $A \bar{\cup} B$?

$$\begin{aligned}A \bar{\cup} B &= \{1, 3, 5, 7, 9\} \bar{\cup} \{1, 2, 3, 5, 7\} \\ &= \{1, 2, 3, 5, 7, 9\} \\ &= \{1 \dots 10\} - \{1, 2, 3, 5, 7, 9\} \\ &= \underline{\underline{\{4, 6, 8, 10\}}}\end{aligned}$$

(d) What is $A \cap B$?

$$\begin{aligned} A \cap B &= \{1, 3, 5, 7, 9\} \cap \{1, 2, 3, 5, 7\} \\ &= \{1, 3, 5, 7\} \\ &= \underline{\underline{\{2, 4, 6, 8, 9, 10\}}} \end{aligned}$$

2 Question 1

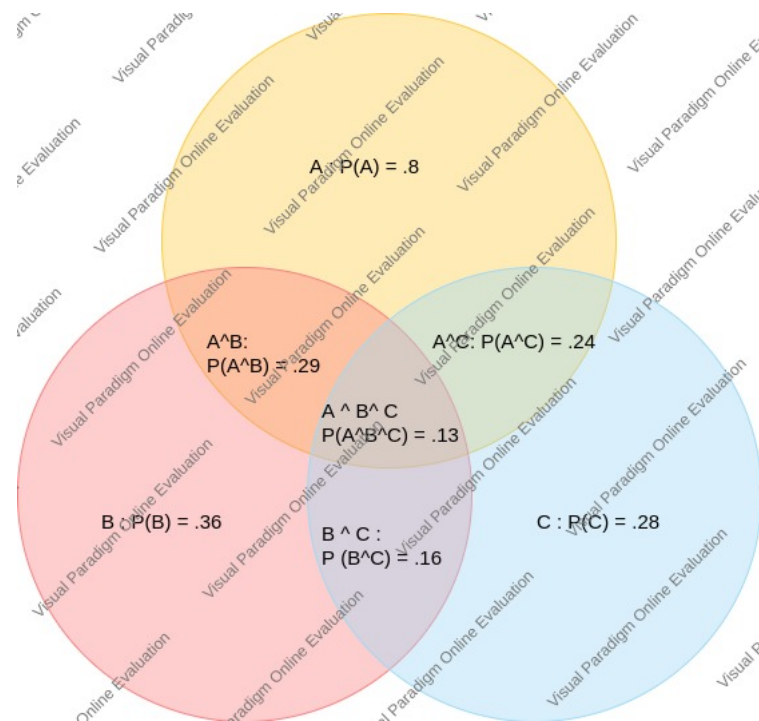
Given :

$$P(A) = .8, P(B) = .36, P(C) = .28$$

$$P(A \cap B) = .29, P(A \cap C) = .24, P(B \cap C) = .16$$

$$P(A \cap B \cap C) = .13$$

(a) Draw the corresponding venn diagram.



(b) What is $P(A \cap B \cap \bar{C})$?

(c) What is $P(A \cap \bar{B} \cap \bar{C})$?

$$\begin{aligned} P(A \cap B) &= P(A \cap B \cap C) + P(A \cap B \cap \bar{C}) \\ P(A \cap B \cap \bar{C}) &= P(A \cap B) - P(A \cap B \cap C) \\ &= .29 - .13 \\ &= \underline{\underline{.16}} \end{aligned}$$

(d) What is $P(\bar{A} \cap \bar{B} \cap \bar{C})$?

$$\begin{aligned} P(A \cap C) &= P(A \cap B \cap C) + P(A \cap \bar{B} \cap C) \\ P(A \cap \bar{B} \cap C) &= P(A \cap C) - P(A \cap B \cap C) \\ &= .24 - .13 \\ &= \underline{\underline{.11}} \end{aligned}$$

$$\begin{aligned} P(A) &= P(A \cap \bar{B} \cap \bar{C}) + P(A \cap B \cap \bar{C}) + P(A \cap \bar{B} \cap C) + P(A \cap B \cap C) \\ P(A \cap \bar{B} \cap \bar{C}) &= P(A) - P(A \cap B \cap \bar{C}) - P(A \cap \bar{B} \cap C) - P(A \cap B \cap C) \end{aligned}$$

$$\begin{aligned} P(A \cap \bar{B} \cap \bar{C}) &= .8 - .16 - .11 - .13 \\ &= \underline{\underline{.40}} \end{aligned}$$

(e) What is $P(\bar{A} \cap \bar{B} \cap \bar{C})$?

$$\begin{aligned} P(\bar{A} \cap \bar{B} \cap \bar{C}) &= 1 - P(A \cup B \cup C) \\ &= 1 - (.8 + .36 + .28 - .29 - .24 - .16 + .13) \\ &= \underline{\underline{.12}} \end{aligned}$$

(f) What is $P(\bar{A} \cap (B \cup C))$?

$$\begin{aligned} P(\bar{A} \cap (B \cup C)) &= P(B \cup C) - P(A \cap (B \cup C)) \\ &= (P(B) + P(C) - P(B \cap C)) - \\ &\quad (P(A \cap B \cap \bar{C}) + P(A \cap \bar{B} \cap C) + P(A \cap B \cap C)) \\ &= (.36 + .28 - .16) - (.16 + .11 + .13) \\ &= \underline{\underline{.08}} \end{aligned}$$

3 Question 7

Given :

$$P(A) = .7, P(B) = .6$$

$$P(\bar{A} \cap B) = .2$$

(a) What is $P(A \cap B)$?

$$\begin{aligned} P(A \cap B) &= P(B) - P(\bar{A} \cap B) \\ &= .6 - .2 \\ &= \underline{\underline{.4}} \end{aligned}$$

(b) Is it possible for A and B to be disjoint events? Why or why not?

For A and B to be disjoint, the intersection of A and B must be the null set, but we know that is not the case from the calculation above. That is, from finding $P(A \cap B) = .4$

(c) What is $P(A \cup \bar{B})$?

$$P(A \cup \bar{B}) = P(A) + P(\bar{B}) - P(A \cap \bar{B})$$

$$\begin{aligned} P(\bar{B}) &= 1 - P(B) \\ &= \underline{\underline{.4}} \end{aligned}$$

$$\begin{aligned} P(A \cap \bar{B}) &= P(A) - P(A \cap B) \\ &= .7 - .4 \\ &= \underline{\underline{.3}} \end{aligned}$$

$$\begin{aligned} P(A \cup \bar{B}) &= P(A) + P(\bar{B}) - P(A \cap \bar{B}) \\ &= .7 + .4 - .3 \\ &= \underline{\underline{.8}} \end{aligned}$$

- (d) Is it possible for A and B to be independent events? Why or why not?

For independence, $P(A) * P(B)$ must equal $P(A \cap B)$.

$$\begin{aligned}P(A \cap B) &= .4 \\P(A) * P(B) &= .7 * .6 \\&= .42\end{aligned}$$

The two values do not match. Therefore, A and B are not independent events.

- (e) What is $P(A|B)$?

$$\begin{aligned}P(A|B) &= \frac{P(A \cap B)}{P(B)} \\&= \frac{.4}{.6} \\&= .67\end{aligned}$$

4 Question 8

Given :

$$P(+|D) = .62, P(-|\bar{D}) = .82$$

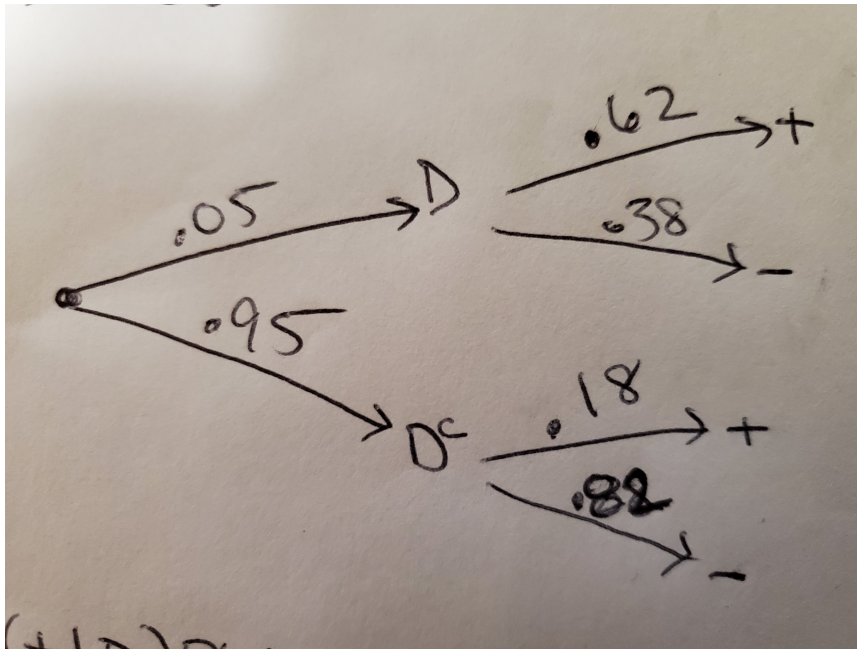
- (a) What is $P(+|\bar{D})$?

$$\begin{aligned}P(+|\bar{D}) &= 1 - P(-|\bar{D}) \\&= 1 - .82 \\&= \underline{\underline{.18}}\end{aligned}$$

- (b) What is $P(-|D)$?

$$\begin{aligned}P(-|D) &= 1 - P(+|D) \\&= 1 - .62 \\&= \underline{\underline{.38}}\end{aligned}$$

(c) Given: $P(D) = .05$. Draw the corresponding tree diagram.



(d) What is $P(+)$?

$$\begin{aligned}
 P(+) &= P(+|D)P(D) + P(+|\bar{D})P(\bar{D}) \\
 &= .62 * .05 + .18 * .95 \\
 &= \underline{\underline{.202}}
 \end{aligned}$$

(e) What is $P(D|+)$?

$$\begin{aligned}
 P(D|+) &= \frac{P(D \cap +)}{P(+)} \\
 &= \frac{P(+|D)P(D)}{P(+)} \\
 &= \frac{.62 * .05}{.202} \\
 &= \underline{\underline{.153}}
 \end{aligned}$$