Overview

- why use CV, EV, CS?
  - ↳ preciser measure of how much better off a consumer may be.
  - ↳ recall: utility tells us how consumers rank bundles. Meaningless otherwise.

CS

- How much you are willing to pay above price.

  new world - old world

EV

- How much money to get you to same utility as in new world while keeping prices same as old world.

CV

- How much money to get you to same utility as in the old world, while keeping prices same as the new world.

	Old utility	New Utility
old Prices	Old world	EV World
new Prices	CV World	New World

There is no SE → only IE.

→ Means prices will be the same

→ adjust income to get us to new/old world Utility.

The EV & CV will not be the same. Why?

Different reference points : [prices]

If EV < 0 or CV > 0 : welfare falls

EV → Starting at old world, I must take money away from you [EV < 0] to make you indifferent to new world.

CV → Starting at new world I must give you money [CV > 0] to make you indifferent to old world.

If EV > 0 or CV < 0 : welfare rises

EV → Starting @ old world, to make you indifferent to new world, I need to give you money.

→ Starting @ new world, I need to take money away to make you indif. to old world.

Area of triangle: $\frac{1}{2}(b)(h)$

Area of trapezoid: $\frac{1}{2}(a+b)(h)$

1 ex) $Q = 400 - 20P$

$P = 15 \rightarrow P = 8$

ΔCS?

Sol: $20P = 400 - Q$

$P = 20 - \frac{1}{20}Q$

@15

$Q = 400 - 20(15)$

$Q = 400 - 300$

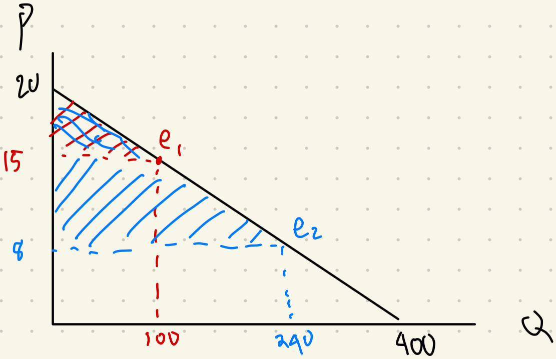
$= 100$

@ 8

$Q = 400 - 20(8)$

$= 400 - 160$

$= 240$



$CS_1 : \frac{1}{2}(20-15)(100-0) = 250$

$CS_2 : \frac{1}{2}(20-8)(240) = 1440$

$\Delta CS = 1440 - 250 = 1190$

old CS: $\frac{1}{2}(20-15)(100-0)$

② C-D

$U = x_1^3 x_2^2$ $\quad P_1 = 8,\ P_2 = 2,\ I = 120$

P_1 falls to 4

1) MRS: $\frac{3x_1^2 x_2^2}{2x_2 x_1^3} \Rightarrow \underbrace{3x_1^{2-3} \quad y^{2-1}}_{2} = \frac{3y}{2x}$

$$\frac{3x_2}{2x_1} = \frac{P_1}{P_2} \Rightarrow x_2 = \frac{2P_1}{3P_2} x_1$$

$$\therefore P_1 X_1 + P_2\left(\frac{2P_1}{3P_2} x_1\right) = I$$

$$P_1 X_1 \left(1 + \frac{2}{3}\right) = I$$

$$X_1 = \frac{3I}{5P_1} \qquad \left[\frac{2\cancel{P_1}}{3P_2}\right]\left[\frac{\cancel{3}I}{5\cancel{P_1}}\right]$$

$$X_2 = \frac{2I}{5P_2}$$

Before

$X_1(8, 2, 120) = \frac{3 \cdot 120}{5 \cdot 8} = 9$

$X_2(8, 2, 120) = \frac{2 \cdot 120}{5 \cdot 2} = 24$

After

$X_1(4, 2, 120) = \frac{3 \cdot 120}{5 \cdot 4} = 18$

$X_2(4, 2, 120) = \frac{2 \cdot 120}{5 \cdot 2} = 24$

EV: old prices with I^E

prices must be the same, so x^e...

$$(x_1^e, x_2^e) = \left(\frac{3I^e}{5P_2^o}, \frac{2I^e}{5P_2^o}\right) = \left(\frac{3I^e}{5 \cdot 8}, \frac{2I^e}{5 \cdot 2}\right) = \frac{3I^e}{40}, \frac{2I^e}{10}$$

new utility: $u' = x_1'^3 \, x_2'^2$
$= 18^3 \cdot 24^2$

Solve w/ I^e: $u(x_1^e, x_2^e) = u'$

$$\left(\frac{3 I^e}{40}\right)^3 \left(\frac{I^e}{5}\right)^2 = 18^3 \cdot 24^2$$

$$I^{e5} \left(\frac{3}{40}\right)^3 \left(\frac{1}{5}\right)^2 = 18^3 \cdot 24^2$$

$$I^{e5} = 18^3 \cdot 24^2 \cdot \left(\frac{3}{40}\right)^{-3} \cdot \left(\frac{1}{5}\right)^{-2}$$

$$I^{e5} = \left(\frac{18 \cdot 40}{3}\right)^3 \cdot (24 \cdot 5)^2$$

raise to $\frac{1}{5}$

$$I^e = (240)^{3/5} \cdot (120)^{2/5}$$

$$I^e = 120 \cdot 2^{3/5}$$

$$I^e \approx 181.86$$

EV: $I^e - I^0$
$\approx 181.86 - 120$
≈ 61.86

Price drop has same effect as giving \$61.86

CV

new prices & I^c

$$4x_1 + 2x_2 = I^c$$

$$(x_1^c, x_2^c) = \left(\frac{3I^c}{5p_1'}, \frac{2I^c}{5p_2'}\right) = \left(\frac{3I^c}{5\cdot 4}, \frac{2I^c}{5\cdot 2}\right) = \left(\frac{3I^c}{20}, \frac{2I^c}{10}\right)$$

$$u^0 = (x_1')^3 (x_2')^2$$

$$= 9^3 \cdot 24^2$$

$$u(x_1^c, x_2^c) = u^0$$

$$\left(\frac{3I^c}{20}\right)^3 \left(\frac{I^c}{5}\right)^2 = 9^3 \cdot 24^2$$

$$I^{c^5} \left(\frac{3}{20}\right)^3 \left(\frac{1}{5}\right)^2 = 9^3 \cdot 24^2$$

$$I^{c^5} = 9^3 \cdot 24^2 \cdot \left(\frac{3}{20}\right)^{-3} \cdot \left(\frac{1}{5}\right)^{-2}$$

$$I^{c^5} = \left(\frac{9 \cdot 20}{3}\right)^3 \cdot (24 \cdot 5)^2$$

$$I^c = \left[60\left(\tfrac{1}{3}\right)\right]^{3/5} \cdot [120]^{2/5}$$

$$\approx 79.17$$

CV: $79.17 - 120 = -40.83$ } Take money away to "compensate" for price drop.

3 Quasi-linear

$U = 4\sqrt{x_1} + 2x_2$

$P_1 = 2$, $I = 10$
$P_2 = 2$, changes to 4

demand: $MRS = \dfrac{4 \cdot \frac{1}{2} x_1^{-1/2}}{2} = \dfrac{P_1}{P_2}$

$$x_1^{-1/2} = \frac{P_1}{P_2}$$

$$x_1 = \left(\frac{P_2}{P_1}\right)^2$$

$$10 = P_1 x + P_2 y$$

$$10 = P_1\left[\frac{P_2}{P_1}\right]^2 + P_2 y$$

$$10 = \frac{P_2^2}{P_1} + P_2 y$$

$$y P_2 = 10 - \frac{P_2^2}{P_1}$$

$$y = \frac{10}{P_2} - \frac{\frac{P_2^2}{P_1}}{P_2}$$

$$y = \frac{10}{P_2} - \frac{P_2}{P_1}$$

$$x_2 = \frac{1}{P_2}\left[I - P_1 x_1(P, I)\right]$$

$$= \frac{I}{P_2} - \frac{P_1}{P_2}\left(\frac{P_2}{P_1}\right)^2$$

$$= \frac{I}{P_2} - \frac{P_2}{P_1}$$

Before

$x_1(2,2,10) = \left(\frac{2}{2}\right) = 1^2 = 1$

$x(2,2,10) = \frac{10}{2} - \frac{2}{2} = 4$

After

$x_1(2,4,10) = 2^2 = 4$

$x_2(2,4,10) = \frac{10}{4} - \frac{4}{2} = \frac{1}{2}$

EV

$$2x_1 + 2x_2 = I^e$$

$$(x_1^e, x_2^e) = \left[\left(\frac{P_2^0}{P_1^0}\right)^2, \frac{I^e}{P_1^0} - \frac{P_2^0}{P_1^0}\right] = \left(\left(\frac{2}{2}\right)^2, \frac{I^e}{2} - \frac{2}{2}\right)$$

$$= 1, \frac{I^e}{2} - 1$$

$$u' = 4\sqrt{x_1'} + 2x_2'$$
$$= 4\sqrt{4} + 2\left(\tfrac{1}{2}\right)$$
$$= 9$$

$$u(x_1^e, x_2^e) = u'$$

$$4\sqrt{1} + 2\left[\frac{I^e}{2} - 1\right] = 9$$

$$4 + I^e - 2 = 9$$

$$I^e = 7$$

$$CV = I^e - I^0$$

$$= 7 - 10$$
$$= -3$$

] take $3 !!

CV

$$2x_1 + 4x_2 = I^c$$

new prices old util

$$x_1^c, x_2^c = \left[\left(\frac{p_2'}{p_1'}\right)^2, \frac{I^c}{p_2'} - \frac{p_2'}{p_1'}\right) = \left[\left(\frac{4}{2}\right)^2, \frac{I^c}{4} - \frac{4}{2}\right]$$

$$\left[4, \frac{I^c}{4} - 2\right]$$

$$u^0 = 4\sqrt{x_1^0} + 2x_2^0$$

$$= 4\sqrt{1} + 2 \cdot 4$$

$$= 12$$

$$u(x_1^c, x_2^c) = u^0$$

$$4\sqrt{4} + 2\left(\frac{I^c}{4} - 2\right) = 12$$

$$8 + \frac{I^c}{2} - 4 = 12$$

$$I^c = 16$$

$$CV = I^c - I^0$$

$$= 16 - 10$$

$$= 6$$] compensate by giving \$6.

numeraire = linear term

Bonus

Numeraire CV & EV [Quasi-linear cont.]

P_1 changes to 4

$$x_1(4,2,10) = \left(\frac{2}{4}\right)^2 = \left(\frac{1}{2}\right)^2 = .25$$

$$x_2(4,2,10) = \frac{10}{2} - \frac{2}{4} = 5 - \frac{1}{2} = 4.5$$

EV: $2x_1 + 2x_2 = I^e$

$$x_1^e, x_2^e = \left(\left(\frac{p_2^0}{p_1^0}\right)^2, \frac{I^e}{p_2^0} - \frac{p_2^0}{p_1^0}\right) = \left(\left(\frac{2}{2}\right)^2, \frac{I^e}{2} - \frac{2}{2}\right)$$

$$= \left[1, \frac{I^e}{2} - 1\right]$$

$$u' = 4\sqrt{x_1'} + 2x_2'$$

$$= 4\sqrt{\tfrac{1}{4}} + 2(4.5)$$

$$= 4\left(\tfrac{1}{2}\right) + 9$$

$$u' = 11$$

$$u(x_1^e, x_2^e) = u'$$

$$4\sqrt{1} + 2\left(\frac{I^e}{2} - 1\right) = 11$$

$$4 + I^e - 2 = 11$$

$$I^e = 9$$

$$EV = u^e - u'$$
$$= 9 - 10$$
$$= -1$$

CV

$$4x_1 + 2x_2 = I^c$$

$$(x_1^c, x_2^c) = \left(\left(\frac{p_2^1}{p_1^1}\right)^2, \frac{I^c}{p_2^1} - \frac{p_2^1}{p_1^1} \right) = \left[\left(\frac{2}{4}\right)^2, \frac{I^c}{2} - \frac{2}{4}\right]$$

$$= \left(\frac{1}{4}, \frac{I^c}{2} - \frac{1}{2}\right]$$

$$u^0 = 4\sqrt{1} + 2 \cdot 4 = 12$$

$$u(x_1^c, x_2^c) = u^0$$

$$4\sqrt{\tfrac{1}{4}} + 2\left[\frac{I^c}{2} - \frac{1}{2}\right] = 12$$

$$2 + I^c - 1 = 12$$

$$I^c = 11$$

$$CV = I^c - I^0$$

$$= 11 - 10$$

$$= -1$$

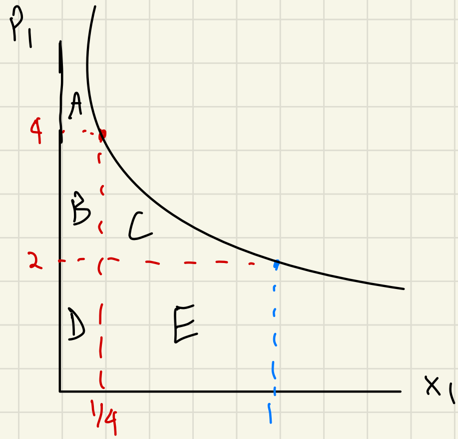
CS

$$X_1 = \left(\frac{P_2}{P_1}\right)^2 \quad \overset{\text{invert}}{\Rightarrow} \quad P_1 = \frac{P_2}{\sqrt{x_1}}$$

go from $P_1 = 2$ to $P_1 = 4$

X changes from 1 to $\frac{1}{4}$

$$\int_{\frac{1}{4}}^{1} P_1(x_1; 2, 10) = \int_{\frac{1}{4}}^{1} \frac{2}{\sqrt{x_1}} = \int_{1/4}^{1} 2x_1^{-\frac{1}{2}}$$



$$= \left[2 \cdot 2 x_1^{\frac{1}{2}}\right]_{1/4}^{1}$$

$$= 4\sqrt{1} - 4\sqrt{\tfrac{1}{4}}$$

$$= 4 - 4\left(\tfrac{1}{2}\right)$$

$$= 4 - 2$$

$$= 2$$

Why not 1?

Δ Gross CS = C+E

- old gross = A+B+C+D+E
- new gross = A+B+D

Δ Net CS = B+C

- old net = A+B+C
- new net = A

$$B = (4-2)\left(\tfrac{1}{4}-0\right) = \tfrac{1}{2}$$

$$E = (2-0)\left(1-\tfrac{1}{4}\right) = 1.5$$

$$\Delta CS_{net} = \Delta CS_{gross} + B - E$$

$$= 2 + \tfrac{1}{2} - 1.5$$

$$= 1$$

④ Perfect Comps

$$u(x_1, x_2) = \min(3x_1, 4x_2)$$

$$P_1 = 1,\ P_2 = 2,\ m = 30$$

$P_1 \uparrow$ to 3.

$$3x_1 = 4x_2$$
$$x_2 = \frac{3}{4}x_1$$

$$\therefore\ P_1x_1 + P_2\left(\frac{3}{4}x_1\right) = I$$

$$x_1(4P_1 + 3P_2) = 4I$$

$$x_1 = \frac{4I}{4P_1 + 3P_2}$$

$$x_2 = \frac{3I}{4P_1 + 3P_2}$$

$$4P_1x_1 + 3P_2x_1 = 4m$$

$$x_1[4p_1 + 3P_2] = 4m$$

$$x = \frac{4m}{4p_1 + 3P_2}$$

before

$$x_1 = \frac{4(30)}{4(1) + 3(2)} = 12$$

$$x_2 = \frac{3 \cdot 30}{4(1) + 3(2)} = 9$$

after

$$x_1 = \frac{4 \cdot 30}{4(3) + 3(2)} = \frac{20}{3}$$

$$x_2 = \frac{3 \cdot 30}{4(3) + 3(2)} = 5$$

EV

$$x_1 + 2x_2 = I^e$$

$$x_1^e, x_2^e = \left[\frac{4I^e}{4p_1^0 + 3p_2^0}, \frac{3I^e}{4p_1^0 + 3p_2^0} \right)$$

$$= \frac{4I^e}{4 \cdot 1 + 3 \cdot 2}, \frac{3I^e}{4 \cdot 1 + 3 \cdot 2}$$

$$= \frac{4I^e}{10}, \frac{3I^e}{10}$$

$$u^1 = \min\left(3 \cdot \frac{20}{30}, 4 \cdot 5\right)$$

$$= (20, 20) \quad = 20$$

$$\min\left\{3 \cdot \frac{4I^e}{10}, 4 \cdot \frac{3I^e}{10}\right\} = 20$$

$$\frac{12 I^e}{10} = 20$$

$$I^e = \frac{200}{12} = \frac{100}{6} \approx 16.67$$

$$EV = I^e - I^0$$

$$= 16.67 - 30$$

$$= -13.33$$

5 Perf. Subs

$u = 2x_1 + x_2$

$P_1 = 3,\ P_2 = 5,\ I = 15$

1 $P_2 \downarrow$ to 3

2 $P_2 \downarrow$ to 1

$MRS = \frac{2}{1} = 2$

$$x_1(P, I) = \begin{cases} 0 & 2 < \frac{P_1}{P_2} \\ \frac{I}{P_1} & 2 > \frac{P_1}{P_2} \\ \in \left[0, \frac{I}{P_1}\right] & 2 = \frac{P_1}{P_1} \end{cases}$$

$$x_2(P, I) = \begin{cases} 0 & 2 > \frac{P_1}{P_2} \\ \frac{I}{P_2} & 2 < \frac{P_1}{P_2} \\ \in \left[0, \frac{I}{P_2}\right] & 2 = \frac{P_1}{P_2} \end{cases}$$

Before	1	2
$\frac{P_1}{P_2} = .6$	$\frac{P_1}{P_2} = 1$	$\frac{P_1}{P_2} = 3$
$x_1 = 5$	$x_1 = 5$	$x_1 = 0$
$x_2 = 0$	$x_2 = 0$	$x_2 = 15$

$3x_1 + 3x_2 = I^c$ (1)

$3x_1 + x_2 = I^c$ (2)

(1)

$x_1^c, x_2^c = \frac{I^c}{p_1'}, 0$

$= \frac{I_c}{3}, 0$

(2)

$x_1^c, x_2^c = \left(0, \frac{I^c}{p_2'}\right)$

$= 0, I^c$

$u(x_1^e, x_2^e) = u^0$ so old util $= 10$

$u^0 = 2x_1^0 + x_2^0$

$= 2(5) + 0$

$= 10$

2. $\frac{I^c}{3} + 0 = 10$

$I^c = 15$

CV: $I^c - I^0$

$= 15 - 15$

$= 0$

didn't change bundles

$u(x_1^c, x_2^c) = u^0$

$2(0) + I^c = 10$

$I^c = 10$

CV: $I^c - I^0$

$= 10 - 15$

$= -5$