Preferences

Economics 100A

Fall 2021

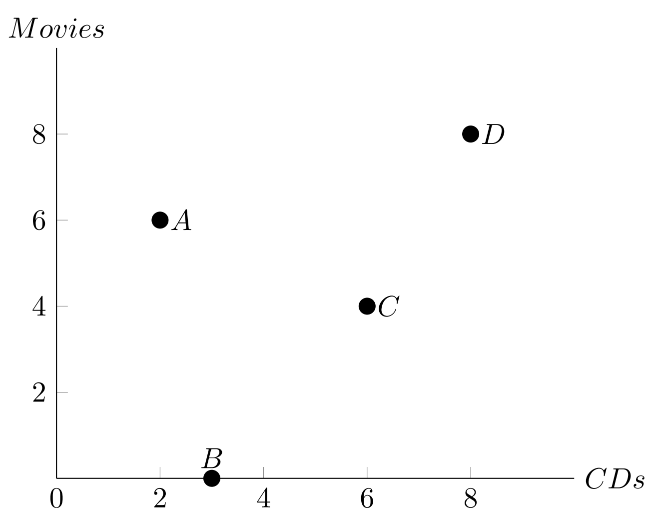
1 Preferences

1.1 Introduction

Econ 100A is all about the consumer side of the market. We will slowly build our intuition for understanding the demand curve. Before we get to that point, however, we should build our understanding of how consumers actually rank different goods. Let's say you go to the grocery store: what determines the ratio in which you purchase apples and oranges? If you have a strong preference for oranges, maybe you consume ten oranges for every one apple. If you are indifferent between the two, maybe you consume them based on whatever is on sale. These are real-life examples we can attempt to model in this class.

Put another way, consumers have certain preferences that determine what they consume. If I gave everyone who attended SI $10 to purchase dinner, everyone would come back with something different. In this example, the difference between what we purchase has to do with our **preferences**.

In the real world, we have to choose between a lot of different goods. In this class, we will keep things simple: we will study binary choices. All this means is that we will take two goods and see how consumers rank different bundles of each good. Consider the following: consumers are asked to rank how many movies and CDs they would like to purchase. Four consumers provide the following answers:



Can you see which person likes CDs much more than movies? Can you see who is indifferent between the two? Who likes CDs slightly more than movies? Think about this based on how each consumer ranked the two goods.

It seems like consumer B has a strong preference for CDs, as they do not want to consume any movies. Conversely, consumer D is indifferent between the two, opting to consume the maximum of each good. Consumer C seems to slightly prefer CDs over movies, but not by a wide margin. Finally, consumer A prefers movies by a pretty significant margin.

1.2 Notation and Properties

Now that we have an understanding of what exactly we want to model, we can start with the math behind preferences. Say we have goods x and y. If a consumer likes x just as much as y, we write $x \succsim y$. If a consumer is indifferent between x and y, we write $x \backsim y$. If a consumer strictly prefers x to y, we write $x \succ y$. To summarize:

1. $\succsim$: Weakly preferred to (”liked at least as much as”)
2. $\backsim$: Indifferent, when $x \succsim y$ <u>and</u> $y \succsim x$
3. $\succ$: Strongly preferred to, when $x \succsim y$ but <u>not</u> $y \succsim x$.

To make this a useful tool, we should institute some basic assumptions. I should probably mention here that these assumptions are **not** realistic or empirically grounded. People routinely violate these assumptions. The first set of assumptions are called **rational preferences**, and the second set is called **well-behaved**. Before I outline them, let's assume the two goods x and y are consumed together in a bundles A and B.

1. <u>**Complete**</u>: for any two bundles A and B, it must be the case that $A \succsim B$ or $B \succsim A$, or both: $A \backsim B$.
2. <u>**Reflexive**</u>: any bundle is at least as good as itself: $A \succsim A$. This is a trivial assumption that is not often mentioned explicitly.
3. <u>**Transitive**</u>: For bundles A, B, C, if $A \succ B$ and $B \succ C$, then $A \succ C$. This essentially allows us to make predictable, unambiguous choices about bundles.

These assumptions fall into the category of **rational preferences**. These are the bare minimum that all preferences must have. The second variety of assumptions are called **well-behaved**. There are two assumptions here:

1. <u>**Monotonicity**</u>: A consumer would always prefer having more of a good. Their preferences never satiate. Expressed formally, for any two bundles $A = (x_1, x_2)$ and $B = (y_1, y_2)$, if $x_1 \geq y_1$, $x_2 \geq y_2$, and $x_i > y_i$ for $i = 1$ or 2, then $A \succ B$.
2. <u>**Convexity**</u>: A consumer prefers to consume averages rather than extremes. Expressed formally: if $x \succsim y$, and $y \succsim z$, then $\lambda x + (1 - \lambda)y \succsim z$ for any $\lambda \in [0, 1]$.

The first three assumptions are necessary for doing basic consumer analysis, but the second two make our lives much easier. Before advancing, let's test our understanding of preferences.

1. From the bundles A, B, C, I have $A \succ B$, $B \succ C$, and $C \succ A$. Are my preferences transitive? Are they complete?
2. I purchase fruit based on their sizes. I prefer bigger fruit. Are my preferences rational?
3. When I have milk tea, I put in a single spoonful of sugar. Any more and I feel sick. Is this preference monotonic?
4. Lucy faces two ways of spending her evenings: violin lessons or walks on the beach. She thinks violin lessons would only be worth the effort if she really committed to taking a lot a week; put another way, she would rather take none than only a few. Are her preferences convex?

Answers below:

1. My preferences are not transitive, because $A \succ B \succ C \succ A$. They are complete because I ranked all bundles.
2. The preferences are indeed rational. I can rank all of the fruits in order from most preferred (the biggest) to least preferred (the smallest), satisfying the completeness and transitivity conditions.
3. The preference is not monotonic because I have a satiation point at one spoonful of sugar. Remember: monotonic preferences state that a consumer is always better off by consuming more.
4. Her preferences are concave. She does not prefer averages of the two goods; she would either fully commit to violin or not commit at all.

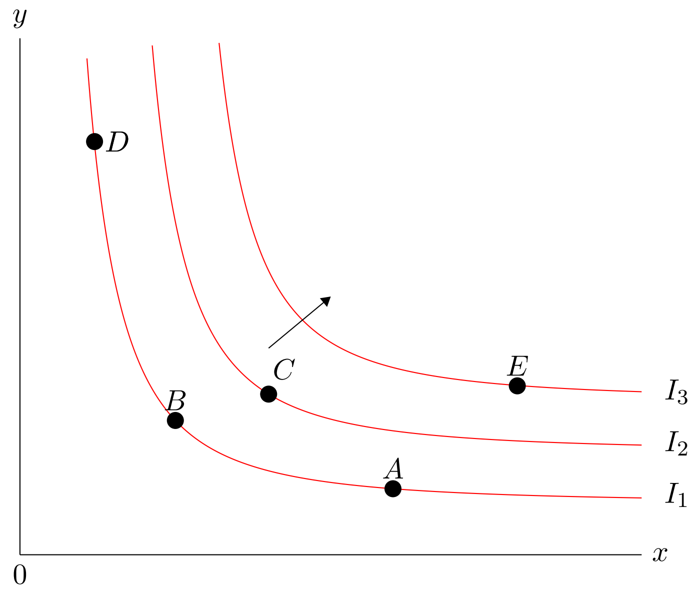
1.3 Indifference Curves

These assumptions allow us to graph consumer preferences using **indifference curves**. An indifference curve is just a graphical representation of a consumer's preferences. Each point on an indifference curve represents a bundle. Indifference curves indicate the set of bundles that the consumer feels indifferent between.

In figure one, we see three different indifference curves. Any point on each curve represents a point of indifference. For example, on I_1, $A \backsim B \backsim D$. However, due to our monotonicity assumptions, we prefer to consume more. So, $I_2 \succ I_1$, and likewise $C \succ B$. It is useful to draw arrows indicating which direction leads to a higher curve. In the absence of an arrow, assume that the farthest northeast point is the best point, or the one that the consumer prefers.

The figure also makes it easy to see how the transitive property works. Point E is on a higher indifference curve than point C. So, $E \succ C$. However, we also know that $C \succ B$. As we can see from the graph, $E \succ B$ as well.

Figure 1: Indifference Curves



To summarize:

1. For any point **above** the indifference curve, the consumer strictly prefers it to any point **on** the indifference curve.

2. For any point **on** the indifference curve, the consumer is indifferent between it and any point **on** the indifference curve.

3. For any point **below** the indifference curve, the consumer is strictly does not prefer it and any point **on** the indifference curve.

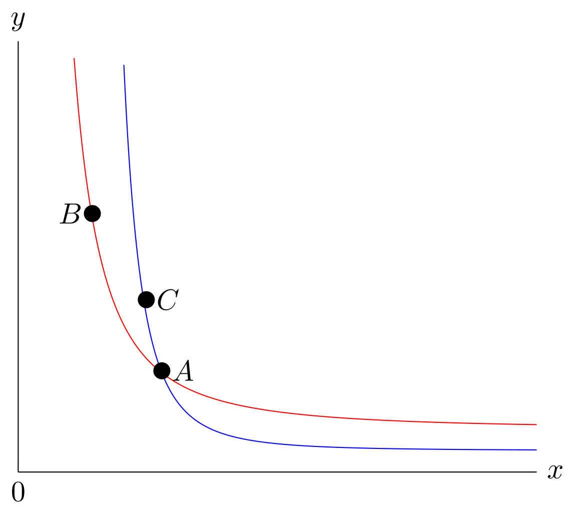
There are a few more points about indifference curves we should think about. They are as follows:

1. **Higher indifference curves are more preferred**: This is a direct result of our monotonicity assumption. We typically draw arrows indicating in which direction indifference curves move. The easiest way to think about this assumption is that people always prefer having a bundle with more goods than one with fewer goods.

2. **Every point has an indifference curve going through it**: This is a result of our completeness assumption. For obvious reasons, we do not draw an infinite number of indifference curves; we draw a few just to get a rough idea of the shape of the preferences. However, since the completeness assumption states that our consumer can rank bundles, which allows us to represent their preferences graphically or algebraically.

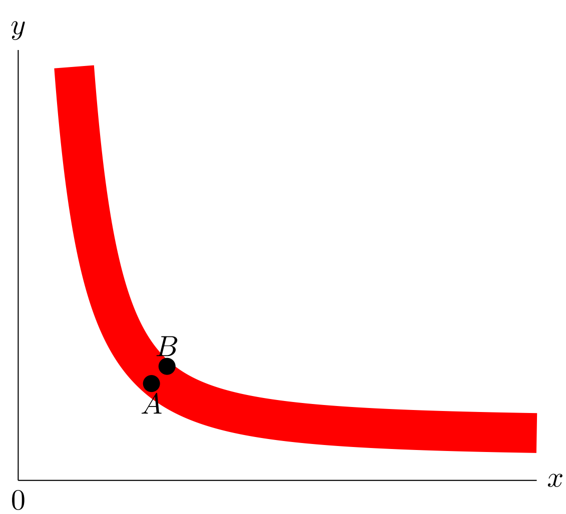
3. **Indifference curves never cross**: Imagine crossing indifference curves in your head. Why might this not be allowed? Well, as you can see below, if we had crossing indifference curves, we would have that $A \backsim B \backsim C$. However, we can clearly see that $C \succ B$ because it is on a

higher indifference curve! We now have that $C \succ B \backsim A \backsim C$. This is a clear violation of our rational preferences, and therefore we should never see crossing indifference curves.



4. **Well-behaved indifference curves are downward sloping**: This comes from our monotonicity assumption. Let's say that you can consume both milk tea and coffee. If I feel generous and give you a cup of coffee, you would be on a higher indifference curve, since your bundle now has more goods. Now, what if I wasn't actually feeling that generous, and I wanted to make you as well off as before. Now, if I gave you coffee, I would have to take away one of your milk teas. I would take away just enough tea to make you as content as before. Let's say x_1 is coffee and x_2 is tea. By giving you coffee and taking away tea, we are increasing x_1 but decreasing x_2, which is the exact definition of a negative slope.

5. **Indifference curves are thin**: This also comes from monotonicity. If you had a thick curve, you could have two points: one above another. If one point represented a larger bundle, yet was on the same indifference curve, it would violate our monotonicity assumption. We can see this in the figure below, as bundle $B \succ A$ despite both being on the same indifference curve, implying $B \backsim A$.



6. **Well-behaved indifference curves are convex**: This comes from the convexity assumption, as the name suggests. This just says that our indifference curves bow in toward the origin, due to the fact that consumers prefer a weighted average of both goods rather than consuming an extreme amount of one good. Indifference curves that curve outward imply irrational preferences.

1.4 Indifference Curves in action

In this section, we are going to think about different indifference curves to suit our preferences. Before we proceed, let's just bear in mind the necessary properties of indifference curves:

- Any point on the same indifference curve must be equally preferred by the consumer
- Higher indifference curves are strictly preferred to lower indifference curves
- We draw arrows indicating the direction in which indifference curves move
- The slope of an indifference curve tells us how consumers substitute each good to retain indifference.
  1. A negative slope means that if I give you a good x_1, I must take take away some amount of your other good x_2 to make you as well off as before. This means both items are economics goods.
  2. A positive slope means that if I give you more of a good x_1, I must also give you more of x_2 to make you indifferent to before. This indicates that x_1 is an economic bad.
  3. A slope of 0 (horizontal line) means that no matter how much additional x_1 you receive, you remain indifferent to before.
  4. A slope of ∞ (vertical line) means that no matter how much additional x_2 you receive, you remain indifferent to before.

With all of that in mind, let's think about a few examples...

(a) Let's start with a simple example. Suppose we have someone who spends part of their income on apples and part on oranges. Assuming that they have well-behaved and rational preferences, their indifference curves should look something like graph (a). This is a standard set of indifference curves which satisfy all of our properties: they slope downward, are convex, and do not cross.

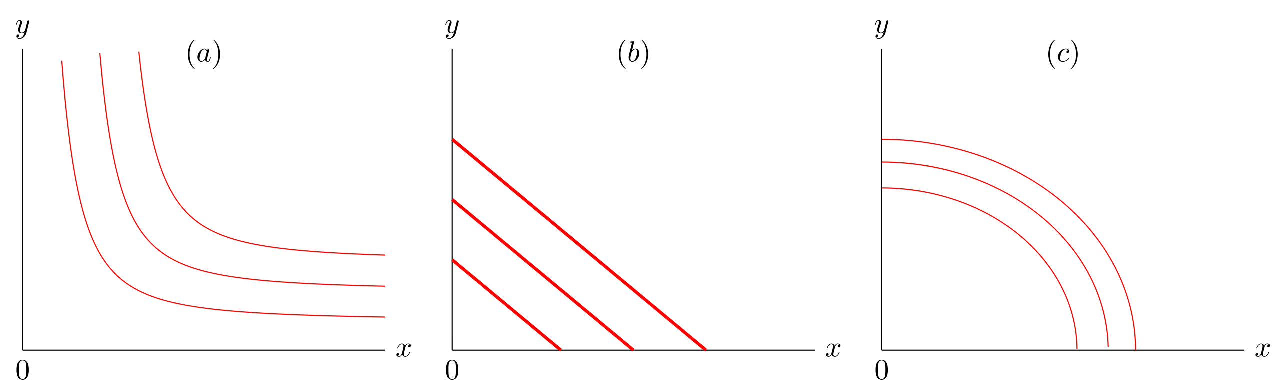
(b) This consumer has indifference curves with a constant slope. We see that they slope downward, but they are not *strictly* convex. We call these indifference curves **perfect substitutes**, because the consumer is willing to substitute goods at a constant rate. Monotonicity is satisfied as well, as the downward-sloping curves move northeast. An example of perfect substitutes in real life could be dimes and nickels: I am always willing to substitute two nickels for one dime, because they have the exact same value.

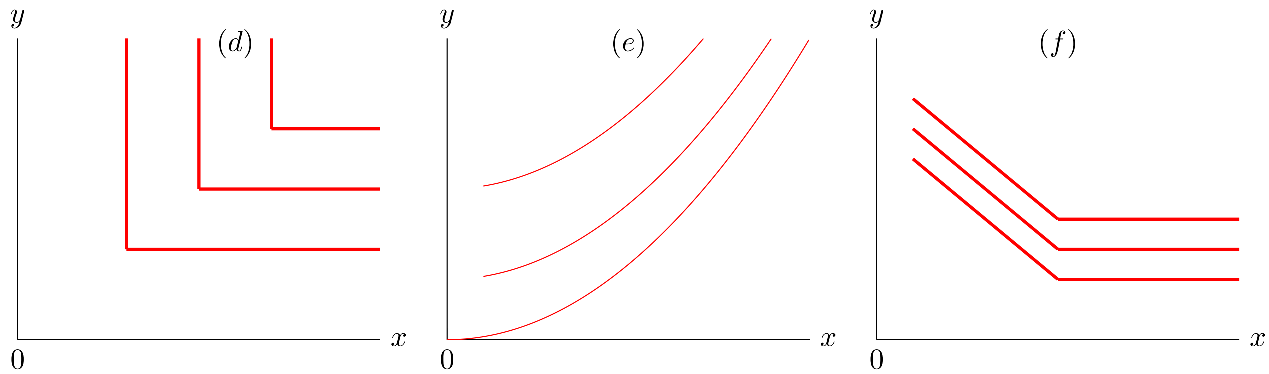
(c) This consumer prefers consuming extreme bundles. The curves are monotonic and downward-sloping, but clearly they are not convex. This is like Lucy's preference for violin lessons and walks on the beach: she would rather have an extreme number of one rather than a mix of both.

(d) These L-shaped indifference curves are technically convex, though they are clearly monotonic. These indifference curves are called **perfect complements**. The point at which the curves seem to bend is call a "kink point." Notice how if you increase x_1 or x_2 in either direction, you remain totally indifferent. Think about these in terms of right shoes and left shoes. If you have three pairs of shoes (so three right shoes and three left shoes), and I give you 100 right shoes, you remain as well off as before. That's kind of a silly example, but the point is that you have to consume each good together. The goods do not need to be consumed in on-to-one ratios, either. Another classic example of perfect complements could be two tires for one bicycle. The key takeaway is that to get to a higher indifference curve, it is not enough to receive more of only one of the goods. The consumer must receive more of both good.

(e) These indifference curves have a positive slope, indicating that one of the goods is actually a "bad." How do we know this? As we give the consumer more x_1, we have to also give them more x_2 in order to make them as well off as before!

(f) These indifference curves are a fun mix of (b) and (d)! We see that there is a kink point: to the left, we have a horizontal line like perfect complements; to the left, we have what appear to be perfect substitutes. These indifference curves are hard to come by in real life, but an example could come from a grading rubric. Say your professor determines your grade as a weighted average of two exams or, if it is higher, only one exam. This would mean that there are a number of different situations in which a student would be indifferent: for example, if their x_1 was 60 and their x_2 was 60, the average of the two are the exact same as x_1, leaving the student indifferent (if this goes over your head, don't worry. We will cover it further in SI).





2 Utility

2.1 A brief note on economic models

This section won't be covered in class, so feel free to skip it if you want. I do think, though, that it is at least responsible to comment on economic models, and how controversial their efficacies are. In the models we study for class, there ends up being a lot of ambiguity about whether the model is being chosen because it actually captures some kind of genuine relationship, or because it has certain desirable formal features.

Most economic models use exogenous variables (i.e., variables outside of the model) to explain endogenous variables (variables within the model). In this class, for example, we will see how consumers react to changes in prices and changes in income. These are all typically given to us, so they are exogenous; the endogenous variables are determined using each model.

The main thing to be aware of in this class is that these models are highly simplified and rely on a number of potentially unrealistic assumptions about consumer behavior. Sometimes they tell us interesting things about the world, but it is always important to be mindful of the shortcomings of these models.

2.2 Representing Utility

Until this point, we have been talking a lot about indifference curves and preferences. We know that we can represent certain preferences using indifference curves, but what if we wanted to be more analytical about ranking preferences? Since preferences are not a function, we have to get creative about representing preferences. We need some way of assigning a number to a certain bundle of goods. We do not really care about the number by itself; this number only has meaning in the context of other numbers generated from different bundles of goods.

In comes the **utility function**. Utility functions are formulas that assign numbers to a bundle of different goods. It normally takes the form $u(x_1, x_2)$, where u is the consumer's utility from consuming goods x_1 and x_2. Now, u can be thought of the amount of happiness a consumer gets from consuming the bundle. If u is higher for one bundle A than for bundle B, we say that the consumer gets more utility from A, or that $A \succ B$. Likewise, if $u_1 = u_2$, then we say that the consumer is indifferent between the two bundles. Written formally, $A \succ B \Leftrightarrow U(A) \succ U(B)$. Likewise, $A \backsim B \Leftrightarrow U(A) = U(B)$.

The important takeaway here is that our utility functions only tell us whether a consumer prefers one bundle to another; it does not tell us the magnitude of the preference. This kind of ordering is **ordinal**, not cardinal. Only the *order* of the preferences matters. This leads to a powerful realization about monotonicity: we can make transformations to functions so long as we preserve the order. Let's think of an example: suppose we have $x \succsim y \succsim z$. By transitivity, $x \succsim z$. If we wanted to write this in terms of our utility functions, we would write $u(x) \succsim u(y) \succsim u(z)$. The important thing here is that the **order** is saved – not the values themselves. The values of utility are completely meaningless. If $u(x) = 3$, $u(y) = 2$, and $u(z) = 1$, the equality is satisfied. However, $u(x) = 10000$, $u(y) = 1000$ and $u(z) = 100$ could also satisfy the equation, despite the much wider differences in utilities.

A positive monotonic transformation is one that transforms the utility function but keeps the order the same. If we, for example, wanted to divide u by 100 to make the numbers a bit nicer, we could do that with no problem. The important thing is that we preserve the order of the preferences. In summary: preference relations do **not** have unique utility representations. They only need to have their orders represented.

2.3 Marginal rate of substitution

As it turns out, our indifference curves correspond nicely with our utility functions. This should not be surprising, but if you graph the utility function, you get the consumer's indifference curves (it would be a pretty bad model if they did not correspond). In any event, our indifference curves show us points of indifference for a certain consumer. This means that bundles with the same utility are on the same indifference curve. $u(x) = u(y)$ implies that $x \backsim y$ which implies that both bundles are on the same curve.

Look back to figure one for a moment. These three indifference curves have no utility labels, but we can clearly see that since $I_3 \succ I_2 \succ I_1$, $u_3 > u_2 > u_1$. Each curve represents a different level of utility.

How should we interpret the slope of the indifference curves? Turns out that the slope has a very specific technical name, **marginal rate of substitution**. I am sure you are familiar with the concept of marginal utility, or the utility from consuming an additional unit of good x. In this case, the MRS just tells us the number of good y the consumer will give up in exchange for one unit of good x. You can write this slope as

$$\frac{\Delta y}{\Delta x} \tag{1}$$

which just says that the change in y over the change in x is the slope. However, since Δy represents a very small change in y, we can actually rewrite the formula for MRS as:

$$\frac{\frac{\delta U(x,y)}{\delta x}}{\frac{\delta U(x,y)}{\delta y}} = -\frac{MU_x}{MU_y} \tag{2}$$

Notice how this formula is really just the ratio of marginal utilities. The marginal utility of good x captures how much my utility changes by consuming a little bit more of good x, holding everything else constant. Notice how the slope is negative. We have covered this a good amount, but it is important once again to understand this intuitively. As x increases by one unit, we must decrease y to stay on the same indifference curve. Therefore, we have a negative relationship!

One final note before getting into examples: MRS is not always constant; in fact, it is basically a function. It takes on different values depending on a consumer's willingness to substitute.

2.4 Examples

2.4.1 Cobb-Douglas

Cobb-Douglas utility functions take on the form

$$u(x,y) = x^{\alpha} y^{\beta} \tag{3}$$

These preferences have well-behaved, standard indifference curves. They never touch the axes. Typically, the exponents represent the share of the consumer's income allocated to each good, meaning that these preferences are convex: consumers prefer mixing. I will go through an example of how to find the MRS of Cobb-Douglas preferences right here:

$$u(x,y) = x^2 y^3$$
$$MU_x = 2xy^3$$
$$MU_y = 3x^2 y^2$$
$$MRS = \frac{MU_x}{MU_y} = \frac{2xy^3}{3x^2y^2}$$
$$MRS = \frac{MU_x}{MU_y} = \frac{2y}{3x}$$

And there you have it. That is how easy it is to find the marginal rate of substitution. These indifference curves are well-behaved because they are monotonic and convex. We know that they are convex because as x increases, y must decrease to remain on the same indifference curve.

2.4.2 Quasi-Linear

Easily the most confusing utility function, quasi-linear functions normally take on the form:

$$u(x,y) = f(x) + c \tag{4}$$

where the first term is a function and the second term is a constant. Popular examples include $u(x,y) = ln(x) + y$. These indifference curves are normally wider than Cobb-Douglas curves, but also only move in one direction: each indifference curve is just a parallel shift of other indifference curves. These preferences are pretty useful for modeling someone who is choosing between one necessity and one luxury good. We will prove this later. However, I can prove right now that the shifts are all parallel. Let's rewrite u such that

$$y = u - f(x)$$

We can see that each new value of u shifts the y-intercept, and the slope is exactly the same. Another way we can do this is by finding the MRS of the general case:

$$MRS = \frac{\delta u(x,y)/\delta x}{\delta u(x,y)/\delta y} = -f'(x)/1 = -f'(x)$$

As you can see, the MRS does not depend on y. Therefore, for the same level of x, the slope of the indifference curve will be the same regardless of whether we move y.

Let's do a more concrete example. Suppose $u(x,y) = ln(x) + y$. We can write out the MRS like so:

$$MRS = -\frac{MU_x}{MU_y} = -\frac{\frac{1}{x}}{1} = -\frac{1}{x}$$

Y is not in the picture at all, affirming once again that the slope is the same if we fix x.

2.4.3 Perfect Substitutes

Perfect substitutes take on the form

$$u(x,y) = \alpha x + \beta y \tag{5}$$

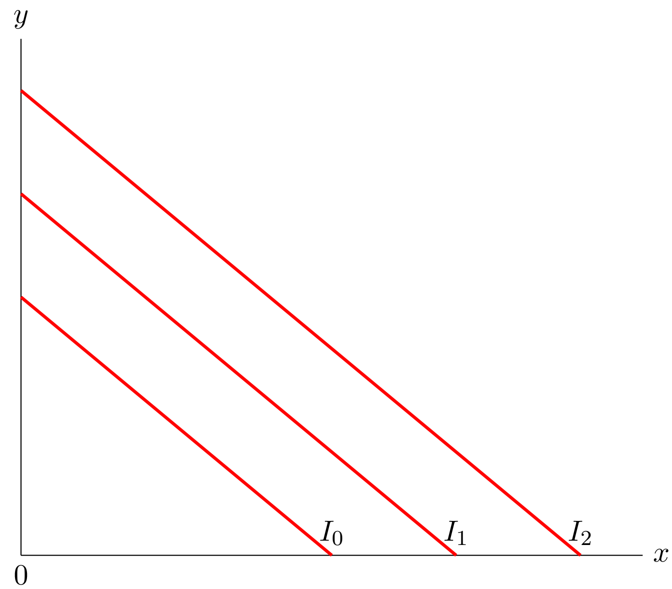
where a consumer is as happy to consume α units of x as they are β units of y. These indifference curves are linear and weakly convex. Let's think of an example: suppose I am indifferent between one dime or two nickels. This would mean that $\alpha = 2$ and $\beta = 1$, giving us the utility function $u(x,y) = 2x + y$.

Finding the MRS of this function is as simple as taking the partial derivatives with respect to x and y. Perfect substitutes are often have the easiest MRS out of all the utility functions we study in this class. Going back to the example of dimes and nickels, the MRS would be

$$MU_x = 2$$

$$MU_y = 1$$
$$MRS = \frac{1}{2}$$

And voila. We now have a constant slope, as we should! I always am willing to trade two nickels for one dime, regardless of how many I have. A key takeaway here is that perfect substitutes have a constant MRS.



2.4.4 Perfect Complements

Perfect complements take on the form

$$u(x, y) = min\{\frac{1}{\alpha}x, \frac{1}{\beta}y\} \tag{6}$$

which produces L-shaped indifference curves. These consumers consume α unites of x for β units of y. The MRS of these functions is hard to find, because we cannot use calculus! Instead, we equate the inside terms, such that $\frac{1}{\alpha}x = \frac{1}{\beta}y$. This is what we call the kink point, or the optimal point of consumption. If you have trouble graphing these, I typically equate the inner terms and solve for y.

$$y = \frac{\beta}{\alpha}x$$

The result is a straight line through the origin on which your kink points will lie. Like the example before, say we have the utility function for a bicycle, $u = min\{\frac{1}{2}x, y\}$. Solving for y yields

$$y = \frac{1}{2}x$$

which is just a straight line containing our kinked points. These functions are not convex in the traditional sense, so we call them weakly convex.