Demand

Economics 100A
Winter 2021

1 Overview

Let's once again restate the utility maximization problem:

$$max u(x, y)$$

$$s.t. p_x x + p_y y = I$$

As you have seen in class, when we solve this problem for general parameters, we find the demand function. Well, as it turns out, this is called the **Marshallian demand**. We tend to write this as a function of prices and income:

$$x^*(p_x, p_y, I)$$

The choice variable here is quantity demanded and the parameters of the demand function are simply prices and income. Turns out that a lot of very smart people before us have played around with these demand functions and found some pretty interesting and confusing results. Turns out if we change individual parameters, we can see some cool properties of demand functions. These notes do just that.

From the outset, I will try to make my notation as clear as possible. I will write functions in terms of their given parameters and the parameter we change. Also, because we fancy ourselves artists of some variety, we will have a lot of graphical interpretations. This means that we have to be careful about the difference between dependent and independent variables. Prices always go on the vertical (y) axis whereas quantities always go on the horizontal axis (except in the case when we graph quantities of x and y together, then those look like our standard graphs).

Let's summarize for sake of clarity:

1. Solve for the consumer's demand as a function of prices and income. In other words, $x(p_x, p_y, I)$.
2. Change *one* parameter of interest while holding all else constant
3. Re-solve the consumer utility maximization problem
4. Observe how x_i changes as the other parameters change. This should give you a new point on the demand curve
5. If you do this for many values of p_x, p_y, and I, you will get an **offer curve!**

2 Own-Price

3 Cross-Price

4 Income

5 Compensated (Hicksian) Demand

5.1 Overview

I am going to rush through this for sake of time. The Hicksian demand is the demand we find when we want to minimize expenditures subject to utility. What does this mean? It means we want to minimize the amount we have to spend in order to reach some fixed utility, U.

The setup is almost the exact same as for Marshallian demand.

1. Find the tangency condition. Set MRS $= \frac{p_x}{p_y}$
2. Solve for either x or y
3. Plug value for x or y into the utility function, $u(x, y)$.
4. Solve for x^c or y^c

5.2 Cobb-Douglas Example

Say we have the following utility function:

$$u = x^2 y$$

Let's just find the compensated demand:

$$MRS = \frac{p_x}{p_y}$$

$$\frac{2y}{x} = \frac{p_x}{p_y}$$

$$x = \frac{2yp_x}{p_y}$$

So now that we have the tangency condition, we have to plug it into our new constraint: the utility function. We know that $u(x, y) = x^2 y$, so we will have to replace x^2.

$$u = (\frac{2yp_x}{p_y})^2 y$$

$$u = (\frac{2p_x}{p_y})y^3$$

$$y^3 = (\frac{up_x}{2p_y})$$

$$y^c = \sqrt[3]{\frac{up_x}{2p_y}}$$

And now we have the compensated demand for good y. To find it for x, we do the same thing as before from the tangency condition. I will leave this as an exercise for you, but you should get:

$$x^c = \sqrt[3]{\frac{u2p_y}{p_x}}$$

5.3 Perfect Compliments Example

Say we have the following utility function:

$$u(x, y) = min3x, y$$

We know that we cannot differentiate this function to find the tangency condition, so what do we do instead? Well, this is where we have to use our economic intuition to get our compensated demand. We know that we consume at the kink point. In other words, our optimal bundle occurs when we equate the inner terms. If $u = min\frac{1}{\alpha}x, \frac{1}{\beta}y$, we must consume where $\frac{1}{\alpha}x = \frac{1}{\beta}y$, otherwise we are inefficiently consuming goods that yield no marginal utility (for further discussion, refer to my optimal choice notes).

So we have that, written out:

$$u = \frac{1}{\alpha}x = \frac{1}{\beta}y$$

This means that our utility is just equal to both terms! Turns out that perfect compliments are much easier in terms of finding compensated demand. Let's refer back to our original problem.

$$u(x, y) = min3x, y$$

$$u = 3x = y$$

Here, we know that we can just solve in terms of x and y.

$$y^c = u$$

and

$$3x = u$$

$$x^c = \frac{u}{3}$$

There we have it! We have just solved for the compensated demand.

5.4 Perfect Substitutes Example

Say we have the following utility function:

$$u(x, y) = 5x + 2y$$

We know that we cannot differentiate this function and solve for a tangency, because these preferences are perfect substitutes. The indifference curves are downward sloping lines, so they will not have points of tangency with the budget constraint. In the case of compensated demand, we are trying to minimize expenditures subject to utility, so we are trying to match the perfect budget constraint subject to utility. This means that we are going to have another corner solution!

How do we find corner solutions for perfect substitutes? We compare marginal utility per dollar! So for this case, we set up the relationship between marginal utilities per dollar.

$$\frac{5}{p_x} = \frac{2}{p_y}$$

So we now have a relationship where we know that the marginal utilities are equal. If $\frac{MUx}{p_x} > \frac{MUy}{p_y}$, we will have a corner solution at the x intercept. But normally, for ordinary demand, this intercept is given by $\frac{I}{p_x}$. Is this still the case for Hicksian (compensated) demand? How has our constraint changed?

Since our constraint is no longer reliant on income, we then have an intercept at $x = \frac{u}{p_x}$. This leads to a very staggering result: if $\frac{MUx}{p_x} > \frac{MUy}{p_y}$, then $x^x = \frac{u}{p_x}$.

Let's solve the given problem with $p_x = 10$. We know that

$$u(x, y) = 5x + 2y$$

And that $p_x = 10$. We can the marginal utilities per dollar to be:

$$\frac{5}{10} = \frac{2}{p_y}$$

We can now see that marginal utility per dollar for good x is just $\frac{1}{2}$. This means that as long as p_y is less than 4, we will only consume good y. Likewise, if $p_y > 4$, we will only consume good x. We then have the following compensated demand:

$$x^c = \begin{cases} \frac{u}{p_x} & if p_y > 4, \\ \in [0, \frac{u}{p_x}] & if p_y = 4, \\ 0 & if p_y < 4. \end{cases}$$

Et voila. These are the cases for x^c.

5.5 Quasi-linear Example

These functions are going to be kind of brutal. Well, they can be. Let's do an example of a simple quasi-linear.

$$u(x, y) = ln(x) + y$$

$$\frac{\frac{1}{x}}{1} = \frac{p_x}{p_y}$$

$$x = \frac{p_y}{p_x}$$

Throw this into our utility function:

$$u = ln(\frac{p_y}{p_x}) + y$$

Rearranging to solve for y:

$$y^c = u - ln(\frac{p_y}{p_x})$$

However, like with all quasi-linear functions, the value of y could be negative. So we have to see whether y is positive. Setting $y = 0$ we can solve for the edge case:

$$u = ln(\frac{p_y}{p_x})$$

So we conclude that the value of utility must be greater than the natural log of the price ratio in order for the consumer to have $y^c > 0$.