

MATH 184A: PROBLEM SET 8

DUE AT 16:00 ON FRIDAY, MARCH 16

- (1) For each permutation π of $1, \dots, N$, let $\text{LIS}(\pi)$ denote the maximal length of an increasing subsequence in π , and let $\text{LDS}(\pi)$ denote the maximal length of a decreasing subsequence in π .
 - (a) Show that $\text{LIS}(\pi)\text{LDS}(\pi) \geq N$ for all π .
 - (b) Let π_N be a *random* permutation of $1, \dots, N$. Show that the expected value of $\text{LIS}(\pi_N)$ is at least \sqrt{N} .
- (2) Show that, in any finite gathering of people, there are at least two people who know the same number of people at the gathering (assume that “knowing” is a mutual relationship).
- (3) Show that, given any five points in a unit square, two of these points are separated by a distance of at most $1/\sqrt{2}$.