4.

Let X be the random variable that maps the result of two fair dice roll to $\{1, 2, 3, 4, 5\}$. Since every dice roll is an independent event, The joint probability that the first roll is a and the second roll is b is $P(a)P(b)$. Since each joint event is independent, the addition rule applies:

$$\begin{aligned} p =& P(X = 1)P(X = 3) + P(X = 1)P(X = 5) + P(X = 2)P(X = 2) + P(X = 2)P(X = 4) \\ &+ P(X = 3)P(X = 1) + P(X = 3)P(X = 3) + P(X = 3)P(X = 5) + P(X = 4)P(X = 2) \\ &+ P(X = 4)P(X = 4) + P(X = 5)P(X = 1) + P(X = 5)P(X = 3) + P(X = 5)P(X = 5) \end{aligned}$$

Sinc the dice is fair, the pdf of the random variable X is uniform. Each face has the probability 1/5 to become the result of the dice roll:

$$p = (\frac{1}{5} \cdot \frac{1}{5}) \cdot 12 = \frac{12}{25}$$

The probability that that sum of the numbers showing on the two fair dice are roll is $\frac{12}{25}$.

5.

Proof.

$$P(A, B) = P(A) + P(B) - P(A \cap B)$$

Since $P(A \cap B)$ represent the probability of getting both A and B, it is at most 1, $P(A \cap B) \leq 1$.

$$\begin{aligned} &P(A, B) = P(A) + P(B) - P(A \cap B) \text{ AND } P(A \cap B) \leq 1 \\ &\Rightarrow P(A) + P(B) - P(A \cap B) \geq P(A) + P(B) - 1 \\ &\Rightarrow P(A, B) \geq P(A) + P(B) - 1 \end{aligned}$$

□

6a.

Let X be the random variable that maps the result of the flip of a single coin to either H for heads or T for tails. Since the three coins are identicle and the flip result in events that are independent from eachother, the the addition rule holds:

$$\begin{aligned} p =& P(X = T)P(X = T)P(X = H) + P(X = T)P(X = H)P(X = T) \\ &+ P(X = T)P(X = T)P(X = T) + P(X = H)P(X = T)P(X = T) \\ p =& 0.7 \cdot 0.3 \cdot 0.7 \cdot 3 + 0.7 \cdot 0.7 \cdot 0.7 \\ p =& 0.441 + 0.343 \\ p =& 0.784 \end{aligned}$$

The probability that the flip result in two or more tails is 0.784

6a.

Let X be the random variable that maps the result of the flip of a single coin to either H for heads or T for tails. Since it is given that one of the coin is already heads, the problem reduces to tossing two coins and getting at least 1 heads.

$$\begin{aligned} p &= P(X = H)P(X = T) + P(X = H)P(X = T) + P(X = H)P(X = H) \\ &= 0.3 \cdot 0.7 \cdot 2 + 0.3 \cdot 0.3 \\ &= 0.51 \end{aligned}$$

The probability that the flip result in two or more heads given at least one head is 0.51

7

Since circuit functional is true if either path works. The probability of the circuit working is therefore the union of the probability that either branch works. The probability that a point does not break is $p^c = 1 - p$ Let η be a normalization factor.

$$
\begin{aligned}
p_{functing} &= \eta(p_{TopWorking} + p_{BottomWorking}) \\
p_{functing} &= \eta((1-p)(1-p) + (1-p)) \\
p_{functing} &= \eta(1 - 2p + pp + 1 - p) \\
p_{functing} &= \eta(2 - 3p + pp)
\end{aligned}
$$

Since probability $\in [0, 1]$, $\eta = \frac{1}{2}$.

$$p_{functing} = \frac{(2 - 3p + pp)}{2}$$

This make sense because if $p = 0$, The probability that the circuit functions is $\frac{2}{2} = 1$. If $p = 1$ the probability that the circuit functions is $\frac{0}{2} = 0$.