1.

Calculating the cost of owning the copiers for 3 year.

Machine A:

$$\begin{aligned}\mathbb{E}(costA) &= 10,000 + \mathbb{E}(costRepair) \\ &= 10,000 + 50 * 12 * 3 \\ &= 10,000 + 1800 \\ &= 11,800\end{aligned}$$

Machine B:

$$\begin{aligned}\mathbb{E}(costB) &= 10,500 + \mathbb{E}(costRepair) \\ &= 10,500 + (1 * 0.25 * 200 + 2 * 0.15 * 200 + 3 * 0.1 * 200) * 3 \\ &= 10,500 + 510 \\ &= 11,010\end{aligned}$$

If the department plan to get rid of the machine after 3 years, go with machine B. The expected cost of repair added with the copier cost is much lower than machine A.

2.

A: $\sum_{n\in[1,4]} \binom{4}{n} 0.25^n 0.75^{4-n} = 0.6836$

B: $\sum_{n\in[2,8]} \binom{8}{n} 0.25^n 0.75^{4-n} = 0.6329$

C: $\sum_{n\in[3,12]} \binom{12}{n} 0.25^n 0.75^{4-n} = 0.6093$

Solution: Based on the above calculations, situation A is most likely.

3.

a.

Plan I $1 - 0.4 * 0.1 = 0.96$

Plan II $1 - 0.1 * 0.4 = 0.96$

b.

Plan I $\mathbb{E}(cost) = 0.6 * 50 + 0.4 * (50 + 80) = 82$

Plan II $\mathbb{E}(cost) = 0.9 * 80 + 0.1 * (50 + 80) = 85$

c.

I would recommend plan I because the expected cost is cheaper while they both have the same probability that the child will be cured.

4.a

$0.85^4 0.15 = 0.0783$

4.b

$0.15 + 0.85 \cdot 0.15 + 0.85^2 0.15 + 0.85^3 0.15 = 0.0783$

4.c

$1 - (0.15 + 0.15 * 0.85) = 0.7225$

4.d

$$\begin{aligned}
\sum_{i=1}^{\text{inf}} 0.85^{2i-1} 0.15 &= 0.15 \sum_{i=1}^{\text{inf}} 0.85^{2i-1} \\
&= 0.15 \frac{0.85}{1 - 0.85^2} \\
&= 0.4595
\end{aligned}$$

4.e

$$\begin{aligned}
\mathbb{E}(3X) &= 3(X) \\
&= 3 \cdot \frac{1}{0.15}
\end{aligned}$$

5

$$\begin{aligned}
T_C(T_F) &= (T_F - 32) \cdot \frac{5}{9} \\
&= \frac{5}{9} \cdot T_F - \frac{32 \cdot 5}{9}
\end{aligned}$$

Since the transform is linear, the transformed standard deviation is $12.2 \cdot \frac{5}{9} = \frac{61}{9}$

6

Since multiplying the distribution by 20 is a linear transformation, the resulting mean (expected) length will be $3 \cdot 20 = 60$, and the standard deviation is $0.1 \cdot 20 = 2$.

7

Let X be the random variable denoting the number on the marble when it is pulled from the bag.

$$\begin{aligned}
\mathbb{E}(X) &= \sum_{i \in [1,n]} i \cdot p(X = i) \\
&= \sum_{i \in [1,n]} i^2 k \\
&= \frac{kn \cdot (n+1) \cdot (2n+1)}{6}
\end{aligned}$$