Abstract Algebra Homework 5

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Section 7

3.

$$\{0, 2, 4, 6, 8, 10, 12, 14, 16\}$$

4.

$$\{0, 6, 12, 18, 24, 30\}$$

5.

$$< 6 >$$

6.

$$< 3 >$$

Section 8

2

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 5 & 6 & 3 \end{pmatrix}$$

3

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 1 & 6 & 2 & 5 \end{pmatrix}$$

5

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 6 & 1 & 5 & 4 & 3 \end{pmatrix}$$

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Since one element in the permutation of 4 choose 4 is fixed, the number of options is therefore $(4-1)! = 6$

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Since one element in the permutation of 5 choose 5 is fixed, the number of options is therefore $(5-1)! = 24$

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No because S_3 is not Abelian but this group is since it is a cyclic group. Group being Abelian is a structural property. Ergo, this group is not isomorphic to S_3

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No f_2 is not a bijection. For example -1 is in $\mathbb{R}$ but not in the range of f_2 therfore violating surjective property.

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Yes because f_3 is a bijection on $\mathbb{R}$.

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Proof. Let the following be two groups in S_n:

$$x = \begin{pmatrix} 1 & 2 & 3 & \cdots & n \\ 2 & 3 & 1 & \cdots & n \end{pmatrix}$$

$$y = \begin{pmatrix} 1 & 2 & 3 & \cdots & n \\ 1 & 3 & 2 & \cdots & n \end{pmatrix}$$

Then the following is true:

$$xy = \begin{pmatrix} 1 & 2 & 3 & \cdots & n \\ 2 & 1 & 3 & \cdots & n \end{pmatrix}$$

$$yx = \begin{pmatrix} 1 & 2 & 3 & \cdots & n \\ 3 & 2 & 1 & \cdots & n \end{pmatrix}$$

Since $xy \neq yx$, the group $S_n,\ n \geq 3$ is not Abelian. □