MATH 184A: PROBLEM SET 2

DUE AT 16:00 ON FRIDAY, JANUARY 26

(1) Show that $\log_2(3)$ is an irrational number.

(2) Let p be a prime. Find a formula for the multiplicity of p in the binomial coefficient $\binom{n}{k}$.

(3) Let p be a prime number.

  (a) Show that any nonzero rational number r can be written uniquely in the form $r = p^k \frac{a}{b}$, where k is an integer, a, b are coprime integers coprime to p, and b is positive.

  (b) With r as above, let us define the "p-adic value" of r by $|r|_p := p^{-k}$. Also define $|0|_p := 0$. This gives a function $|\cdot|_p \colon \mathbb{Q} \to \mathbb{R}_{\geq 0}$ which has properties similar to those of the absolute value function. Indeed, verify the following:

    (i) $|x|_p = 0$ if and only if $x = 0$;

    (ii) $|xy|_p = |x|_p |y|_p$;

    (iii) $|x + y|_p \leq \max\{|x|_p, |y|_p\}$.

  (c) Show that $|x + y|_p \leq |x|_p + |y|_p$.

  (d) Show that $|x + y|_p = \max\{|x|_p, |y|_p\}$ whenever $|x|_p \neq |y|_p$.