

MATH 184A: PROBLEM SET 1

DUE AT 16:00 ON FRIDAY, JANUARY 19

- (1) Let p be a prime number. Show that \sqrt{p} is irrational.
- (2) (a) Let n be a positive integer, and $n = p_1^{e_1} p_2^{e_2} \dots p_k^{e_k}$ its decomposition into primes. Find a formula for the number of divisors of n .
- (b) Which number/numbers among $1, 2, 3, \dots, 33$ has/have the most divisors?
- (3) Fix a real number $x \geq 1$, and let \mathbb{N}_x denote the set of positive integers with no prime factor exceeding x . Prove the inequality

$$\sum_{\substack{m \in \mathbb{N} \\ m \leq x}} \frac{1}{m} \leq \sum_{m \in \mathbb{N}_x} \frac{1}{m},$$

which was used in lecture to connect the functions $\log(x)$ and $\pi(x)$.

- (4) (a) Show that

$$\binom{2n}{0} < \binom{2n}{1} < \dots < \binom{2n}{n} > \dots > \binom{2n}{2n-1} > \binom{2n}{2n}.$$

- (b) Deduce from the above that $\binom{2n}{n} \geq \frac{4^n}{2n}$.