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1. (10 points) Two players have two boards 8×8 and 9×9, one by one they put rooks on these boards such that none of the rooks attack each other (on each turn a player can put a rook on only one board). Who is the winner in this game.

Solution: The first player is always the winner of this game. The first player will take the center of the 9×9 board on the first turn. For any subsequent turns, the first player will place rooks symetric to the second player along either diagonal on the same board. Since the center is claimed on the 9×9 board and 8×8 board does not have a center point, this is possible for the first player. Since rook attack all horizontal and vertical squares, diagonal mirroring positions never attack eachother therefore if the second player has a move, the first player will always have a move.

2. Compute the Grundy function for states of the subtraction game with two piles of chips where players may subtract 1, 2 or 5 chips from one of the piles on their turn.

Solution:

x	0	1	2	3	4	5	6
g(x)	0	1	2	0	1	2	0

Assume $g(x) = x \ mod \ 3$ for $x \leq k$. Show that $g(k+1)$ holds.

k+1 mod 3	0	1	2
mex{g(x-1), g(x-2), g(x-5)}	mex{2, 1, 1}=0	mex{2, 0, 2}=1	mex{1, 0, 0}=2

Hence shown $g(x) = x \ mod \ 3$ is the correct Grundy function for one subtraction game.
By the Grundy theorem, the grundy function for two subtraction game would be the following:

$$g_{G_1,G_2}(x_1, x_2) = (x_1 \ mod \ 3) \oplus (x_2 \ mod \ 3)$$

x_2 / x_1	0	1	2
0	0	1	2
1	1	0	3
2	2	3	0

3. Let G_1 be the subtraction game where on their turn a player may remove 1 or 2 coins, and where there are 10 coins initially. Let G_2 be the game of Nim with three piles, of sizes 1, 6, 7. List all winning moves in $G_1 + G_2$

Solution:

x	0	1	2	3	4	5	6
g(x)	0	1	2	0	1	2	0

Assume $g_{G_1}(x) = x \; mod \; 3$ for $x \leq k$. Show that $g(k+1)$ holds.

k+1 mod 3	0	1	2
mex{g(x-1), g(x-2)}	mex{2, 1}=0	mex{2, 0,}=1	mex{1, 0}=2

Hence shown $g_{G_1}(x) = x \; mod \; 3$ is the correct Grundy function for one subtraction game.

Winning move would be one that move both games to p positions. This is because if both games are in p positions after the first move, the first player can win any game that the second player make a play on for every move. In this case, the game of Nim is already in p position since $1 \oplus 6 \oplus 7 = 0$. The players only winning move would be taking 1 from the subtraction game, since $g_{G_1}(9) = 9 \; mod \; 3 = 0$, the condition is met for p position for both games.

Note: the converse strategy (making both games n-positions) does not work, since sending both games to n positions does not result in winning since an n position can go to another n position, but a p position can only go to n position. The second player can mave a move such that a n-position game remains in n-position.