Math 152 Homework 1 April 6, 2018

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1. Alice and Bob play the following game.

   - Initially, there are 20 numbers: 10 numbers 1 and 10 numbers 2.
   - On each step one of the players select two numbers; and if they were the same, replace them by 2; otherwise, replace them by 1.
   - Alice make the first move and they do moves one after another.

   Who is the winner?

   Solution: Alice wins. This is a parity split problem. Regardless of what move was chosen, the result is the total amount of numbers is subtracted by one. Since there are a total of 20 numbers, and a move is not possible when there is only 1 number left, the problem becomes subtracting 1 from 19 until 0 is reached. This means that the first person wins since 19 is odd.

2. In the subtraction game where players may subtract 1, 2 or 5 chips on their turn, identify the N and P positions.

Solution: Let c denote the number of chips in the pile.
The positions follows the patter PNN starting from terminal (left).

Proof. proof by induction:

base case

$$\begin{bmatrix} c & 0 & 1 & 2 \\ pos & P & N & N \end{bmatrix}$$

Induction step: Assume given

$$\begin{bmatrix} c & k & k+1 & k+2 \\ pos & P & N & N \end{bmatrix}$$

By the definition of P and N states, the following holds:

$$\begin{bmatrix} c & k+6 & k+7 & k+8 \\ pos & P & N & N \end{bmatrix}$$

□

3. Is the Nim position $(1, 3, 5)$ an N-position (explain your answer)?

Solution: $(1_{10}, 3_{10}, 5_{10}) = (01_2, 11_2, 101_2)$, Since $01_2 \oplus 11_2 \oplus 101_2 = 111 \neq 0$, the position $(1, 3, 5)$ is an N position.

4. Consider the Misére subtraction game where players may subtract 1, 5 or 6 chips on their turn, identify the N and P positions.

Solution: The positions follows the patter $PNPNPNNNNNN$ starting from terminal position (left).

Proof. proof by induction:

base case

$$\begin{bmatrix} c & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ pos & P & N & P & N & P & N & N & N & N & N & N \end{bmatrix}$$

Induction step: Assume given

$$\begin{bmatrix} c & k & k+1 & k+2 & k+3 & k+4 & k+5 & k+6 & k+7 & k+8 & k+9 & k+10 \\ pos & P & N & P & N & P & N & N & N & N & N & N \end{bmatrix}$$

By the definition of P and N states, and the step sizes, the following holds:

$$\begin{bmatrix} c & k+11 & k+12 & k+13 & k+14 & k+15 & k+16 & k+17 & k+18 & k+19 & k+20 & k+21 \\ pos & P & N & P & N & P & N & N & N & N & N & N \end{bmatrix}$$

Hence prooven the pattern works. $\square$