1a

$$\begin{aligned} X &\sim Exp(\frac{1}{10}) \\ p(X < 4.5) &= \int_0^{4.5} \frac{1}{10} e^{-\frac{x}{10}} dx \\ &= -e^{-0.1x}\Big|_0^{4.5} \\ &= -e^{-0.1 \cdot 4.5} + 1 \\ &= 0.362 \end{aligned}$$

1c

$$\begin{aligned} 0.8 &= \int_0^{a} \frac{1}{10} e^{-\frac{x}{10}} dx \\ 0.8 &= -e^{-0.1x}\Big|_0^{a} \\ 0.8 &= -e^{-0.1a} + 1 \\ 0.2 &= e^{-0.1a} \\ a &= \frac{\ln 0.2}{-0.1} \\ a &= 16.0944 \end{aligned}$$

1d

$$\begin{aligned} X &\sim Exp(\frac{1}{10}) \\ \mathbb{E}(X) &= \frac{1}{\lambda} \\ &= \frac{1}{0.1} \\ &= 10 \end{aligned}$$

2a

$$\begin{aligned} \int_0^1 12x^2(1-x)dx &= \int_0^1 12x^2 - 12x^3 dx \\ &= 4x^3 - 3x^4\Big|_0^1 \\ &= 4 - 3 \\ &= 1 \end{aligned}$$

2b

$$
\begin{aligned}
\mathbb{E}(X) &= \int_0^1 x \cdot 12x^2(1-x)dx \\
&= \int_0^1 12x^3 - 12x^4 dx \\
&= 3x^4 - \frac{12}{5}x^5\Big|_0^1 \\
&= 3 - \frac{12}{5} \\
&= \frac{3}{5}
\end{aligned}
$$

$$
\begin{aligned}
\mathbb{E}(X^2) &= \int_0^1 x^2 \cdot 12x^2(1-x)dx \\
&= \int_0^1 12x^4 - 12x^5 dx \\
&= \frac{12}{5}x^5 - 2x^6\Big|_0^1 \\
&= \frac{12}{5} - 2 \\
&= \frac{2}{5}
\end{aligned}
$$

$$
\begin{aligned}
Var(X) &= \mathbb{E}(X^2) - \mathbb{E}(X)^2 \\
&= \frac{2}{5} - \frac{3}{5}^2 \\
&= \frac{1}{25}
\end{aligned}
$$

$$
\begin{aligned}
SD(X) &= \sqrt{Var(X)} \\
&= \sqrt{\frac{1}{25}} \\
&= \frac{1}{5}
\end{aligned}
$$

3

Since the probability density function of the uniform distribution is a constant $\frac{1}{4-0} = 0.25$, propogating this distribution through the polynomial can be done through convolution:

$$
\begin{aligned}
\int_0^4 0.25(3x^2 + 4x - 1)dx &= 0.25\left[x^3 + 2x^2 - x\right]_0^4 \\
&= 0.25(4^3 + 2 \cdot 4^2 - 3) \\
&= 23
\end{aligned}
$$

4

$$
\begin{aligned}
0.75 &= \int_0^a \lambda e^{-\lambda x} \\
0.75 &= -e^{-\lambda x}\Big|_0^a \\
0.75 &= -e^{-\lambda a} + 1 \\
0.25 &= e^{-\lambda a} \\
a &= \frac{-\ln 0.25}{\lambda}
\end{aligned}
$$

5

$$
\begin{aligned}
Var(X) &= \mathbb{E}[X^2] - \mathbb{E}[X]^2 \\
&= \int_a^b \frac{1}{b-a} x^2 dx - (\frac{a+b}{2})^2 \\
&= \frac{1}{b-a} \int_a^b x^2 dx - (\frac{a+b}{2})^2 \\
&= \frac{1}{b-a} \left[\frac{x^3}{3}\right]_a^b - (\frac{a+b}{2})^2 \\
&= \frac{b^3 - a^3}{3(b-a)} - (\frac{a+b}{2})^2 \\
&= \frac{a^2 + ab + b^2}{3} - \frac{a^2 + 2ab + b^2}{4} \\
&= \frac{a^2 - 2ab + b^2}{12} \\
&= \frac{(b-a)^2}{12}
\end{aligned}
$$

6

$$
\begin{aligned}
p(X > 3) &= \lim_{n\to\infty} \int_3^n \frac{1}{5} e^{-0.2x} dx \\
&= \lim_{n\to\infty} \left[-e^{-0.2x}\right]_3^n \\
&= \lim_{n\to\infty} [-e^{-0.2n} + e^{-0.2\cdot 3}] \\
&= e^{-0.6} \\
&= 0.549
\end{aligned}
$$

$$
\begin{aligned}
0.549 * 10 &= 5.49 \\
\mathbb{E}(Y) &= 5.49
\end{aligned}
$$