1a

$$
\begin{aligned}
X &\sim Exp(\frac{1}{10}) \\
p(X < 4.5) &= \int_0^{4.5} \frac{1}{10} e^{-\frac{x}{10}} dx \\
&= -e^{-0.1x}\Big|_0^{4.5} \\
&= -e^{-0.1 \cdot 4.5} + 1 \\
&= 0.362
\end{aligned}
$$

1c

$$
\begin{aligned}
0.8 &= \int_0^{a} \frac{1}{10} e^{-\frac{x}{10}} dx \\
0.8 &= -e^{-0.1x}\Big|_0^{a} \\
0.8 &= -e^{-0.1a} + 1 \\
0.2 &= e^{-0.1a} \\
a &= \frac{\ln 0.2}{-0.1} \\
a &= 16.0944
\end{aligned}
$$

1d

$$
\begin{aligned}
X &\sim Exp(\frac{1}{10}) \\
\mathbb{E}(X) &= \frac{1}{\lambda} \\
&= \frac{1}{0.1} \\
&= 10
\end{aligned}
$$

2a

$$
\begin{aligned}
\int_0^1 12x^2(1-x)dx &= \int_0^1 12x^2 - 12x^3 dx \\
&= 4x^3 - 3x^4\Big|_0^1 \\
&= 4 - 3 \\
&= 1
\end{aligned}
$$

2b

$$\begin{aligned}
\int_0^1 x \cdot 12x^2(1-x)dx &= \int_0^1 12x^3 - 12x^4 dx \\
&= 3x^4 - \frac{12}{5}x^5\Big|_0^1 \\
&= 3 - \frac{12}{5} \\
&= \frac{3}{5}
\end{aligned}$$

3

Since the probability density function of the uniform distribution is a constant $\frac{1}{4-0} = 0.25$, propogating this distribution through the polynomial can be done through convolution:

$$\begin{aligned}
\int_0^4 0.25(3x^2 + 4x - 1)dx &= 0.25\left[x^3 + 2x^2 - x\right]_0^4 \\
&= 0.25(4^3 + 2 \cdot 4^2 - 3) \\
&= 23
\end{aligned}$$

4

$$\begin{aligned}
0.75 &= \int_0^a \lambda e^{-\lambda x} \\
0.75 &= -e^{-\lambda x}\Big|_0^a \\
0.75 &= -e^{-\lambda a} + 1 \\
0.25 &= e^{-\lambda a} \\
a &= \frac{-\ln 0.25}{\lambda}
\end{aligned}$$

5

$$\begin{aligned}
Var(X) &= \mathbb{E}[X^2] - \mathbb{E}[X]^2 \\
&= \int_a^b \frac{1}{b-a} x^2 dx - (\frac{a+b}{2})^2 \\
&= \frac{1}{b-a}\int_a^b x^2 dx - (\frac{a+b}{2})^2 \\
&= \frac{1}{b-a}\left[\frac{x^3}{3}\right]_a^b - (\frac{a+b}{2})^2 \\
&= \frac{b^3 - a^3}{3(b-a)} - (\frac{a+b}{2})^2 \\
&= \frac{a^2 + ab + b^2}{3} - \frac{a^2 + 2ab + b^2}{4} \\
&= \frac{a^2 - 2ab + b^2}{12} \\
&= \frac{(b-a)^2}{12}
\end{aligned}$$

6

$$
\begin{aligned}
p(X > 3) &= \lim_{n\to\infty} \int_3^n \frac{1}{5} e^{-0.2x} dx \\
&= \lim_{n\to\infty} \left[-e^{-0.2x}\right]_3^n \\
&= \lim_{n\to\infty} [-e^{-0.2n} + e^{-0.2\cdot 3}] \\
&= e^{-0.6} \\
&= 0.549
\end{aligned}
$$