1.

Lies\Gender	A	B	C	Margin
Lies				
Truth				
Margin	0.42	0.3	0.28	1.00

Given Conditional probabilities: $p(lies|A) = 0.20$, $p(lies|B) = 0.30$, $p(lies|C) = 0.40$

Given Marginal probabilities: $p(A) = 0.42$, $p(B) = 0.30$, $p(C) = 0.28$

Find marginal probability $p(lies)$
By law of total probability and conditioning on Gender:

$$\begin{aligned}
p(lies) &= \sum_{g \in \{A,B,C\}} p(lies, g) \\
&= \sum_{g \in \{A,B,C\}} p(lies|g)p(g) \\
&= p(lies|A)p(A) + p(lies|B)p(B) + p(lies|C)p(C) \\
&= 0.20 \cdot 0.42 + 0.30 \cdot 0.30 + 0.28 \cdot 0.40 \\
&= 0.286
\end{aligned}$$

The probability that a randomly selected member of the civilization lies is 0.286

2.

Random variables:

$$\begin{aligned}
X_1 &= \text{selecting a ball from the first box} \\
X_2 &= \text{selecting a ball from the second box}
\end{aligned}$$

Each of the Random variables can take on $\{R, G\}$ to denote the color of the selected ball.

Let the probability of the secrete transfer being green is denoted as $p(X_1 = G) = \frac{2}{5} = 0.40$. Let the probability of the secrete transfer being red is denoted as $p(X_1 = R) = \frac{3}{5} = 0.60$.
Given the secrete transer being green, the conditional probability of getting a green ball would be $p(X_2 = G|X_1 = G) = \frac{4}{5} = 0.8$ since it simply add another green ball into the mix.
Given the secrete transer being red, the conditional probability of getting a green ball would be $p(X_2 = G|X_1 = R) = \frac{3}{5} = 0.6$ since it simply add another red ball into the mix.
By law of total probability we can get the probability of drawing a green ball from the second box as follows:

$$\begin{aligned}
p(X_2 = G) &= \sum_{c \in \{R,G\}} p(X_2 = G|X_1 = c)p(X_1 = c) \\
&= p(X_2 = G|X_1 = R)p(X_1 = R) + p(X_2 = G|X_1 = G)p(X_1 = G) \\
&= 0.6 * 0.6 + 0.8 * 0.4 \\
&= 0.68
\end{aligned}$$

The probability of getting a green box from the second box is 0.68

3a.

Let O denote the older child love chocolate.
Let Y denote the younger child love chocolate.
Since the events are independent, the following holds:

$$\begin{aligned} p(O, Y|O) &= \frac{p(O)p(Y)}{P(O)} \\ &= p(Y) = 0.5 \end{aligned}$$

3b.

4b

$$p(Some|Younger) = \frac{P(Some, Younger)}{P(Younger)} = \frac{0.15}{0.3} = 0.5$$

4c

$$\begin{aligned} &p(Younger|None) + p(Younger|Some) \\ =&\frac{Younger, None}{None} + \frac{Younger, Some}{Some} = \frac{0.1}{0.2} + \frac{0.15}{0.35} \\ =&0.5 + 0.429 = 0.929 \end{aligned}$$

The probability of someone is Younger given they did not watch Lots of Olympics is 0.929

5a

Find probability that first three questions are guessed incorrectly and the 4th question is guessed correctly. Let random variable X denote the event of guessing on an question. X can take on $\{right, wrong\}$. Since the events are independent, the joint probability is as follows:

$$\begin{aligned} p &= p(X = wrong)p(X = wrong)p(X = wrong)p(X = right) \\ &= \left(\frac{4}{5}\right)^3 \frac{1}{5} = 0.512 * 0.2 = 0.1024 \end{aligned}$$

The probability of the first question guessed right being number 4 is 0.1024

5b

Since each event is independent and the probability of getting an answer correct is uniformly $\frac{1}{5} = 0.2$. The probability of getting at least one right is therefore $0.2 \cdot 10 = 2$. Since probability must be between 0 and 1, the probability of getting at least one right is normalized to 1.

6

$$\begin{aligned} &max\{P\left((O \text{ } or \text{ } M \text{ } or \text{ } G)^C\right)\} \\ =&1 - min\{P(O \text{ } or \text{ } M \text{ } or \text{ } G)\} \\ =&1 - min\{P(O) + P(M) + P(G) - P(O \cap M) - P(O \cap G) - P(M \cap G) - 2 \cdot P((O \cap M) \cap G)\} \\ =&1 - min\{P(O) + P(M) + P(G) - (P(O \cap M) + P(O \cap G) + P(M \cap G) + 2 \cdot P((O \cap M) \cap G))\} \end{aligned}$$

Let C denote the the sum of $P(O)$, $P(M)$, $P(G)$. Since C is fixed, the minimization problem becomes maximizing the negative variable term:

$$
\begin{aligned}
&1-(C-max\{P(O\cap M)+P(O\cap G)+P(M\cap G)+2\cdot P((O\cap M)\cap G)\})\\
=&1-(C-(min\{P(O),P(M)\}+min\{P(O),P(G)\}+min\{P(M),P(G)\}-min\{P(O),P(M),P(G)\}))\\
=&1-((0.2+0.15+0.25)-(0.15+0.2+0.15-0.15))\\
=&1-0.25=0.75
\end{aligned}
$$

$max\{P\left((O\ or\ M\ or\ G)^C\right)\}=0.75$

7

Given $p(pass|D) = 0.85$, $p(pass|Q) = 0.75$, $p(D) = 0.3$, $p(Q) = 0.7$

Solving $p(Q|pass)$

$$\begin{aligned}
p(Q|pass) &= \frac{p(pass|Q)p(Q)}{\sum_{s\in\{D,Q\}} p(pass|s)p(s)} \\
&= \frac{p(pass|Q)p(Q)}{p(pass|D)p(D) + p(pass|Q)p(Q)} \\
&= \frac{0.75 \cdot 0.7}{0.8 \cdot 0.3 + 0.75 \cdot 0.7} \\
&= 0.686
\end{aligned}$$

The probability of "you" having professor Q given the course was passed is 0.686.

R5

$$p(HighSATV|highSATM) = \frac{p(HighSATV\ and\ highSATM)}{p(highSATM)}$$