Abstract Algebra Homework 6

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Section 8

1.

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 6 & 5 & 4 \end{pmatrix}$$

7.

$$\tau^2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 2 & 1 & 5 & 6 \end{pmatrix}$$

$$(\tau^2)^2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{pmatrix}$$

$< \tau^2 >= \{\tau^2,\ e\} \qquad | < \tau^2 > | = 2$

8.

$$\sigma^6 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{pmatrix}$$

$$\sigma^{100} = (\sigma^6)^{16}\sigma^4 = \sigma^4 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 2 & 1 & 3 & 4 \end{pmatrix}$$

33.

f_4 is not a permutation on $\mathbb{R}$ because f_4 is not a surjective. -1 is in $\mathbb{R}$ but it is not in the image of f_4.

34.

f_5 is not a permutation on $\mathbb{R}$ because f_5 is not a injective $f_5(2) = f_5(-1) = 0$.

41.

No because b is a particular element in B, $\sigma(b) \in B$. This means that it could be possible to have another function θ in this set that sends $\sigma(b)$ outside of B. This means $\theta(\sigma(b)) \notin B$ is not in the set. This means that it is not closed.

42.

No, the inverse is not strictly enforced. Lets say $\sigma(b) = b + 1$ and $\sigma[B] = \mathbb{Z}^+$. $\sigma^{-1}(1)$ would be undefined in this case.

48.

Proof. Let c be the element shared between $\mathcal{O}_{a,\sigma}$ and $\mathcal{O}_{b,\sigma}$. This means that $\sigma^m(a) = c$, $\sigma^n(b) = c$. Wants to show $\sigma^z(a) = \sigma^{z+k}(b) \mid z \in \mathbb{Z}$.

$$\begin{aligned}\sigma^{m-n}(a) &= \sigma^{-n}(\sigma^m(a)) \\ &= \sigma^{-n}(c) \\ &= b\end{aligned}$$

With this, we can subsitute $b = \sigma^{m-n}(a)$ into $\sigma^z(b)$ to try to get an relationship to $\sigma^z(a)$.

$$\begin{aligned}\sigma^z(b) &= \sigma^z(\sigma^{m-n}(a)) \\ &= \sigma^{z+m-n}(a)\end{aligned}$$

Since $m-n$ is constant, let $k = m-n$ the definition holds $\sigma^z(a) = \sigma^{z+k}(b) \mid z \in \mathbb{Z}$. Hence shown $\mathcal{O}_{a,\sigma} = \mathcal{O}_{b,\sigma}$ when both orbits share an common element. □

49.

Let $A = \{a_1, a_2, \cdots a_n\}$. Let $\sigma(a_i) = a_{i+1 \ mod \ n}$. $< \sigma >$ defines a group that sends element in A to the next element. Let $H =< \sigma >$, $|A| = |H| = n$. This satisfies the transitive property becasue given a_i and a_j, let $i < j$, $\sigma^{j-i}(a_i) = a_j \wedge \sigma^{i-j}(a_j) = a_i$ (basically composing multiple "move by one" functions to send any a to destination element).

52.

Proof. Given permutation $\rho_a : G \mapsto G$, where $\rho_a(x) = xa \mid a \in G \wedge x \in G$. Let $H = \{\rho_a \mid a \in G\}$. Closed under permutation multiplication: let $a, b \ inG$

$$\begin{aligned}(\rho_a\rho_b)(x) &= \rho_a(\rho_b(x)) \\ &= \rho_a(xb) \\ &= xba \\ &= \rho_{ba}(x)\end{aligned}$$

$\rho_{ba}(x)$ is in H because $ba \in G$. Identity element would be $\rho_e(x) \mid x \in G$. Inverse exists as the following holds true $\rho_a\rho_{-a} = \rho_e \mid a, -a \in G$. Hence H is a group.

Define isomorphism ϕ, such that $\phi(a) = \rho_a$. This function is trivially one to one. Since it is shown that $\rho_{ba} = \rho_a\rho_b$, the following homomorphic property holds:

$$\begin{aligned}\phi(ab) &= \rho_{ab} \\ &= \rho_a\rho_b \\ &= \phi(a)\phi(b)\end{aligned}$$

Hence shown H is a isomorphic to G. □