Name: _____

Pid: _____

1. Compute Grundy function for states of the subtraction game where players may subtract 1, 2 or 5 chips on their turn.

Solution:

$$g_G(0) = 0$$
$$g_G(1) = 1$$
$$g_G(2) = 2$$
$$g_G(3) = 0$$
$$g_G(4) = 1$$
$$g_G(5) = 2$$
$$g_G(6) = 0$$
$$g_G(7) = 1$$
$$g_G(8) = 2$$

Assume by strong induction, assume p position for $n = 0 \ (mod\ 3)$, and n position for $n = 1 \ (mod\ 3)$, $n = 2 \ (mod\ 3)$. Check the following cases for k+1:

$k + 1 = 0 \ (mod\ 3)$ - k+1 is p position because k, k-1 are n positions

$k + 1 = 1 \ (mod\ 3)$ - k+1 is n position because k is p positions

$k + 1 = 2 \ (mod\ 3)$ - k+1 is n position because k-1, is p positions

Hence shown the pattern of the game is pnn.

2. Compute Grundy function for the Nim position $(1, 3, 5)$.

Solution:

$$\begin{aligned} g_G((1,3,5)) &= g_G(1) \oplus g_G(3) \oplus g_G(5) \\ &= 1 \oplus 3 \oplus 5 \qquad\qquad = 7_{10} \end{aligned}$$

Since 7 is not 0, (1, 3, 5) is a n position in game of Nim.

3. Consider the following game: Alice and Bob are writing from left to right digits of a 11-digit number one by one. Alice wins if the number divides by 7 and Bob wins otherwise (Alice writes the first the digit). Determine who is the winner.

Solution: