Abstract Algebra Homework 7

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Section 9:

6.

$$\{3n \mid n \in \mathbb{Z}\},\ \{3n+1 \mid n \in \mathbb{Z}\},\ \{3n+2 \mid n \in \mathbb{Z}\}$$

9.

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & 4 & 3 & 7 & 8 & 6 & 2 & 1 \end{pmatrix}$$

23.

a. False.
$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix} = (1,2)(3,4)$$

b. True. A cycle can be written in permutation presentation.

c. False. The even and odd properties must be mutually exclusive.

d. False. Counter example: $H = \{(123), (123)(123)\}$

e. False. $\frac{5!}{2} = 60 \neq 120$

f. False. S_1 contains only the identity therefore it is trivially cyclic.

g. True.

h. True.

i. True.

j. False. Odd permutations in S_8 is not closed. Permutation multiplication of two odd permutation result in a even permutation.

29.

Let $\omega \in H$ be an odd permutation, $\epsilon \in H$ be an even permutation. We know ϵ exists if ω exists because odd permutation is not a closed, since odd permutation becomes even under permutation multiplication. Wants to show there exists bijection $f(\epsilon) = \sigma\epsilon$. This function maps even permutation to odd permutation. f is a bijection because $f^{-1}(\tau) = \sigma^1\tau$. τ^{-1} exists because τ is a odd permutation in H which is bijective.
This shows that f is a bijection that maps even permutations in H to odd permutations in H. This means the cardinality of odd permutation in H must equal to the cardinality of even permutation if there exists a odd permutation. Since even permutations are closed under permutation multiplicaiton, every subgroup H of S_n for $n \geq 2$, either all the permutations in H are even or exactly half of them are even (cardinality are the same between even permutaitons and odd permutaitons in H).

34.

Let $\tau = (a_1\ a_2\ a_3 \cdots a_{2k}\ a_{2k+1})$

$$\tau^2 = (a_1\ a_3\ a_5 \cdots a_{2k+1}\ a_2\ a_4 \cdots a_{2k})$$

Hence shown square of an odd length cycle is a cycle.

36.

Wants to show $\lambda_a(g) = ag$ is a bijection from G to G.
$\lambda_a^{-1}(g') = a^{-1}g'$ is the inverse. It is well defined because G is a group therefore $a \in G$ implies $a^{-1} \in G$