MATH 184A: PROBLEM SET 5

(1) Show that the vertices of *any* graph can be coloured black and white in such a way that each white vertex has at least as many black neighbours as white neighbours, and vice versa.

(2) An r-step walk on a graph $G = (V, E)$ is a sequences of vertices $v_0, v_1, \ldots, v_r \in V$ such that $\{v_{i-1}, v_i\} \in E$ for each $1 \leq i \leq r$. The walk is said to begin at v_0, and end at v_r. Find a formula for the number of r-step walks on the complete graph K_n which begin and end at the vertex 1.

(3) Let $G = (V, E)$ be a *triangle free* graph — that is, there are no three vertices $\{u, v, w\} \subseteq V$ such that $\{\{u, v\}, \{v, w\}, \{u, w\}\} \subseteq E$.

   (a) Suppose that G is a connected planar graph with at least three vertices, and no vertices of degree 1. Prove that $|E| \leq 2|V| - 4$.

   (b) Prove that $K_{3,3}$ is not a planar graph.