CSE 130 Homework 3

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1 Type embodiment and parametricity

1.1 The Algebra of Datatypes

1. How many inhabitants does `Animal` have? Give an example
   Answer: $|Animal| = 3$

$$myCat = Cat :: Animal$$

2. How many inhabitants does `AnimalPair` have? Give an example
   Answer: $|AnimalPair| = 9$

$$myCatPair = AnimalPair\ myCat\ myCat :: AnimalPair$$

3. How many inhabitants does `Maybe Animal` have? Give an example
   Answer: $|MaybeAnimal| = 4$

$$Just\ Cat$$

4. How many inhabitants does `Maybe` have? Give your answer in terms of a.
   Answer: $|Maybe| = |a| + 1$

5. How many inhabitants does `Pair a b` have? Give your answer in terms of a and b.
   Answer: $|Pairab| = |a| \cdot |b|$

6. How many inhabitants does `Either (Maybe Animal) (Pair (Pair Animal Animal) Animal)` have?
   Answer: $|\text{Either (Maybe Animal) (Pair (Pair Animal Animal) Animal)}| = 4 + ((3 \cdot 3) \cdot 3) = 31$

1.2 Types and Lambda Calculus

1. $a \to a$
   Answer: 1 function. $\lambda x.x$

2. $a \to b$
   Answer: ∞ functions. $\lambda x.e \mid type(e) \neq type(x)$

3. $a \to b \to a$
   Answer: 1 function. $\lambda xy.x$

4. $a \to b \to b$
   Answer: 1 function. $\lambda xy.y$

5. $(a \to b \to c) \to (a \to b) \to a \to c$
   Answer: 1 function. $\lambda fga.f\ a\ (g\ a)$

6. $a \to \mathbb{Z}_6$
   Answer: 6 function.

$$\lambda x.0$$
$$\lambda x.1$$
$$\lambda x.2$$
$$\lambda x.3$$
$$\lambda x.4$$
$$\lambda x.5$$

1.3 Type Tetris

$$(\$) :: (a \to b) \to a \to b$$
$$(.) :: (b \to c) \to (a \to b) \to (a \to c)$$
$$flip :: (a \to b \to c) \to (b \to a \to c)$$
$$map :: (a \to b) \to [a] \to [b]$$
$$concat :: [[a]] \to [b]$$

1. Give a definition for $bog :: (a \to [b]) \to [a] \to [b]$
   Answer:

```
bog :: concat . map
```

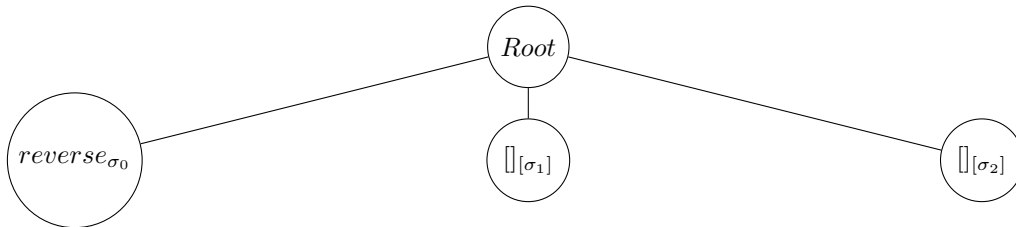2. **WIP**Give a definition for $zog :: a \to [a \to b] \to [b]$
   Answer:

```
zog ::
```

2 Type Inference
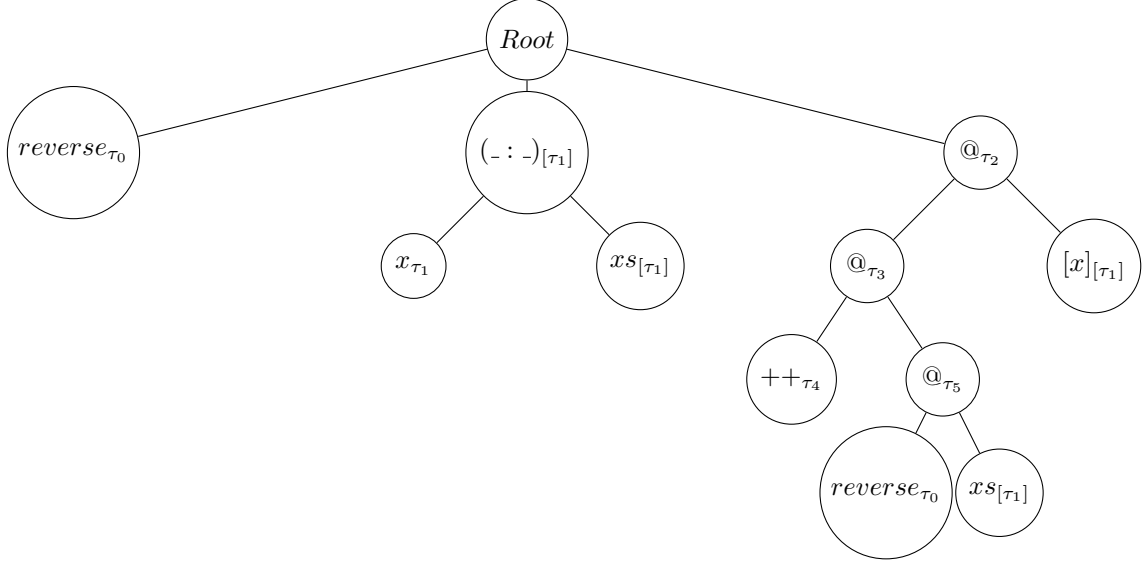
2.1 Infer the type of `reverse`:

Answer:

```
reverse [] = []
```


$$\sigma_0 = [\sigma_1] \to [\sigma_2] \tag{0}$$

```
reverse (x:xs) = reverse xs ++ [x]
               = ((++) (reverse xs)) [x]
```

$$\tau_0 = [\tau_1] \to \tau_2 \tag{1}$$
$$\tau_0 = [\tau_1] \to \tau_5 \tag{2}$$
$$\tau_4 = \tau_5 \to \tau_3 \tag{3}$$
$$\tau_3 = [\tau_1] \to \tau_2 \tag{4}$$
$$++ :: [a] \to [a] \to [a] \tag{5}$$

$$\text{Unify}(1, 2)$$

$$[\tau_1] \to \tau_2 = [\tau_1] \to \tau_5 \Rightarrow \tau_5 = \tau_2 \tag{6}$$

$$\text{Subsitute 6 and 4 into 3}$$

$$\tau_4 = \tau_2 \to ([\tau_1] \to \tau_2) \tag{7}$$

$$\text{Unify}(5, 7)$$

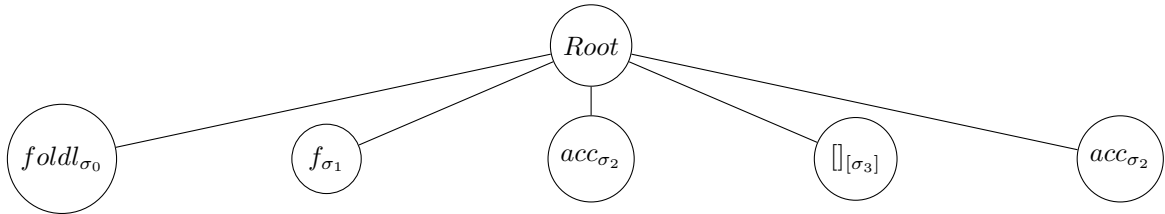$$\tau_2 \to ([\tau_1] \to \tau_2) = [a] \to [a] \to [a] \Rightarrow \tau_2 = [\tau_1] \tag{8}$$
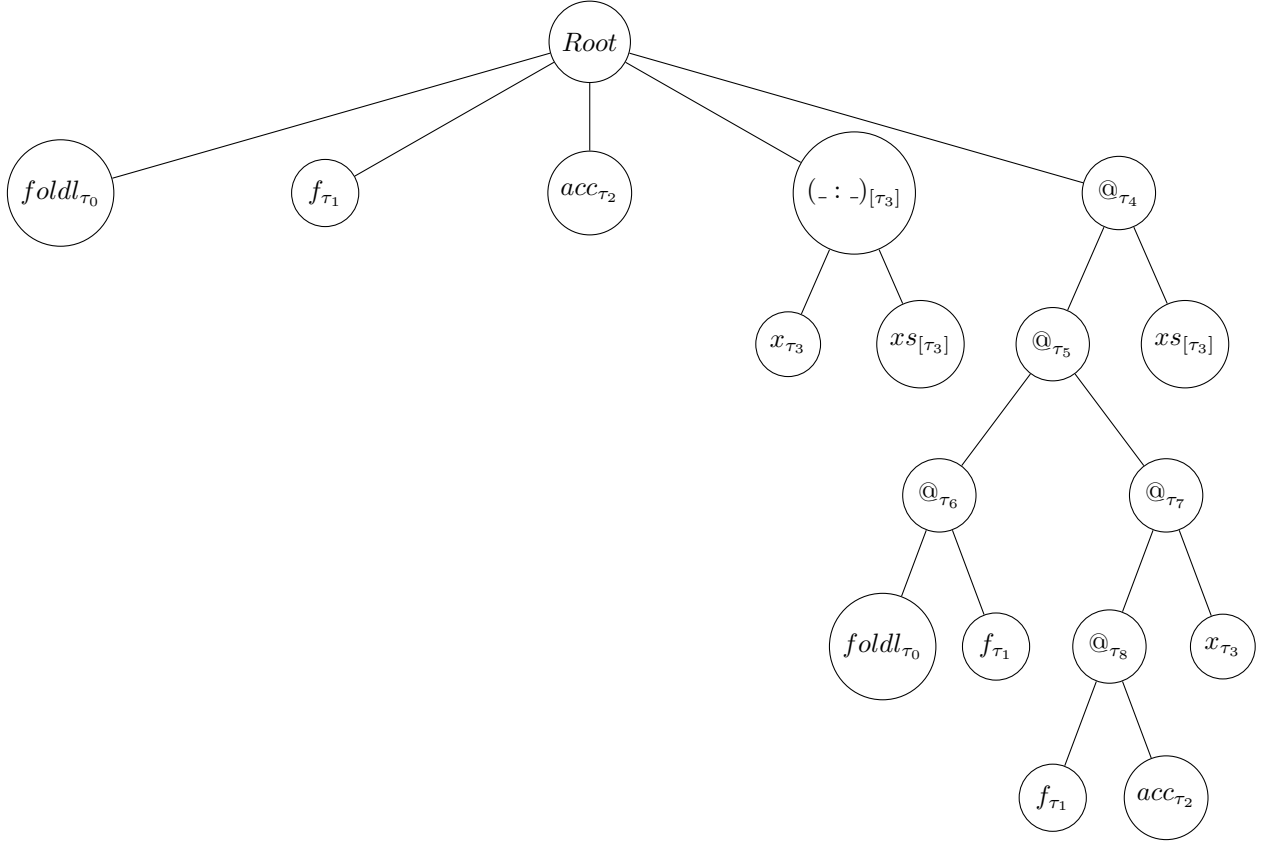
Solution:

$$\tau_0 = [\tau_1] \to [\tau_1]$$

2.2 Infer the type of `foldl`:

Answer:

```
foldl f acc [] = acc
```


$$\sigma_0 = \sigma_1 \to \sigma_2 \to [\sigma_3] \to \sigma_2 \tag{0}$$

3

```
foldl f acc (x:xs) = foldl f (f acc x) xs
                   = ((foldl f) ((f acc) x)) xs
```


$$\tau_0 = \tau_1 \to \tau_2 \to [\tau_3] \to \tau_4 \tag{1}$$

$$\tau_5 = [\tau_3] \to \tau_4 \tag{2}$$

$$\tau_6 = \tau_7 \to \tau_5 \tag{3}$$

$$\tau_0 = \tau_1 \to \tau_6 \tag{4}$$

$$\tau_8 = \tau_3 \to \tau_7 \tag{5}$$

$$\tau_1 = \tau_2 \to \tau_8 \tag{6}$$

Unify(1, 4)

$$\tau_1 \to \tau_2 \to [\tau_3] \to \tau_4 = \tau_1 \to \tau_6 \Rightarrow \tau_6 = \tau_2 \to [\tau_3] \to \tau_4 \tag{7}$$

Subsitute 6 to 1

$$\tau_0 = (\tau_2 \to \tau_8) \to \tau_2 \to [\tau_3] \to \tau_4 \tag{8}$$

Subsitute 5 to 8

$$\tau_0 = (\tau_2 \to \tau_3 \to \tau_7) \to \tau_2 \to [\tau_3] \to \tau_4 \tag{9}$$

Subsitute 2 to 3

$$\tau_6 = \tau_7 \to ([\tau_3] \to \tau_4) \tag{10}$$

$$\text{Unify}(7, 10)$$

$$\tau_7 \to ([\tau_3] \to \tau_4) = \tau_2 \to [\tau_3] \to \tau_4 \Rightarrow \tau_7 = \tau_2 \tag{11}$$

$$\text{Subsitute 11 to 9}$$

$$\tau_0 = (\tau_2 \to \tau_3 \to \tau_2) \to \tau_2 \to [\tau_3] \to \tau_4 \tag{12}$$

$$\text{Unify}(0, 12)$$

$$(\tau_2 \to \tau_3 \to \tau_2) \to \tau_2 \to [\tau_3] \to \tau_4 = \sigma_1 \to \sigma_2 \to [\sigma_3] \to \sigma_2 \Rightarrow \tau_4 = \tau_2 \tag{13}$$

$$\text{Subsitute 13 to 12}$$

$$\tau_0 = (\tau_2 \to \tau_3 \to \tau_2) \to \tau_2 \to [\tau_3] \to \tau_2 \tag{14}$$

Solution:

$$\tau_0 = (\tau_2 \to \tau_3 \to \tau_2) \to \tau_2 \to [\tau_3] \to \tau_2$$