Abstract Algebra Homework 8

Huize(Jack) Shi - A92122910

March 8, 2018

Section 9

2.

$$(1\ 5\ 8\ 7)\ (2\ 6\ 3)$$

3.

$$(1\ 2\ 3\ 5\ 4)\ (7\ 8)$$

4.

$$(\cdots\ -1\ 0\ 1\ 2\ \cdots)$$

30.

n elements are moved because any element that is not in the cycle are not moved by σ. The ith elements in the cycle are mapped to the $(i+1 mod n)th$ element of the cycle therefore moved by σ.

31.

Show $H \leq S_n$:

Closure: Let $\sigma_1 \in H$ moves n elements (n is finite). Let $\sigma_2 \in H \mid \sigma_2$ moves m elements (m is finite). The largest possible movement as a result from permutation multiplication would be the case when σ_1 and σ_2 are disjoint. In which case the size is $n+m$ which is finite. Therefore H is closed.

Identity: Let σ_0 be a orbit of length 0. Since 0 is finite, and σ_0 is the identity, H contains the identity element.

Inverse: Given any $\sigma \in H \mid \sigma = a_1,\ a_2\ \cdots\ a_n$, $\sigma^{-1} \in H \mid \sigma = a_n,\ a_{n-1}\ \cdots\ a_2,\ a_1$.
Hence shown $H \leq S_n$.

Section 10

4.

$$< 4 >= (\{0,\ 4,\ 8\},\ +_n)$$

Since $\mathbb{Z}_{12}$ is Abelian, left coset and right coset are equivalent.

$$0+<4>=\{0,\ 4,\ 8\}$$
$$1+<4>=\{1,\ 5,\ 9\}$$
$$2+<4>=\{2,\ 6,\ 10\}$$
$$3+<4>=\{3,\ 7,\ 11\}$$

5.

Since $\mathbb{Z}_{36}$ is Abelian, left coset and right coset are equivalent.
$i+<18>=\{0+i,\ 18+i\} \mid i \in \mathbb{Z}, 0 \leq i < 17$

6.

$\rho_0,\ \mu_2,\ \rho_1,\ \delta_2,\ \rho_2,\ \mu_1,\ \rho_3,\ \delta_1$

19.

a. True, $aH, H \leq G, \forall a \in G$ is always well definined.

b. True, proved in class.

c. True, proved in class.

d. False, each partition can be infinite.

e. True, subgroups are closed.

f. False, the cosets could be infinite.

g. True

h. True, of course!

i. False

j. True

20.

Impossible. A subgroup H of an abelian group satisfies $a * H = H * a$, which means left right cosets are the same.

23.

Impossible. Since each cell has to be non-empty and disjoint, it is impossible to have a 12 cells partition for an group of order 6.

24.

Impossible. The number of cells must divide the order of the group.

30.

Not true. Given $G = S_3$, $H = \{\rho_0, \mu_1\}$, $a = \rho_1$, $b = \mu_3$. $aH = \{\rho_1, \mu_3\} = bH$, $Ha = \{\rho_1, \mu_2\}$, $Hb = \{\rho_2, \mu_3\}$, $Ha \neq Hb$.

33.

$$
\begin{aligned}
G &= S_3 \\
H &= \{e,\ (1\ 3)\} = (1,3)H = eH \\
(1\ 2\ 3)H &= \{(1\ 2\ 3),\ (2\ 3)\} \\
(1\ 2\ 3)H &= (2\ 3)H \\
\big((1\ 2\ 3)^2 H = (1\ 3\ 2)H\big) &\neq \big((2\ 3)^2 H = eH\big)
\end{aligned}
$$

Hence shown $aH = bH \not\Leftrightarrow a^2 H = b^2 H$.