

MATH 184A: PROBLEM SET 6

DUE AT 16:00 ON FRIDAY, MARCH 2

- (1) Let (Ω, p) be a finite probability space, and let $X: \Omega \rightarrow \mathbb{R}$ be a random variable.

(a) If X is integer-valued, show that

$$\mathbf{E}[X] = \sum_{k \in \mathbb{Z}} k p(X = k).$$

- (b) The *variance* of X is by definition the number $\mathbf{V}[X] = \mathbf{E}[(X - \mathbf{E}[X])^2]$. Show that

$$\mathbf{V}[X] = \mathbf{E}[X^2] - \mathbf{E}[X]^2.$$

- (c) Show that, for any $k \geq 0$,

$$p(|X - \mathbf{E}[X]| \geq k) \leq \frac{\mathbf{V}[X]}{k^2}.$$