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1. Alice and Bob play the following game.

- Initially, there are 20 numbers: 10 numbers 1 and 10 numbers 2.
- On each step one of the players select two numbers; and if they were the same, replace them by 2; otherwise, replace them by 1.
- Alice make the first move and they do moves one after another.

Who is the winner?

Solution: Alice wins. This is a parity split problem. Regardless of what move was chosen, the result is the total amount of numbers is subtracted by one. Since there are a total of 20 numbers, and a move is not possible when there is only 1 number left, the problem becomes subtracting 1 from 19 until 0 is reached. This means that the first person wins since 19 is odd.

2. In the subtraction game where players may subtract 1, 2 or 5 chips on their turn, identify the N and P positions.

Solution: Let c denote the number of chips in the pile.

Given a P position p , $p + 1 = N$, $p + 2 = N$, $p + 3 = P$, $p + 4 = N$, $p + 5 = N$

Proof. proof by induction:

base case

$$\begin{bmatrix} c & 0 & 1 & 2 & 3 & 4 & 5 \\ pos & P & N & N & P & N & N \end{bmatrix}$$

Induction step: Assume given

$$\begin{bmatrix} c & k & k+1 & k+2 & k+3 & k+4 & k+5 \\ pos & P & N & N & P & N & N \end{bmatrix}$$

By the definition of P and N states, the following holds:

$$\begin{bmatrix} c & k+6 & k+7 & k+8 & k+9 & k+10 & k+11 \\ pos & P & N & N & P & N & N \end{bmatrix}$$

□

3. Is the Nim position $(1, 3, 5)$ an N-position (explain your answer)?

Solution:

4. Consider the Misère subtraction game where players may subtract 1, 5 or 6 chips on their turn, identify the N and P positions.

Solution: