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Discussion: A04
Homework: 3

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1. (a) Show that $\mathbb{Z}[w] = \{a + bw \mid a, b \in \mathbb{Z}\}$ is a subring of $\mathbb{C}$ where $w = \frac{-1+\sqrt{-3}}{2}$

Addition:

$$\begin{aligned}
&\left(a + b\left(\frac{-1+\sqrt{-3}}{2}\right)\right) - \left(c + d\left(\frac{-1+\sqrt{-3}}{2}\right)\right) \\
&= \left((a-c) + (b+d)\left(\frac{-1+\sqrt{-3}}{2}\right)\right)
\end{aligned}$$

Multiplication:

$$\begin{aligned}
&\left(a + b\left(\frac{-1+\sqrt{-3}}{2}\right)\right) \cdot \left(c + d\left(\frac{-1+\sqrt{-3}}{2}\right)\right) \\
&= ac + (ad+bc)\left(\frac{-1+\sqrt{-3}}{2}\right) + bd\left(\frac{-1+\sqrt{-3}}{2}\right)^2 \\
&= (ac - \frac{ad}{2} - \frac{bc}{2} - \frac{bd}{2}) + (ad+bc+bd)\frac{\sqrt{-3}}{2} \\
&= ac - (ad+bc+bd)\frac{1}{2} + (ad+bc+bd)\frac{\sqrt{-3}}{2} \\
&= ac + (ad+bc+bd)\frac{-1+\sqrt{-3}}{2}
\end{aligned}$$

Hence shown $\mathbb{Z}[w]$ is a subring of $\mathbb{C}$.

(b) Show that the field of fraction of $\mathbb{Z}[w]$ is $\mathbb{Q}[w] = \{a + bw \mid a, b \in \mathbb{Q}\}$.