結果

```
In [15]: %runfile 'C:/Users/jacks/OneDrive/Desktop/E94111229_石茂宏_HW1/untitled0.py' --wdir
== 題目一 ==
複合梯形法結果: 0.39614759221490675
複合 Simpson 法結果: 0.3856635960237503
複合 Midpoint 法結果: 0.3808047983772932

== 題目二 ==
Gaussian Quadrature (n=3): 0.19225937725687903
Gaussian Quadrature (n=4): 0.19225935780486314
精確值: 0.19225935773279604

== 題目三 ==
Simpson's Rule Approximation: 0.5119875440121252
Gaussian Quadrature Approximation: 0.5118655399452959
Exact Value (dblquad): 0.5118446353109126

== 題目四 ==
Approximation for part (a): 0.5259312819330654
Approximation for part (b): 0.27448167954263997
```

第一題說明：套三個公式即可

a. Composite Trapezoidal rule

$$\int_a^b f(x)dx = \frac{h}{2}[f(a)+2\sum_{i=1}^{2n-1} f(x_i)+f(b)]-\frac{b-a}{12}h^2 f''(\xi)$$

b. Composite Midpoint rule

$$\int_a^b f(x)dx = 2h[f(x_1)+f(x_3)+\ldots+f(x_{2n-1})]+\frac{b-a}{6}h^2 f''(\xi)$$

c. Composite Simpson's rule

$$\int_a^b f(x)dx = \int_{x_0}^{x_{2n}} f(x)dx = \sum_{i=1}^{n}\int_{x_{2(i-1)}}^{x_{2i}} f(x)dx = \sum_{i=1}^{n}\frac{h}{3}[f(x_{2i-2})+4f(x_{2i-1})+f(x_{2i})]$$

$$= \frac{h}{3}[f(x_0)+2\sum_{i=2}^{n} f(x_{2i-2})+4\sum_{i=1}^{n} f(x_{2i-1})+f(x_{2n})]-\frac{b-a}{180}h^4 f^{iv}(\xi)$$

然後不用管後面誤差，題目沒問

第二題：

$$\text{Then}\quad \int_a^b f(x)dx = \frac{b-a}{2}\int_{-1}^{1} f[\frac{a+b}{2}+\frac{b-a}{2}\eta]d\eta$$

$$\int_a^b f(x)dx = \frac{b-a}{2}\sum_{i=1}^{n} c_i f[\frac{a+b}{2}+\frac{b-a}{2}\eta_i]$$

N 帶 3,4 各算一個答案，並把正確的值算出來即可

第三題：

跟 1.2 題差不多，不贅述

第四題：要注意不要除以 0，python 會炸掉，要設定一個很小的數當起點，再帶公式即可