1.

```
In [5]: %runfile C:/Users/jacks/OneDrive/Desktop/hw12.py --wdir
x     y    u(x,y)
0.000  0.000   1.000000
0.314  0.000   0.951057
0.628  0.000   0.809017
0.942  0.000   0.587785
1.257  0.000   0.309017
1.571  0.000   0.000000
1.885  0.000  -0.309017
2.199  0.000  -0.587785
2.513  0.000  -0.809017
2.827  0.000  -0.951057
3.142  0.000  -1.000000
0.000  0.314   0.951057
0.314  0.314   0.753225
0.628  0.314   0.555907
0.942  0.314   0.333230
1.257  0.314   0.085805
1.571  0.314  -0.173198
1.885  0.314  -0.424259
2.199  0.314  -0.645221
2.513  0.314  -0.814475
2.827  0.314  -0.915788
3.142  0.314  -0.951057
0.000  0.628   0.809017
0.314  0.628   0.564621
0.628  0.628   0.347639
0.942  0.628   0.132645
1.257  0.628  -0.086865
1.571  0.628  -0.305635
1.885  0.628  -0.511153
2.199  0.628  -0.686179
2.513  0.628  -0.809949
2.827  0.628  -0.858894
3.142  0.628  -0.809017
0.000  0.942   0.587785
0.314  0.942   0.368084
0.628  0.942   0.176346
0.942  0.942  -0.004977
1.257  0.942  -0.182349
1.571  0.942  -0.353914
1.885  0.942  -0.511647
2.199  0.942  -0.642022
2.513  0.942  -0.724391
2.827  0.942  -0.725487
3.142  0.942  -0.587785
0.000  1.257   0.309017
0.314  1.257   0.172806
0.628  1.257   0.053085
0.942  1.257  -0.058883
1.257  1.257  -0.166750
1.571  1.257  -0.269911
1.885  1.257  -0.364163
2.199  1.257  -0.441313
2.513  1.257  -0.486322
2.827  1.257  -0.467875
3.142  1.257  -0.309017
0.000  1.571   0.000000
0.314  1.571   0.000000
0.628  1.571   0.000000
0.942  1.571   0.000000
1.257  1.571   0.000000
1.571  1.571   0.000000
1.885  1.571   0.000000
2.199  1.571   0.000000
2.513  1.571   0.000000
2.827  1.571   0.000000
3.142  1.571   0.000000
```

2.

```
==== (a) Forward Difference ====

r →          0.5           0.6    ...          0.9     1.0
t ↓                              ...
0.0    0.000000e+00   2.000000e+01  ...   8.000000e+01   100.0
0.5    3.358974e+02   4.366667e+02  ...   3.577778e+02   120.0
1.0   -1.266307e+04  -1.646199e+04  ...  -2.927764e+04   140.0
1.5    2.212421e+06   2.876147e+06  ...   7.462671e+06   160.0
2.0   -5.804072e+08  -7.545294e+08  ...  -2.332296e+09   180.0
2.5    1.280170e+11   1.664221e+11  ...   7.859857e+11   200.0
3.0   -1.409005e+13  -1.831707e+13  ...  -2.787825e+14   220.0
3.5   -7.157282e+15  -9.304467e+15  ...   1.032891e+17   240.0
4.0    7.279129e+18   9.462868e+18  ...  -3.977586e+19   260.0
4.5   -4.470372e+21  -5.811483e+21  ...   1.583770e+22   280.0
5.0    2.356254e+24   3.063131e+24  ...  -6.482897e+24   300.0
5.5   -1.157018e+27  -1.504123e+27  ...   2.712287e+27   320.0
6.0    5.466311e+29   7.106204e+29  ...  -1.153747e+30   340.0
6.5   -2.524168e+32  -3.281418e+32  ...   4.968090e+32   360.0
7.0    1.149127e+35   1.493865e+35  ...  -2.158131e+35   380.0
7.5   -5.183979e+37  -6.739172e+37  ...   9.433107e+37   400.0
8.0    2.324754e+40   3.022180e+40  ...  -4.141000e+40   420.0
8.5   -1.038448e+43  -1.349982e+43  ...   1.823265e+43   440.0
9.0    4.626531e+45   6.014490e+45  ...  -8.044204e+45   460.0
9.5   -2.057618e+48  -2.674903e+48  ...   3.554052e+48   480.0
10.0   9.140329e+50   1.188243e+51  ...  -1.571732e+51   500.0

[21 rows x 6 columns]

==== (b) Backward Difference ====

r →          0.5           0.6    ...          0.9     1.0
t ↓                              ...
0.0    0.000000e+00   2.000000e+01  ...   8.000000e+01   100.0
0.5    4.184921e-01   5.440397e-01  ...   9.638855e-01   120.0
1.0    9.228183e-03   1.199664e-02  ...   1.643249e-02   140.0
1.5    1.899455e-04   2.469292e-04  ...   3.140457e-04   160.0
2.0    3.824663e-06   4.972062e-06  ...   6.191073e-06   180.0
2.5    7.649072e-08   9.943794e-08  ...   1.230696e-07   200.0
3.0    1.526646e-09   1.984640e-09  ...   2.452027e-09   220.0
3.5    3.045135e-11   3.958675e-11  ...   4.888508e-11   240.0
4.0    6.072932e-13   7.894812e-13  ...   9.747781e-13   260.0
4.5    1.211067e-14   1.574387e-14  ...   1.943827e-14   280.0
5.0    2.415081e-16   3.139605e-16  ...   3.876286e-16   300.0
5.5    4.816074e-18   6.260896e-18  ...   7.729936e-18   320.0
6.0    9.604045e-20   1.248526e-19  ...   1.541475e-19   340.0
6.5    1.915204e-21   2.489765e-21  ...   3.073953e-21   360.0
7.0    3.819230e-23   4.965000e-23  ...   6.129965e-23   380.0
7.5    7.616171e-25   9.901022e-25  ...   1.222415e-24   400.0
8.0    1.518789e-26   1.974426e-26  ...   2.437696e-26   420.0
8.5    3.028714e-28   3.937328e-28  ...   4.861165e-28   440.0
9.0    6.039752e-30   7.851677e-30  ...   9.693960e-30   460.0
9.5    1.204425e-31   1.565753e-31  ...   1.933134e-31   480.0
10.0   2.401821e-33   3.122368e-33  ...   3.854986e-33   500.0

[21 rows x 6 columns]
```

```
==== (c) Crank-Nicolson ====

r →        0.5         0.6         0.7         0.8         0.9      1.0
t ↓
0.0      0.000000   20.000000   40.000000   60.000000   80.000000  100.0
0.5    -13.746815  -17.870859  -36.015621  -55.184769  -76.208956  120.0
1.0     12.250766   15.925996   32.362191   50.738118   72.671072  140.0
1.5    -10.885018  -14.150523  -29.013404  -46.631518  -69.367393  160.0
2.0      9.639024   12.530732   25.945023   42.838663   66.280426  180.0
2.5     -8.503073  -11.053995  -23.134714  -39.335298  -63.394031  200.0
3.0      7.468221    9.708687   20.561898   36.099053   60.693315  220.0
3.5     -6.526229   -8.484098  -18.207615  -33.109306  -58.164530  240.0
4.0      5.669516    7.370371   16.054396   30.347035   55.794989  260.0
4.5     -4.891097   -6.358426  -14.086148  -27.794703  -53.572981  280.0
5.0      4.184544    5.439907   12.288041   25.436134   51.487695  300.0
5.5     -3.543938   -4.607119  -10.646415  -23.256413  -49.529152  320.0
6.0      2.963831    3.852980    9.148688   21.241785   47.688137  340.0
6.5     -2.439208   -3.170970   -7.783266  -19.379561  -45.956143  360.0
7.0      1.965453    2.555089    6.539471   17.658042   44.325312  380.0
7.5     -1.538319   -1.999815   -5.407468  -16.066436  -42.788388  400.0
8.0      1.153897    1.500066    4.378195   14.594790   41.338669  420.0
8.5     -0.808590   -1.051167   -3.443310  -13.233922  -39.969965  440.0
9.0      0.499090    0.648817    2.595128   11.975364   38.676554  460.0
9.5     -0.222353   -0.289058   -1.826570  -10.811305  -37.453151  480.0
10.0    -0.024421   -0.031748    1.131121    9.734538   36.294873  500.0
```

3.

```
In [6]: %runfile C:/Users/jacks/OneDrive/Desktop/hw12-3.py --wdir
r      θ     T(r,θ)
0.500  0.000   0.000000
0.550  0.000   0.000000
0.600  0.000   0.000000
0.650  0.000   0.000000
0.700  0.000   0.000000
0.750  0.000   0.000000
0.800  0.000   0.000000
0.850  0.000   0.000000
0.900  0.000   0.000000
0.950  0.000   0.000000
1.000  0.000   0.000000
0.500  0.105   50.000000
0.550  0.105   28.787943
0.600  0.105   21.818115
0.650  0.105   20.090891
0.700  0.105   21.021831
0.750  0.105   23.934897
0.800  0.105   28.961666
0.850  0.105   36.844364
0.900  0.105   49.104997
0.950  0.105   68.517539
1.000  0.105   100.000000
0.500  0.209   50.000000
0.550  0.209   40.377451
0.600  0.209   35.929686
0.650  0.209   35.312185
0.700  0.209   37.488237
0.750  0.209   42.001417
0.800  0.209   48.773186
0.850  0.209   57.936382
0.900  0.209   69.672831
0.950  0.209   83.950388
1.000  0.209   100.000000
0.500  0.314   50.000000
0.550  0.314   45.949110
0.600  0.314   44.326027
0.650  0.314   45.245116
0.700  0.314   48.417775
0.750  0.314   53.566945
0.800  0.314   60.473148
0.850  0.314   68.925028
0.900  0.314   78.644482
0.950  0.314   89.208556
1.000  0.314   100.000000
0.500  0.419   50.000000
0.550  0.419   48.641498
0.600  0.419   48.754252
0.650  0.419   50.709863
0.700  0.419   54.460224
0.750  0.419   59.805653
0.800  0.419   66.486489
0.850  0.419   74.202623
0.900  0.419   82.613244
0.950  0.419   91.341812
1.000  0.419   100.000000
0.500  0.524   50.000000
0.550  0.524   49.456796
0.600  0.524   50.136260
0.650  0.524   52.441687
0.700  0.524   56.376040
0.750  0.524   61.760255
0.800  0.524   68.330335
0.850  0.524   75.776005
0.900  0.524   83.760748
0.950  0.524   91.942352
1.000  0.524   100.000000
0.500  0.628   50.000000
```

```
0.500    0.628    50.000000
0.550    0.628    48.641498
0.600    0.628    48.754252
0.650    0.628    50.709863
0.700    0.628    54.460224
0.750    0.628    59.805653
0.800    0.628    66.486489
0.850    0.628    74.202623
0.900    0.628    82.613244
0.950    0.628    91.341812
1.000    0.628    100.000000
0.500    0.733    50.000000
0.550    0.733    45.949110
0.600    0.733    44.326027
0.650    0.733    45.245116
0.700    0.733    48.417775
0.750    0.733    53.566945
0.800    0.733    60.473148
0.850    0.733    68.925028
0.900    0.733    78.644482
0.950    0.733    89.208556
1.000    0.733    100.000000
0.500    0.838    50.000000
0.550    0.838    40.377451
0.600    0.838    35.929686
0.650    0.838    35.312185
0.700    0.838    37.488237
0.750    0.838    42.001417
0.800    0.838    48.773186
0.850    0.838    57.936382
0.900    0.838    69.672831
0.950    0.838    83.950388
1.000    0.838    100.000000
0.500    0.942    50.000000
0.550    0.942    28.787943
0.600    0.942    21.818115
0.650    0.942    20.090891
0.700    0.942    21.021831
0.750    0.942    23.934897
0.800    0.942    28.961666
0.850    0.942    36.844364
0.900    0.942    49.104997
0.950    0.942    68.517539
1.000    0.942    100.000000
0.500    1.047    0.000000
0.550    1.047    0.000000
0.600    1.047    0.000000
0.650    1.047    0.000000
0.700    1.047    0.000000
0.750    1.047    0.000000
0.800    1.047    0.000000
0.850    1.047    0.000000
0.900    1.047    0.000000
0.950    1.047    0.000000
1.000    1.047    0.000000
```

4.

```
In [7]: %runfile C:/Users/jacks/OneDrive/Desktop/hw12-4.py --wdir
```

x →	0.0	0.1	0.2	0.3	...	0.7	0.8	0.9	1.0
t ↓					...				
0.0	1.0	0.809017	0.309017	-0.309017	...	-0.309017	0.309017	0.809017	1.0
0.1	1.0	1.023825	0.847566	0.347566	...	-0.847566	-0.347566	0.285192	2.0
0.2	1.0	1.038549	1.062374	0.871391	...	-1.062374	-0.871391	0.843417	2.0
0.3	1.0	1.038549	1.062374	1.062374	...	-0.871391	0.128609	0.843417	2.0
0.4	1.0	1.023825	1.038549	1.038549	...	0.843417	0.843417	1.285192	2.0
0.5	1.0	1.000000	1.000000	1.000000	...	2.000000	2.000000	2.000000	2.0
0.6	1.0	0.976175	0.961451	0.961451	...	3.156583	3.156583	2.714808	2.0
0.7	1.0	0.961451	0.937626	0.937626	...	3.871391	3.871391	3.156583	2.0
0.8	1.0	0.961451	0.937626	2.128609	...	4.062374	3.871391	3.156583	2.0
0.9	1.0	0.976175	2.152434	2.652434	...	3.847566	3.347566	2.714808	2.0
1.0	1.0	2.190983	2.690983	3.309017	...	3.309017	2.690983	2.190983	2.0