TCJA Through RBC: Testing Tax Shocks

Jackson Mejia*

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1 An RBC Model

Here, in an attempt to tease out some of the effects of TCJA on the economy, we sketch out a basic RBC model with two exogenous processes: productivity shocks z_t and a proportional tax on capital income τ_t. Following Hansen (1985), we specify a quasilinear utility function $u(c_t, n_t) = \ln(c_t) - \gamma n_t$. Here, the key assumption behind the derivation of this utility function is that households can work a fixed number of hours or none at all, which follows from the observation that households tend not to choose a certain number of hours to work per week, but rather whether to work at all. The linear (rather than logarithmic) entry of hours worked in the utility function allows for greater substitutability between hours worked at different dates and therefore provide for a closer match between the variability in hours worked in the model and what we observe in the data.[1]

This model has a representative household, a representative firm, and a government forced to have a balanced budget every period financed by distortionary taxes on capital. The government produces no public goods and all taxes are assumed to be for the purpose of government consumption.

1.1 Household

Here, we have a representative household which owns capital and leases it to firms. The household chooses current-period consumption labor, and next-period capital:

$$\max_{c_t, n_t, k_{t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\ln(c_t) - \gamma n_t\right) \tag{1}$$

subject to the sequence of flow budget constraints

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[1]See Hansen (1985) King and Rebelo (2000) for further explanation and analysis, as well as McGrattan (1994a) for a comparison in performance between Kydland and Prescott's (1982) benchmark model and Hansen's indivisible labor case, as well as a comparison between McGrattan's own model with fiscal shocks and Hansen's model.

$$c_t + i_t + \tau_t \left(r_t - \delta\right) k_t \leq w_t n_t + r_t k_t \tag{2}$$

where c_t is consumption in period t, i_t is investment in period t, τ_t is the proportional tax rate on capital produced in period t, r_t is the real interest rate, δ is the depreciation rate, k_t is capital, w_t is wages, and n_t is the proportion of time spent working. Agents are aware of the law of motion for capital, namely that

$$k_{t+1} = i_t + (1 - \delta) k_t \tag{3}$$

The above equation also says that investment in period t is given by

$$i_t = k_{t+1} - k_t \tag{4}$$

which is known by households.

1.2 Government

Per-capita government spending g_t is constrained by the amount of taxes it collects:

$$\tau_t \left(r_t - \delta\right) k_t = g_t \tag{5}$$

Note that if the tax rate is lowered to zero, then there is no government spending. Note also that government spending does not enter the utility function, which in practice is a function of the fact that government consumption has little effect on utility (McGrattan, 1994b). It would have been possible to include government spending in the utility function either as part of the logarithmic component (see, e.g., McGrattan (1994a)) or as an additive component. In the latter case, since g_t is not chosen by households, g_t would not show up in the first-order conditions for the household anyway. Furthermore, the government does not produce public goods or invest, something which may not accord well with what we observe in reality, but is a necessary simplification for the model. Finally, the government cannot buy or sell bonds; it must run a balanced budget. This simplification allows us to overlook issues relating to Ricardian equivalence and intertemporal budgeting.

1.3 The Firm

There is a representative firm which solves the following problem:

$$\max_{k_t, n_t} \pi = z_t f(k_t, n_t) - (w_t n_t + r_t k_t) \tag{6}$$

where

$$f(k_t, n_t) = k_t^{\theta} n_t^{(1-\theta)} \tag{7}$$

Each firm rents capital and hires labor from households. In equilibrium, it must be that $\pi = 0$, i.e., that $z_t f(k_t, n_t) = (w_t n_t + r_t k_t)$. This allows us to rewrite (2) as

$$c_t + i_t + \tau_t \left(r_t - \delta\right) k_t \leq y_t \tag{8}$$

1.4 Exogenous Processes

There are two exogenous processes, both of which evolve according to an AR(1) process:

$$z_{t+1} = (1-\rho_z)\overline{z} + \rho_z z_t + \epsilon^z_{t+1},\ \epsilon^z_{t+1} \sim \mathcal{N}\left(0, \sigma^2_z\right) \tag{9}$$

$$\tau_{t+1} = (1-\rho_\tau)\overline{\tau} + \rho_\tau \tau_t + \epsilon^\tau_{t+1},\ \epsilon^\tau_{t+1} \sim \mathcal{N}\left(0, \sigma^2_\tau\right) \tag{10}$$

Each of the ρ_i for $i \in \{\tau, z\}$ is an AR(1) coefficient of the respective process. Each of the variables with a bar above it denotes that variable in the steady state, so that $\overline{z}$ is the steady-state TFP, which is normalized to 1.

2 Equilibrium

A competitive equilibrium is a set of decision functions over consumption, capital, and labor for the household $\{c(\cdot), k(\cdot), n(\cdot)\}$, a set of decision functions for the firm over output, capital, and labor $\{y(\cdot), k(\cdot), n(\cdot)\}$, pricing functions $r(\cdot), w(\cdot)$, a law of motion for per capita capital stock and a balanced government budget constraint such that the following hold:

- The household's decision functions are optimal given the pricing functions, the law of motion for per capita capital stock, and distortionary taxes on capital τ.
- The government satisfies its budget constraint with equality in each period.
- The firm's decision functions are optimal given pricing functions
- Markets clear for labor, capital, and goods.
- Expectations are rational

The competitive equilibrium can then be characterized as the following set of equations:

$$(1-\theta)\frac{y_t}{n_t} = \gamma c_t \tag{11}$$

$$\frac{1}{c_t} = \beta \mathbb{E}\left[\frac{1}{c_{t+1}}\left(1 + (1-\tau_{t+1})\left(r_{t+1} - \delta\right)\right)\right] \tag{12}$$

$$i_t = k_{t+1} - (1-\delta)k_t \tag{13}$$

$$c_t + i_t + \tau_t\left(r_t - \delta\right)k_t = y_t \tag{14}$$

$$r_t = \theta\frac{y_t}{k_t} \tag{15}$$

$$y_t = z_t k_t^\theta n_t^{(1-\theta)} \tag{16}$$

And a set of exogenous stochastic processes for productivity and taxes:

$$z_{t+1} = (1-\rho_z)\overline{z} + \rho_z z_t + \epsilon^z_{t+1},\ \epsilon^z_{t+1} \sim \mathcal{N}\left(0, \sigma^2_z\right) \tag{17}$$

$$\tau_{t+1} = (1-\rho_\tau)\overline{\tau} + \rho_\tau \tau_t + \epsilon^\tau_{t+1},\ \epsilon^\tau_{t+1} \sim \mathcal{N}\left(0, \sigma^2_\tau\right) \tag{18}$$

Parameter	Value
β	0.99
$\overline{n}$	$\frac{1}{3}$
θ	0.359
δ	0.023
$\overline{z}$	1
$\overline{\tau}$	0.2
σ_z^2	0.0096
σ_τ^2	0.0108
ρ_z	0.97
ρ_τ	0.97

Table 1: Parameters for solving the model

To actually solve for the equilibrium, it is necessary to first obtain steady-state values, then log-linearize the equilibrium equations, then follow the method of Uhlig (1999) to find impulse responses. Below are the parameter values used to compute steady-state values:

Since we are taking steady-state $\overline{n}$ as given, we can compute the remaining steady-state values as follows:

$$\overline{r} = \frac{\beta^{-1} - 1}{1 - \overline{\tau}} + \delta \tag{19}$$

$$\frac{\overline{y}}{\overline{k}} = \frac{\overline{r}}{\theta} \tag{20}$$

$$\overline{y} = \left(\frac{\overline{y}}{\overline{k}}\right)^{\frac{-\theta}{1-\theta}} \overline{n} \tag{21}$$

$$\overline{k} = \left(\frac{\overline{y}}{\overline{k}}\right)^{-1} \overline{y} \tag{22}$$

$$\overline{c} = \overline{y} + (1 - \delta)\overline{k} - \overline{k} - \overline{\tau}\overline{r}\overline{k} \tag{23}$$

$$\gamma = \left(\frac{1}{\overline{c}}\right)\left(\frac{(1-\theta)\overline{y}}{\overline{n}}\right) \tag{24}$$

$$\overline{i} = \delta\overline{k} \tag{25}$$

Next, we log-linearize the equilibrium conditions. For any variable x_t, let $\tilde{x}_t = \ln\left(\frac{x_t}{\overline{x}}\right)$ be the log-deviation of x_t from its steady state, so that $100\tilde{x}_t$ is approximately the percent deviation of x_t from $\overline{x}$.[2] Thus we have

$$x_t = \overline{x}e^{\tilde{x}_t} \approx \overline{x}\left(1 + \tilde{x}_t\right)$$

[2]Note that we can only do this for small deviations from $\overline{x}$ since for $x \approx 0$, $e^x \approx 1 + x$.

Using this, we proceed to log-linearize:

$$0 = (1-\theta)\frac{\overline{y}}{\overline{n}}\left(\tilde{y}_t - \tilde{n}_t\right) - \gamma\overline{c}\tilde{c}_t \tag{26}$$

$$0 = -\overline{r}\tilde{r}_t + \theta\frac{\overline{y}}{\overline{k}}\left(\tilde{y}_t - \tilde{k}_t\right) \tag{27}$$

$$0 = \overline{y}\tilde{y}_t - \overline{c}\tilde{c}_t - \overline{i}\tilde{i}_t - \left(\overline{\tau}\overline{k}(\overline{r}-\delta)\tilde{\tau}_t + \overline{\tau}\overline{k}\overline{r}\tilde{r}_t + \overline{\tau}\overline{k}(\overline{r}-\delta)\tilde{k}_t\right) \tag{28}$$

$$0 = \overline{k}\tilde{k}_{t+1} - (1-\delta)\overline{k}\tilde{k}_t - \overline{i}\tilde{i}_t \tag{29}$$

$$0 = \tilde{z}_t + \theta\tilde{k}_t + (1-\theta)\tilde{n}_t - \tilde{y}_t \tag{30}$$

$$0 = \tilde{c}_t + \beta\mathbb{E}\left[\overline{r}\tilde{r}_t - \overline{\tau}(\overline{r}+\delta)\tilde{\tau}_{t+1}\right] - \mathbb{E}\left[\tilde{c}_{t+1}\right] \tag{31}$$

$$z_{t+1} = (1-\rho_z)\overline{z} + \rho_z z_t + \epsilon^z_{t+1},\ \ \epsilon^z_{t+1} \sim \mathcal{N}\left(0, \sigma^2_z\right) \tag{32}$$

$$\tau_{t+1} = (1-\rho_\tau)\overline{\tau} + \rho_\tau \tau_t + \epsilon^\tau_{t+1},\ \ \epsilon^\tau_{t+1} \sim \mathcal{N}\left(0, \sigma^2_\tau\right) \tag{33}$$

We can compute the solution to this numerically. Define as x_t the vector of endogenous states, y_t the vector of endogenous controls, and z_t the vector of exogenous stochastic processes:

$$x_t = [\text{capital}] = \begin{bmatrix}\tilde{k}_{t+1}\end{bmatrix}$$

$$y_t = \begin{bmatrix}\text{consumption}\\ \text{output}\\ \text{labor}\\ \text{interest}\\ \text{investment}\end{bmatrix} = \begin{bmatrix}\tilde{c}_t\\ \tilde{y}_t\\ \tilde{n}_t\\ \tilde{r}_t\\ \tilde{i}_t\end{bmatrix}$$

$$z_t = \begin{bmatrix}\text{taxes}\\ \text{productivity}\end{bmatrix} = \begin{bmatrix}\tilde{\tau}_t\\ \tilde{z}_t\end{bmatrix}$$

Given these vectors, we can rewrite the general structure of the log-linearized system as follows.

$$Ax_t + Bx_{t-1} + Cy_t + Dz_t = 0$$

$$\mathbb{E}_t\left[Fx_{t+1} + Gx_t + Hx_{t-1} + Jy_{t+1} + Ky_t + Lz_{t+1} + Mz_t\right] = 0$$

$$\mathbb{E}_t z_t + 1 = Nz_t$$

where

$$A = \begin{bmatrix}0\\0\\0\\ \overline{k}\\0\end{bmatrix},\ B = \begin{bmatrix}0\\ -\theta\frac{\overline{y}}{\overline{k}}\\ -\overline{\tau}\overline{k}(\overline{r}-\delta)\\ -(1-\delta)\overline{k}\\ \theta\end{bmatrix},\ D = \begin{bmatrix}0 & 0\\ 0 & 0\\ -\overline{\tau}\overline{k}(\overline{r}-\delta) & 0\\ 0 & 0\\ 0 & 1\end{bmatrix}$$

$$C = \begin{bmatrix} -\gamma\overline{\tau} & (1-\theta)\frac{\overline{y}}{\overline{n}} & 0 & 0 & \\ 0 & \theta\frac{\overline{y}}{\overline{k}} & 0 & -\overline{r} & 0 \\ -\overline{\tau} & \overline{y} & 0 & -\overline{\tau}\overline{r}\overline{k} & -\overline{i} \\ 0 & 0 & 0 & 0 & -\overline{i} \\ 0 & -1 & 1-\theta & 0 & 0 \end{bmatrix}$$

$$F = [0],\ H = [0],\ J = \begin{bmatrix} -1 \\ 0 \\ 0 \\ \beta\overline{r} \\ 0 \end{bmatrix}^T,\ K = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T,\ L = \begin{bmatrix} 0 \\ 0 \end{bmatrix}^T,\ M = \begin{bmatrix} -\beta\overline{\tau}\overline{r} + \delta \\ 0 \end{bmatrix}^T,\ N = \begin{bmatrix} \rho_\tau & 0 \\ 0 & \rho_z \end{bmatrix}$$

Using these matrices, we guess recursive equilibrium laws of motion of the form

$$x_t = Px_{t-1} + Qz_t y_t = Rx_{t-1} + Sz_t$$

where P, Q, R, and S are matrices such that the computed equilibrium is stable. Using code provided by Harald Uhlig, we get the following results:

$$\tilde{k}_{t+1} = 0.9408\tilde{k}_t - 0.0239\tilde{\tau}_t + 0.1296\tilde{z}_t \tag{34}$$

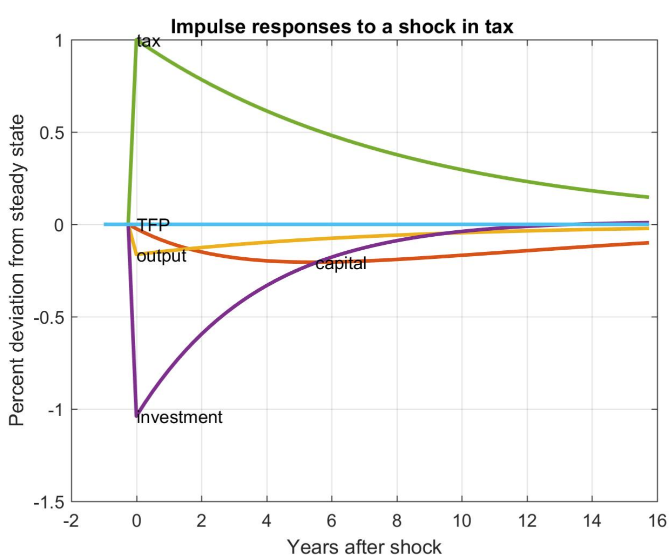
$$\tilde{c}_t = 0.5692\tilde{k}_t + 0.0914\tilde{\tau}_t + 0.5438\tilde{z}_t \tag{35}$$

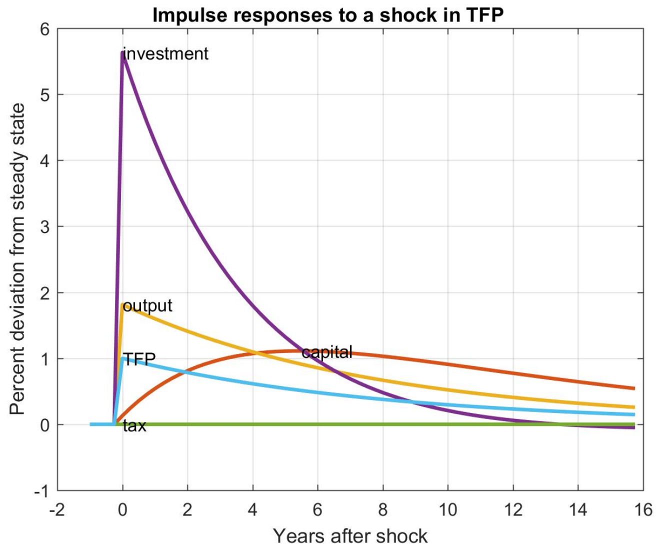
$$\tilde{y}_t = -0.0163\tilde{k}_t - 0.1633\tilde{\tau}_t + 1.8146\tilde{z}_t \tag{36}$$

$$\tilde{n}_t = -0.5855\tilde{k}_t - 0.2547\tilde{\tau}_t + 1.2708\tilde{z}_t \tag{37}$$

$$\tilde{i}_t = -1.5758\tilde{k}_t - 1.0384\tilde{\tau}_t + 5.6332\tilde{z}_t \tag{38}$$

This yields the impulse functions:





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