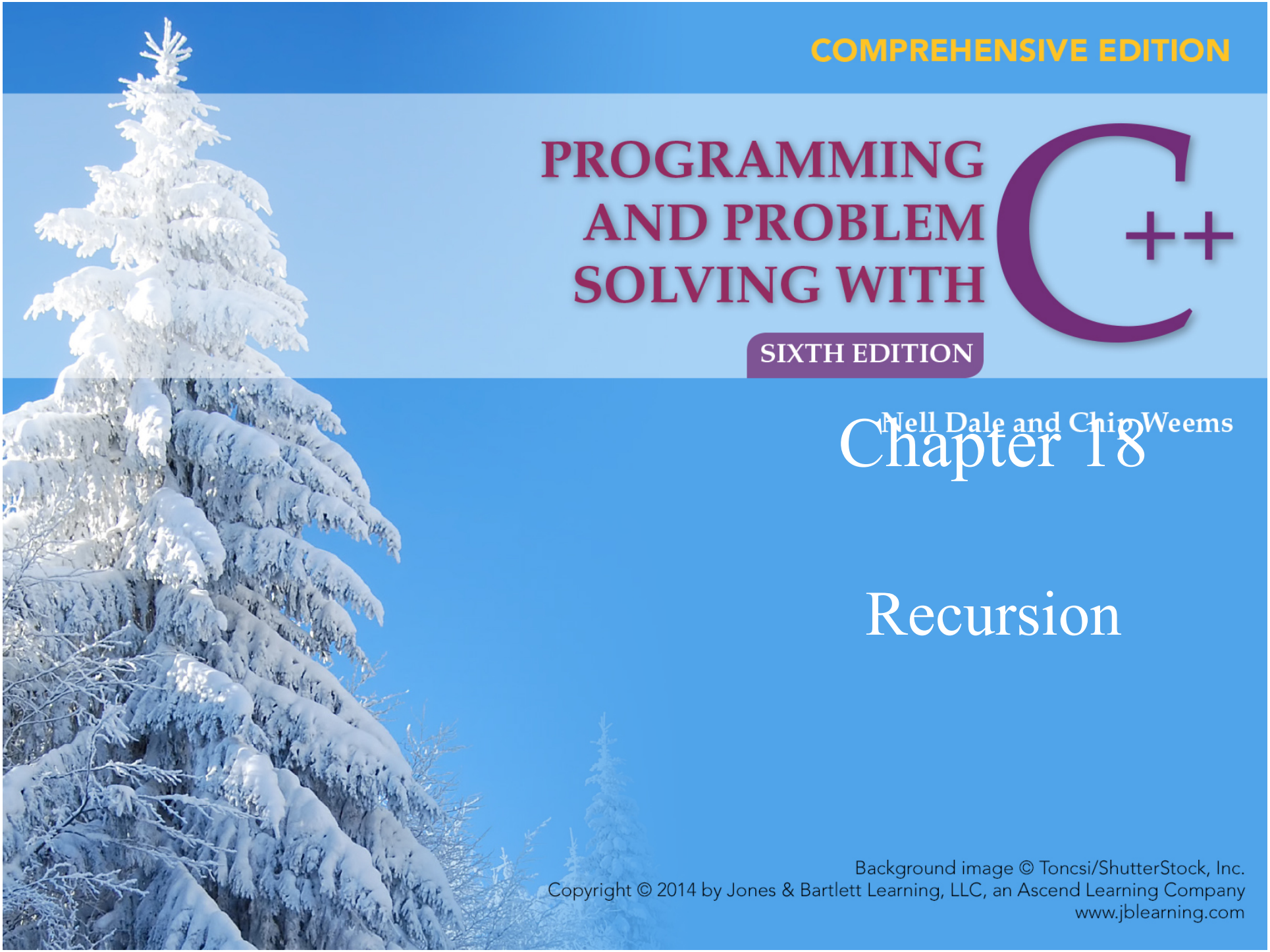


Chapter 18 Topics

- **Meaning of Recursion**
- **Base Case and General Case in Recursive Function Definitions**
- **Writing Recursive Functions with Simple Type Parameters**

Chapter 18 Topics

- **Writing Recursive Functions with Array Parameters**
- **Writing Recursive Functions with Pointer Parameters**
- **Understanding How Recursion Works**

Recursive Function Call

- A **recursive call** is a function call in which the called function is the same as the one making the call
- In other words, *recursion occurs when a function calls itself!*
- But we need to avoid making an infinite sequence of function calls **(infinite recursion)**

Finding a Recursive Solution

- A recursive solution to a problem must be written carefully
- The idea is for each successive recursive call to bring you one step closer to a situation in which the problem can easily be solved
- This easily solved situation is called the base case
- Each recursive algorithm must have at least one base case, as well as a general (recursive) case

General Format for Many Recursive Functions

```
if   (some easily-solved condition)     // Base case

     solution statement

else                                    // General case

     recursive function call
```

Some examples . . .

Writing a Recursive Function to Find the Sum of the Numbers from 1 to n

DISCUSSION

The function call Summation(4) should have value 10, because that is 1 + 2 + 3 + 4

For an easily-solved situation, the sum of the numbers from 1 to 1 is certainly just 1

So our base case could be along the lines of

```
if  (n == 1)
        return 1;
```

Writing a Recursive Function to Find the Sum of the Numbers from 1 to n

Now for the general case. . .

The sum of the numbers from 1 to n, that is,
1 + 2 + . . . + n can be written as

n + the sum of the numbers from 1 to (n - 1),
that is, n + 1 + 2 + . . . + (n - 1)

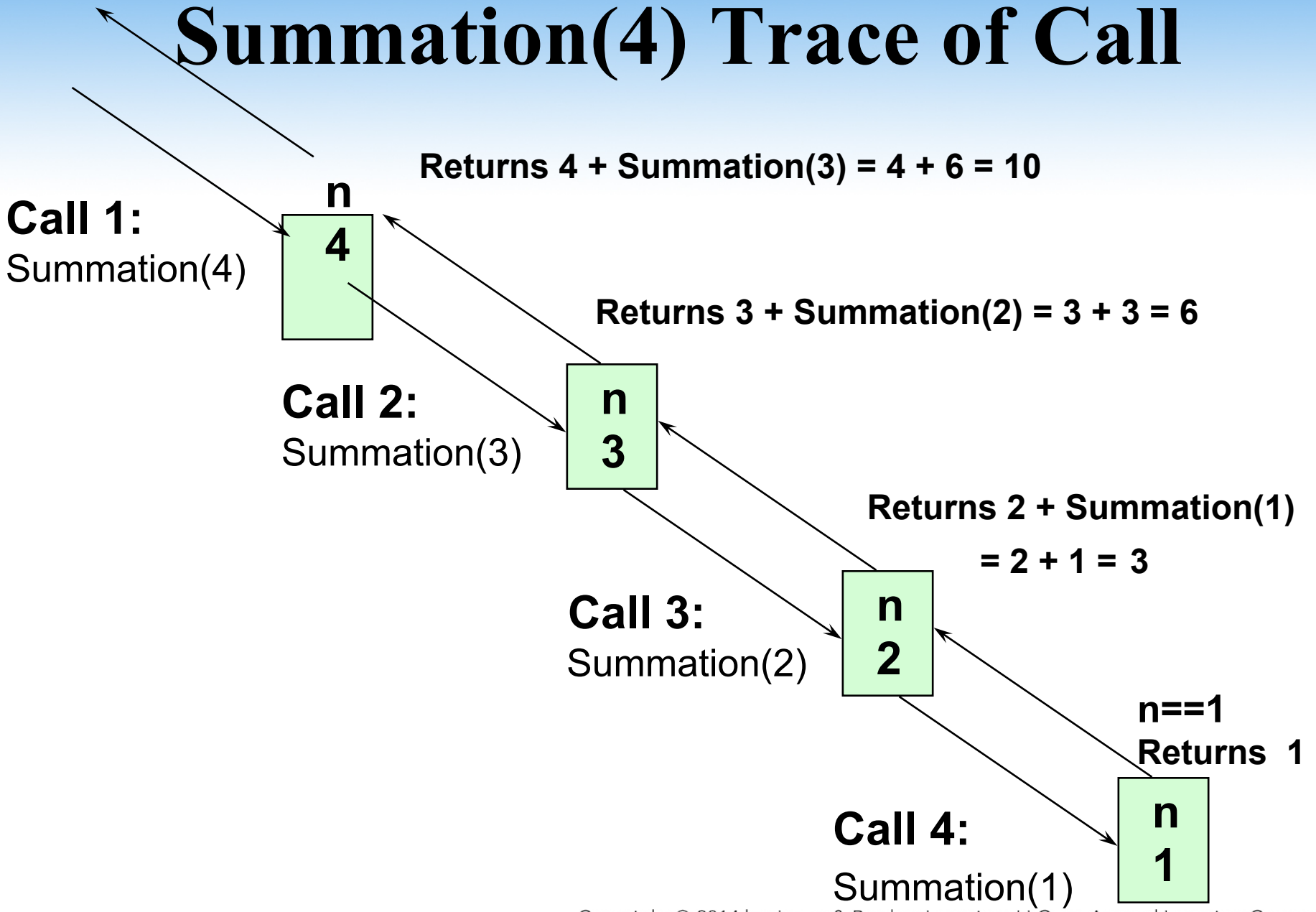
or, n + Summation(n - 1)

And notice that the recursive call Summation(n - 1) gets us "closer" to the base case of Summation(1)

Finding the Sum of the Numbers from 1 to n

```
int   Summation (/* in */  int   n)
// Computes the sum of the numbers from 1 to
//  n by adding n to the sum of the numbers
//  from 1 to (n-1)
// Precondition: n is assigned && n > 0
// Postcondition: Return value == sum of
//  numbers from 1 to n
{
    if  (n == 1)     // Base case
          return  1;
     else                      // General case
          return (n + Summation (n - 1));
}
```

Summation(4) Trace of Call



Writing a Recursive Function to Find n Factorial

DISCUSSION

The function call Factorial(4) should have value 24, because that is 4 * 3 * 2 * 1

For a situation in which the answer is known, the value of 0! is 1

So our base case could be along the lines of

```
if  (number == 0)
        return 1;
```

Writing a Recursive Function to Find Factorial(n)

Now for the general case . . .

The value of Factorial(n) can be written as n * the product of the numbers from (n - 1) to 1, that is,

n * (n - 1) * . . . * 1

or, n * Factorial(n - 1)

And notice that the recursive call Factorial(n - 1) gets us "closer" to the base case of Factorial(0)

Recursive Solution

```
int   Factorial ( int   number)
// Pre: number is assigned and number >= 0
{
    if  (number == 0)      // Base case
          return  1;
     else                          // General case
          return
           number + Factorial (number - 1);
}
```

Another Example Where Recursion Comes Naturally

- From mathematics, we know that

  $2^0 = 1$ and $2^5 = 2 * 2^4$

- In general,

  $x^0 = 1$ and $x^n = x * x^{n-1}$
  for integer x, and integer n > 0

- Here we are defining x^n recursively, in terms of x^{n-1}

```
// Recursive definition of power function
int  Power ( int   x,   int   n)

// Pre:  n >= 0; x, n are not both zero
// Post: Return value == x raised to the
//  power n.

{
    if  (n == 0)
        return  1; // Base case

    else            // General case
        return ( x * Power (x, n-1))
}
```

Of course, an alternative would have been to use an iterative solution instead of recursion

Extending the Definition

- ***What is the value of 2^{-3} ?***
- **Again from mathematics, we know that it is**

  $2^{-3} = 1 / 2^3 = 1 / 8$

- **In general,**

  $x^n = 1/ x^{-n}$

  for non-zero x, and integer n < 0

- **Here we again defining x^n recursively, in terms of x^{-n} when n < 0**

```
// Recursive definition of power function
float  Power ( /* in */  float   x,
               /* in */  int   n)
// Pre:  x  !=  0 && Assigned(n)
// Post: Return value == x raised to the power n

{
    if  (n == 0)       // Base case

        return  1;

    else  if  (n > 0) // First  general  case

        return ( x * Power (x, n - 1));
    else                    // Second general case

        return ( 1.0 / Power (x, - n));

}
```

The Base Case Can Be "Do Nothing"

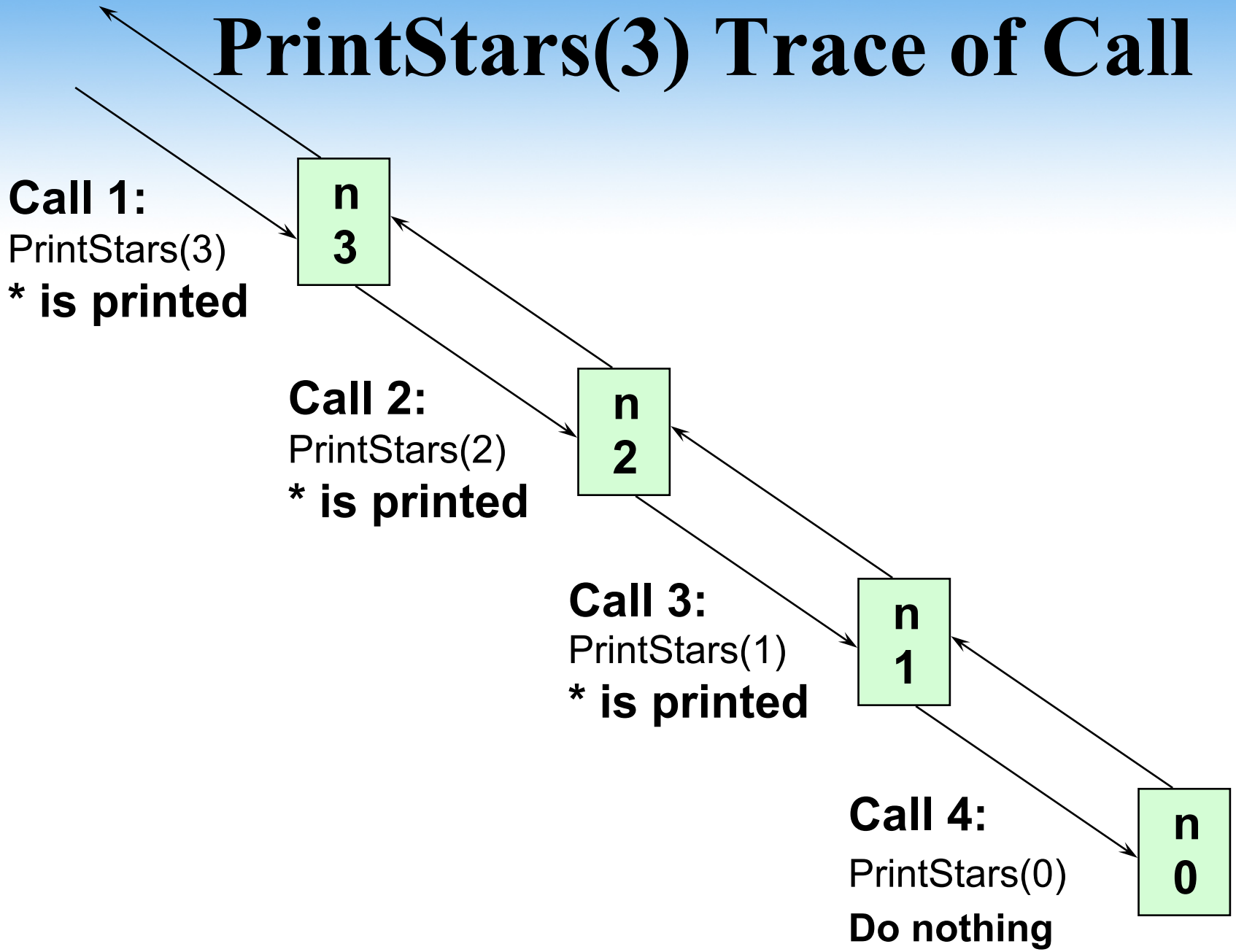
```
void   PrintStars (/* in */  int   n)
// Prints n asterisks, one to a line
// Precondition:     n is assigned
// Postcondition:
//    IF n <= 0, n stars have been written
//    ELSE call PrintStars
{
    if  (n <= 0)  // Base case: do nothing
    else
    {
        cout   <<  '*'  <<  endl;
        PrintStars (n - 1);
    }
}
```

// Can rewrite as . . .

Recursive Void Function

```
void  PrintStars (/* in */  int   n)
//   Prints n asterisks, one to a line
//   Precondition:  n is assigned
//   Postcondition:
//        IF n > 0, call PrintStars
//        ELSE n stars have been written
{
    if  (n > 0)  // General case
    {
        cout  <<  '*'  <<  endl;
        PrintStars (n - 1);
    }
     // Base case is empty else-clause
}
```

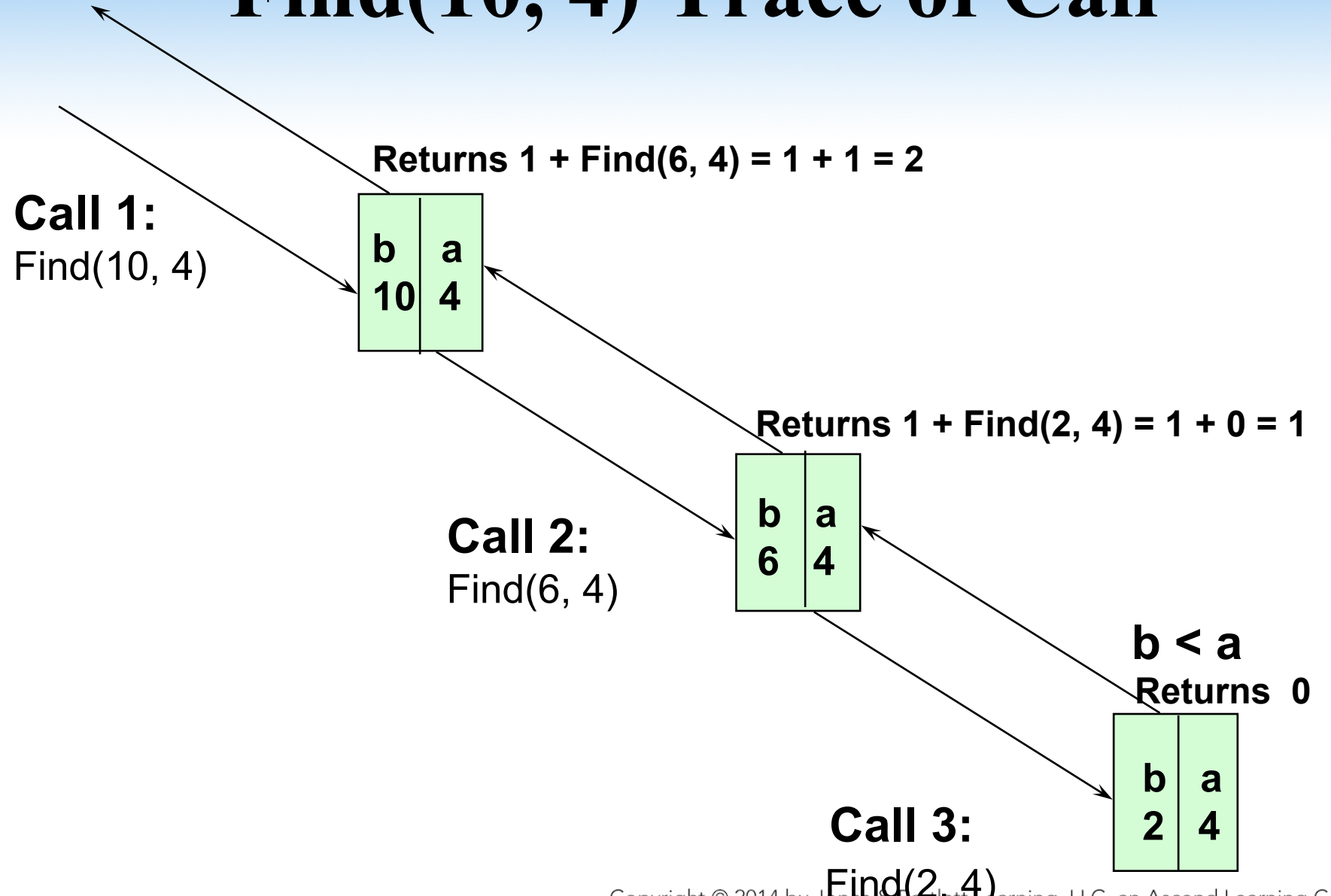
PrintStars(3) Trace of Call



Recursive Mystery Function

```
int Find(/* in */ int b, /* in */ int a)
// Simulates a familiar integer operator
// Precondition: a is assigned && a > 0
// &&  b is assigned  &&    b >= 0
// Postcondition: Return value  ==   ???
{
    if  (b < a)              // Base case
        return  0;
    else                      // General case
        return (1 + Find (b - a, a));
}
```

Find(10, 4) Trace of Call

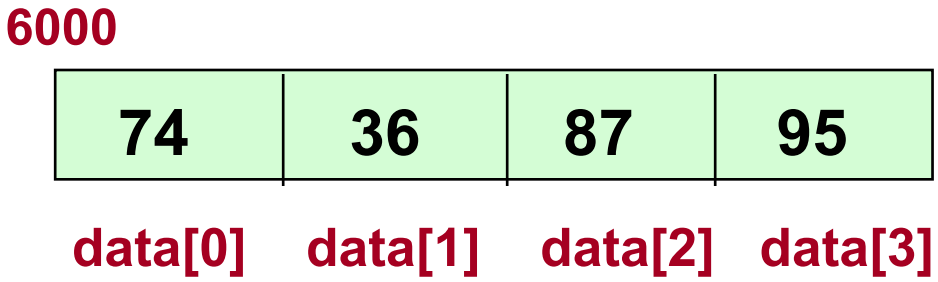


Writing a Recursive Function to Print Array Elements in Reverse Order

DISCUSSION

For this task, we will use the prototype:

```
void PrintRev(const  int  data[ ], int  first,  int  last);
```



The call
PrintRev (data, 0, 3);
should produce this output: 95 87 36 74

Base Case and General Case

- A base case may be a solution in terms of a "smaller" array
  - Certainly for an array with 0 elements, there is no more processing to do
- The general case needs to bring us closer to the base case situation
  - If the length of the array to be processed decreases by 1 with each recursive call, we eventually reach the situation where 0 array elements are left to be processed

Base Case and General Case, cont. . .

In the general case, we could print either the first element, that is, data[first] or we could print the last element, that is, data[last]

Let's print data[last]: After we print data[last], we still need to print the remaining elements in reverse order

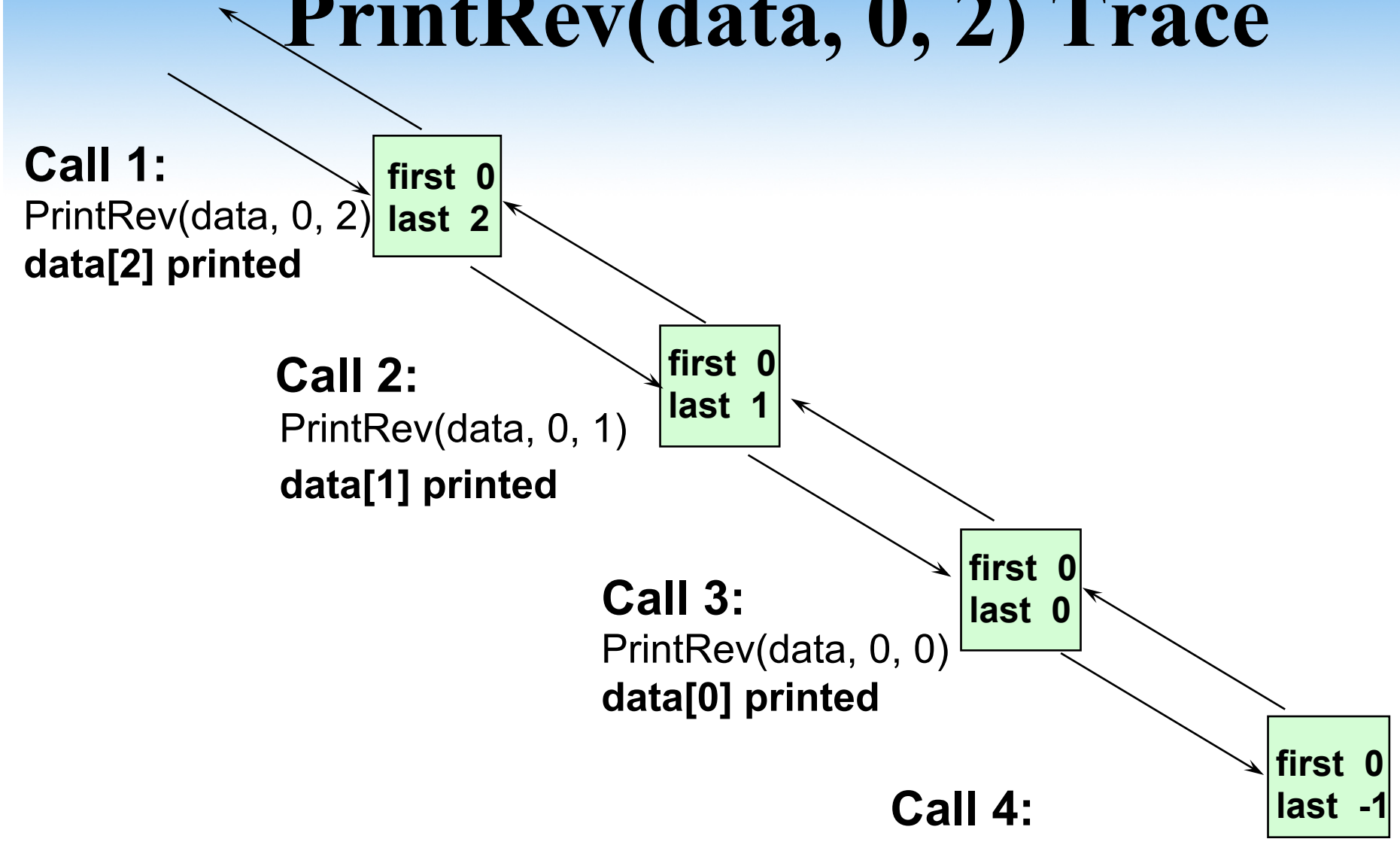
Using Recursion with Arrays

```
int PrintRev (
  /* in */ const int data [ ],// Array to be printed
  /* in */       int  first,  // Index of first element
   /* in */       int  last ) // Index of last element
// Prints items in data [first..last] in reverse order
// Precondition: first assigned  &&  last assigned
//     && if first <= last, data [first..last] assigned
```

Using Recursion with Arrays

```
{
    if  (first  <=  last)  // General case
    {
        cout << data[last]  << “  “; // Print last
        PrintRev(data, first, last - 1); //Print rest
    }
    // Base case is empty else-clause
}
```

PrintRev(data, 0, 2) Trace



NOTE: data address 6000 is also passed

Why use recursion?

- **These examples could all have been written more easily using iteration**

- **However, for certain problems the recursive solution is the most natural solution**

- **This often occurs when structured variables are used**

Why use recursion?

Remember The iterative solution uses a loop, and the recursive solution uses a selection statement

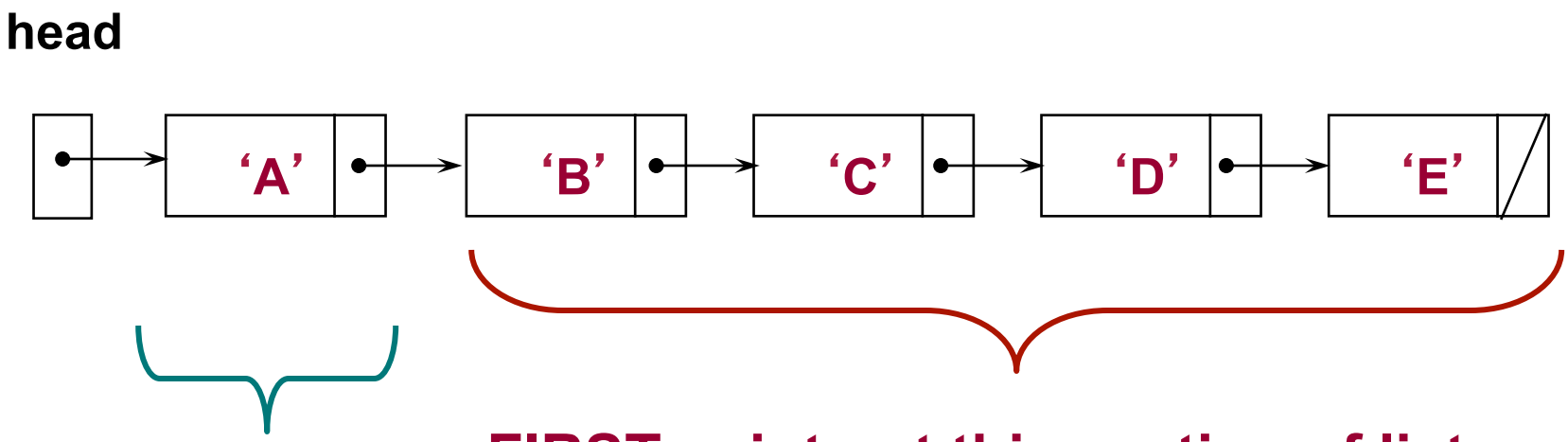
Recursion with Linked Lists

- For certain problems the recursive solution is the most natural solution
- This often occurs when pointer variables are used

struct NodeType

```
typedef  char  ComponentType;

struct  NodeType
{
    ComponentType  component;
    NodeType*      link;
}

NodeType*  head;
```

RevPrint(head);



Base Case and General Case

- A base case may be a solution in terms of a "smaller" list
  - Certainly for a list with 0 elements, there is no more processing to do
- Our general case needs to bring us closer to the base case situation
  - If the number of list elements to be processed decreases by 1 with each recursive call, the smaller remaining list will eventually reach the situation where 0 list elements are left to be processed

Base Case and General Case

In the general case, we print the elements of the (smaller) remaining list in reverse order and then print the current element

Using Recursion with a Linked List

```
void  RevPrint (NodeType*   head)

// Pre:  head points to an element of a list
// Post: All elements of list pointed to by head have
//   been printed in reverse order.
```

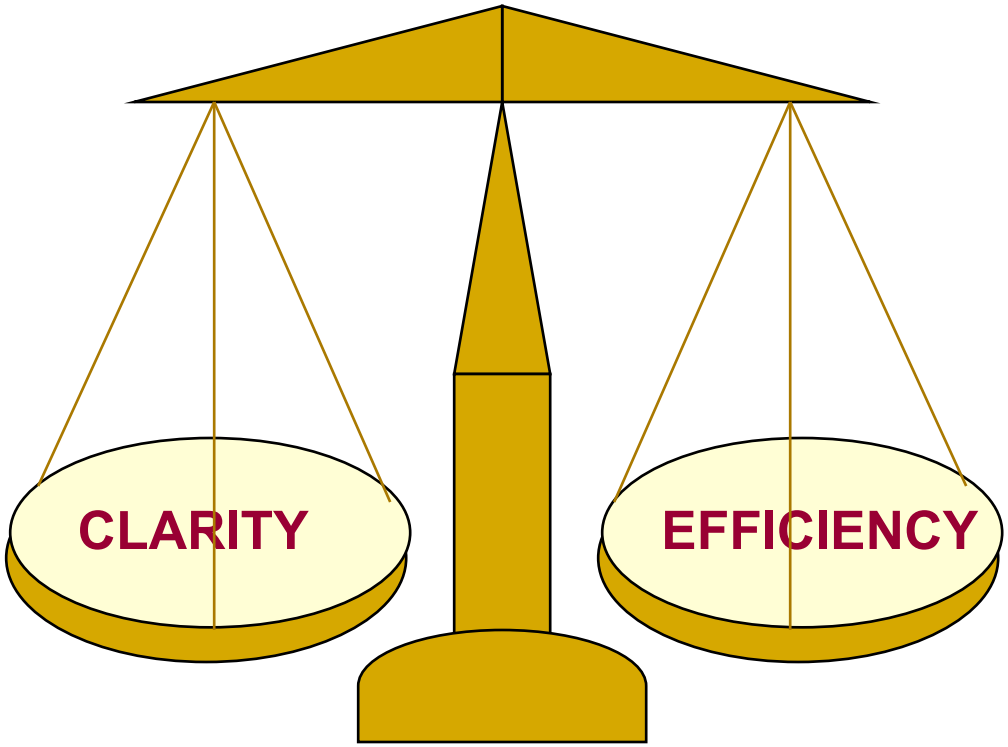
Using Recursion with a Linked List

```
{
    if (head != NULL) // General case
    {
        RevPrint (head-> link); // Process the rest
        // Print current
        cout << head->component << endl;
    }
    // Base case : if the list is empty, do nothing
}
```

Recall that . . .

- **Recursion occurs when a function calls itself (directly or indirectly)**
- **Recursion can be used in place of iteration (looping)**
- **Some functions can be written more easily using recursion**

Recursion or Iteration?



What is the value of `Rose(25)`*?*

```
int  Rose (int  n)
{
    if  (n == 1)  // Base case
        return  0;
    else              // General case
        return (1 + Rose(n / 2));
}
```

Finding the Value of `Rose(25)`

Rose(25)	***the original call***
= 1 + Rose(12)	***first recursive call***
= 1 + (1 + Rose(6))	***second recursive call***
= 1 + (1 + (1 + Rose(3)))	***third recursive call***
= 1 + (1 + (1 + (1 + Rose(1))))	***fourth recursive call***
= 1 + 1 + 1 + 1 + 0	
= 4	

Writing Recursive Functions

- There must be at least one base case and at least one general (recursive) case--the general case should bring you "closer" to the base case
- The arguments(s) in the recursive call cannot all be the same as the formal parameters in the heading
- Otherwise, infinite recursion would occur

Writing Recursive Functions

- In function `Rose()`, the base case occurred when `(n == 1)` was true
- The general case brought us a step closer to the base case, because in the general case the call was to `Rose(n/2)`,
- And the argument `n/2` was closer to 1 (than `n` was)

When a function is called...

- A transfer of control occurs from the calling block to the code of the function
- It is necessary that there be a return to the correct place in the calling block after the function code is executed
- This correct place is called the return address
- When any function is called, the run-time stack is used--activation record for the function call is placed on the stack

Stack Activation Record

- The activation record (stack frame) contains:
- the return address for this function call;
- the parameters;
- local variables;
- and space for the function's return value (if non-void)

Stack Activation Record

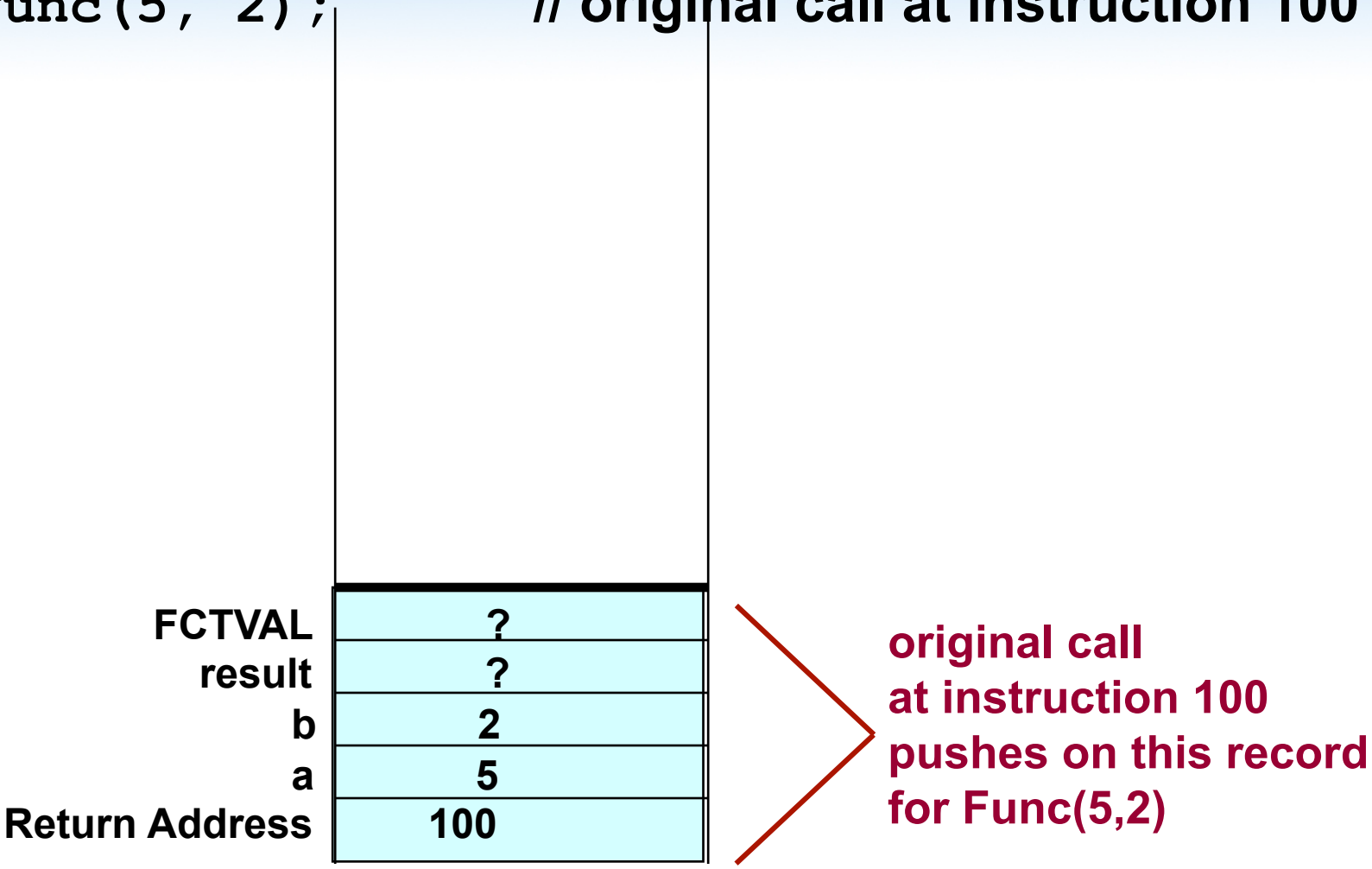
- The activation record for a particular function call is popped off the run-time stack when final closing brace in the function code is reached,
- Or when a return statement is reached in the function code
- At this time the function's return value, if non-void, is brought back to the calling block return address for use there

```
// Another recursive function

int Func (/* in */  int  a, /* in */  int  b)

//  Pre:  Assigned(a)  &&  Assigned(b)
//  Post: Return value == ??
```

```
{
    int  result;

    if  (b == 0)        // Base case

        result = 0;

    else if  (b > 0) // First general case

        result = a + Func (a, b - 1));
          // Say location 50

    else                // Second general case
        result = Func (- a, - b);
          // Say location 70

    return  result;
}
```

Run-Time Stack Activation Records

```
x = Func(5, 2);
```
// original call at instruction 100



Run-Time Stack Activation Records

```
x = Func(5, 2);
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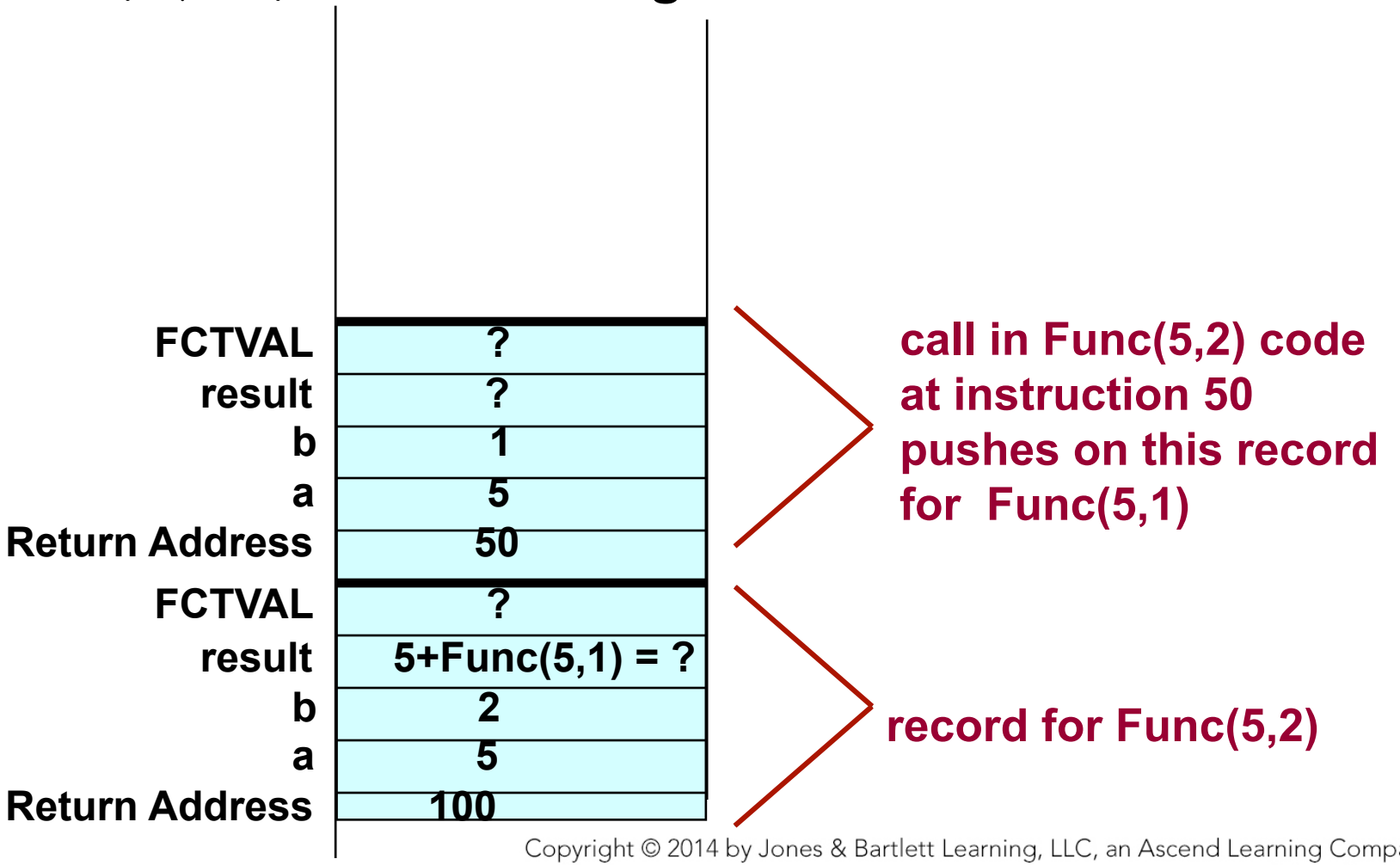

Run-Time Stack Activation Records

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x = Func(5, 2);
```
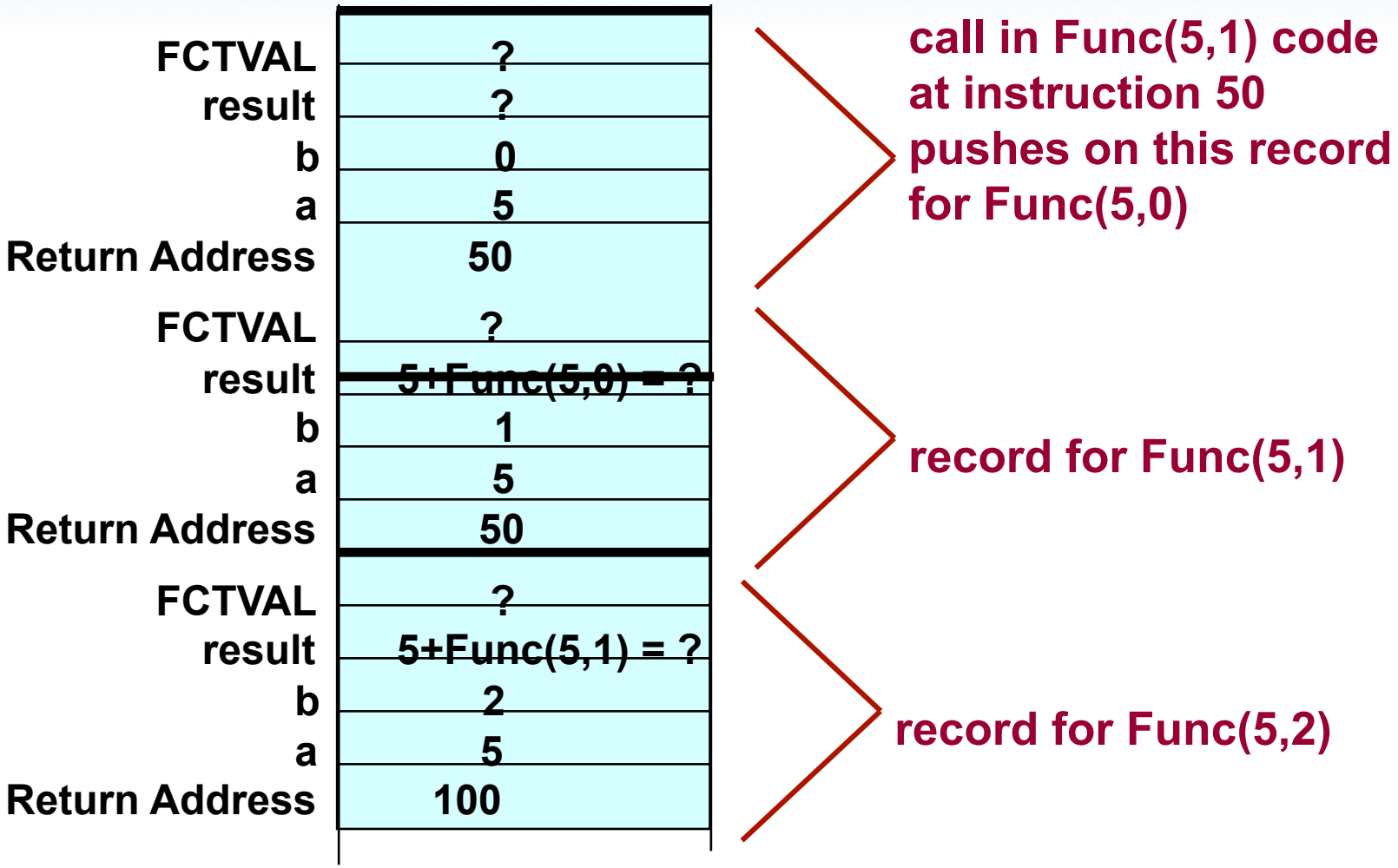
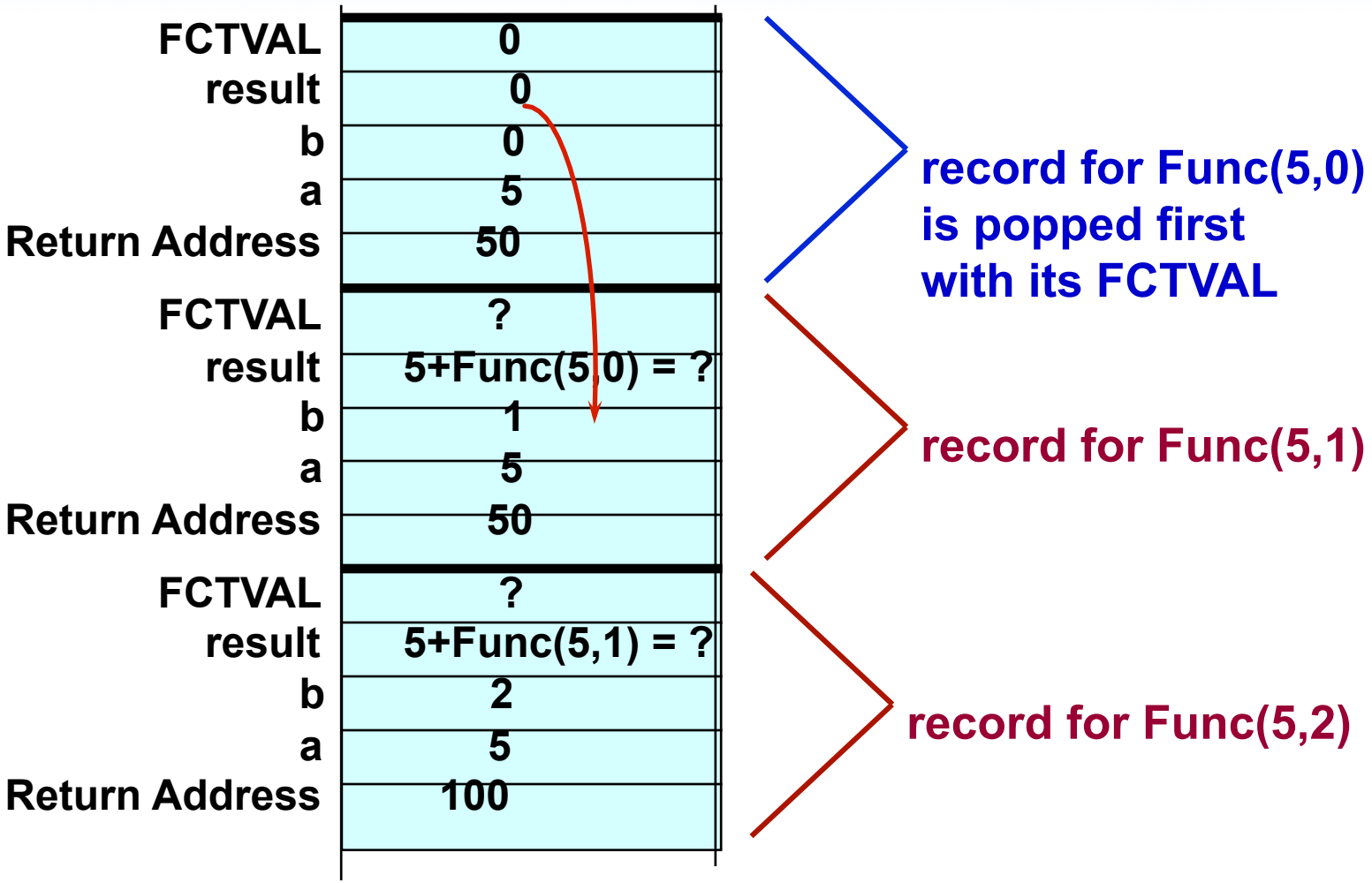
// original call at instruction 100



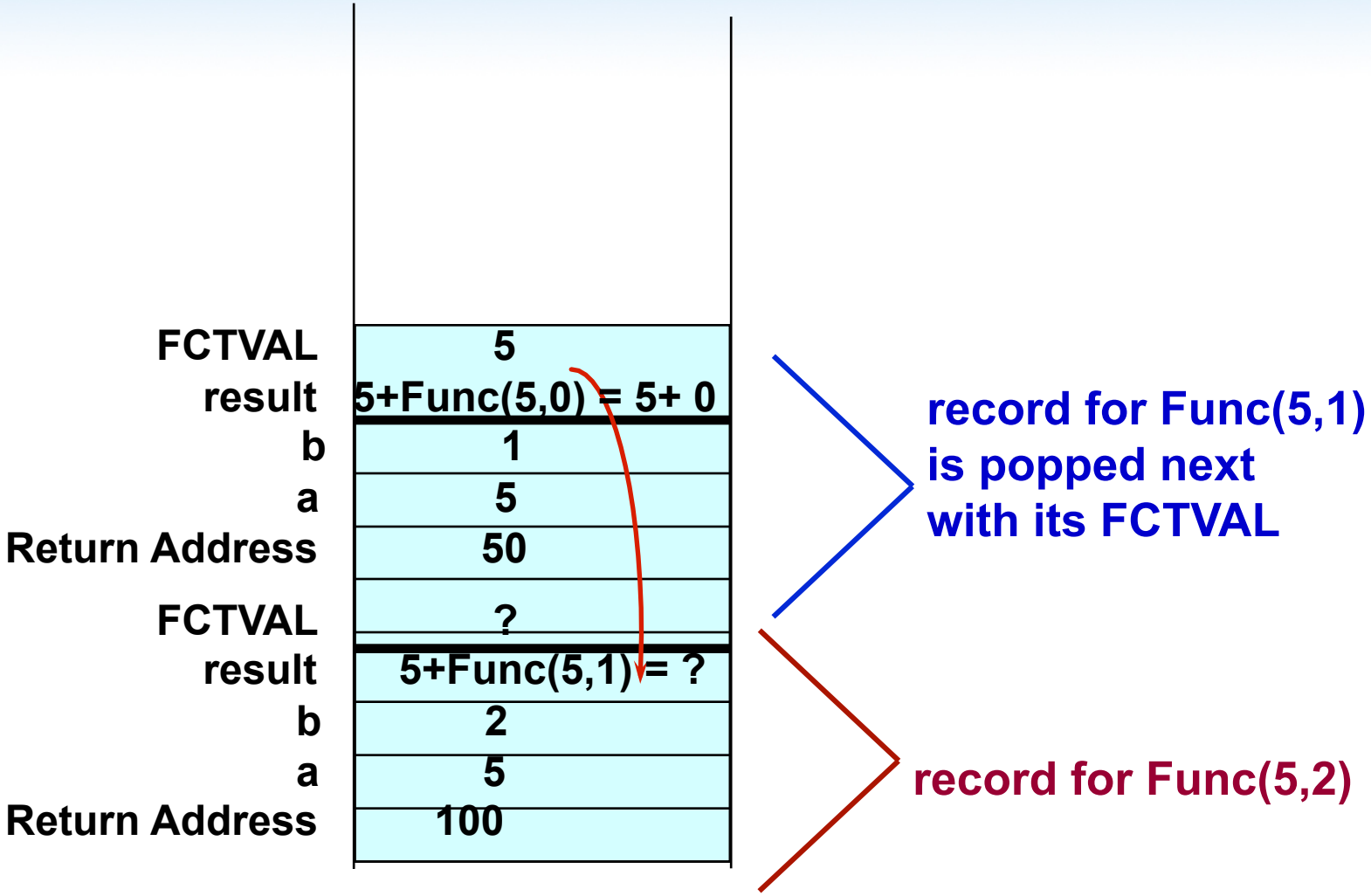
Run-Time Stack Activation Records

```
x = Func(5, 2);
```
// original call at instruction 100



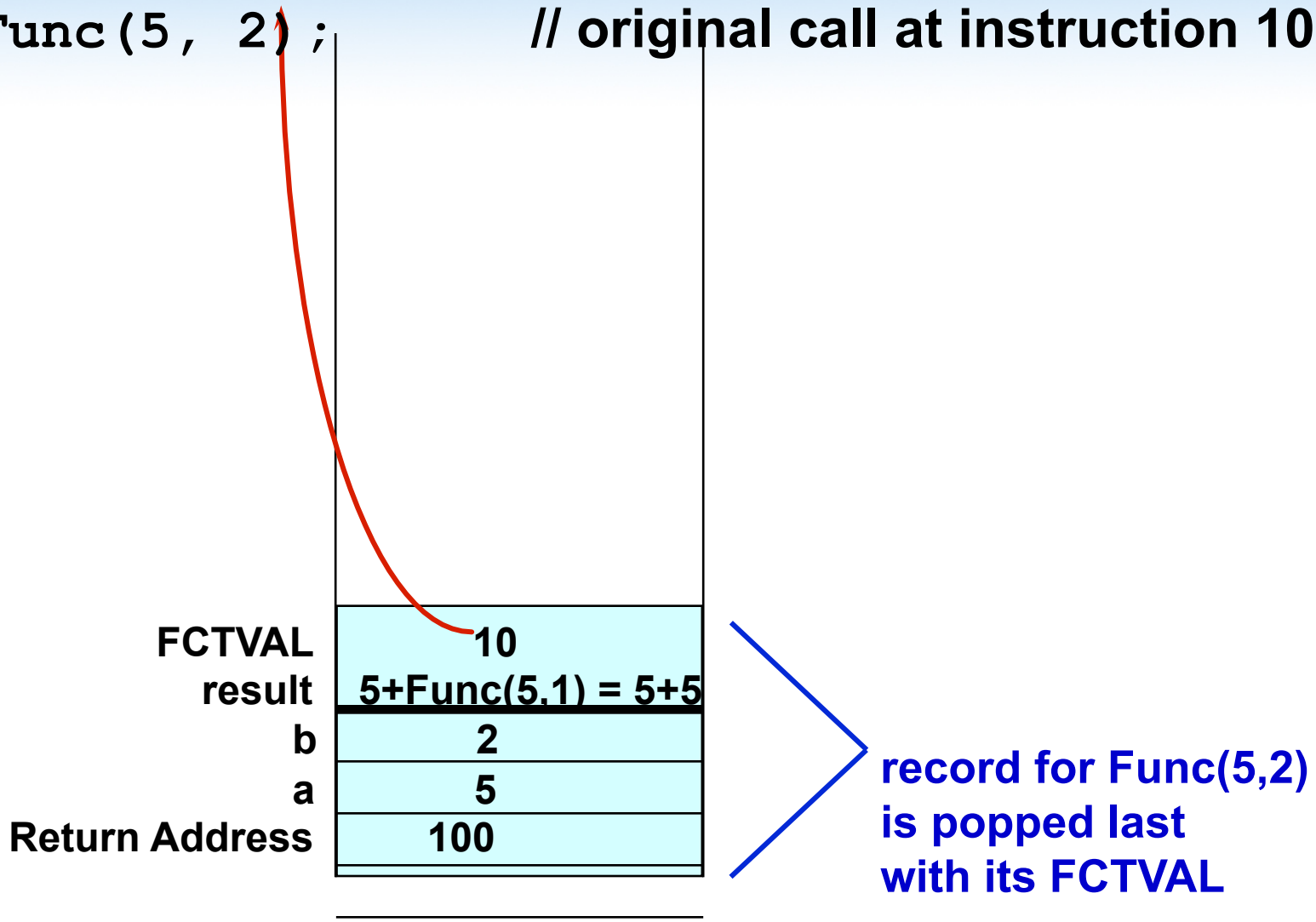
Run-Time Stack Activation Records

```
x = Func(5, 2);
```
// original call at instruction 100



Run-Time Stack Activation Records

```
x = Func(5, 2);
```
// original call at instruction 100



Show Activation Records for these calls

x = Func(- 5, - 3);

x = Func(5, - 3);

What operation does Func(a, b) simulate?

Write a function . . .

- Write a function that takes an array `a` and two subscripts, `low` and `high` as arguments, and returns the sum of the elements

```
a[low]+..+ a[high]
```

- Write the function two ways - - one using iteration and one using recursion

- For your recursive definition's base case, for what kind of array do you know the value of `Sum(a, low, high)` right away?

```
// Recursive definition

int  Sum ( /* in */  const  int  a[ ],
           /* in */    int   low,
           /* in */    int   high)

//  Pre:  Assigned(a[low..high]) && low <= high
//  Post: Return value == sum of items a[low..high]
{
    if  (low == high) // Base case

        return  a [low];


    else                    // General case

        return  a [low] + Sum(a, low + 1, high);

}
```

```
// Iterative definition


int  Sum ( /* in */  const  int  a[ ],
           /* in */   int   high)

//  Pre:  Assigned(a[0..high])
//  Post: Return value == sum of items a[0..high]
{
    int sum = 0;
    for (int index= 0; index <= high; index++)

        sum = sum + a[index];
    return sum;

}
```

Write a function . . .

- Write a LinearSearch that takes an array `a` and two subscripts, `low` and `high`, and a `key` as arguments R

- It returns true if `key` is found in the elements `a[low...high]`; otherwise, it returns false

Write a function . . .

- **Write the function two ways - - using iteration and using recursion**

- **For your recursive definition's base case(s), for what kinds of arrays do you know the value of `LinearSearch(a, low, high, key)` right away?**

```
// Recursive definition

bool LinearSearch
    (/* in */ const int a[ ],
      /* in */   int   low,
      /* in */   int   high,
      /* in */   int   key)
// Pre: Assigned(a[low..high])
// Post: IF (key in a[low..high])
//           Return value is true,
//       else return value is false
```

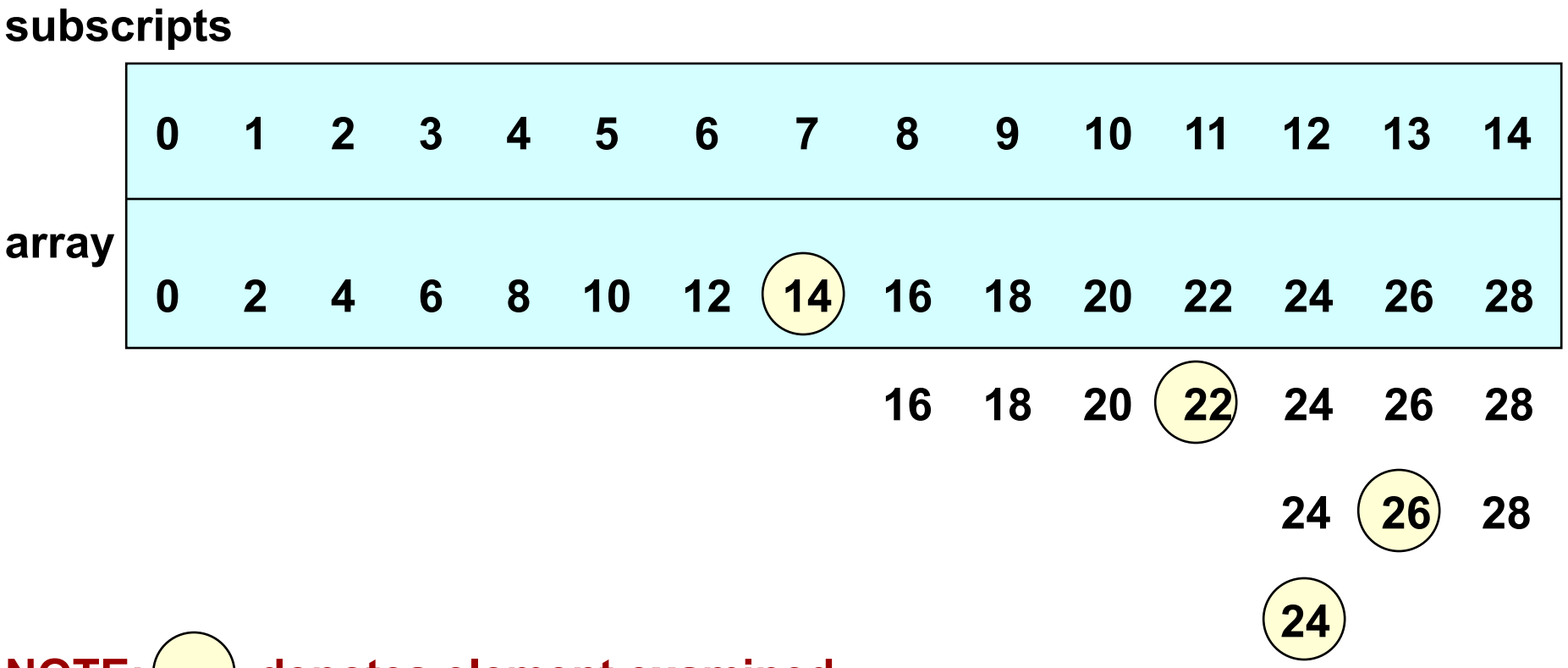
```
{
    if  (a [ low ] == key) // Base case
        return true;
    else if (low == high) // Second base case
        return  false;
    else                        // General case
        return
          LinearSearch(a, low + 1, high, key);
}
```

Function `BinarySearch()`

- `BinarySearch` that takes sorted array `a`, and two subscripts, `low` and `high`, and a `key` as arguments
- It returns true if `key` is found in the elements `a[low...high]`, otherwise, it returns false
- `BinarySearch` can also be written using iteration or recursion, but it is an inherently recursive algorithm

```
x = BinarySearch(a, 0, 14, 25);
```

low high key



NOTE: ◯ denotes element examined

```
// Recursive definition
bool BinarySearch (/* in */ const int  a[ ],
                   /* in */ int  low,
                   /* in */ int  high,
                   /* in */ int   key)
// Pre:  a[low .. high] in ascending order && Assigned
   (key)
// Post: IF (key in a[low . . high]), return value is true
//   otherwise return value is false
```

```
{
    int  mid;
    if  (low > high)
        return false;
    else
    {
        mid = (low + high) / 2;
        if (a [ mid ] == key)
            return  true;
        else if (key < a[mid]) // Look in lower half
            return BinarySearch(a, low, mid-1, key);
        else  // Look in upper half
            return BinarySearch(a, mid+1, high, key);
    }
}
```