Quadratic Machine Homework

Perceptron Configuration:

- **Inputs:** x, y, xy, x^2, y^2
- **Weights:** w_0 (bias), w_1 (x), w_2 (y), w_3 (xy), w_4 (x^2), w_5 (y^2)
- **Activation Function:** Output z = 1 if net > 0, else z = 0
- **Learning Rate (c):** 0.5
- **Initial Weights:** All weights are initialized to 0.

Training Set:

Pattern	x	y	Target (t)
1	0	0.4	0
2	-0.1	1.2	1
3	0.5	0.8	0

Weight Update Rule:

$\Delta wi = c \times (t - z) \times xi$

- For the bias (w_0), xi = 1.

Training Process:

Initial Weights:

$w = [w_0, w_1, w_2, w_3, w_4, w_5] = [0, 0, 0, 0, 0, 0]$

Pattern 1: x = 0, y = 0.4, Target t = 0

1. **Compute Inputs:**

$Input_0 = 1$
$Input_1 = x = 0$
$Input_2 = y = 0.4$
$Input_3 = xy = 0 \times 0.4 = 0$
$Input_4 = x^2 = 0^2 = 0$
$Input_5 = y^2 = 0.4^2 = 0.16$

2. **Calculate Net Input:**

   $net = w_0 \times 1 + w_1 \times 0 + w_2 \times 0.4 + w_3 \times 0 + w_4 \times 0 + w_5 \times 0.16 = 0$

3. **Determine Output (z):**

   $z = 0$ (since $net \leq 0$)

4. **Calculate Error:**

   $t - z = 0 - 0 = 0$

5. **Update Weights:** Since the error is 0, no weights are updated.
6. **Weights After Pattern 1:**

   $w = [0, 0, 0, 0, 0, 0]$

Pattern 2: x = -0.1, y = 1.2, Target t = 1

1. **Compute Inputs:**

   $Input_0 = 1$
   $Input_1 = x = -0.1$
   $Input_2 = y = 1.2$
   $Input_3 = xy = -0.1 \times 1.2 = -0.12$
   $Input_4 = x^2 = (-0.1)^2 = 0.01$
   $Input_5 = y^2 = 1.2^2 = 1.44$

2. **Calculate Net Input:**

   $net = 0 + 0 \times (-0.1) + 0 \times 1.2 + 0 \times (-0.12) + 0 \times 0.01 + 0 \times 1.44 = 0$

3. **Determine Output (z):**

   $z = 0$

4. **Calculate Error:**

   $t - z = 1 - 0 = 1$

5. **Update Weights:**

   $\Delta w_0 = 0.5 \times 1 \times 1 = 0.5$
   $\Delta w_1 = 0.5 \times 1 \times (-0.1) = -0.05$
   $\Delta w_2 = 0.5 \times 1 \times 1.2 = 0.6$
   $\Delta w_3 = 0.5 \times 1 \times (-0.12) = -0.06$
   $\Delta w_4 = 0.5 \times 1 \times 0.01 = 0.005$
   $\Delta w_5 = 0.5 \times 1 \times 1.44 = 0.72$

   Updated Weights:

   $w_0 = 0 + 0.5 = 0.5$
   $w_1 = 0 - 0.05 = -0.05$
   $w_2 = 0 + 0.6 = 0.6$
   $w_3 = 0 - 0.06 = -0.06$
   $w_4 = 0 + 0.005 = 0.005$
   $w_5 = 0 + 0.72 = 0.72$
6. **Weights After Pattern 2:**

   w = [0.5, -0.05, 0.6, -0.06, 0.005, 0.72]

Pattern 3: x = 0.5, y = 0.8, Target t = 0

1. **Compute Inputs:**

   $Input_0 = 1$
   $Input_1 = x = 0.5$
   $Input_2 = y = 0.8$
   $Input_3 = xy = 0.5 \times 0.8 = 0.4$
   $Input_4 = x^2 = 0.5^2 = 0.25$
   $Input_5 = y^2 = 0.8^2 = 0.64$
2. **Calculate Net Input:**

   $net = w_0 \times 1 + w_1 \times 0.5 + w_2 \times 0.8 + w_3 \times 0.4 + w_4 \times 0.25 + w_5 \times 0.64$
   $net = 0.5 \times 1 + (-0.05) \times 0.5 + 0.6 \times 0.8 + (-0.06) \times 0.4 + 0.005 \times 0.25 + 0.72 \times 0.64$
   $net = 0.5 - 0.025 + 0.48 - 0.024 + 0.00125 + 0.4608 = 1.39305$
3. **Determine Output (z):**

   z = 1 (since net = 1.39305 > 0)

4. **Calculate Error:**

   $t - z = 0 - 1 = -1$

5. **Update Weights:**

   $\Delta w_0 = 0.5 \times (-1) \times 1 = -0.5$
   $\Delta w_1 = 0.5 \times (-1) \times 0.5 = -0.25$
   $\Delta w_2 = 0.5 \times (-1) \times 0.8 = -0.4$
   $\Delta w_3 = 0.5 \times (-1) \times 0.4 = -0.2$
   $\Delta w_4 = 0.5 \times (-1) \times 0.25 = -0.125$
   $\Delta w_5 = 0.5 \times (-1) \times 0.64 = -0.32$

   Updated Weights:

   $w_0 = 0.5 - 0.5 = 0$
   $w_1 = -0.05 - 0.25 = -0.3$
   $w_2 = 0.6 - 0.4 = 0.2$
   $w_3 = -0.06 - 0.2 = -0.26$
   $w_4 = 0.005 - 0.125 = -0.12$
   $w_5 = 0.72 - 0.32 = 0.4$

6. **Weights After Pattern 3:**

   $w = [0, -0.3, 0.2, -0.26, -0.12, 0.4]$

Summary of Weight Updates After Each Pattern:

After Pattern	w_0	w_1	w_2	w_3	w_4	w_5
Initial	0	0	0	0	0	0
After Pattern 1	0	0	0	0	0	0
After Pattern 2	0.5	-0.05	0.6	-0.06	0.005	0.72
After Pattern 3	0	-0.3	0.2	-0.26	-0.12	0.4

Final Weights After One Epoch:

$w = [0, -0.3, 0.2, -0.26, -0.12, 0.4]$

These weights will be used in the next epoch for further training.