Backpropogation Homework

Updated Weights After First Pattern:

- $w_{14} = w_{15} = 0.4896$
- $w_{24} = w_{25} = 0.4931$
- $w_{34} = w_{35} = 0.4885$
- $w_{47} = w_{57} = 0.3964$
- $w_{67} = 0.3667$

Second Pattern Input and Target:

- Inputs: $x_1 = -1$, $x_2 = 0.4$
- Target Output: $t = 0.2$

Forward Pass:

1. **Compute Net Inputs to Hidden Nodes:**

$$\begin{aligned}\text{net}_4 &= x_1 w_{14} + x_2 w_{24} + 1 \times w_{34} \\ &= (-1)(0.4896) + (0.4)(0.4931) + 1 \times 0.4885 \\ &= -0.4896 + 0.19724 + 0.4885 \\ &= 0.19614\end{aligned}$$

   Similarly, $\text{net}_5 = \text{net}_4 = 0.19614$

2. **Compute Activations of Hidden Nodes:**

$$\begin{aligned}z_4 &= \frac{1}{1 + e^{-\text{net}_4}} = \frac{1}{1 + e^{-0.19614}} = 0.54888 \\ z_5 &= z_4 = 0.54888\end{aligned}$$

3. **Compute Net Input to Output Node:**

$$\begin{aligned}\text{net}_7 &= z_4 w_{47} + z_5 w_{57} + 1 \times w_{67} \\ &= (0.54888)(0.3964) + (0.54888)(0.3964) + 0.3667 \\ &= 0.2176 + 0.2176 + 0.3667 \\ &= 0.8019\end{aligned}$$

4. **Compute Activation of Output Node:**

$$z_7 = \frac{1}{1 + e^{-\text{net}_7}} = \frac{1}{1 + e^{-0.8019}} = 0.6904$$

Backward Pass:

1. **Compute Error at Output Node:**

$$\begin{aligned}\delta_7 &= (t - z_7) \times z_7(1 - z_7) \\ &= (0.2 - 0.6904) \times 0.6904 \times (1 - 0.6904) \\ &= (-0.4904) \times 0.6904 \times 0.3096 \\ &= -0.1048\end{aligned}$$

2. **Compute Errors at Hidden Nodes:**

$$\begin{aligned}\delta_4 &= \delta_7 w_{47} \times z_4(1 - z_4) \\ &= (-0.1048)(0.3964) \times 0.54888 \times 0.45112 \\ &= -0.01029 \\ \delta_5 &= \delta_4 = -0.01029\end{aligned}$$

Weight Updates:

1. **Update Weights from Input to Hidden Layer:**
   - $\Delta w_{14} = \eta \delta_4 x_1 = 1 \times (-0.01029) \times (-1) = 0.01029$

$$\begin{aligned}w_{14}^{\text{new}} &= w_{14} + \Delta w_{14} \\ &= 0.4896 + 0.01029 \\ &= 0.49989\end{aligned}$$

   - $\Delta w_{24} = \eta \delta_4 x_2 = 1 \times (-0.01029) \times 0.4 = -0.004116$

$$\begin{aligned}w_{24}^{\text{new}} &= w_{24} + \Delta w_{24} \\ &= 0.4931 - 0.004116 \\ &= 0.4890\end{aligned}$$

   - $\Delta w_{34} = \eta \delta_4 \times 1 = -0.01029$

$$\begin{aligned}w_{34}^{\text{new}} &= w_{34} + \Delta w_{34} \\ &= 0.4885 - 0.01029 \\ &= 0.47821\end{aligned}$$

   - Similarly for w_{15}, w_{25}, and w_{35}:

$$
\begin{aligned}
w_{15}^{\text{new}} &= 0.49989 \\
w_{25}^{\text{new}} &= 0.4890 \\
w_{35}^{\text{new}} &= 0.47821
\end{aligned}
$$

2. **Update Weights from Hidden to Output Layer:**
   - $\Delta w_{47} = \eta \delta_7 z_4 = 1 \times (-0.1048) \times 0.54888 = -0.0575$

$$
\begin{aligned}
w_{47}^{\text{new}} &= w_{47} + \Delta w_{47} \\
&= 0.3964 - 0.0575 \\
&= 0.3389
\end{aligned}
$$

   - $\Delta w_{57} = \Delta w_{47}$

$$
\begin{aligned}
w_{57}^{\text{new}} &= w_{57} + \Delta w_{57} \\
&= 0.3964 - 0.0575 \\
&= 0.3389
\end{aligned}
$$

   - $\Delta w_{67} = \eta \delta_7 \times 1 = -0.1048$

$$
\begin{aligned}
w_{67}^{\text{new}} &= w_{67} + \Delta w_{67} \\
&= 0.3667 - 0.1048 \\
&= 0.2619
\end{aligned}
$$

Updated Weights After Second Pattern:

- $w_{14}^{\text{new}} = w_{15}^{\text{new}} = 0.49989$
- $w_{24}^{\text{new}} = w_{25}^{\text{new}} = 0.4890$
- $w_{34}^{\text{new}} = w_{35}^{\text{new}} = 0.47821$
- $w_{47}^{\text{new}} = w_{57}^{\text{new}} = 0.3389$
- $w_{67}^{\text{new}} = 0.2619$