Naïve Bayes Homework

1. Statistics Needed for Naïve Bayes

Prior Probabilities:

- $P(P) = \frac{\text{Number of Positive Instances}}{\text{Total Number of Instances}} = \frac{4}{7}$
- $P(N) = \frac{\text{Number of Negative Instances}}{\text{Total Number of Instances}} = \frac{3}{7}$

Conditional Probabilities:

Given Output = P:

- $P(\text{Size} = L|P) = \frac{2}{4} = 0.5$
- $P(\text{Size} = S|P) = \frac{2}{4} = 0.5$
- $P(\text{Color} = R|P) = \frac{1}{4} = 0.25$
- $P(\text{Color} = B|P) = \frac{3}{4} = 0.75$
- $P(\text{Color} = G|P) = \frac{0}{4} = 0$

Given Output = N:

- $P(\text{Size} = L|N) = \frac{2}{3} \approx 0.6667$
- $P(\text{Size} = S|N) = \frac{1}{3} \approx 0.3333$
- $P(\text{Color} = R|N) = \frac{1}{3} \approx 0.3333$
- $P(\text{Color} = B|N) = \frac{1}{3} \approx 0.3333$
- $P(\text{Color} = G|N) = \frac{1}{3} \approx 0.3333$

2. Best Output for Small and Blue

We compute the posterior probabilities for each class:

- **For Output = P:**

$$\begin{aligned}\text{Score}_P &= P(P) \times P(\text{Size} = S|P) \times P(\text{Color} = B|P) \\ &= \frac{4}{7} \times 0.5 \times 0.75 \\ &= \frac{4}{7} \times 0.375 \\ &= \frac{1.5}{7} \approx 0.2143\end{aligned}$$

- **For Output = N:**

$$\begin{aligned}\text{Score}_N &= P(N) \times P(\text{Size} = S|N) \times P(\text{Color} = B|N) \\ &= \frac{3}{7} \times 0.3333 \times 0.3333 \\ &= \frac{3}{7} \times 0.1111 \\ &= \frac{0.3333}{7} \approx 0.0476\end{aligned}$$

Since $\text{Score}_P > \text{Score}_N$, the best output for a new instance which is Small and Blue is **P**.

3. True Probability for Each Output Class

Compute the normalized posterior probabilities:

- **Total Score:**

$$\text{Total} = \text{Score}_P + \text{Score}_N = \frac{1.5}{7} + \frac{0.3333}{7} = \frac{1.8333}{7}$$

- **Probability for Output = P:**

$$P(P|\text{S,B}) = \frac{\text{Score}_P}{\text{Total}} = \frac{\frac{1.5}{7}}{\frac{1.8333}{7}} = \frac{1.5}{1.8333} \approx 0.8182$$

- **Probability for Output = N:**

$$P(N|\text{S,B}) = \frac{\text{Score}_N}{\text{Total}} = \frac{\frac{0.3333}{7}}{\frac{1.8333}{7}} = \frac{0.3333}{1.8333} \approx 0.1818$$

Therefore,

- $P(P|\text{Small, Blue}) \approx 81.82\%$

- $P(N|\text{Small, Blue}) \approx 18.18\%$