Internal Assessment: The Impact of the Sphere's Radius on the Sphere's Angular Velocity

IB Physics II Period 6, Dr. Petach

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1 Exploration

1.1 Research

The aim of the experiment is to investigate the relationship between radius and angular velocity for the linear motion of a sphere unraveling from a string at a fixed height. This will be done by changing the radius of the sphere that is being dropped through the use of various sizes balls. They are then unraveled from the string and fall through a photogate, which measures their linear velocity. The purpose of the string is to cause the ball to rotate while falling, due to the nature of its unraveling motion.

Prior to the experiment, I derived a relationship between the radius and the angular velocity of the rotational motion of the falling ball by using the law of conservation of energy. Section 2.1 will more specifically detail the derivation. The derived relationship predicted that radius and angular velocity will follow an inverse relationship. The experiment itself was to test whether Newtonian physics upheld this inverse relationship with tangible experimentation.

1.2 Personal Engagement

I have always been a fan of yo-yo's. One day while playing with a yo-yo, I noticed how quickly the yo-yo was spinning, which was made especially conspicuous by the bright design on the side of the yo-yo. I became curious, did the size of the yo-yo affect how quickly it spun? I took a smaller yo-yo and noticed that it seemed to spin more quickly. However, this was just a casual observation, not performed under standard experimentation conditions. Interested in seeing if this was truly the case, I came up with my research question: does the radius of a sphere affect its angular velocity?

1.3 Variables

The independent variable is the radius of the sphere. The dependent variable is the angular velocity of the ball as it passes through the photogate. The controlled variables include the height of the initial position of the ball that is attached to the clamp, material of the string, the photogate, and the properties of the surrounding environment.

1.4 Apparatus

- Photogate
- Four different sized balls that are relatively uniform spheres
- String
- DataWorks software that reads data from the photogate

- Ring stand with clamp
- Meter stick

1.5 Procedure

1. Attach a clamp to a ringstand
2. Place a photogate toward the bottom of the ringstand so that the ball will drop through it.
3. Connect the photogate to the DataWorks software, such that it reads the linear velocity of the falling ball
4. Measure the height difference between the placement of the clamp and the photogate along the ring stand using a meter stick
5. Pick a ball, measure its radius using a meter stick
6. Tie one end of the string to a clamp attached to a ring stand, and the other end around the ball
7. Carefully wrap the string around the ball until the ball is level with the clamp
8. Drop the ball so that it falls through the photogate
9. Record the linear velocity that is measured by the photogate
10. Repeat steps 6 to 9 for six trials
11. Repeat steps 5 through 10 for the four different balls

1.6 Experiment Setup

Figure 1: Picture of the experiment setup

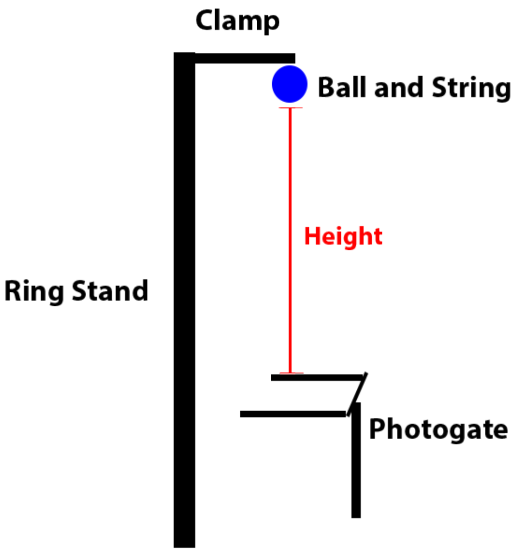

Figure 2: Diagram of the experiment setup

1.7 Data Collection

Trial	Radius (cm)	Linear Velocity (m/s)
1	2.34 ± 0.05	1.86
2	2.34 ± 0.05	1.81
3	2.34 ± 0.05	3.10
4	2.34 ± 0.05	2.50
5	2.34 ± 0.05	2.87
6	2.34 ± 0.05	2.20

Table 1: Data for six trials with ball radius of 2.34 cm and height of 50.00 ± 0.05 cm.

Trial	Radius (cm)	Linear Velocity (m/s)
1	1.90 ± 0.05	2.12
2	1.90 ± 0.05	2.38
3	1.90 ± 0.05	2.98
4	1.90 ± 0.05	2.23
5	1.90 ± 0.05	2.35
6	1.90 ± 0.05	2.23

Table 2: Data for six trials with ball radius of 1.90 cm and height of 50.00 ± 0.05 cm.

Trial	Radius (cm)	Linear Velocity (m/s)
1	1.69 ± 0.05	3.32
2	1.69 ± 0.05	3.18
3	1.69 ± 0.05	2.22
4	1.69 ± 0.05	3.91
5	1.69 ± 0.05	2.85
6	1.69 ± 0.05	3.63

Table 3: Data for six trials with ball radius of 1.69 cm and height of 50.00 ± 0.05 cm.

Trial	Radius (cm)	Linear Velocity (m/s)
1	1.40 ± 0.05	2.91
2	1.40 ± 0.05	3.62
3	1.40 ± 0.05	3.62
4	1.40 ± 0.05	2.31
5	1.40 ± 0.05	3.67
6	1.40 ± 0.05	3.41

Table 4: Data for six trials with ball radius of 1.40 cm and height of 50.00 ± 0.05 cm.

2 Analysis

2.1 Deriving the relationship between radius and angular velocity

A relationship can be found between radius and angular velocity by using the law of conservation of energy. Effects of thermal energy (friction) from the string are negligible.

$$GPE = KE_T + KE_R$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}m(R\omega)^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\omega^2$$

$$mgh = \frac{1}{2}mR^2\omega^2 + \frac{1}{5}mR^2\omega^2$$

The masses can be eliminated:

$$gh = \frac{7}{10}R^2\omega^2$$

$$\omega^2 = \frac{10gh}{7R^2}$$

$$\omega = \sqrt{\frac{10gh}{7R^2}}$$

$$\boxed{\omega = \frac{1}{R} \cdot \sqrt{\frac{10gh}{7}}} \tag{1}$$

2.2 Averaging the six trials for each ball size and finding uncertainty

Uncertainty for linear velocity can be calculated by dividing the linear velocity range by 2. Calculations for the average linear velocity and uncertainty for the four ball sizes will be shown below:

For ball with radius 2.34 cm:

$$\frac{3.10 - 1.81}{2} = \pm 0.65$$

$$v = \frac{1.86 + 1.81 + 3.10 + 2.50 + 2.87 + 2.20}{6} = 2.39 \pm 0.65\frac{m}{s}$$

For ball with radius 1.90 cm:

$$\frac{2.98 - 2.12}{2} = \pm 0.43$$

$$v = \frac{2.12 + 2.38 + 2.98 + 2.23 + 2.35 + 2.23}{6} = 2.38 \pm 0.43\frac{m}{s}$$

For ball with radius 1.69 cm:

$$\frac{3.91 - 2.22}{2} = \pm 0.85$$

$$v = \frac{3.32 + 3.18 + 2.22 + 3.91 + 2.85 + 3.63}{6} = 3.19 \pm 0.85\frac{m}{s}$$

For ball with radius 1.40 cm:

$$\frac{3.67 - 2.31}{2} = \pm 0.68$$

$$v = \frac{2.91 + 3.62 + 3.62 + 2.31 + 3.67 + 3.41}{6} = 3.26 \pm 0.68\frac{m}{s}$$

2.3 Calculating angular velocity

Using the data from the experiment, the calculated angular velocity can be found from the linear velocity that is measured by the photogate, as shown in Equation 2.

$$\omega_{calc} = \frac{v}{r} \tag{2}$$

The theoretical angular velocity can be found using Equation 1. Calculations for these two ω values will be shown below:

For ball with radius 2.34 cm:

$$\omega_{theoret} = \frac{1}{0.0234} \cdot \sqrt{\frac{10 * 9.8 * 0.5000}{7}} = 113.1 \frac{rad}{s}$$

$$\omega_{calc} = \frac{2.39 \pm 0.65}{0.0234} = 100 \pm 30 \frac{rad}{s}$$

For ball with radius 1.90 cm:

$$\omega_{theoret} = \frac{1}{0.0190} \cdot \sqrt{\frac{10 * 9.8 * 0.5000}{7}} = 139.3 \frac{rad}{s}$$

$$\omega_{calc} = \frac{2.38 \pm 0.43}{0.0190} = 130 \pm 20 \frac{rad}{s}$$

For ball with radius 1.69 cm:

$$\omega_{theoret} = \frac{1}{0.0169} \cdot \sqrt{\frac{10 * 9.8 * 0.5000}{7}} = 156.6 \frac{rad}{s}$$

$$\omega_{calc} = \frac{3.19 \pm 0.85}{0.0169} = 190 \pm 50 \frac{rad}{s}$$

For ball with radius 1.40 cm:

$$\omega_{theoret} = \frac{1}{0.0140} \cdot \sqrt{\frac{10 * 9.8 * 0.5000}{7}} = 189.0 \frac{rad}{s}$$

$$\omega_{calc} = \frac{3.26 \pm 0.68}{0.0140} = 230 \pm 50 \frac{rad}{s}$$

Radius (m)	v **(m/s)**	ω_{calc} **(rad/s)**	$\omega_{theoret}$ **(rad/s)**
0.0234 ± 0.0005	2.39 ± 0.65	100 ± 30	113.1
0.0190 ± 0.0005	2.38 ± 0.43	130 ± 20	139.3
0.0169 ± 0.0005	3.19 ± 0.85	190 ± 50	156.6
0.0140 ± 0.0005	3.26 ± 0.68	230 ± 50	189.0

Table 5: Combined table with the average linear velocities, calculated and theoretical angular velocities

2.4 Analyzing experiment evidence for angular velocity versus radius

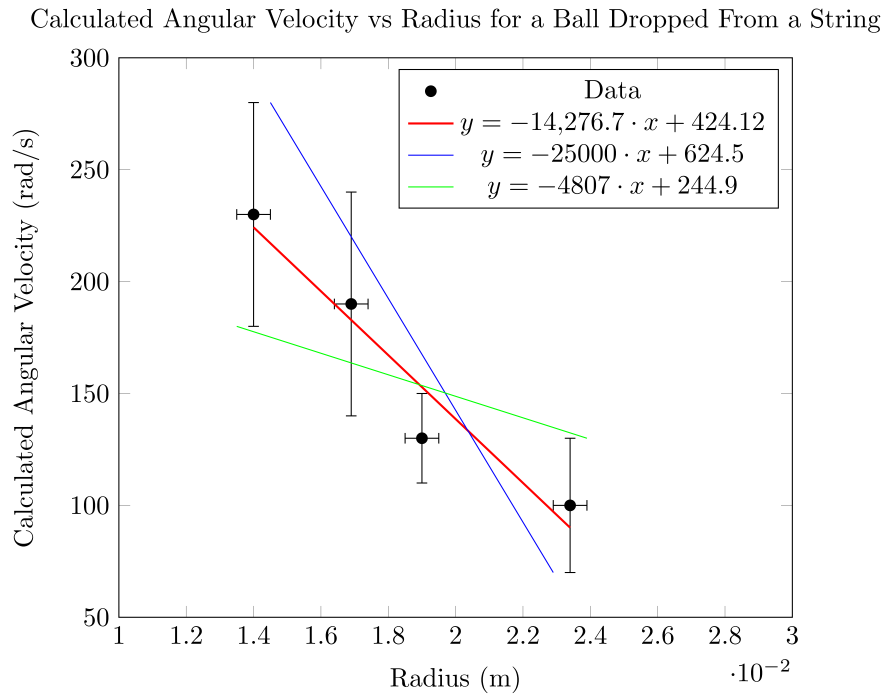

Figure 3: Graph of ω_{calc} vs radius with line of best fit, maximum slope, and minimum sloped lines.

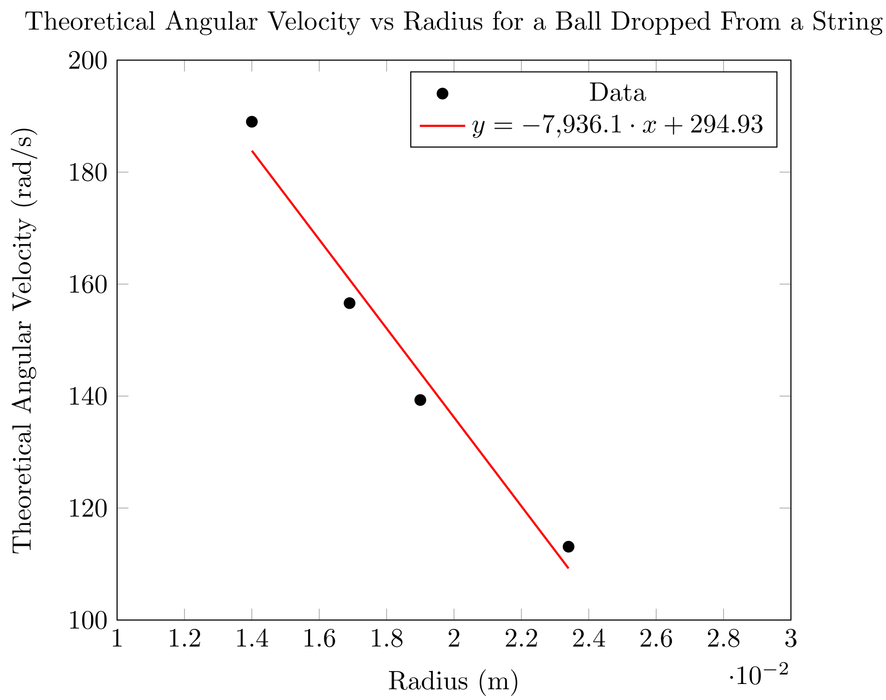

Figure 4: Graph of $\omega_{theoret}$ vs radius with line of best fit.

From Figure 3, the steepest, or maximum, slope is -25000 and the minimum -4807. The best fit line slope is -14277.

The uncertainty above the best fit line is $25000 - 14277 = 10723$. The negative can be disregarded because it is unnecessary when calculating uncertainty.
The uncertainty below the best fit line is $4807 - 14277 = -9470$

The slope of the set of data points, including uncertainty is $-14277 \cdot \frac{10723}{-9470} = \boxed{-14000 \pm 1000}$

3 Evaluation

3.1 Percent Error Analysis

The slope means that an increasing radius decreases the angular velocity by a proportionality factor of $14000 \frac{m \cdot s}{rad}$ with an uncertainty of ± 1000.

According to the slope of the best fit line in Figure 4, which is based on $\omega_{theoret}$ values from Table 5, the true proportionality factor should be 8000. Therefore a percent error analysis can be performed on the slope:

$$\frac{|8000 - 14000 \pm 1000|}{8000} \cdot 100 = 80 \pm 10\%$$

3.2 Conclusion

The relationship between radius and angular velocity is supported by the data, since the theoretical value for the slope of angular velocity versus radius is between the minimum and maximum values of the experiment. However, the errors involved were rather significant due to the large percent error, 80% .

A potential source is error is the shape of the balls that were used in the experiment. Some of the balls used were not completely spherical in shape, and had noticeable edges on the surface. This would have impacted the angular velocity, because the inertia used in the calculation was based on a spherical object. Secondly, the string that the ball unraveled from was uneven in width throughout its length due to the uneven distribution of mass. Some parts of the string also began to be worn out as the experiment continued. This would have affected the ball's path as it unraveled by changing the ball's trajectory and ultimately the height it fell. Finally, another source of error may stem from the software and photogate not measuring the angular velocity at fine enough time intervals. The graphs generated by the software displayed a spike when the ball passed through the photogate's sensors, but the point was very discrete. The lack of continuity makes it difficult to tell where precisely the spike was measured, and if it was measured in the most accurate location possible.

3.3 Improvements

There are many improvements that could be made to this experiment to fix many of the sources of error identified in Section 3.2. Firstly, balls that are visibly uniform should be picked for future experiments. This would mitigate the irregularity in ball shape, which would effect the angular velocity. In addition, a thinner string made out of a different material could be used to minimize uneven widths of the string at different points along its length. Furthermore, this experiment ignored the effects of thermal energy when deriving a relationship between radius and angular velocity, but future experimentation could take frictional force into consideration to provide even more accurate results. Finally, a different photogate could have been used to read data at much shorter intervals, and to ultimately allow for a more continuous time versus velocity graph.