Machine Learning (Homework 1)

Due date : 2022/10/21 23:59:59

1 Bayesian Linear Regression (20%)

Given the training data $\mathbf{x}$ and the corresponding label data $\mathbf{t}$, we want to predict the label t of new test point x. In other words, we want to evaluate the predictive distribution $p(t|x, \mathbf{x}, \mathbf{t})$.

A linear regression function can be expressed as below where the $\boldsymbol{\phi}(x)$ is a basis function:

$$y(x, \mathbf{w}) = \mathbf{w}^\top \boldsymbol{\phi}(x)$$

In order to make prediction of t for new test data x from the learned $\mathbf{w}$, we will

- multiply the likelihood function of new data $p(t|x, \mathbf{w})$ and the posterior distribution of training set with label set.
- take the integral over $\mathbf{w}$ to find the predictive distribution

$$\begin{aligned} p(t|x, \mathbf{x}, \mathbf{t}) &= \int_{-\infty}^{\infty} p(t, \mathbf{w}|x, \mathbf{x}, \mathbf{t}) d\mathbf{w} \\ &= \int_{-\infty}^{\infty} p(t|\mathbf{w}, x, \mathbf{x}, \mathbf{t}) p(\mathbf{w}|x, \mathbf{x}, \mathbf{t}) d\mathbf{w} \\ &= \int_{-\infty}^{\infty} p(t|x, \mathbf{w}) p(\mathbf{w}|\mathbf{x}, \mathbf{t}) d\mathbf{w}. \end{aligned}$$

Prove that the predictive distribution just mentioned is the same with the form

$$p(t|x, \mathbf{x}, \mathbf{t}) = \mathcal{N}(t|m(x), s^2(x))$$

where

$$m(x) = \beta \boldsymbol{\phi}(x)^\top \mathbf{S} \sum_{n=1}^{N} \boldsymbol{\phi}(x_n) t_n$$

$$s^2(x) = \beta^{-1} + \boldsymbol{\phi}(x)^\top \mathbf{S} \boldsymbol{\phi}(x).$$

Here, the matrix $\mathbf{S}^{-1}$ is given by $\mathbf{S}^{-1} = \alpha \mathbf{I} + \beta \sum_{n=1}^{N} \boldsymbol{\phi}(x_n) \boldsymbol{\phi}(x_n)^\top$. (20%) (hint: $p(\mathbf{w}|\mathbf{x}, \mathbf{t}) \propto p(\mathbf{t}|\mathbf{x}, \mathbf{w}) p(\mathbf{w})$ and you may use the formulas shown in page 93.)

2 Linear Regression (80%)

In this homework, you need to predict the red wine quality based on a set of features. Two learning objectives are implemented:

- Maximum likelihood approach
- Maximum *a posteriori* approach

You are given a dataset (X.csv, T.csv) to train your own linear regression model! Dataset provides total 1599 data with 11 features. Can you use these features to predict the Red Wine Quality? One might consider the following steps to start the work:

1. download and check for the dataset
2. create a new Colab or Jupyter notebook file
3. divide the dataset into training and validation

Dataset descriptions

- X.csv contains 11 different features serving as the inputs fixed acidity, volatile acidity, citric acid, residual sugar, chlorides, free sulfur dioxide, total sulfur dioxide, density, pH, sulphates, alcohol
- T.csv contains the red wine quality regarding as the targets

Specifications

- For those problems with **Code Result** at the end, you must show your result in your .ipynb file or you will get no points.
- For those problems with **Explain** at the end, you must have a clear explanation or you will get low points.
- You are also encouraged to have some discussions on those problems which are not marked as **Explain**.

2.1 Feature selection

In real-world applications, the dimension of data is usually more than one. In the training stage, please fit the data by applying a polynomial function of the form

$$y(\mathbf{x}, \mathbf{w}) = w_0 + \sum_{i=1}^{D} w_i x_i + \sum_{i=1}^{D} \sum_{j=1}^{D} w_{ij} x_i x_j \quad (M = 2)$$

and minimizing the error function

$$E(\mathbf{w}) = \frac{1}{2N} \sum_{n=1}^{N} \{y(x_n, \mathbf{w}) - t_n\}^2$$

(a) In the feature selection stage, please apply polynomials of order $M = 1$ and $M = 2$ over the input data with dimension $D = 11$. Please evaluate the corresponding RMS error on the training set and valid set. (15%) **Code Result**

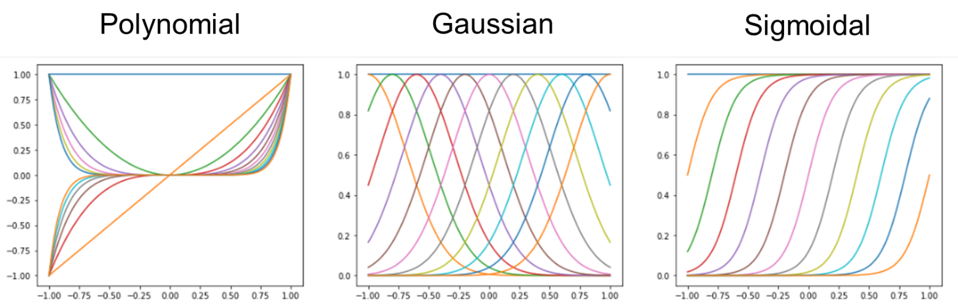
(b) How will you analysis the weights of polynomial model $M = 1$ and select the most contributive feature? **Code Result, Explain** (10%)

2.2 Maximum likelihood approach

(a) Which basis function will you use to further improve your regression model, polynomial, Gaussian, Sigmoid, or hybrid? **Explain** (5%)

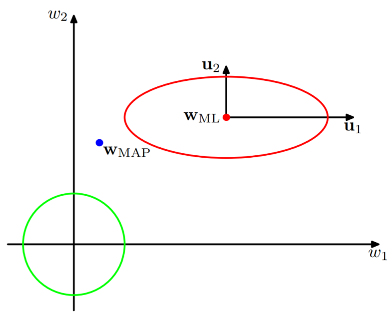
(b) Introduce the basis function you just decided in (a) to linear regression model and analyze the result you get. (Hint: You might want to discuss about the phenomenon when model becomes too complex.) **Code Result, Explain** (10%)

$$\phi(x) = [\phi_1(x), \phi_2(x), \ldots, \phi_N(x), \phi_{\text{bias}}(x)]$$



(c) Apply N-fold cross-validation in your training stage to select at least one hyperparameter (order, parameter number, ...) for model and do some discussion (underfitting, overfitting). **Code Result, Explain** (10%)

2.3 Maximum *a posteriori* approach



(a) What is the key difference between maximum likelihood approach and maximum *a posteriori* approach? **Explain** (10%)

(b) Use maximum *a posteriori* approach method to retest the model in 2.2 you designed. You could choose Gaussian distribution as a prior. **Code Result** (10%)

(c) Compare the result between maximum likelihood approach and maximum *a posteriori* approach. Is it consistent with your conclusion in (a)? **Explain** (10%)

3 Rules

- Please name the assignment as hw1_StudentID.zip (e.g. hw1_0123456.zip).
- Only **Numpy and Pandas** can be used for the Python library.
- In your submission, it needs to contain three files.
  - **.ipynb** file which contains all the results and codes for this homework. Also, it should contain the description or explanation for this homework.

  - **.py** file which is downloaded from the .ipynb file.
  - **.pdf** file which contains the handwriting parts.
- Implementation will be graded by
  - Completeness
  - Algorithm Correctness
  - Model description
  - Discussion
- Only Python implementation is acceptable.
- DO NOT PLAGIARIZE. (We will check program similarity score.)