Chapter 1 - All you need is lambda

Intermission

1. $\lambda xy.xz$
   $\lambda x.(\lambda y.xz)$
   (b)

2. $\lambda xy.xxy$
   $\lambda x.(\lambda y.xxy)$
   (c)

3. $\lambda xyz.zx$
   (b)

1.11 Exercises

Combinators:

1. Yes, all three x's in the body of the lambda are bound by the x in the head.
2. No, the z in the body of the lambda is not bound in the head.
3. Yes, each variable in the head appears in the body.
4. Yes, for the same reason as 3.
5. No, the z in the body does not appear in the head.

Normal form or diverge:

1. Normal form because the lambda $\lambda x.xxx$ is not applied to any arguments.
2. $(\lambda z.zz)(\lambda y.yy)$
   $[z := (\lambda y.yy)]$
   $(\lambda y.yy)(\lambda y.yy)$
   } Diverges. Applying the argument to the lambda $(\lambda z.zz)$ yields an expression that is alpha equivalent to the original expression, and the argument can be applied indefinitely.
3. Normal form, because the expression reduces to zzz which cannot be further applied.

Beta reduce:

1. $(\lambda abc.cba)zz(\lambda wv.w)$
   $(\lambda a.\lambda b.\lambda c.cba)zz(\lambda w.\lambda v.w)$
   $(\lambda b.\lambda c.cbz)z(\lambda w.\lambda v.w)$
   $(\lambda c.czz)(\lambda w.\lambda v.w)$
   $(\lambda w.\lambda v.w)zz$
   $(\lambda v.z)z$
   z

2. $(\lambda x.\lambda y.xyy)(\lambda a.a)b$
   $\lambda y.(\lambda a.a)yy)b$
   $(\lambda a.a)bb$
   bb

3. $(\lambda y.\, y)(\lambda x.\, xx)(\lambda z.\, zq)$
$(\lambda x.\, xx)(\lambda z.\, zq)$
$(\lambda z.\, zq)(\lambda z.\, zq)$
$(\lambda z.\, zq)\, q$
qq

alpha equivalent

4. $(\lambda z.\, z)(\lambda z.\, zz)(\lambda z.\, zy)$
$(\lambda z.\, zz)(\lambda z.\, zy)$
$(\lambda z.\, zy)(\lambda z.\, zy)$
$(\lambda z.\, zy)\, y$
yy

5. $(\lambda x.\, \lambda y.\, xyy)(\lambda y.\, y)\, y$
$(\lambda y.\, (\lambda y.\, y)yy)\, y$
$(\lambda y.\, y)yy$
yy

6. $(\lambda a.\, aa)(\lambda b.\, ba)c$
$(\lambda b.\, ba)(\lambda b.\, ba)\, c$
$(\lambda b.\, ba)\, ac$
aac

7. $(\lambda xyz.\, xz(yz))(\lambda x.\, z)(\lambda x.\, a)$
$(\lambda x.\, \lambda y.\, \lambda z.\, xz\,(yz))\,(\lambda x.\, z)(\lambda x.\, a)$
$(\lambda y.\, \lambda z.\, (\lambda x.\, z1)z(yz))(\lambda x.\, a)$
$(\lambda z.\, (\lambda x.\, z1)z((\lambda x.\, a)z))$
$(\lambda z.\, z1((\lambda x.\, a)z))$
$(\lambda z.\, z1a)$

Note: $z \neq z1$.