In[1]:=
```
$PrePrint = If[MatrixQ[#], MatrixForm[#], #] &;
Import[NotebookDirectory[] <> "../DiffGeoLib.m"]
```

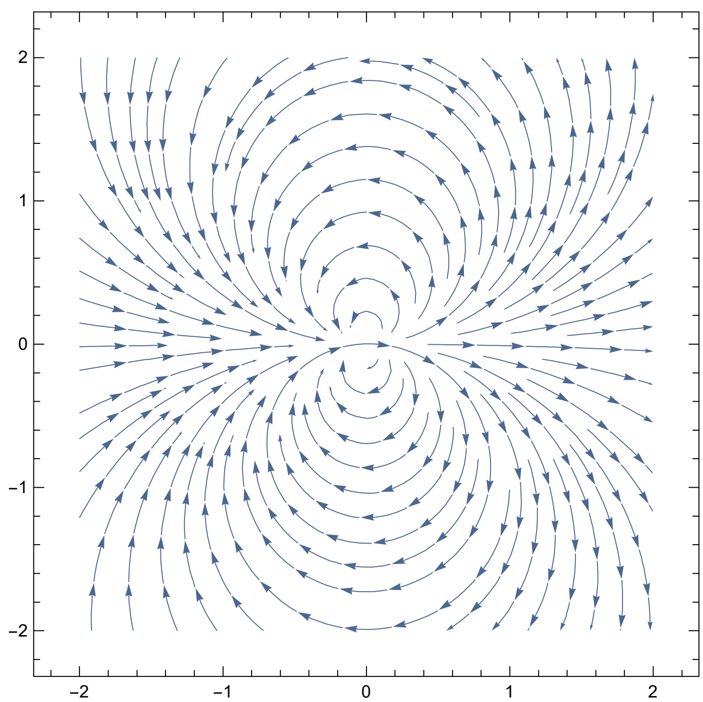
Q1.

In[ ]:=
```
gPol = {{1, 0}, {0, r^2}};
gCart = {{1, 0}, {0, 1}};
VPol = {A Cos[θ], B Sin[θ] / r};
VPolDown = Table[Sum[gPol[[i, j]] VPol[[j]], {j, 1, 2}], {i, 1, 2}]
ExampleConsts = {A → 1, B → 1};
polCo = {r, θ};
cartCo = {x, y};
polInCart = {r → Sqrt[x^2 + y^2], r Cos[θ] → x, r Sin[θ] → y, θ → ArcTan[x, y]};
cartInPol = {x → r Cos[θ], y → r Sin[θ]};
```

Out[ ]= $\{A\,\mathrm{Cos}[\theta],\ B\,r\,\mathrm{Sin}[\theta]\}$

In[ ]:=
```
(*a*)
VCart = FullSimplify[ConvertVector[VPolDown, polCo /. polInCart, cartCo] /. polInCart]
StreamPlot[VCart /. ExampleConsts, {x, -2, 2}, {y, -2, 2}]
```

Out[ ]= $\left\{\frac{A\,x^2 - B\,y^2}{x^2 + y^2},\ \frac{(A + B)\,x\,y}{x^2 + y^2}\right\}$

Out[ ]=



```
(*b*)
```

In[ ]:=
```
CovDPol = Table[CovariantD[VPol, {aa}, {}, gPol, polCo], {aa, {1, 2}}]
```

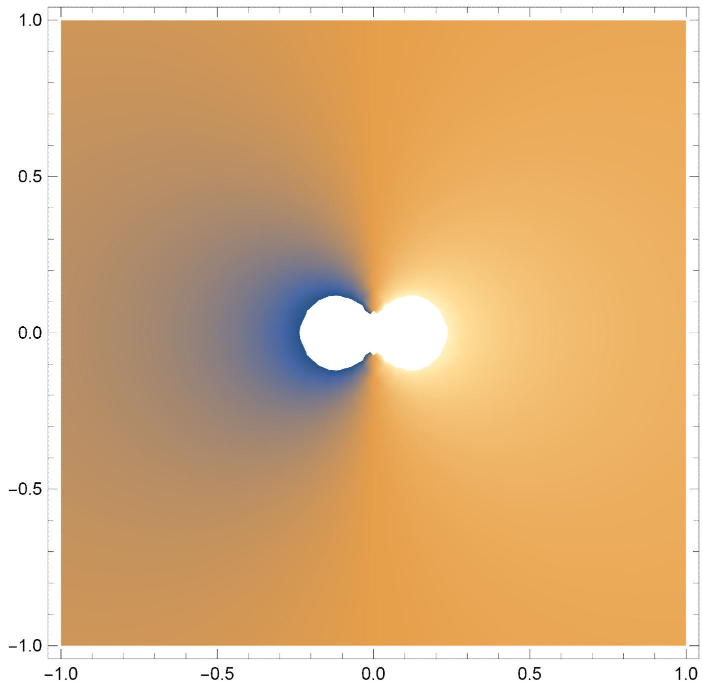
Out[ ]= $\begin{pmatrix} 0 & -A\,\mathrm{Sin}[\theta] - B\,\mathrm{Sin}[\theta] \\ 0 & \frac{A\,\mathrm{Cos}[\theta]}{r} + \frac{B\,\mathrm{Cos}[\theta]}{r} \end{pmatrix}$

In[ ]:=
```
CovDCart = FullSimplify[Table[CovariantD[VCart, {}, {aa}, gCart, cartCo], {aa, {1, 2}}]]
```

Out[ ]=
$$\begin{pmatrix} \frac{2\,(A+B)\,x\,y^2}{(x^2+y^2)^2} & -\frac{2\,(A+B)\,x^2\,y}{(x^2+y^2)^2} \\ \frac{(A+B)\,y\,(-x^2+y^2)}{(x^2+y^2)^2} & \frac{(A+B)\,x\,(x-y)\,(x+y)}{(x^2+y^2)^2} \end{pmatrix}$$

In[ ]:=
```
(*Plot the trace (divergence)*)
FullSimplify[Sum[CovDCart[[ii, ii]], {ii, 1, 2}]]
DensityPlot[% /. ExampleConsts, {x, -1, 1}, {y, -1, 1}]
```

Out[ ]=
$$\frac{(A+B)\,x}{x^2+y^2}$$

Out[ ]=


In[ ]:=
```
(*This checks if the two covariant derivatives are
 the same when converted to the same coordinate system*)

CovDPolDown = Table[Sum[CovDPol[[j, i]] gPol[[k, j]], {j, 1, 2}], {k, 1, 2}, {i, 1, 2}]

CovDCart ==
 FullSimplify[Convert2Tensor[CovDPolDown, polCo /. polInCart, cartCo] /. polInCart]
```

Out[ ]=
$$\begin{pmatrix} 0 & -A\,\mathrm{Sin}[\theta] - B\,\mathrm{Sin}[\theta] \\ 0 & r^2\left(\frac{A\,\mathrm{Cos}[\theta]}{r} + \frac{B\,\mathrm{Cos}[\theta]}{r}\right) \end{pmatrix}$$

Out[ ]= True

```
(*c*)
```

The covariant derivative is zero if B=-A. Because the zero tensor is the same in all basis, the same A and B values give zero in both polar and Cartesian basis.

Problem 4

In[ ]:=
```
ClearAll["Global`*"]
```

In[3]:=
```
g = A^2 {{1, 0}, {0, Sin[θ]^2}};
coords = {θ, ϕ};
```

In[5]:=
```
(*a*)
Γ[g, coords]
```

Out[5]= {{{0, 0}, {0, -Cos[θ] Sin[θ]}}, {{0, Cot[θ]}, {Cot[θ], 0}}}

In[6]:=
```
(*b*)
(*my code calcuates R_abc^d, so I contract weith the metric to lower the d.*)
FullSimplify[Table[Sum[g[[i, j]] RiemannTensor[g, coords][[a, b, c, j]], {j, 1, 2}],
  {i, 1, 2}, {a, 1, 2}, {b, 1, 2}, {c, 1, 2}]]
```

Out[6]= $\{\{\{\{0, 0\}, \{0, -A^2 \sin[\theta]^2\}\}, \{\{0, A^2 \sin[\theta]^2\}, \{0, 0\}\}\}, \{\{\{0, 0\}, \{A^2 \sin[\theta]^2, 0\}\}, \{\{-A^2 \sin[\theta]^2, 0\}, \{0, 0\}\}\}\}$

In[7]:=
```
(*c*)
FullSimplify[RicciTensor[g, coords]]
FullSimplify[R[g, coords]]
```

Out[7]= $\begin{pmatrix} 1 & 0 \\ 0 & \sin[\theta]^2 \end{pmatrix}$

Out[8]= $\frac{2}{A^2}$

```
(*d*)
```

The Weyl tensor is zero, because the Riemann tensor contains all the information about the curvature.