Functions and Cardinality

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From last time: Prove the Following

1. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
2. $\overline{A \cup B} = \bar{A} \cap \bar{B}$
3. $A \cap \bar{A} = \emptyset$
4. $A \cap B \subseteq A$
5. If $A \cup B = A$, then $B \subseteq A$.

   $\overline{A \cap B} = \bar{A} \cup \bar{B}$

Functions

Let A and B be sets. A *function* from A to B is an assignment of exactly one element of B to each element of A.

We write f(a) = b if b is the unique element of B assigned by the function f to the element a of A.

If f is a function from A to B, we write f : A -> B.

Function Vocab

- Let f: A -> B.
- A is called the *domain* of f
- B is called the *codomain* of f
- The *image* (or *range*) of f is the set
  {b in B | exists a in A with f(a) = b}

Injective Functions

A function f is said to be *injective (or 1-1)*, if and only if f(x) = f(y) implies x = y for all x and y.

Surjective Functions

A function f from A to B is called *surjective (or onto)*, if and only if for every element b in B, there exists an element a in A with f(a) = b.

Bijective Functions

A functions that is both injective and surjective is said to be *bijective*.

Inverse Functions

Let f be a bijection from A to B. The inverse of f is the function g from B to A such that g(b) = a, if and only if, f(a) = b.

The function g is often written as f^{-1}.

Why is it important that f be a bijection?

Cardinality

Two finite sets are said to have the same *cardinality (or size)* if they have the same number of elements.

Two infinite sets are said to have the same *cardinality (or size)* if there is a bijective function between them.

Cardinality Examples

- The sets **N, Z** and **Q** have the same cardinality
- A set and its power set never have the same cardinality!