Probability I

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Basic Definitions

An **experiment** is some process which yields a single outcome from a set of possible outcomes. The set of possible outcomes is called the **sample space**. An **event** is a subset of the sample space.

Probability

The **probability** of an event *E*, which is a subset of a finite same space *S* of equally likely outcomes, is

$$P(E) = \frac{|E|}{|S|}$$

Algebra of Probability

- Complements: $P(\bar{E}) = 1 - P(E)$
- Unions: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Example 1

What is the probability that in a room with 15 people two people have the same birthday (month/day)?

25 people? 35 people?

Example 2

Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats.

You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat.

He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?

Probability Assignment

A **probability assignment** on a sample space *S* is a function *p* from S to the real numbers such that:

$$0 \leq p(s) \leq 1 \quad \text{and} \quad \sum_{s \in S} p(s) = 1$$

Biased Coins

Given a biased coin (where you do not know the bias) how can you simulate an unbiased coin?

Hint: Construct an experiment whose same space has to events with equal probability.

Random Variables

A **random variable** is a function from the sample space of an experiment to the set of real numbers, i.e., a random variable assigns to each outcome a real number.

Expected Value

The **expected value** of a random variable *X* of a sample space S is equal to:

$$E(X) = \sum_{s \in S} p(s) X(s)$$