Equivalence Relations

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Relations

A **relation** on a set *A* is a subset of the cartesian cross *AxA*. Alternatively, a relation can be thought of as a function *R:AxA -> {true, false}.*

Equivalence Relations

A relation *R* is said to be an **equivalence** relation if the following three properties hold

1. for all *a, aRa* (every element is related to itself)
2. for all *a* and *b*, *aRb* implies *bRa* (the relation is symmetric)
3. for all *a*, *b* and *c*, *aRb* and *bRc* implies *aRc*. (the relation is transitive)

Examples of Equivalence Relations

1. Equals (in all its forms)
2. Modular equivalence
3. Functional equality of source code
4. Students with respect to major

Equivalence Classes

Given an equivalence relation ~ on a set *A* the **equivalence class** of an element *a* in *A* is the set of all elements *x* in *A* with *a~x.*

Partitions of a Set

A **partition** of a set *A* is a collection of disjoint subset of *A* whose union is all of *A.*

Partitions = Equivalence Relations

The set of all equivalence classes of a set *A* with respect to an equivalence relation ~ is a partition of *A.*

Given an partition *P* of *A* there exists an equivalence relation ~ on *A* such that the set of equivalence classes is equal to *P.*

Canonical Surjection

The **canonical surjection** from a set *A* with an equivalence relation ~ onto its set of equivalence classes takes an element of *A* to its equivalence class.

First Isomorphism Theorem

Given any surjection *f* from a set *A* onto a set *B* there exists an equivalence relation ~ on *A* such that there is a **natural bijection** from the equivalence classes of A onto B.

Applications to Counting

To count the size of a set *A* we can introduce an equivalence relation ~, count the number of elements in each equivalence class and sum up all of the results.

If all of the equivalence classes have the same size, then we count the number of elements in a single equivalence class and multiply by the number of equivalence classes.