Probability II

CS 55 - Spring 2016 - Pomona College
Michael J Bannister

Probability Assignment

A **probability assignment** on a sample space *S* is a function *p* from S to the real numbers such that:

$$0 \leq p(s) \leq 1 \quad \text{and} \quad \sum_{s \in S} p(s) = 1$$

Conditional Probability

The probability of *E* given *F*.

$$P(E \mid F) = \frac{P(E \cap F)}{P(F)}$$

Basically, restricting the sample space to *F* !

Reversing Conditional Probability

How are the probabilities

$$P(E \mid F) \qquad \text{and} \qquad P(F \mid E)$$

related?

Independence

The events E and F are said to independent if

$$P(E \mid F) = P(E) \quad \text{and} \quad P(F \mid E) = P(F)$$

It is enough for just one of the conditions above to hold. Why?

Independence

Two events E and F are independent if and only if

$$P(E \cap F) = P(E)P(F)$$

Bernoulli Trials

A **Bernoulli Trial** is an experiment with only two possible outcomes, usually called **success** and **failure**.

Independent Bernoulli Trials

The probability of k successes in n independent Bernoulli trials, with probability of success p and probability of failure $q = 1-p$, is:

$$\binom{n}{k} p^k q^{n-k}$$

Random Variables

A **random variable** is a function from the sample space of an experiment to the set of real numbers, i.e., a random variable assigns to each outcome a real number.

Expected Value

The **expected value** of a random variable *X* of a sample space S is equal to:

$$E(X) = \sum_{s \in S} p(s)X(s)$$

Linearity of Expectation

If *X* and *Y* are random variables and *a* is a real number, then

$$E(aX + Y) = aE(X) + E(Y)$$

Independent Random Variables

Two random variables *X* and *Y* are said to be independent if

$$P(X = r \text{ and } Y = s) = P(X = r)P(X = s)$$

for all r and s.

If X and Y are independent, then

$$E(XY) = E(X)E(Y)$$