Counting I

CS 55 - Spring 2016 - Pomona College
Michael J Bannister

Review Question

Let *A* be a set with *n* and *B* be a set with *m* elements with n < m.

How many functions from *A* to *B* are not 1-1?

Counting with Sums: “or”

If a *first task* can be done in N ways and a *second task* can be done in M ways, and the tasks have no overlap, then there are N+M ways to do the first task OR the second task.

$$|A \cup B| = |A| + |B|$$

Counting with Products: “and”

If there are N ways to do a first task and M ways to do a second task (after the first task), then there are NM ways to do the first task and the second task.

$$|A \times B| = |A||B|$$

Inclusion-Exclusion: Over Counting

When tasks overlap the following identity may allow you to quickly solve the problem.

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Passwords

Assume we are working with a computer system that allows passwords to consist of the following:

- Lower: a,b,c,…,x,y,z
- Upper: A,B,C,…,X,Y,Z
- Digits: 0,1,2,3,4,5,6,7,8,9
- Special: !,@,#,$,%,^,&,*,(,)

Password Examples

How many ways can you pick a pass satisfying:

1. length 8
2. length ≤ 8
3. 6 ≤ length ≤ 8

More Password Examples

How many ways can you pick a password of exactly length 8 satisfying:

1. No duplicate characters
2. Exactly one special character
3. No special characters
4. At least one special character

Counting Subsets

Given a set *A* of *n* elements how many subsets are there of A of size *k* ?

$$C(n,k) = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

A Useful Identity

$$\binom{n}{k} = \binom{n}{n-k}$$

Counting All Subsets

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n$$

Pascal's Identity

$$\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$$