Coloring

CS 55 - Spring 2016 - Pomona College
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Two Important Examples

The graphs $K_{3,3}$ and K_5 are not planar!

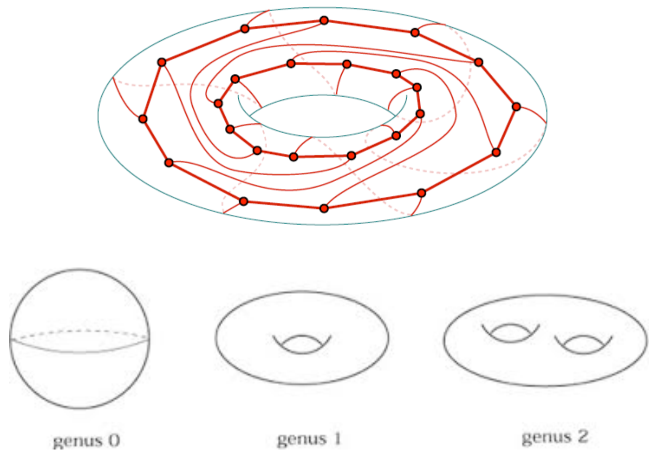
Graph Minors

A graph *H* is called a **minor** of the graph *G* if *H* can be formed from *G* by deleting edges, deleting vertices and by contracting edges.

Wagner's Theorem

A graph is planar if and only if it does not contain either $K_{3,3}$ or K_5 as a minor.

Higher Genus Graphs



The Coloring Problem

A **coloring** of a graph is an assignment of colors (often represented using natural numbers) to the vertices of a graph such that no two adjacent vertices have the same color.

Chromatic Number

The **chromatic number** of a graph is the minimal number of colors needed to produce a valid coloring.

$$\chi(G) = \text{“The chromatic number of } G\text{”}$$

Graph Coloring

The following are believed to require exponential time to solve algorithmically:

- determine chromatic number
- determine if chromatic number is 3
- approximating the chromatic number

Coloring is a canonical example of a very difficult algorithmic problem.

Degeneracy

The **degeneracy** of a graph G is the value of k after running the following algorithm:

```
Let k equal 1
while G still has vertices:
   while G still has vertices of degree ≤ k:
      remove a vertex from G of degree k along with all
      of its incident edges
   increment k
```

The order is which the vertices were removed is called a **degeneracy ordering**.

k-cores

The **k-cores** of a graph are the connected components of a graph that are left after iteratively removing all vertices of degree less than *k*.

The k-cores are implicitly found when using the degeneracy algorithm on the previous slide.

The concept of a k-core was first introduced to study clustering of communities in social networks.

Coloring in Degeneracy Order

Greedily coloring in degeneracy order will produce a coloring using k + 1, where k is the degeneracy.

Thus the chromatic number of a graph is at most one more than the degeneracy.

This coloring is rarely optimal, but is often the best we can do.