Introduction to Graphs

CS 55 - Spring 2016 - Pomona College
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Subgraphs

A graph *H = (W, F)* is said to be a **subgraph** of a graph *G = (V, E)* if $W \subseteq V$ and $F \subseteq E$. Furthermore, we require that all endpoints of edges in *F* are in *W* !

The definition given last time is unnecessarily restrictive in general.

Isomorphism Formally

The graphs *G = (V, E)* and *H = (W, F)* are said to be **isomorphic** if there is a bijection *f* from *V* to *W* with the property that whenever vertices *u* and *v* are adjacent in *G* if and only if *f(u)* and *f(v)* are adjacent in *H*.

The function *f* is called an **isomorphism**.

Terms related to paths in graphs

- path / path length
- simple paths
- cycle / cycle length
- simple cycle
- acyclic graph
- connected
- connected component
- strongly connected
- strongly connected component
- k-connected
- k-connected component

Euler cycle

An **Euler cycle** in a graph *G* is a cycle containing every edge of G exactly one time.

An **Euler path** in a graph *G* is a path containing every edge of G exactly one time.

Theorem on Euler Cycle/Paths

A connected graph has an Euler circuit if and only if each of its vertices have even degree.

A connected graph has an Euler path if and only if it has exactly two vertices of odd degree.

Hamiltonian Cycles/Paths

A cycle in a connected graph is said to be **hamiltonian** if it contains every vertex in the graph exactly once and has no repeated edges.

A hamiltonian path is defined similarly.

Theorems on Hamiltonian Graphs

There is no general theorem on when a graph has a hamiltonian cycle and determining if a graph has a hamiltonian cycle is a computationally difficult task.

However, may results are known for specific graphs, and some of these examples will be covered in the homework.