# Sequences & Series

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Michael J Bannister

## Review Questions

- Find a set $A$ where $A \cap P(A) \neq \emptyset$ .
- If $f$ and $g$ are surjective, does it imply $f \circ g$ is surjective? Why? How about if we replace surjective with injective?

## Cardinality

Two finite sets are said to have the same *cardinality (or size)* if they have the same number of elements.

Two infinite sets are said to have the same *cardinality (or size)* if there is a bijective function between them.

## Cardinality Examples

- The sets **N, Z** and **Q** have the same cardinality
- A set and its power set never have the same cardinality!

# Sequence

A *sequence* is a function whose domain is a subset of the integers.

Typically the domain is either:
**N** = {0,1,2,3,…} or {1,2,3,4,….}

# Summation Notation

We use the following *summation notation* to express the sum of a sequence:

$$\sum_{k=m}^{n} a_k = a_m + a_{m+1} + \cdots + a_{n-1} + a_n$$

# Product Notation

We use the following *product notation* to express the product of a sequence:

$$\prod_{k=m}^{n} a_k = a_m \times a_{m+1} \times \cdots \times a_{n-1} \times a_n$$

# Gauss Sum

The following formula is called the Gauss sum:

$$\sum_{k=1}^{n} = \frac{n(n+1)}{2}$$

Proof: On board.

# Geometric Sum

The following formula is called the geometric sum:

$$\sum_{k=0}^{n} ar^k = \frac{ar^{n+1} - a}{r - 1}$$

Proof: On board.