Introduction to Graphs

CS 55 - Spring 2016 - Pomona College
Michael J Bannister

Informally Graphs
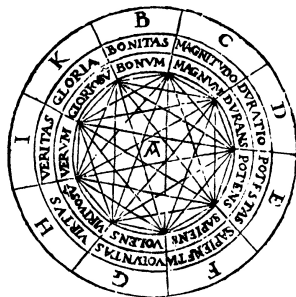
A graph consists of a collection of entities together with a binary "relation".
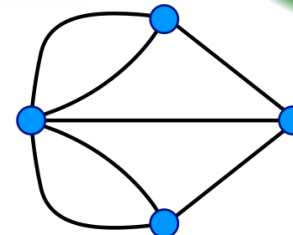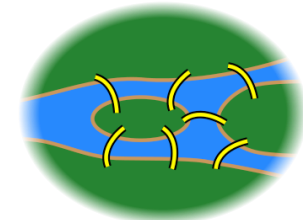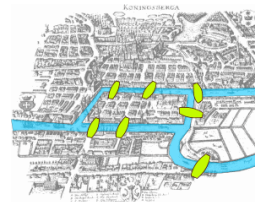
Node-Link Diagrams

Graphs are often communicated best with a drawing. The most common such drawing is a node-link diagram, where vertices are drawn as dots and edges are drawn as lines connecting the dots.
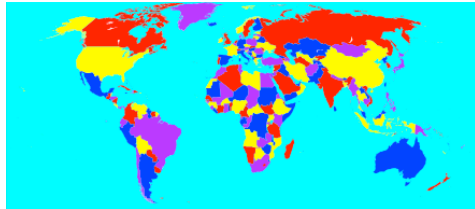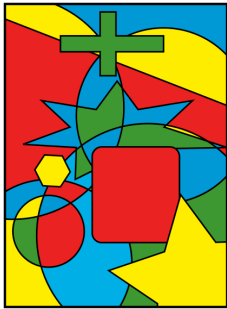
Ramon Llull (circa ~1300) and his drawing
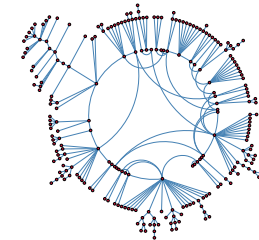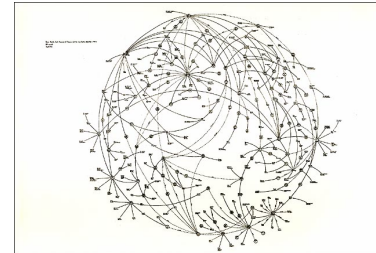
Seven Bridges of Königsberg

The Four Color Theorem

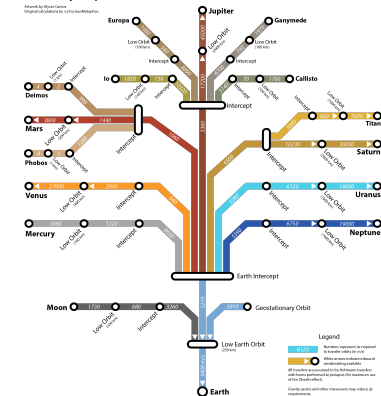Any map can be colored using only 4 colors!

Social Interaction Graphs

Communication Networks

Transportation Networks
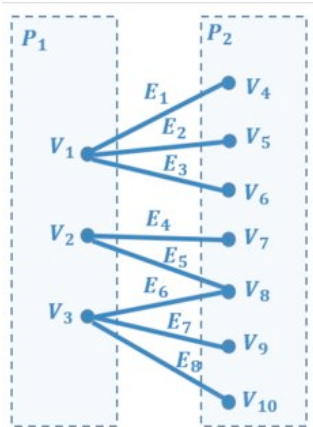
Advertising

Formally Graphs

A **(simple) graph** is a pair of sets *(V, E)* where *V* is called the set of **vertices** and *E* is called the set of edges and consists of a subset of the unordered pairs of elements in *V*.

Alternatively, we can think of a graph as a set V with a symmetric (but not reflexive) relation *E*.

Directed Graphs

A **directed graph (digraph)** is an ordered pair *(V, E)* where *V* is the set of vertices and *E* is the set of edges, where edges are now ordered pairs over distinct vertices.

Alternatively, we can think of E as a non-reflexive relation.

Extensions of Graphs

We will often want to consider graph with **data** on the vertices or edges. We can think of vertex data as a function with domain *V*, and edge data as a function with domain *E*.

On occasion we will want to allow **parallel edges** and **self loops** in our graphs, but this is rare.

Glossary

Common graph terms:

- $n = |V|$ and $m = |E|$

- Adjacent / neighbors

- Endpoints (source/destination for digraphs)

- Degree (in/out for digraphs)

- Complete graph (clique)

- Subgraph