Counting II

CS 55 - Spring 2016 - Pomona College
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Review Question

Given a set of *10* numbered balls how many ways can you pick out *5* and place them in a

1. Bag?

2. Seven numbered boxes?

3. Seven numbered box where each box contains at most one ball?

Binomial Theorem

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

Counting Subsets Again

$$\sum_{k=0}^{n} \binom{n}{k} = (1+1)^n = 2^n$$

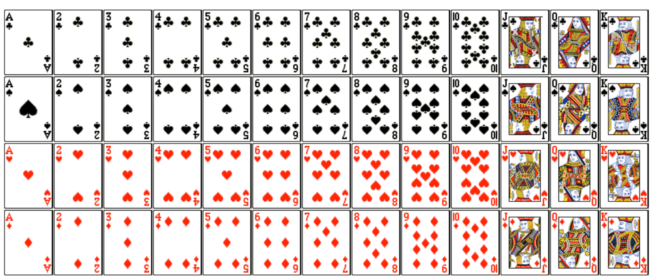
Alternating Sum on Binomials

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} = ?$$

Another Binomial Sum

$$\sum_{k=0}^{n} 2^k \binom{n}{k} = ?$$

Canonical Example: Cards



- 52 Cards
  - 4 Suites: Clubs, Spades, Hearts, Diamonds
  - Rank: 2,3,4,5,6,7,8,9,T,J,K,Q,A

Four of a Kind

How many ways are there to choose 5 cards such that 4 of them have the same rank?

Full House

How many ways are there to choose 5 cards such that 3 of the cards have the same rank and the remaining two cards also have the same rank?

Straight

How many ways can you choose 5 cards such that the ranks are sequential? For this problem and Ace counts as both 1 and 13.

One Pair

How many ways are there to choose 5 cards such that exactly two of the cards have the same rank?

Pigeonhole Principle

If $k+1$ or more balls are placed into k bins, then there is at least one box containing two or more balls.

Example 1

How many students must be in a class to guarantee that at least two have the same birthday (month and day)?

What if we also want the year to match? Make some reasonable assumptions to solve this one.

Extending the Principle

If you are placing k balls into n bins, how big must k be to guarantee that there exists a bin with two balls in it?

Monotone Subsequences

Every sequence of $n^2 + 1$ distinct real numbers contains a subsequence of length $n+1$ that is either strictly increasing or strictly decreasing.