Welcome to CS 55 Discrete Mathematics

Introduction and Propositional Logic

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Course Webpage

See course webpage for important information!
Please ask questions whenever the information is confusing or ambiguous!

Propositional Logic

Propositions

A **proposition** is a statement that is either true or false, but not both. The **truth value** of a proposition is true (denoted **T**) if the proposition is true, and false (denoted **F**) otherwise

Common Logical Operators

p	$\neg p$
T	F
F	T

negation

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

and

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

or

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

exclusive or

Conditionals Operators

The proposition $q \rightarrow p$ is called the **converse** of $p \rightarrow q$.

The proposition $\neg q \rightarrow \neg p$ is called the **contrapositive** of $p \rightarrow q$.

The proposition $\neg p \vee q$ is equivalent to the conditional $p \rightarrow q$.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Conditional

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Bi-conditional

Translating From English

1. You can use the quantum computer on campus only if you are a computer science major or you are not a freshman.
2. You cannot use the secret tunnels if you are taller than six foot unless you can crawl long distances.

Propositional Equivalences

Tautology and Contradiction

A propositional expression that is always true is called a **tautology**.

A propositional expression that is always false is called **contradiction**.

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

Logical Equivalence

The propositions p and q are called **logically equivalent** if $p \leftrightarrow q$ is a tautology. The notation $p \Leftrightarrow q$ denoted that p and q are logically equivalent.

Example: $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$

Common Logical Equivalences 1

$$p \wedge \mathrm{T} \Leftrightarrow p \qquad p \vee \mathrm{T} \Leftrightarrow \mathrm{T} \qquad p \vee p \Leftrightarrow p$$

$$p \vee \mathrm{F} \Leftrightarrow p \qquad p \wedge \mathrm{F} \Leftrightarrow \mathrm{F} \qquad p \wedge p \Leftrightarrow p$$

$$\neg(\neg p) \Leftrightarrow p$$

Common Logical Equivalences 2

$$p \vee q \Leftrightarrow q \vee p \qquad (p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$$

$$p \wedge q \Leftrightarrow q \wedge p \qquad (p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$$

$$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$$

$$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$$

De Morgan's Laws

$$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$$

$$\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$$

Additional Examples