October 29, 2018

CSCI/MATH 3180
Lab Assignments #8

Construct the natural cubic for $f(x) = \frac{1}{(x^2 + 1)}$ using the data points at x = -8,-6, -4, -2, 0, 2, 4, 6, 8.

Part I. Use Visual Studio 2015 to do the following task.

1. Create a C++ console application project in Visual Studio 2015 and name your project **YourLastName8**.
2. Write a program that implements the systematic procedure for constructing a natural spline discussed in class.
3. The program must use **single dimensional arrays only**; no two dimensional arrays are allowed.
4. Program input: not required.
5. Program output:
   - Data points, (t_i, y_i), $0 \le i \le n$
   - Tri-diagonal system for z_i, $1 \le i \le n$-1 before Forward Elimination
   - Tri-diagonal system for z_i, $1 \le i \le n$-1 after Forward Elimination
   - Solution for z_i, $0 \le i \le n$ after Back Substitution
   - Coefficients A_i, B_i, C_i, D_i, $0 \le i \le n$-1 for Natural Cubic Spline in nested form
   - Natural Cubic Spline, S_i, $0 \le i \le n$-1 in nested form
   - Table containing x, $f(x)$, $S(x)$, and $|f(x)-S(x)|$ for x = -8, -6, -5, -4, -3,-2, -1, 0, 1, 2, 3, 4, 5, 6, 8.

Part II. Use Maple to do the following task as shown in the handout given in class.

1. Create the nine data points using $f(x)$.
2. Plot f(x) only.
3. Create the Newton form of the interpolating polynomial of degree 8, $P_8(x)$ using the data set.
4. Plot $P_8(x)$ only.
5. Plot S(x) only using the output from **Part I**.
6. Plot f(x), $P_8(x)$, and S(x) in the same graph.

Submission

1. Delete the following from your Visual Studio project folder.
   - *Debug* sub-folder
   - *Debug* sub-sub-folder under your project folder(second level down)
   - *sdf* file.
2. Save the following in a compressed (zipped) folder.

   PartI : **YourLastName8** -- *main project folder*

   PartII : **YourLastNameMaple8.mw**

3. Submit the compressed folder to D2L.
4. **Confirm** your submission.
   - **Download** the zipped folder which you have submitted and **check the contents**.
   - Multiple submissions are allowed, but the last submission will be graded.

NOTE: LABS MUST BE YOUR ORIGINAL AND INDEPENDENT WORK.

LAB #8 EVALUATION RUBRIC

Part I. Programming Project		
1	Solve the assigned problem using methods described in program description.	__/4
2	Compilation/Execution ➢ Compile without errors when tested in the CSCI computer lab. ➢ Execute without crashing when tested in the CSCI computer lab.	__/2
3	Produce correct answers. • Data points, (t_i, y_i), $0 \le i \le n$ • Tri-diagonal system for z_i, $1 \le i \le n$-1 before Forward Elimination • Tri-diagonal system for z_i, $1 \le i \le n$-1 after Forward Elimination • Solution for z_i, $0 \le i \le n$ after Back Substitution • Coefficients A_i, B_i, C_i, D_i, $0 \le i \le n$-1 for Natural Cubic Spline in nested form • Natural Cubic Spline, S_i, $0 \le i \le n$-1 in nested form • Table for x, $f(x)$, $S(x)$, and $\lvert f(x)-S(x)\rvert$ for x = -8,-7,-6,-5,-4,-3,-2,-1,0,1,2,3,4,5,6,7,8.	__/7
4	The program output well formatted. The program output properly labeled and identified. The program output meets the problem specification.	__/3
5	Main Comment Block includes the following. file name due date author course # program description input output	__/0.5
6	Documentation, indentation, and white space usage ✓ Meaning variable names are used and they are briefly described. ✓ Each section of statements in the program is well documented. ✓ Proper INDENTATION is used to make the program easier to read. ✓ WHITE SPACES are used in appropriate places for readability.	__/0.5
7	Contents of zipped folder ✓ Zip folder contains the two items described above. ✓ The project folder does NOT contain the following. ❖ Debug sub-folder ❖ Debug sub-sub-folder ❖ .sdf file	
Part II. Maple worksheet		
1	✓ Maple worksheet meets the requirements described above.	__/3
	TOTAL	__/20

Output for five data points at x = -4, -2, 0, 2, 4

```
Data points (ti, yi)
(t0, y0) = (-4,0.0588235)
(t1, y1) = (-2,0.2)
(t2, y2) = (0,1)
(t3, y3) = (2,0.2)
(t4, y4) = (4,0.0588235)

Tri-diagonal system for z1 through z(n-1)
         8         2         0   1.97647
         2         8         2      -4.8
         0         2         8   1.97647

Tri-diagonal system after Forward Elimination
         8         2         0   1.97647
         0       7.5         2  -5.29412
         0         0   7.46667   3.38824

Solutions for S''(ti) = zi
---------------------------
z0:0
z1:0.453782
z2:-0.826891
z3:0.453782
z4:0

Coefficients for Natural Cubic Spline in nested form
-------------------------------------------------------------------
A0:  0.0588235   B0: -0.0806723   C0:           0   D0:  0.0378151
A1:        0.2   B1:   0.373109   C1:   0.226891   D1:  -0.106723
A2:          1   B2:          0   C2:  -0.413445   D2:   0.106723
A3:        0.2   B3:  -0.373109   C3:   0.226891   D3: -0.0378151

Natural Cubic Spline in nested form
-------------------------------------------------------------------
S0 :  0.0588235+(x+4)*(-0.0806723+(x+4)*(0+(x+4)*(0.0378151)))
S1 :  0.2+(x+2)*(0.373109+(x+2)*(0.226891+(x+2)*(-0.106723)))
S2 :  1+(x-0)*(0+(x-0)*(-0.413445+(x-0)*(0.106723)))
S3 :  0.2+(x-2)*(-0.373109+(x-2)*(0.226891+(x-2)*(-0.0378151)))

Evaluation of Original and Spline functions and the absolute errors
-------------------------------------------------------------------
 i        xi           f(xi)          S(xi)     |f(xi) -S(xi)|
-------------------------------------------------------------------
 0   -4.000000       0.058824       0.058824       0.000000
 1   -3.000000       0.100000       0.015966       0.084034
 2   -2.000000       0.200000       0.200000       0.000000
 3   -1.000000       0.500000       0.693277       0.193277
 4    0.000000       1.000000       1.000000       0.000000
 5    1.000000       0.500000       0.693277       0.193277
 6    2.000000       0.200000       0.200000       0.000000
 7    3.000000       0.100000       0.015966       0.084034
 8    4.000000       0.058824       0.058824       0.000000
```