Lab 09
MATH 3180: Numerical Analysis

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Contents

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CSCI/MATH 3180
Lab Assignment #9

1. Create a C++ console application project in Visual Studio 2015 and name your project YourLastName9.
2. Write a program that implements 1) the Bisection Method and 2) Secant Method for approximating a zero of a function, $f(x) = x^3 - 2x^2 - 5x + 6$.
3. Write a separate function for each of the following.
   - Evaluating $f(x)$
   - Bisection Method
   - Secant Method
4. Use the following parameters for both methods.

```
double x0:  starting approximation 0
double x1:  starting approximation 1
int maxIterations: maximum number of approximations generated
double xTolerance: max distance between last 2 approximations
double yTolerance: max distance from f(last approximation) to 0
```

   Function should iterate until both stopping criteria are met or it exceeds the maximum number of iterations.

5. Test your program using the following function calls.

```
secant(0, 4, 20, 0.001, 0.00001);
secant(0, 2, 20, 0.001, 0.00001);
secant(2, 4, 20, 0.001, 0.00001);
secant(0, 3, 20, 0.001, 0.00001);
secant(1, 2, 20, 0.001, 0.00001);
secant(2, 30, 20, 0.001, 0.00001);
secant(10, 30, 20, 0.001, 0.00001);
```

```
bisection(0, 4, 20, 0.001, 0.00001);
bisection(0, 2, 20, 0.001, 0.00001);
bisection(2, 4, 20, 0.001, 0.00001);
bisection(0, 3, 20, 0.001, 0.00001);
bisection(1, 2, 20, 0.001, 0.00001);
bisection(2, 30, 20, 0.001, 0.00001);
bisection(10, 30, 20, 0.001, 0.00001);
```

Make sure your program produces the results similar to the screen as shown below.

6. Analyze your output and write a short report (YourLastNameReport9.pdf) including the following
   - Output of the program
   - Comparison of the two methods along with the advantages and disadvantages based on your experiment.
   - Your conclusion and/or your recommendation

7. Submission
   - Delete the following from your project folder.
     - ➢ *Debug* sub-folder
     - ➢ *Debug* sub-sub-folder under your project folder(second level down)
     - ➢ *ipch* sub-folder
     - ➢ *sdf* file.
   - Save the following in a compressed (zipped) folder and submit it to D2L.
     - ➢ *main project folder* (YourLastName9)
     - ➢ *report* (YourLastNameReport9.pdf) on the experiment
   - Submit the compressed folder to D2L.

NOTE: <u>PROGRAMS MUST BE INDEPENDENT WORK.</u>

C:\Windows\system32\cmd.exe

```
Interval: [0.000000, 4.000000]
Secant Method
---------------------------------------------------------------
Iteration       Approx. root    x_tolerance     y_tolerance
        1       -2.000000       6.000000        0.000000
        Exact root found at -2.000000
        Number of iterations: 1
Bisection Method
        found no root on the interval



Interval: [0.000000, 2.000000]
Secant Method
---------------------------------------------------------------
Iteration       Approx. root    x_tolerance     y_tolerance
        1       1.200000        0.800000        1.152000
        2       0.876404        0.323596        0.754961
        3       1.004515        0.128111        0.027070
        4       1.000081        0.004435        0.000483
        5       1.000000        0.000081        0.000000
        Approximated root: 1.000000
        Number of iterations: 5
        x_tolorence: 0.000081
        y_tolorence 0.000000

Bisection Method
---------------------------------------------------------------
Iteration       Approx. root    x_tolerance     y_tolerance
        1       1.000000        1.000000        0.000000
        Exact root found at 1.000000
        Number of iterations: 1



Interval: [2.000000, 4.000000]
Secant Method
---------------------------------------------------------------
Iteration       Approx. root    x_tolerance     y_tolerance
        1       2.363636        1.636364        3.786627
        2       2.648045        0.284408        2.696043
        3       3.351133        0.703089        4.417689
        4       2.914509        0.436624        0.804372
        5       2.981764        0.067255        0.180039
        6       3.001158        0.019394        0.011591
        7       2.999985        0.001173        0.000149
        8       3.000000        0.000015        0.000000
        Approximated root: 3.000000
        Number of iterations: 8
        x_tolorence: 0.000015
        y_tolorence 0.000000

Bisection Method
---------------------------------------------------------------
Iteration       Approx. root    x_tolerance     y_tolerance
        1       3.000000        1.000000        0.000000
        Exact root found at 3.000000
        Number of iterations: 1



Interval: [0.000000, 3.000000]
Secant Method
        Exact root found at 3.000000
        Number of iterations: 0
Bisection Method
        Exact root found at 3.000000
        Number of iterations: 0



Interval: [1.000000, 2.000000]
Secant Method
        Exact root found at 1.000000
        Number of iterations: 0
Bisection Method
        Exact root found at 1.000000
        Number of iterations: 0
```

C:\Windows\system32\cmd.exe

```
Interval: [2.000000, 30.000000]
Secant Method
------------------------------------------------------------------
Iteration        Approx. root     x_tolerance      y_tolerance
        1        2.004469         27.995531        4.004389
        2        2.008943         0.004473         4.008622
        3        -2.227552        4.236495         3.839298
        4        -98.286744       96.059192        968301.004265
        5        -2.227171        96.059573        3.832140
        6        -2.226791        0.000380         3.824998
        7        -2.023185        0.203606         0.352080
        8        -2.002543        0.020641         0.038199
        9        -2.000031        0.002512         0.000467
        10       -2.000000        0.000031         0.000001
        Approximated root: -2.000000
        Number of iterations: 10
        x_tolorence: 0.000031
        y_tolorence 0.000001

Bisection Method
------------------------------------------------------------------
Iteration        Approx. root     x_tolerance      y_tolerance
        1        16.000000        14.000000        3510.000000
        2        9.000000         7.000000         528.000000
        3        5.500000         3.500000         84.375000
        4        3.750000         1.750000         11.859375
        5        2.875000         0.875000         1.142578
        6        3.312500         0.437500         3.839111
        7        3.093750         0.218750         0.999847
        8        2.984375         0.109375         0.154545
        9        3.039063         0.054688         0.401366
        10       3.011719         0.027344         0.118150
        11       2.998047         0.013672         0.019505
        12       3.004883         0.006836         0.048995
        13       3.001465         0.003418         0.014663
        14       2.999756         0.001709         0.002441
        15       3.000610         0.000854         0.006106
        16       3.000183         0.000427         0.001831
        17       2.999969         0.000214         0.000305
        18       3.000076         0.000107         0.000763
        19       3.000023         0.000053         0.000229
        20       2.999996         0.000027         0.000038
        Exceeded the maximum number of iterations.
        Approximated root: 2.999996
        was found at Iteration 20
        with x_tolorence: 0.000027
        with y_tolorence 0.000038



Interval: [10.000000, 30.000000]
Secant Method
------------------------------------------------------------------
Iteration        Approx. root     x_tolerance      y_tolerance
        1        9.377778         20.622222        607.932927
        2        8.864979         0.512798         501.179151
        3        6.457535         2.407445         159.590418
        4        5.332775         1.124759         74.115223
        5        4.357501         0.975275         28.976272
        6        3.731438         0.626063         11.450703
        7        3.322386         0.409052         3.984897
        8        3.104054         0.218333         1.117452
        9        3.018969         0.085085         0.192212
        10       3.001293         0.017676         0.012940
        11       3.000017         0.001276         0.000170
        12       3.000000         0.000017         0.000000
        Approximated root: 3.000000
        Number of iterations: 12
        x_tolorence: 0.000017
        y_tolorence 0.000000

Bisection Method
        found no root on the interval
```

LAB #9 EVALUATION RUBRIC

1	Solve the assigned problem using methods described in program description.	__/3
2	Compilation/Execution ✓ Compile without errors. ✓ Execute without crashing. ✓ Work for all data and produce correct answers. ✓ The program output well formatted and properly labeled.	__/3
3	Main Comment Block includes the following. file name due date author course # program description input output	__/0.5
4	Documentation, indentation, and white space usage ✓ Meaning variable names are used and they are briefly described. ✓ Each section of statements in the program is well documented. ✓ Proper INDENTATION is used to make the program easier to read. ✓ WHITE SPACES are used in appropriate places for readability.	__/0.5
5	Contents of zipped folder ✓ Zip folder contains the project folder and the report. ✓ The project folder does NOT contain the following. ❖ Debug sub-folder ❖ Debug sub-sub-folder ❖ ipch sub-folder ❖ .sdf file	
6	Contents of report ✓ Output of the program ✓ Conclusion	__/3
	TOTAL	__/10

1 Program Output

```
####################################################################################################
Interval: [0.000000, 4.000000]
Secant Method
--------------------------------------------------------------------------------
   Iteration    Approximate Root              x_error             y_error
--------------------------------------------------------------------------------
           1           -2.000000             6.000000            0.000000
Exact root found at -2.000000
Number of iterations: 1


Interval: [0.000000, 4.000000]
Bisection Method
Found no root on the interval


####################################################################################################
Interval: [0.000000, 2.000000]
Secant Method
--------------------------------------------------------------------------------
   Iteration    Approximate Root              x_error             y_error
--------------------------------------------------------------------------------
           1            1.200000             0.800000            1.152000
           2            0.876404             0.323596            0.754961
           3            1.004515             0.128111            0.027070
           4            1.000081             0.004435            0.000483
           5            1.000000             0.000081            0.000000
Approximated root: 1.000000
Number of iterations: 5
x_error: 0.000081
y_error: 0.000000


Interval: [0.000000, 2.000000]
Bisection Method
--------------------------------------------------------------------------------
   Iteration    Approximate Root              x_error             y_error
--------------------------------------------------------------------------------
           1            1.000000             2.000000            0.000000
Exact root found at 1.000000
Number of iterations: 1


####################################################################################################
Interval: [2.000000, 4.000000]
Secant Method
--------------------------------------------------------------------------------
   Iteration    Approximate Root              x_error             y_error
--------------------------------------------------------------------------------
           1            2.363636             1.636364            3.786627
           2            2.648045             0.284408            2.696043
           3            3.351133             0.703089            4.417689
           4            2.914509             0.436624            0.804372
           5            2.981764             0.067255            0.180039
           6            3.001158             0.019394            0.011591
           7            2.999985             0.001173            0.000149
```

```
          8           3.000000            0.000015            0.000000
Approximated root: 3.000000
Number of iterations: 8
x_error: 0.000015
y_error: 0.000000


Interval: [2.000000, 4.000000]
Bisection Method
--------------------------------------------------------------------------------
   Iteration    Approximate Root             x_error             y_error
--------------------------------------------------------------------------------
          1           3.000000            2.000000            0.000000
Exact root found at 3.000000
Number of iterations: 1


####################################################################################################
Interval: [0.000000, 3.000000]
Secant Method
Exact root found at 3.000000
Number of iterations: 0


Interval: [0.000000, 3.000000]
Bisection Method
Exact root found at 3.000000
Number of iterations: 0


####################################################################################################
Interval: [1.000000, 2.000000]
Secant Method
Exact root found at 1.000000
Number of iterations: 0


Interval: [1.000000, 2.000000]
Bisection Method
Exact root found at 1.000000
Number of iterations: 0


####################################################################################################
Interval: [2.000000, 30.000000]
Secant Method
--------------------------------------------------------------------------------
   Iteration    Approximate Root             x_error             y_error
--------------------------------------------------------------------------------
          1           2.004469           27.995531            4.004389
          2           2.008943            0.004473            4.008622
          3          -2.227552            4.236495            3.839298
          4         -98.286744           96.059192       968301.003974
          5          -2.227171           96.059573            3.832140
          6          -2.226791            0.000380            3.824998
          7          -2.023185            0.203606            0.352080
          8          -2.002543            0.020641            0.038199
```

```
         9          -2.000031             0.002512             0.000467
        10          -2.000000             0.000031             0.000001
Approximated root: -2.000000
Number of iterations: 10
x_error: 0.000031
y_error: 0.000001


Interval: [2.000000, 30.000000]
Bisection Method
--------------------------------------------------------------------------------
   Iteration    Approximate Root              x_error              y_error
--------------------------------------------------------------------------------
         1          16.000000            28.000000          3510.000000
         2           9.000000            14.000000          3510.000000
         3           5.500000             7.000000           528.000000
         4           3.750000             3.500000            84.375000
         5           2.875000             1.750000            11.859375
         6           3.312500             0.875000             1.142578
         7           3.093750             0.437500             3.839111
         8           2.984375             0.218750             0.999847
         9           3.039062             0.109375             0.154545
        10           3.011719             0.054688             0.401366
        11           2.998047             0.027344             0.118150
        12           3.004883             0.013672             0.019505
        13           3.001465             0.006836             0.048995
        14           2.999756             0.003418             0.014663
        15           3.000610             0.001709             0.002441
        16           3.000183             0.000854             0.006106
        17           2.999969             0.000427             0.001831
        18           3.000076             0.000214             0.000305
        19           3.000023             0.000107             0.000763
        20           2.999996             0.000053             0.000229
Exceeded maximum number of iterations.
Approximated root: 2.999996
was found at 20
x_error: 0.000053
y_error: 0.000229


####################################################################################################
Interval: [10.000000, 30.000000]
Secant Method
--------------------------------------------------------------------------------
   Iteration    Approximate Root              x_error              y_error
--------------------------------------------------------------------------------
         1           9.377778            20.622222           607.932927
         2           8.864979             0.512798           501.179151
         3           6.457535             2.407445           159.590418
         4           5.332775             1.124759            74.115223
         5           4.357501             0.975275            28.976272
         6           3.731438             0.626063            11.450703
         7           3.322386             0.409052             3.984897
         8           3.104054             0.218333             1.117452
         9           3.018969             0.085085             0.192212
        10           3.001293             0.017676             0.012940
        11           3.000017             0.001276             0.000170
```

```
          12            3.000000            0.000017            0.000000
Approximated root: 3.000000
Number of iterations: 12
x_error: 0.000017
y_error: 0.000000


Interval: [10.000000, 30.000000]
Bisection Method
Found no root on the interval
```

2 Comparison of the two methods

Based on this experiment, the better method for general purpose computation of the zeros of some function is the secant method. Bisection has the advantage that *if* the function being examined is continuous over some interval and its values change sign over that interval, then it will always be able to find the zero on that interval. However, it fails not only if the interval given does not contain a root, but also when the function values do not change sign when evaluated at the ends. This is in contrast to the secant method, which evidently has very little trouble working towards the nearest root outside of the interval initially guessed. According to the text, it is also much slower, which we can see in this experiment on the interval $[2, 30]$, where it is clear that the convergence on the root is slower in comparison to the secant method.

3 Conclusion

Based on this experiment, it seems as though the secant method must naturally be a better general purpose root-finding method. It can be applied more generally, can be interpreted geometrically and analytically from Newton's method, and has a faster convergence than bisection. If an interval is known to have a within it, bisection is perfectly valid, but if the problem at hand suggest that a root may exist in a certain region of the function, the secant method seems more appropriate.