PHYS 3180: Lab06

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Create the nine data points:

```
x_list := [-8, -6, -4, -2, 0, 2, 4, 6, 8]:
data := []:
f := unapply(1/(x^2+1), x):
for x_val in x_list do data := [op(data), [x_val, f(x_val)]] end do:
print(data)
```

$$\left[\left[-8, \frac{1}{65}\right], \left[-6, \frac{1}{37}\right], \left[-4, \frac{1}{17}\right], \left[-2, \frac{1}{5}\right], [0, 1], \left[2, \frac{1}{5}\right], \left[4, \frac{1}{17}\right], \left[6, \frac{1}{37}\right], \left[8, \frac{1}{65}\right]\right] \tag{1.1}$$

Create the divided difference table using the data set:

```
with(Student[NumericalAnalysis]):
p := PolynomialInterpolation(data, independentvar='x', method=newton):
dd_table := DividedDifferenceTable(p):
print(dd_table)
```

$$\left[\left[\frac{1}{65}, 0, 0, 0, 0, 0, 0, 0, 0\right], \tag{2.1}$$
$$\left[\frac{1}{37}, \frac{14}{2405}, 0, 0, 0, 0, 0, 0, 0\right],$$
$$\left[\frac{1}{17}, \frac{10}{629}, \frac{103}{40885}, 0, 0, 0, 0, 0, 0\right],$$
$$\left[\frac{1}{5}, \frac{6}{85}, \frac{43}{3145}, \frac{76}{40885}, 0, 0, 0, 0, 0\right],$$
$$\left[1, \frac{2}{5}, \frac{7}{85}, \frac{36}{3145}, \frac{49}{40885}, 0, 0, 0, 0\right],$$
$$\left[\frac{1}{5}, -\frac{2}{5}, -\frac{1}{5}, -\frac{4}{85}, -\frac{23}{3145}, -\frac{174}{204425}, 0, 0, 0\right],$$
$$\left[\frac{1}{17}, -\frac{6}{85}, \frac{7}{85}, \frac{4}{85}, \frac{1}{85}, \frac{6}{3145}, \frac{47}{204425}, 0, 0\right],$$
$$\left[\frac{1}{37}, -\frac{10}{629}, \frac{43}{3145}, -\frac{36}{3145}, -\frac{23}{3145}, -\frac{6}{3145}, -\frac{1}{3145}, -\frac{8}{204425}, 0\right],$$
$$\left[\frac{1}{65}, -\frac{14}{2405}, \frac{103}{40885}, -\frac{76}{40885}, \frac{49}{40885}, \frac{174}{204425}, \frac{47}{204425}, \frac{8}{204425}, \frac{1}{204425}\right]$$

Create the Newton form of the interpolating polynomial of degree 8, P8(x) using the data set:

```
p8 := Interpolant(p):
print(P8(x) = p8)
```

$$\frac{149}{2405}+\frac{14\,x}{2405}+\frac{103\,(x+8)\,(x+6)}{40885}+\frac{76\,(x+8)\,(x+6)\,(x+4)}{40885}+\frac{49\,(x+8)\,(x+6)\,(x+4)\,(x+2)}{40885}-\frac{174\,(x+8)\,(x+6)\,(x+4)\,(x+2)\,x}{204425}+\frac{47\,(x+8)\,(x+6)\,(x+4)\,(x+2)\,x\,(x-2)}{204425}-\frac{8\,(x+8)\,(x+6)\,(x+4)\,(x+2)\,x\,(x-2)\,(x-4)}{204425}+\frac{(x+8)\,(x+6)\,(x+4)\,(x+2)\,x\,(x-2)\,(x-4)\,(x-6)}{204425}=\frac{149}{2405}+\frac{14\,x}{2405}+\frac{103\,(x+8)\,(x+6)}{40885}+\frac{76\,(x+8)\,(x+6)\,(x+4)}{40885}+\frac{49\,(x+8)\,(x+6)\,(x+4)\,(x+2)}{40885}-\frac{174\,(x+8)\,(x+6)\,(x+4)\,(x+2)\,x}{204425}+\frac{47\,(x+8)\,(x+6)\,(x+4)\,(x+2)\,x\,(x-2)}{204425}-\frac{8\,(x+8)\,(x+6)\,(x+4)\,(x+2)\,x\,(x-2)\,(x-4)}{204425}+\frac{(x+8)\,(x+6)\,(x+4)\,(x+2)\,x\,(x-2)\,(x-4)\,(x-6)}{204425} \tag{3.1}$$

```
P8 := unapply(p8, x):
```

Evaluate f(x), P8(x), and |f(x) - P8(x)| for the 17 data points at x = -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8:

```
# Setting up lists
x_list_new := [-8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8]:
f_soln := [ ]:
 p_soln := [ ]:
abs_diff := [ ]:

 # Appending new values to the appropriate list
for x_val in x_list_new do
    f_soln := [op(f_soln), f(x_val)]:
    p_soln := [op(p_soln), P8(x_val)]:
    abs_diff :=[op(abs_diff), |f(x_val) - P8(x_val)|]:
 end do:

# Printing header
printf("   f(x)          P8(x)       |f(x) - P8(x)|      \n");
```

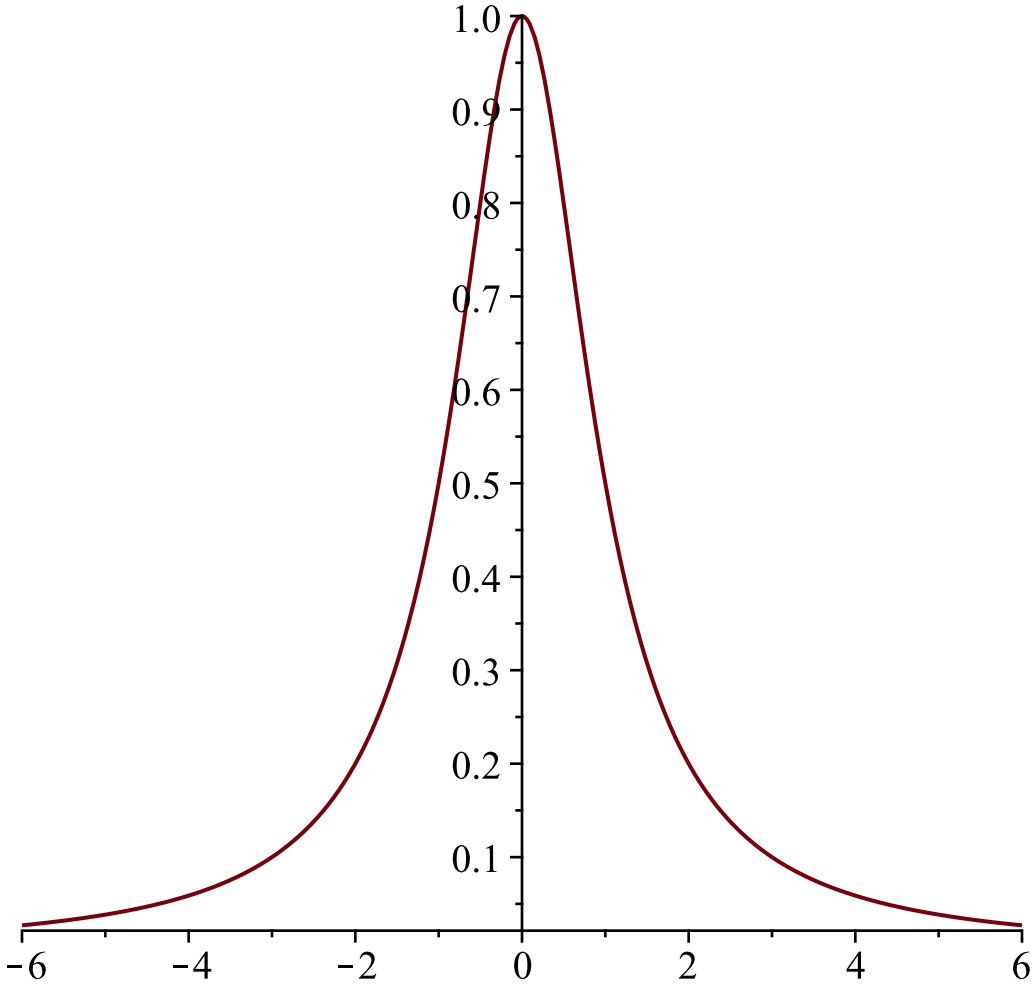
```
printf("-----------------------------------------------\n");

 # Printing table
for i from 1 to nops(f_soln) do
    #print(f_soln[i])
    printf(" %05f       %05f       %05f \n", f_soln[i],  p_soln[i],  abs_diff[i] )
 end do:
```

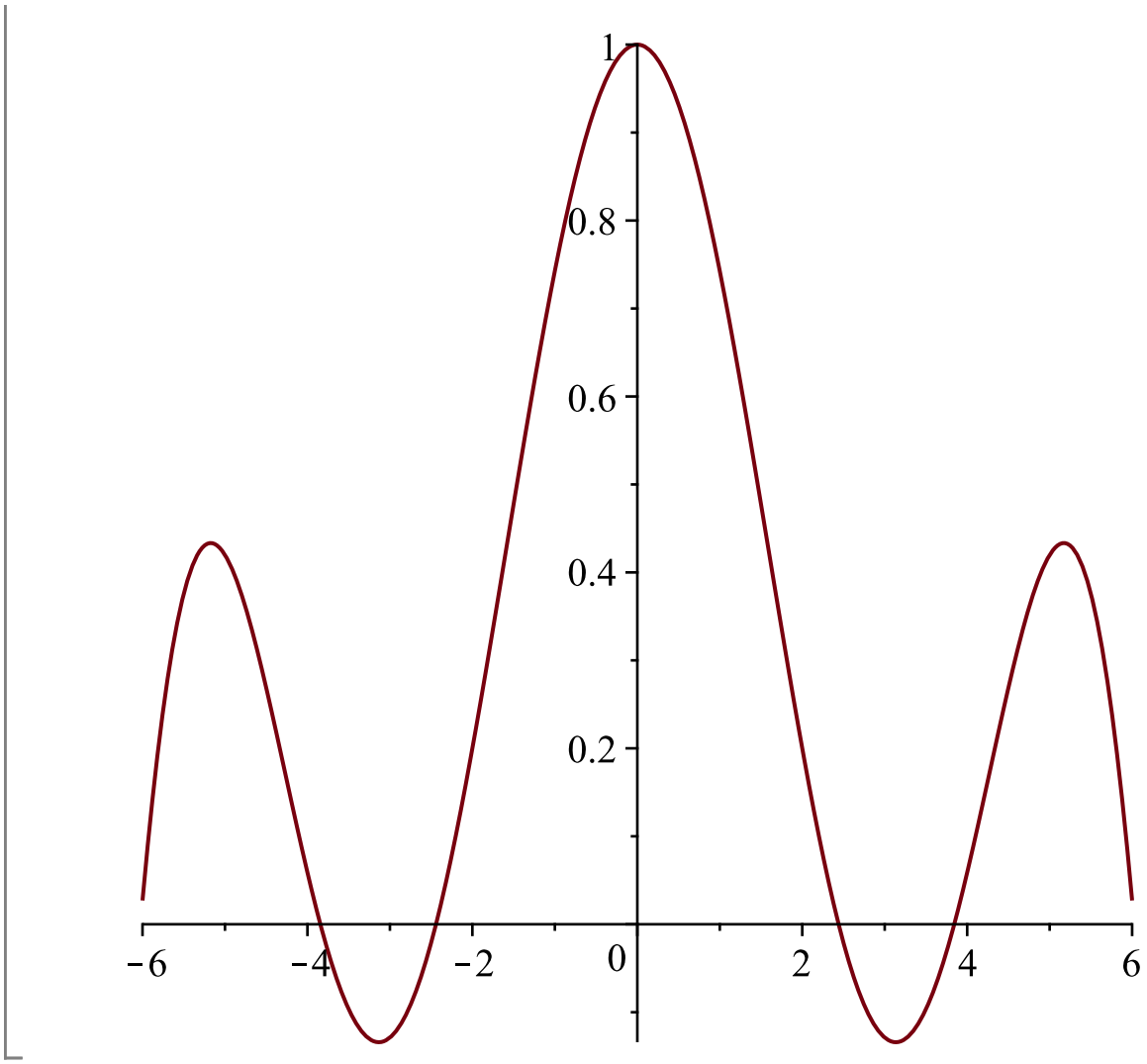
```
    f(x)              P8(x)         |f(x)  -  P8(x)|
-----------------------------------------------------
 0.015385          0.015385          0.000000
 0.020000          -1.368203          1.388203
 0.027027          0.027027          0.000000
 0.038462          0.419836          0.381375
 0.058824          0.058824          0.000000
 0.100000          -0.128825          0.228825
 0.200000          0.200000          0.000000
 0.500000          0.742693          0.242693
 1.000000          1.000000          0.000000
 0.500000          0.742693          0.242693
 0.200000          0.200000          0.000000
 0.100000          -0.128825          0.228825
 0.058824          0.058824          0.000000
 0.038462          0.419836          0.381375
 0.027027          0.027027          0.000000
 0.020000          -1.368203          1.388203
 0.015385          0.015385          0.000000
```

Plot f(x) and P8(x):

```
plot(f, -6.0..6.0);
```




```
plot(P8,-6..6);
```


1
0.8
0.6
0.4
0.2
−6
−4
−2
0
2
4
6