

MAD 3105 Assignment 01
Relations and Their Properties

NAME: _____
DUE: Thursday, January 19th (11:59pm EST)

Directions: See Canvas. Scan this page to the front of your work.

(1) (3 points/3 points/2 points/2 points)

For the relation $R = \{(1, 1), (1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4), (4, 1)\}$ on the set $A = \{1, 2, 3, 4\}$, list **all** elements that are **missing** from the relation in order for R to satisfy the following properties: (a) reflexive, (b) symmetric, (c) transitive.
(d) Explain whether or not R is *antisymmetric*.

(2) (2 points each)

Let the sets be relations on the real numbers: $R_1 = \{(a, b) \in \mathbb{R}^2 \mid a \geq b\}$, the “greater than or equal to” relation & let $R_2 = \{(a, b) \in \mathbb{R}^2 \mid a \neq b\}$, the “unequal to” relation.
Find:

(a) $R_1 \cap R_2$ (write out the relation in the *set notation*, as R_1 and R_2 were written)

(b) $R_1 \oplus R_2$ (write out the relation in the *set notation*, as R_1 and R_2 were written).

Note: \oplus is XOR (exclusive or).

(3) (3 points/3 points)

For the set $A = \{a, b, c\}$, answer the following:

(a) How many *elements* are in the set $A \times A$?

(b) How many *binary relations* are there on the set?

(4) (5 points)

If $R = \{(1, 1), (1, 2), (2, 4), (3, 1), (3, 0)\}$, $S = \{(1, 2), (2, 0), (3, 1), (0, 0), (4, 3)\}$
find $S \circ R$, with elements listed as above.

(5) (5 points each)

R is the relation represented by the matrix $\mathbf{M}_R = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, find the *matrix*

representations for:

(a) R^{-1} (R inverse),

(b) \bar{R} (R complement),

(c) $R \circ R$ (i.e., R^2 , R composed with R).

(6) (4 points/2 points/4 points)

(a) The relation R is on $\{1, 2, 3\}$. Represent the relation

$R = \{(1, 1), (2, 1), (2, 2), (2, 3), (3, 2)\}$ with a *matrix*.

(b) By looking at the matrix, is the relation R *reflexive*? *Why or why not?*

(c) Draw the *directed graph* that represents the relation R .