

# One Nine

October 18, 2024

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## 1 Recall

College algebra  $f \circ g(x) = f(g(x))$

## 2 Objectives

Compose linear transformations Compute the compositions of reflection and rotation transformations Compute the inverse of a transformation

### 3 Notes

Given a linear transformation  $TA : \mathbb{R}^n \rightarrow \mathbb{R}^m$  and  $T_B : \mathbb{R}^m \rightarrow \mathbb{R}^k$  define the composition  $(T_B \circ T_A)(x)$  to be the function  $T_B(TA(x))$

- In fact  $T_B \circ T_A = T_{BA}$

### 4 Examples

find the standard matrix for each of the following transformation, and find the standard matrix for the composite functions  $T_1 \circ T_2$  and  $T_2 \circ T_1$

$$T_1(e_1) = T_1(1, 0) = (1 + 0, 1 - 0) = (1, 1)$$

$$T_1(e_2) = T_1(0, 1) = (0 + 1, 0 - 1) = (1, -1)$$

$$T_1 \Rightarrow A = [T(e_1) \quad T(e_2)] = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$T_2(e_1) = T_2(1, 0) = (2(1), -3(0)) = (2, 0) \quad T_2(e_2) = T_2(0, 1) = (2(0), -3(1)) = (0, -3)$$

$$T_2 \Rightarrow B = [T_2(e_1) \quad T_2(e_2)] = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}$$

$$T_1 \circ T_2 \Rightarrow AB = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 2 & 3 \end{bmatrix}$$

$$T_2 \circ T_1 \Rightarrow BA = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -3 & 3 \end{bmatrix}$$

Transpose of each other

### 5 Example

$T_1 : \mathbb{R}^2 \rightarrow \mathbb{R}^3$  by  $T_1(x, y) = (-2, 2y, y)$

$$T_1(e_1) = T_1(1, 0) = (-1, 0, 0), T_1(e_2) = T_1(0, 1) = (0, 2, 1) \Rightarrow T_1 \Rightarrow$$

$$A = [T_1(e_1) \quad T_1(e_2)] = \begin{bmatrix} -1 & 0 \\ 0 & 2 \\ 0 & 1 \end{bmatrix} \quad T_2(e_1) = T_2(1, 0, 0) = (-1, 0, 0), T_2(e_2) =$$

$$T_2(0, 1, 0) = (0, -1, 1), T_2(e_3) = T_2(0, 0, 1) = (0, 0, 1) \Rightarrow T_2 \Rightarrow B = [T_2(e_1) \quad T_2(e_2) \quad T_2(e_3)] =$$

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

## 6 Prove that the composition of two rotations in $\mathbb{R}^2$ is commutative

- $\alpha + \beta$  degrees

Recall, supposed we rotate the vector  $\alpha$  &  $\beta$  ccw

$$\alpha = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \beta = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix}$$

$$\alpha + \beta \Rightarrow \begin{bmatrix} \cos(\alpha + \beta) & -\sin(\alpha + \beta) \\ \sin(\alpha + \beta) & \cos(\alpha + \beta) \end{bmatrix}$$

wts  $AB = BA$

$$\$ = [\cos(\alpha+\beta) \& -\sin(\alpha+\beta) \backslash \sin(\alpha+\beta) \& \cos(\alpha+\beta)] \$$$

## 7 Reflecting a Vector across the line $y=x$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \text{ Transpose if you switch the order of the mat mul}$$

## 8 If $T_A : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a matrix operator whose standard matrix $A$ is invertible, then we say that $T_A$ is invertible, and we define the inverse $T_A$ as

$$T_A^{-1} = T_{A^{-1}}$$

1. Given the operator  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  defined by

$$w_1 = 2x_1 + 5x_2, w_2 = -x_1 + 7x_2$$

compute  $T^{-1}(w_1, w_2)$

WTF  $A^{-1}$ . So we need to find  $A$  first.  $T(1,0) = (2(1)+5(0), -1+7(0)) = (2,-1)$   
 $T(0,1) = (5,7)$   $\begin{bmatrix} 2 & 5 \\ -1 & 7 \end{bmatrix} \Rightarrow \frac{1}{19} \begin{bmatrix} 7 & -5 \\ 1 & 2 \end{bmatrix} (\frac{7}{19}w_1 - \frac{5}{19}w_2, \frac{1}{19}w_1 + \frac{2}{19}w_2)$