Hw7

Jackson Mowry

October 20, 2024

1 1

A = $\{0^n\#0^{2n}\#0^{3n} \mid n \geq 0\}$. Hint: consider s = $0^p\#0^{2p}\#0^{3p}$.

Assume that A is a context free language and let p be the pumping length for the language. Choose the string s = $0^p\#0^{2p}\#0^{3p} \in A$ so that clearly $|s| > p$. By condition 1 of the PL for CFLs, we can partition s = uvxyz such that for all $i \geq 0$, $uv^ixy^iz \in A$. In order to show that s cannot be pumped, let's consider the ramifications of Condition 1 of the PL for CFLs for the contents of v and y.

If either v or y contain the '#' symbol. S' = $uv^2xy^2z \notin A$ as the resulting string would now contain greater than 2 '#' symbols, therefore s' condition 1 of the PL for CFLs is violated.

By condition 3 of the PL for CFLs we know that $|vxy| \leq p$, therefore we can choose vxy to consist of entirely 0's in the second segment (0^{2p}). This means that both v and y also consist entirely of 0's. If we then pump up uv^2xy^2z, the resulting string s' $\notin A$, as the second segment would now contain more than 2n 0's, while the other sections (including '#' symbols would remain the same), violating the constraint originally laid out ($0^n\#0^{2n}\#0^{3n}$). We can also consider the case where either v or y contains the '#' symbol, keeping the same s' = uv^2xy^2z we now violate the strict ordering of 0's and #'s by introducing additional # symbols between 0's. Therefore s' $\notin A$ and condition 1 of the PL for CFLs is violated, and we can conclude that A is not a CFL.#

2 2

C = {w | the number of 1s equals the number of 2s and the number of 3s equals the number of 4s} and Σ = {1,2,3,4}. Hint: consider s = $1^p3^p2^p4^p$.

Assume that C is a context free language and let p be the pumping length for the language. Choose the string s = $1^p3^p2^p4^p \in$ C so that clearly |s| > p. By condition 1 of the PL for CFLs, we can partition s = uvxyz such that for all i $\geq$ 0, $uv^ixy^iz \in$ A. In order to show that s cannot be pumped, let's consider the ramifications of Condition 1 of the PL for CFLs for the contents of v and y.

When both v and y contain only one type of symbol (terminal), then s' = uv^2xy^2z cannot contain equal numbers of 1's and 2's, or 3's and 4's, because one symbol would be present more times than the other in its pairing. Therefore s' $\notin$ C and condition 1 of the PL for CFLs is violated, and we can conclude that C is not a CFL.#

3 3

B = {w#t | w is a substring of t, where w, t $\in$ {a, b}}. Hint: consider s = $a^pb^p\#a^pb^p$.

Assume that B is a context free language and let p be the pumping length for the languge. Choose the string s = $a^pb^p\#a^pb^p \in$ B so that clearly |s| > p. By condition 1 of the PL for CFLs, we can partition s = uvxyz such that for all i $\geq$ 0, $uv^ixy^iz \in$ B. In order to show that s cannot be pumped, let's consider the ramifications for Condition 1 of the PL for CFLs for the contents of v and y.

When v and y contain ony one type of symbol (not '#'), and v/y occupy the begining of the string (substring w), then s' = $uv^2xy^2z \notin$ B because the w would no longer be a substring of t as it contains either more a's or b's and t. Therefore s' $\notin$ B and condition 1 of the PL for CFLs is violated.

When either v or y contain the '#' symbol (terminal). S' = $uv^2xy^2z \notin$ B due to the fact that the '#' symbol is now repeated more than once. Hence s' $\notin$ B, because condition 1 of the PL for CFLs is violated.

Since one of the above case must apply for any s $\in$ B, we can conclude that B is not a CFL.#