Three Two

October 18, 2024

Contents

1 Norm

For a vector in R^n, the length (or magnitude) of a vector is called the norm (magnitude | length) of u and is denoted ||u|| and defined by

- $||u|| = \sqrt{u_1^2 + u_2^2 + ... + u_n^2}$

1.1 Examples

- $||\mathrm{v}|| \geq 0$
  - let $v = (v_1, ..., v_n) \Rightarrow ||v|| = \sqrt{v_1^2 + ... + v_n^2} \geq 0$
- $||\mathrm{v}|| = 0$ iff $\mathrm{v} = 0$
  - $\$||\mathrm{v}|| = \sqrt{}\{v_1{}^2 + \ldots + v_\mathrm{n}{}^2\} = 0 \iff v_1{}^2 + \ldots + v_\mathrm{n}{}^2 \Rightarrow \mathrm{v}_1=\mathrm{v}_2=\mathrm{v}_\mathrm{n}\$\$$
- $||\mathrm{kv}|| = |\mathrm{k}|||\mathrm{v}||$

- $kv = (kv_1, kv_2, ..., kv_n)$
- $||kv|| = \sqrt{(kv_1)^2 + ... + (kv_n)^2}$
- $vk^2v_1^2 + ... + k^2v_n^2$
- $\sqrt{k^2(v_1^2 + ... + v_n^2)} = \sqrt{k^2}\sqrt{v_1^2 + ... + v_n^2}$

2 Unit Vector

- A vector of norm 1 is called a unit vector. To construct a unit vector from any nonzero vector v, in the same direction multiply v by the reciprocal of its length.
- $v \Rightarrow unitvectoru = \frac{v}{||v||}$
- $u = \frac{(1,2,3,4)}{\sqrt{1^2+2^2+3^2+4^2}} = (\frac{1}{\sqrt{30}}, \frac{2}{\sqrt{30}}, \frac{3}{\sqrt{30}}, \frac{4}{\sqrt{30}})$
- $R^2 \Rightarrow e_1 = (1, 0), e_2 = (0, 1)$
- $v = (v_1, v_2, ...v_n)$
- $v = v_1 \times e_1 + v_2 \times e_2 + ... + v_n \times e_n$

3 Distance

$d(u, v)$

- $\sqrt{(v_1 - u_1)^2 + ... + (u_n - u_n)^2}$
- The distance between the 2 tips

4 Dot Product

For u and v two nonzero vectors in R^2, or R^3, position the vectors so that their initial points coincide. The angle θ between u and v is the angle that satisfies $0 \leq \theta\ \pi$

4.1 Definition

If u and v are vectors in R^2, or R^3, and if θ is the angle between u and v, then the dot product or Euclidean inner product.

- $u * v = ||u||||v||cos\theta$

  - Only use this definition to find the angle θ

- If u = 0, or v = 0, then define u*v = 0

4.2 Easy

- $u * v = u_1v_1 + ... + u_nv_n$

This is really a mat mul of u * v^T