

Two One

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1 Obj

Use cofactor expansion on any row or col to compute the determinant of a square matrix Use diagonals to copute the determinants of 2x2 and 3x3 matrices

2 Definition

ad-bc

- 1. 0
- 2. 50

3 Definition Minors and Cofactors

If A is a square matrix, then the minor M_{ij} of the entry a_{ij} is the determinant of the matrix obtained by deleting the i th row and the j th column of A . The cofactor C_{ij} of the entry a_{ij} is $C_{ij} = (-1)^{i+j}M_{ij}$.

Find the minors and the cofactors for $|A| = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 1 & 5 \\ 4 & 6 & 10 \end{bmatrix}$

3.1 Minors

$$\begin{aligned} M_{11} &= \begin{bmatrix} 1 & 5 \\ 6 & 10 \end{bmatrix} = -20, C \rightarrow (-1)^{1+1}(-20) = -20 \quad M_{12} = \begin{bmatrix} 3 & 5 \\ 4 & 10 \end{bmatrix} = 10, C \rightarrow \\ &(-1)^{1+2}(10) = -10 \quad M_{13} = \begin{bmatrix} 3 & 1 \\ 4 & 6 \end{bmatrix} = 14, C \rightarrow (-1)^{1+3}(14) = 14 \quad M_{21} = \begin{bmatrix} 2 & 4 \\ 6 & 10 \end{bmatrix} \\ &= -4, C \rightarrow (-1)^{2+1}(-4) = 4 \quad M_{22} = \begin{bmatrix} 1 & 4 \\ 4 & 10 \end{bmatrix} = -6, -6 \quad M_{23} = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix} = -2, 2 \\ M_{31} &= \begin{bmatrix} 2 & 4 \\ 1 & 5 \end{bmatrix} = 6, 6 \quad M_{32} = \begin{bmatrix} 1 & 4 \\ 3 & 5 \end{bmatrix} = -7, 7 \quad M_{33} = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} = -5, -5 \end{aligned}$$

4 Definition of Determinant

For A a square matrix of order $n \geq 2$, the determinant of A is the sum of the entries in the first row of A (or any row or column of A), multiplied by their respective cofactors.

$$\text{Det}(A) = |A| = \sum_{j=1}^n a_{1j}C_{1j} = a_{11}C_{11} + a_{12}C_{12} + \dots + a_{1n}C_{1n}.$$

4.1 Compute

$$|A| = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 1 & 5 \\ 4 & 6 & 10 \end{bmatrix}$$

Find the column or row with the most zero entries

$$\text{Pick the second row} \Rightarrow |A| = a_{21}C_{21} + a_{22}C_{22} + a_{23}C_{23} = 3 \cdot 4 + 1 \cdot (-6) + 5 \cdot 2 = 16$$

5 Triangle Determinant

Product of main diagonal entries

6 Trick

$$\begin{aligned} & \left[\begin{smallmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{smallmatrix} \right] \begin{smallmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{smallmatrix} \begin{smallmatrix} a_{11} & a_{22} & a_{33} \\ a_{12} & a_{23} & a_{33} \\ a_{13} & a_{21} & a_{32} \end{smallmatrix} \\ & - (a_{12}a_{21}a_{33} + a_{11}a_{23}a_{32} + a_{12}a_{21}a_{33}) \end{aligned}$$