Three One

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A vector is characterized by 2 quantities, length and direction. Geometrically a vector in the plane is represented by a direct line segments with its initial point at the origin and its terminal point (x,y). We will consider vectors as objects of Vector spaces.

By R^{n} we mean all vectors with n componenets, and in each component is a real number.

1 Alternative Notation for Vectors

Comma delimited form $\mathrm{v} = (\mathrm{v}_1,\mathrm{v}_2,\mathrm{v}_3,\ldots \mathrm{v}_\mathrm{n})$ Row-vector form $\mathrm{v} = [\mathrm{v}_1\ \mathrm{v}_2\ \mathrm{v}_3\ \ldots\ \mathrm{v}_\mathrm{n}]$ Column vector form $\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \ldots \\ v_n \end{bmatrix}$

1.1 Facts

- Vectors with the same length and directions are equivalent
- Zero vector, inital and terminal points coincide
- Parallel and colinear mean the same thing when applied to vectors
  - colinear = One vector is a multiple of the other
- Vectors whose inital point is not at the origin:
  - $P_1P_2^{\Rightarrow}$ denotes the initial point $P_1(x_1,y_1)$ and terminal point $P_2(x_2,y_2)$. The components are $P_1P_2^{\Rightarrow} = (x_2\text{-}x_1,y_2\text{-}y_1)$
- The linear combination of vectors v_1,v_2,v_n is $k_1v_1 + k_2v_2 + k_nv_n$, where the k_i's are scalars, or real numbers

2 Length (Magnitude)

Sqrt of the sum of each component squared

3 Unit vector in the same direction

$\frac{u}{||u||}$

4 Write vector as a linear combination of (1,0) & (0,1)

$v^{\rightarrow} = a \times (1,0) + b \times (0,1)$ for some real numbers a,b So, $(2,5) = a(1,0) + b(0,1)$, where a = 2, and b = 5

5 For vectors (1,5,4) and (2,7,-4) find all vectors w such that w can be written as a linear combination of u and v

$w = (w_1, w_2, w_3)$ $w = a(1,5,4) + b(2,7,-4)$ $w_1 = a + 2b$ $w_2 = 5a + 7b$ $w_3 = 4a-4b$ $\begin{bmatrix} 1 & 2 & w_1 \\ 5 & 7 & w_2 \\ 4 & -4 & w_3 \end{bmatrix}$ $r_2-5r_1 \rightarrow r_2$ $r_3-4r_1 \rightarrow r_3$ $\begin{bmatrix} 1 & 2 & w_1 \\ 0 & -3 & w_2 - 5w_1 \\ 0 & -12 & w_3 - 4w_1 \end{bmatrix}$

$$r_3 - 4r_2 \to r_3 \begin{bmatrix} 1 & 2 & w_1 \\ 0 & -3 & w_2 - 5w_1 \\ 0 & 0 & w_3 - 4w_1 - 4(w_2 - 5w_1) \end{bmatrix}$$

- To have solution we need to make the RHS of row 3 = 0

$w_3 - 4w_1 - 4(w_2 - 5w_1) = 0$ $w_3 + 16w_1 - 4w_2 = 0$
let y = w_2, x = w_1 $w_3 = 4y - 16x$ So, $w^{\to}$ = any vector such that (x,y,4y-16x) where x,y $\in$ R

6 Notes

Cannot do u*v

- $\begin{bmatrix} 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 4 & 6 \end{bmatrix}$

Can do u * v^T

- $\begin{bmatrix} 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$