One Nine

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Contents

1 Recall

College algebra $f \circ g(x) = f(g(x))$

2 Objectives

Compose linear transformations Compute the compositions of reflection and rotation transformations Compute the inverse of a transformation

3 Notes

Given a linear transofmration $TA : \mathbb{R}^n \to m$ and $T_B : \mathbb{R}^m \to \mathbb{R}^k$ define the composition $(T_B \circ T_A)(x)$ to be the function $T_B(TA(x))$

- In fact $T_B \circ T_A = T_{BA}$

4 Examples

find the standard matrix for each of the following transformation, and find the standard matrix for the compositie functions $T_1 \circ T_2$ and $T_2 \circ T_1$

$$T_1(e_1) = T_1(1,0) = (1+0, 1-0) = (1,1)$$
$$T_1(e_2) = T_1(0,1) = (0+1, 0-1) = (1,-1)$$

$T_1 \Rightarrow A = \begin{bmatrix} T(e_1) & T(e_2) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$T_2(e_1) = T_2(1,0) = (2(1), -3(0)) = (2,0)$ $T_2(e_2) = T_2(0,1) = (2(0), -3(1)) = (0,-3)$

$T_2 \Rightarrow B = \begin{bmatrix} T_2(e_1) & T_2(e_2) \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}$

$T_1 \circ T_2 \Rightarrow AB = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 2 & 3 \end{bmatrix}$

$T_2 \circ T_1 \Rightarrow BA = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -3 & 3 \end{bmatrix}$

Transpose of each other

5 Example

$T_1 : \mathbb{R}^2 \to \mathbb{R}^3$ by $T_1(x,y) = (-2, 2y, y)$

$T_1(e_1) = T_1(1,0) = (-1,0,0), T_1(e_2) = T_1(0,1) = (0,2,1) \Rightarrow T_1 \Rightarrow A = \begin{bmatrix} T_1(e_1) & T_1(e_2) \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 2 \\ 0 & 1 \end{bmatrix}$ $T_2(e_1) = T_2(1,0,0) = (-1,0,0), T_2(e_2) = T_2(0,1,0) = (0,-1,1), T_2(e_3) = T_2(0,0,1) = (0,0,1) \Rightarrow T_2 \Rightarrow B = \begin{bmatrix} T_2(e_1) & T_2(e_2) & T_3(e_3) \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

6 Prove that the composition of two rotations in $\mathbb{R}^2$ is commutative

- $\alpha + \beta$ degrees

Recall, supposed we rotate the vector α & β ccw

$\alpha = \begin{bmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{bmatrix}$ $\beta = \begin{bmatrix} cos\beta & -sin\beta \\ sin\beta & cos\beta \end{bmatrix}$

$\alpha + \beta \Rightarrow \begin{bmatrix} cos(\alpha+\beta) & -sin(\alpha+\beta) \\ sin(\alpha+\beta) & cos(\alpha+\beta) \end{bmatrix}$

wts $AB = BA$

\$ = $\big[cos(\alpha+\beta)$&-sin(α+β)\sin(α+β)&cos(α+β)$\big]$\$

7 Reflecting a Vector across the line y=x

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ Transpose if you switch the order of the mat mul

8 If $T_A : \mathbb{R}^n \to \mathbb{R}^n$ is a matrix operator whose standard matrix A is invertible, then we say that $\mathbf{T_A}$ is invertible, and we define the inverse $\mathbf{T_A}$ as

$$T_A^{-1} = T_{A^{-1}}$$

1. Given the operator $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by

$$w_1 = 2x_1 + 5x_2 w_2 = -x_1 + 7x_2$$

compute $\mathrm{T}^{-1}(\mathrm{w}_1,\mathrm{w}_2)$

WTF A^{-1}. So we need to find A first. T(1,0)=(2(1)+5(0),-1+7(0) = (2,-1) T(0,1) = (5,7) $\begin{bmatrix} 2 & 5 \\ -1 & 7 \end{bmatrix} \Rightarrow \frac{1}{19}\begin{bmatrix} 7 & -5 \\ 1 & 2 \end{bmatrix}$ $(\frac{7}{19}w_1 - \frac{5}{19}w_2, \frac{1}{19}w_1 + \frac{2}{19}w_2)$