Ec Vector Spaces

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1 6

The set V = {(x,y,z) | x,y,z are integers}, with regular component addition and scalar multiplication as the operations

1. ku $\in$ V (Axiom 6)

    - Scalar multiples do not have to be integers $\therefore$ ku $\notin$ V
    - $u = (1, 1, 1), k = 4.5 \therefore ku = (4.5, 4.5, 4.5) \notin V$

2 7

The set V of all continuous functions that are differentiable on (-inf(), inf)

1. u + v $\in$ V

    - Both u and v are continuous and differentiable, u+v must also be continuous and differentiable (as the sum of continuous and differentiable functions respectively)
    - u + v $\in$ V

2. u + v = v + u

- Function addition is known to be commutative
- Therefore u + v = v + u

3. u + (v + w) = (u + v) + w

   - Function addition is known to be associative
   - Therefore u + (v + w) = (u + v) + w

4. There exists a zero 0 $\vec{0} such that 0 + u = u + 0 = u$

   5. The zero function f(x) = 0 for all x is continuous and differentiable, thus the zero vector is in V

For each u in V, there exists -u such that u + -u = 0

- If a function f is continuous and differentiable, then -f must also be continuous and differentiable, thus -f $\in$ V

ku $\in$ V

- If u is continuous and differentiable, then ku is continuous (since scalar multiples of continuous functions are continuous) and differentiable (since scalar multiples of differentiable functions are differentiable)
- $\therefore$ ku $\in$ V

k(u + v) = ku + kv

- This follows the basic properties of scalar multiplication and function addition

- $\therefore$ this property holds for our vector space

(k+m)u = ku + mu

- This is the same property as scalar multiplication
- $(k + m)u$
- $uk + um$
- $ku + mu$

(km)u = k(mu)

- This follows the same properties as scalar multiplication as proven above

1 × u = u

- We can simply substitute the value 0 in for '1' in the original axiom
  - Or the zero function
- This is treated as an addition of the identity
- $f(x) + 0 = f(x)$