Exam Review 2

October 18, 2024

I 1.8

1. Determine if a given transformation is linear/non-linear

   - Given T: R $\rightarrow$ R^n, want to determine if T is linear or non-linear

     – or x $\rightarrow$ *

   - Calculate T(u), T(v), and T(u + v)
   - Calculate T(ku) and kT(u)
   - Examples at the very bottom of completed lecture notes for 1.8

2. Given a linear transformation T, find the associated matrix A such that $T = T_A$

   - $\begin{bmatrix} T(e_1) & T(e_2) & ... & T(e_n) \end{bmatrix}$
   - Example at the very bottom of page 2
   - Practice HW problem similar to this

II 1.9

1. Find a standard matrix given T_1, T_2 to find standard matrix for the transformation

   - Given a linear transformation TA: $R^n \rightarrow m$ and T_B: $R^m \rightarrow R^k$ define the composition $(T_B \cdot T_A)(x)$ to be the function $T_B(T_A(x))$

     – $T_B \cdot T_A = T_{BA}$

   - Do the mat mul for $T_1 \cdot T_2 = T_A T_B$ and $T_2 \cdot T_1 = T_B T_A$

     – Example 1 and 2

2. Construct the standard matrix for a linear transformation to

- Reflect across the line y=x

  - $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} b \\ a \end{bmatrix}$

- Reflect across the x axis

  - $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a \\ -b \end{bmatrix}$

- Reflect across the y axis

  - $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -a \\ b \end{bmatrix}$

III 2.1

1. Cofactor Expansion across any row or column (ONLY 3x3!) to find determinant

- $det(\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix})$
- Choose a row or col that has the most 0 entries
- Remember $(-1)^{i+j}$

IV 2.2

1. Using elementary row operations to compute the determinant

- For a square matrix A, if A has a row or column of zeros, then det(A) = 0
- For a square matrix A, det(A) = det(A^T)
- Swapping 2 rows of A to make B, then det(B) = -det(A)
- Multiplying a row by a number to make B, then det(B) = kdet(a)
- Adding a multiple of another row to make B, then det(B) = det(A)

V 2.3

1. Prove det(A^{-1}) = $1\frac{}{det(A)}$

    2. We'll need to work this out from start to finish

    3. "Just a couple of lines" - Dr. Le

Prove det(ABC) = det(A)det(B)det(C)

- det(ABC) = det[(AB)C]
- = det(AB)det(C)
- = det(A)det(B)det(C)

Use a determinant to determine if a matrix is singular or non-singular (singular meaning non)

- $\neq 0$ means invertible or non-singular
- = 0 means singular or non-invertible

VI 3.1

1. draw vectors (a,b)

    - Basically just graph them from the origin

2. Write vector as a linear combination of other vectors

    - Given $u^{\rightarrow}$ and $v^{\rightarrow}$ find $w^{\rightarrow}$ such that $w^{\rightarrow}$ can be written as a linear combination of $u^{\rightarrow}$ and $v^{\rightarrow}$
    - $w^{\rightarrow} = au^{\rightarrow} + bv^{\rightarrow}$
    - Then solve the corresponding linear system for a & b
    - Example 3
    -] [

VII 3.2

1. Find the distance between 2 vectors $u^{\rightarrow} = (u_1, u_2, u_3)$ and $v^{\rightarrow} = (v_1, v_2, v_3)$

   - $d(u^{\rightarrow}, v^{\rightarrow})$
   - $\sqrt{(v_1 - u_1)^2 + (v_2 - u_2)^2 + (v_3 - u_3)^2}$

2. Angle between 2 vectors

   - $cos\theta = \frac{u \cdot v}{||u||||v||}$
     - Norm (|| ||) is $\sqrt{u_1^2 + u_2^2 + ... + u_n^2}$
   - $\theta = cos^{-1}$(see above)

VIII 3.3

1. Equation of a plane given a point (x_0, y_0, z_0) and normal vector (a, b, c)

   - $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

2. Given u and a Vector component of u along a

   - $w_1 = proj_a(u) = \frac{u \cdot a}{a \cdot a} \times a$
   - $w_2 = proj_a(u) = u - w_1$