One Five

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1 Recall

Multiply a row by a nonzero constant c Interchange 2 rows Add a constant c times one row to another

2 Definition 1

Matricies A and B are said to be row equivalent if either (hence each) can be obtained from the other by a sequence of elementary row operations

3 Definition 2

A matrix E is called an elementary matrix if it can be obtained from an identity matrix by performing a single elementary row operation

3.1 Example 1

Listed below are four elementary matricies and the oerations that product them

$$\begin{bmatrix}1 & 0\\0 & -3\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & 0\\0 & 0 & 0 & 1\\0 & 0 & 1 & 0\\0 & 1 & 0 & 0\end{bmatrix}\begin{bmatrix}1 & 0 & 3\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

1. Multiply the second row of I_2 by -3
2. Interchange the second and fourth rows of I_4
3. Add 3 times the third row of I_3 to the first row
4. Multiply the first row of I_3 by 1

Every elementary matrix is invertible, and the inverse is also an elementary matrix

- any elementary row operation we want to do on Matrix A, we could do it on the identity matrix, and left multiply it to matrix A to get what we want

For example, given $\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\end{bmatrix}$

Find the elementary matrix E, so that

1. $EA = \begin{bmatrix}3 & 6 & 9\\4 & 5 & 6\\7 & 8 & 9\end{bmatrix}$
2. $EA = \begin{bmatrix}7 & 8 & 9\\4 & 5 & 6\\1 & 2 & 3\end{bmatrix}$
3. $EA = \begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\8 & 9 & 10\end{bmatrix}$

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a = [1,0,0;0,1,0;1,0,1]
b = [1,2,3;4,5,6;7,8,9]

a * b
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3.2 Theorem

If a is an n x n matrix, then the following statements are equivlant, that is all true or all false

- A is invertible
- Ax = o has only the trivial solution
- The reduced row echelon form of A is I_n
- A is expressible as a product of elementary matricies

4 A Method for Inverting Matricies

To find the inverse of an invertible matrix A, find a sequence of elementary row operations that reduces A to the identity and then perform that same sequence of operations on I_n to obtain A^{-1}

4.1 Procedure

We want to reduce A to the identity matrix by row operations and simultaneously apply these operations to I to produce A^{-1}. To accomplist this we will adjoin the identity matrix to the right side of A, thereby producing a partitioned matrix of the form

$$[A|I]$$

Then we will apply row operations to this matrix until the left side is reduced to I; these operations will convert the right side to A^{-1}, so the final matrix will have the form

$$[I|A^{-1}]$$

4.2 Example 3

Find the inverse of A if it exists

1. $A = \begin{bmatrix} 1 & 2 & 3 & & & \\ 2 & 5 & 3 & & & \\ 1 & 0 & 8 \| 1 & 2 & 3 \end{bmatrix} \text{]\$[]\$[]\$[]\$[]\$[]\$[]\$[]\$[}$

2. $A = \begin{bmatrix} 1 & 6 & 4 \\ 2 & 4 & -1 \\ -1 & 2 & 5 \end{bmatrix}$