

# HW6

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## 1 1

Starting with

- $A \rightarrow BAB \mid B \mid 1 \mid \epsilon$
- $B \rightarrow 00 \mid \epsilon$

Add a new start state 'S'

- $S \rightarrow A$
- $A \rightarrow BAB \mid B \mid 1 \mid \epsilon$
- $B \rightarrow 00 \mid \epsilon$

## 2 2

Removal of  $\epsilon$  rules

### 2.1 a

$\Sigma = \{a,b,c\}$  R:  $S \rightarrow A \mid A \rightarrow AaB \mid B \rightarrow b \mid C \mid \epsilon \mid C \rightarrow CC \mid c \mid \epsilon$

- Step 2 removing Epsilon Rules
  - Removing  $B \rightarrow \epsilon$ 
    - \*  $S \rightarrow A$
    - \*  $A \rightarrow AaB \mid Aa$
    - \*  $B \rightarrow b \mid C$
    - \*  $C \rightarrow CC \mid c \mid \epsilon$

- Removing  $C \rightarrow \epsilon$ 
  - \*  $S \rightarrow A$
  - \*  $A \rightarrow AaB \mid Aa$
  - \*  $B \rightarrow b \mid C$
  - \*  $C \rightarrow CC \mid c$

## 2.2 b

- $S \rightarrow A$
- $A \rightarrow AA \mid AB \mid B \mid a$
- $B \rightarrow BB \mid b \mid \epsilon$

The only epsilon rule we see is  $B \rightarrow \epsilon$

- $S \rightarrow A$
- $A \rightarrow AA \mid A \mid AB \mid B \mid a$
- $B \rightarrow BB \mid B \mid b$

## 3 3

### 3.1 a

- $S \rightarrow A$
- $A \rightarrow AA \mid AB \mid A \mid B \mid aB$
- $B \rightarrow BB \mid Bb \mid b$

Start by removing  $A \rightarrow A$

- $S \rightarrow A$
- $A \rightarrow AA \mid AB \mid B \mid aB$
- $B \rightarrow BB \mid Bb \mid b$

Replace  $A \rightarrow B$

- $S \rightarrow A$
- $A \rightarrow AA \mid AB \mid BB \mid Bb \mid b \mid aB$

- $B \rightarrow BB \mid Bb \mid b$

Replace  $S \rightarrow B$

- $S \rightarrow AA \mid AB \mid BB \mid Bb \mid b \mid aB$
- $A \rightarrow AA \mid AB \mid BB \mid Bb \mid b \mid aB$
- $B \rightarrow BB \mid Bb \mid b$

### 3.2 b

$\Sigma = \{a,b,c\}$  R:

- $S \rightarrow A \mid \epsilon$
- $A \rightarrow BC$
- $B \rightarrow BD \mid bb$
- $C \rightarrow CD \mid cc$
- $D \rightarrow B \mid C$

Step 3 Removal of unit rules (remember we can have epsilon because the empty string is in the language)

- $S \rightarrow \mathbf{A} \mid \epsilon$
- $A \rightarrow BC$
- $B \rightarrow BD \mid bb$
- $C \rightarrow CD \mid cc$
- $\mathbf{D} \rightarrow \mathbf{B} \mid \mathbf{C}$

Removing  $S \rightarrow A$

- $S \rightarrow BC \mid \epsilon$
- $A \rightarrow BC$
- $B \rightarrow BD \mid bb$
- $C \rightarrow CD \mid cc$
- $D \rightarrow B \mid C$

Removing  $B \rightarrow D$

- $S \rightarrow BC \mid \epsilon$
- $A \rightarrow BC$
- $B \rightarrow BD \mid bb$
- $C \rightarrow CD \mid cc$
- $D \rightarrow BD \mid bb \mid C$

Removing  $D \rightarrow C$

- $S \rightarrow BC \mid \epsilon$
- $A \rightarrow BC$
- $B \rightarrow BD \mid bb$
- $C \rightarrow CD \mid cc$
- $D \rightarrow BD \mid bb \mid CD \mid cc$

A is now redundant

## 4 4

- $S \rightarrow AAB \mid aBb \mid ABB \mid Ab$
- $A \rightarrow AAB \mid aBb \mid ABB \mid Ab$
- $B \rightarrow BB \mid Bb \mid b$

Start by identifying rules not in CNF (bold)

- $S \rightarrow \mathbf{AAB} \mid \mathbf{aBb} \mid \mathbf{ABB} \mid \mathbf{Ab}$
- $A \rightarrow \mathbf{AAB} \mid \mathbf{aBb} \mid \mathbf{ABB} \mid \mathbf{Ab}$
- $B \rightarrow BB \mid \mathbf{Bb} \mid b$

Create  $A_1$  which produces AA

- $S \rightarrow A_1B \mid \mathbf{aBb} \mid \mathbf{ABB} \mid \mathbf{Ab}$
- $A \rightarrow A_1B \mid \mathbf{aBb} \mid \mathbf{ABB} \mid \mathbf{Ab}$

- $B \rightarrow BB \mid \mathbf{Bb} \mid b$

- $A_1 \rightarrow AA$

Create  $B_1$  which produces  $BB$

- $S \rightarrow A_1B \mid \mathbf{aBb} \mid AB_1 \mid \mathbf{Ab}$

- $A \rightarrow A_1B \mid \mathbf{aBb} \mid AB_1 \mid \mathbf{Ab}$

- $B \rightarrow BB \mid \mathbf{Bb} \mid b$

- $A_1 \rightarrow AA$

- $B_1 \rightarrow BB$

Create  $B_2$  which produces  $b$

- $S \rightarrow A_1B \mid \mathbf{aBb} \mid AB_1 \mid AB_2$

- $A \rightarrow A_1B \mid \mathbf{aBb} \mid AB_1 \mid AB_2$

- $B \rightarrow BB \mid BB_2 \mid b$

- $A_1 \rightarrow AA$

- $B_1 \rightarrow BB$

- $B_2 \rightarrow b$

Create  $A_2$  which produces  $a$  &  $C_1$  which produces  $A_2B$

- $S \rightarrow A_1B \mid C_1B_2 \mid AB_1 \mid AB_2$

- $A \rightarrow A_1B \mid C_1B_2 \mid AB_1 \mid AB_2$

- $B \rightarrow BB \mid BB_2 \mid b$

- $A_1 \rightarrow AA$

- $A_2 \rightarrow a$

- $B_1 \rightarrow BB$

- $B_2 \rightarrow b$

- $C_1 \rightarrow A_2B$

5 5

