Four One

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1 Objectives

- Vector space and subspace

2 Vector Space

A vector space is a set V of objects called vectors with a field of scalars, we will use the field of real numbers, and operation denoted by + that combines vectors, and an operation denoted * that combines a scalar and a vector, such that for u,v,w, objects in V and k,l, and m scalars in R

1. u+v $\in$ V

2. u+v = v+u
3. u+(v+w) = (u+v) + w
4. There exists a zero 0, vector such that 0+u = u+0 = u
5. For each u in V, there exists -u such that u + -u = 0
6. ku $\in$ V
7. k(u+v) = ku + kv
8. (k+m)u = ku + mu
9. (km)u = k(mu)
10. 1 * u = u

2.1 Examples

2.1.1 The zero vector space

Nothing R = {0} R^2 = {(0,0)}

2.1.2 R^n with componentwise addition and scalar multiplication

$v + u = (v_1 + u_1 + \ldots + v_n + u_n)$ $ku = (kv_1, kv_2, \ldots, kv_n)$

2.1.3 $M_{m \times n}$ with matrix addition and scalar multiplication

2.1.4 P_n the set of all polynomials of degree n or less with regular polynomial addition and scalar multiplication

Any polynomial of degree n or less

2.1.5 C(-inf,inf): all functions continuous on R

2.2 Determine if this is a Vector Space

1. The set V of all 2 $\times$ 2 matricies, and the vector operation to be matrix multiplication
    - Vector operation in this case A + B = AB
    - Not a vector space

2. Determine if the set of positive Real numbers: R^+ with the operations of u+v = uv real number multiplication and ku = u^k is a vector space

    - A + B = AB
    - kA = A^k
    - Not a vector space

3 Theorem

Let V be a vector space, u a vector in V and k a scalar;

- 0u = 0
- k $\overrightarrow{0} = 0(-1)u = -u$

- If ku = 0 then k = 0 or u = 0