One Three

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1 Objectives

- Add and subtract matricies
- Multiply matricies
- Writing a system of linear equations as a matrix product
- Write a matrxi product as the linear combination of the columns
- Compute the tract and transpost of a matrix

2 Consistent Factors

A matrix with m rows and n columns is called an m by n matrix Every location in a matrix A is double subscripted The $\mathrm{a}_{\mathrm{i,j}}$th number is found in the /i/th row and /j/th column 3 by 4 matrix 5, 9, 3 Row matricies have one row for example, column matricies have one column, also called vector

3 Matrix Arithmetic

3.1 Defintion

Two matricies A and B are equal, if they have the same dimension (m x n), and their corresponding entries are identical. $(\mathrm{A})_{\mathrm{ij}} = = \mathrm{a}_{\mathrm{ij}} = \mathrm{b}_{\mathrm{ij}} = (\mathrm{B})_{\mathrm{ij}}$ for $1 <= i <=$ m, and $1 <= j <= n$

3.2 Addition/Subtraction

Two matricies have a sumer or different if and only if they have the same dimensions (m x n), and their sum is defined to be $(\mathrm{A} + \mathrm{B})_{\mathrm{ij}} = \mathrm{a}_{\mathrm{ij}} + \mathrm{b}_{\mathrm{ij}}$

1	2	4
1	3	9

-

$$\begin{matrix} 2 & 3 & 6 \\ 4 & 5 & 20 \end{matrix}$$

=

$$\begin{matrix} 3 & 5 & 10 \\ 5 & 8 & 29 \end{matrix}$$

$$\begin{matrix} 1 & 3 \\ 4 & 7 \\ 2 & -1 \end{matrix}$$

- $$\begin{matrix} 4 & 2 \\ 1 & 3 \\ 0 & 6 \end{matrix}$$

=

$$\begin{matrix} (-3) & 1 \\ 3 & 4 \\ 2 & -7 \end{matrix}$$

3.3 Definition

If A is an m x n matrix and c is a scalar, then the **scalar multiple** of A by c is the m x n matric given by cA = (ca_{ij})

1.

$$\begin{matrix} 7 & 63 \\ 56 & -14 \end{matrix}$$

1.

$$\begin{matrix} ka & kb & kc \\ kd & ke & kf \end{matrix}$$

1.

$$\begin{matrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{matrix}$$

1.

$$\begin{matrix} 16 & 5 \\ 2 & 13 \end{matrix}$$

3.4 Definition

Two matricies A and B can be multiplied if and only if the number of columns of A matches the number of rows of B

$A_{m\ x\ n}B_{n\ x\ p} = C_{m\ x\ p}$ and $(AB)_{ij}$ = k=1n$a_{ik}b_{kj} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots + a_{in}b_{nj}$

If A is 3x4 and B is 4x5 => 3x5 If A is 3x4 and B is 3x4 => invalid multiplication

3.4.1 How-to Matrix Multiply

Row i times column j

1. 1

```
a = [1,3]
b = [a,b;c,d]
a * b
```

row1 = [1,3] & column1 = [a;c] turns into 1(a) + 3(c) = a+3c

row1 = [1,3] & column2 = [b;d] turns into 1(b) + 3(d) = b+3d

row1*colum1	row1*column2
a+3c	b+3d

2. 2

```
a = [0,-1;1,0]
b = [5;1]

a * b
```

0(5)+(-1)1
1(5)+0(1)

(-1)
5

3. 3

```
a = [1,2;3,4;5,6]
b = [4,3;5,1]

a * b
```

1(4)+2(5)	1(3)+2(1)
3(4)+4(5)	3(3)+4(1)
5(4)+6(5)	5(3)+6(1)

14	5
32	13
50	21

4. 4

a_{11}	a_{12}
a_{21}	a_{22}

and

b_{11}	b_{12}
b_{21}	b_{22}

$a_{11}(b_{11}) + a_{22}(b_{21})$	$a_{11}(b_{12}) + a_{12}(b_{22})$
$a_{21}(b_{11}) + a_{22}(b_{21})$	$a_{21}(b_{12}) + a_{22}(b_{22})$

5. 5

1(5)+2(6)	1(-1)+2(7)
3(5)+4(6)	3(-1)+4(7)

17	13
39	25

6. 6

5(1)+-1(3)	5(2)+-1(4)
6(1)+7(3)	6(2)+7(4)

2	6
27	40

7. 7

1(1)+3(4)+5(6)	1(6)+3(1)+5-1)

4 Coefficient Matrix

[A|b] where A are the left hand side, b is the right hand side of the equals sign

5 Linear Combination

Matrix product as linear combination

5.1 Theorem

If $A_{m \text{ x } n}$ and $x_{n \text{ x } 1}$, then Ax can be expressed as a linear combination of the column vectors of A in which coefficients are the entires of X(vector)

In general: A_x = [column 1]x_1 + [column 2]x_2 + ... + [column n]x_n

6 Transpose a Matrix

If any m x n matrix is transposed then it's dimensions will become n x m, row of A becomes column of A_T

$$(A^T)_{ij} = (A)_{ji}$$

7 Trace of a Matrix

If A is a square matrix, then the tract of A, denoted by **tr(A)** is defined to be the sum of the entries on the main diagonal of **A**. The trace of A is undefined if A is not a square matrix.

8 Matrix Polynomials

Given $2x^2-3x+4$ find f(a) We need to add an 'I' that is the same dimensions as the matrix, in this case $4I_2$

```
A = [4,1;3,5]

2 * (A * A) - (3*A) + 4*eye(2)

2*[19,9;27,28]-[12,3;9,15]+[4,0;0,4]
```

9 The Inverse of a Square Matrix

Any nxn matrix A is said to be invertible, or nonsingular, if there exists a matrix B such that AB = I = Ba. The inverse is written as A^{-1} If A has no inverse it is said to be singular

Need to check if AB = I && BA = I

If you multiply a matrix and its inverse you should get the identity matrix of the same size

9.1 Given A = [5,1;4,1] and B = [1,-1;-4,5], check if A and B are inverses

We take [a,b;c,d] -> [d,-b;-c,a]

```
a = [5,1;4,1]
b = [1,-1;-4,5]

b*a
```

9.2 Given A = [1,3;1,6] and B = [2,-1;-1/3,1/3] check if A and B are inverse

```
A = [1,3;1,6]
B = [2,-1;-1/3,1/3]

A*B
```

9.3 Theorem

A matrix A is only invertible if $ad - bc \neq 0$, in which case the inverse is given by the formula

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$ad - bc$ is called the determinant of the matrix A (cross multiply)

9.4 Practice

```
a = [1,2;3,4]
b = [-1,2;3,-2]
c = [2,-1;-4,2]
```

```
inv(a)
```

9.5 Why Inverse Matrix?

Helps us to solve a linear system

Given a matrix equation A $\rightarrow_{\mathrm{x}}$ = b$^{\rightarrow}$, we could solve the equation by applying Aˆ$_{\text{-1}}$ to both sides to get

$$A^{-1}Ax^{\rightarrow} = A^{-1}b^{\rightarrow}$$

$$x^{\rightarrow} = A^{-1}b^{\rightarrow}$$

Note, if A$^{\text{-1}}$ does not exist, the the equation has no solutions

9.5.1 Examples

Solve the system by matrix inversion

$$x_1 + 2x_2 = 43x_1 + 4x_2 = 10$$

\[$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 10 \end{bmatrix}$ \]

$$A^{-1} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & \frac{-1}{2} \end{bmatrix}$$

9.6 Properties of Inverses