

# Assignment 6B: Graphs

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## Overview

In this assignment you will explore different approaches to analyzing Graphs via Markov chains. You can find the dataset for this assignment from the course website:

<https://users.cs.utah.edu/~jeffp/teaching/DM/A/A6B-Graphs.pdf>

## 1 Anomalies

We will consider three ways to find  $q_* = M^t q_0$  as  $t \rightarrow \infty$ .

State Propagation: Iterate  $q_{i+1} = M * q_i$  for some large enough number  $t$  iterations.

Random Walk: Starting with a fixed state  $q_0 = [0, 0, \dots, 1, \dots, 0, 0]^T$  where there is only a 1 at the  $i^{\text{th}}$  entry, and then transition to a new state with only a 1 in the  $j^{\text{th}}$  entry by choosing a new location proportional to the values in the  $i^{\text{th}}$  column of  $M$ . Iterate this some large number  $t_0$  of steps to get state  $q'_0$ . (This is the burn-in period.)

Now make  $t$  new steps starting at  $q'_0$  and record the location after each step. Keep track of how many times you have recorded each location and estimate  $q_*$  as the normalized version (recall  $\|q_*\|_1 = 1$ ) of the vector of these counts.

Eigen-Analysis: Compute `LA.eig(M)` and take the first eigenvector after it has been  $L_1$ -normalized.

**A (30 points):** Run each method (with  $t = 1024$ ,  $q_0 = [1, 0, 0, \dots, 0]^T$  and  $t_0 = 100$  when needed) and report the answers.

**State propagation:**

[0.05103, 0.04374, 0.12806, 0.18613, 0.08748, 0.11726, 0.0885, 0.08319, 0.11726, 0.09735]

**Random walk:**

[0.05859, 0.05664, 0.10449, 0.15234, 0.06738, 0.10059, 0.11719, 0.09375, 0.14551, 0.10352]

**Random walk:**

[0.05103, 0.04374, 0.12806, 0.18613, 0.08748, 0.11726, 0.0885, 0.08319, 0.11726, 0.09735]

**B (10 points):** Rerun the State Propagation techniques with  $q_0 = [0.1, 0.1, \dots, 0.1]^T$ . What value of  $t$  is required to get as close to the true answer as the older initial state?

**t = 683 steps**

**C (10 points):** Is the Markov chain ergodic? Explain why or why not.

The Markov chain is ergodic because all the entries in the first eigenvector (the one with eigenvalue 1) are >0, meaning a random walk over the graph settles into a stable state as the number of steps approaches infinity.

# 1 Bonus: Graph Embedding (2 points)

Use a method of your choice to embed the graph in 2 dimensions to draw it. It should show vertices and edges.

