

Assignment 6B: Graphs

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Overview

In this assignment you will explore different approaches to analyzing Graphs via Markov chains. You can find the dataset for this assignment from the course website:

<https://users.cs.utah.edu/~jeffp/teaching/DM/A/A6B-Graphs.pdf>

1 Anomalies

We will consider three ways to find $q_* = M^t q_0$ as $t \rightarrow \infty$.

State Propagation: Iterate $q_{i+1} = M * q_i$ for some large enough number t iterations.

Random Walk: Starting with a fixed state $q_0 = [0, 0, \dots, 1, \dots, 0, 0]^T$ where there is only a 1 at the i^{th} entry, and then transition to a new state with only a 1 in the j^{th} entry by choosing a new location proportional to the values in the i^{th} column of M . Iterate this some large number t_0 of steps to get state q'_0 . (This is the burn-in period.)

Now make t new steps starting at q'_0 and record the location after each step. Keep track of how many times you have recorded each location and estimate q_* as the normalized version (recall $\|q_*\|_1 = 1$) of the vector of these counts.

Eigen-Analysis: Compute $\text{LA.eig}(M)$ and take the first eigenvector after it has been L_1 -normalized.

A (30 points): Run each method (with $t = 1024$, $q_0 = [1, 0, 0, \dots, 0]^T$ and $t_0 = 100$ when needed) and report the answers.

State propagation:

[0.05103, 0.04374, 0.12806, 0.18613, 0.08748, 0.11726, 0.0885, 0.08319, 0.11726, 0.09735]

Random walk:

[0.05859, 0.05664, 0.10449, 0.15234, 0.06738, 0.10059, 0.11719, 0.09375, 0.14551, 0.10352]

Random walk:

[0.05103, 0.04374, 0.12806, 0.18613, 0.08748, 0.11726, 0.0885, 0.08319, 0.11726, 0.09735]

B (10 points): Rerun the State Propagation techniques with $q_0 = [0.1, 0.1, \dots, 0.1]^T$. What value of t is required to get as close to the true answer as the older initial state?

t = 683 steps

C (10 points): Is the Markov chain ergodic? Explain why or why not.

The Markov chain is ergodic because all the entries in the first eigenvector (the one with eigenvalue 1) are >0, meaning a random walk over the graph settles into a stable state as the number of steps approaches infinity.

1 Bonus: Graph Embedding (2 points)

Use a method of your choice to embed the graph in 2 dimensions to draw it. It should show vertices and edges.

