Worksheet 20

October 30, 2018

1. Calculate the inverse of these matrices:

   a.
   $$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

   b.
   $$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

   c.
   $$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

   d.
   $$\begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}$$

   Use the equation for finding the inverse of any 2×2 matrix.

   a.
   $$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

   b.
   $$-1\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

   c.
   $$\frac{1}{-2}\begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}$$

   d.
   $$\frac{1}{-2}\begin{pmatrix} 1 & -3 \\ -2 & 4 \end{pmatrix}$$

2. **True or false:** The determinant of the identity matrix is 1.

   This is **true**.

3. **True or false:** Let A, B, and C be square matrices. Then $(AB)\,C = A\,(BC)$ is not always true.

   This is always **true**.

4. Calculate the determinant of the following matrix:

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 6 & 7 \end{pmatrix}$$

   One way to calculate this is as follows:

$$1 \cdot \det \begin{pmatrix} 4 & 5 \\ 6 & 7 \end{pmatrix} + 0 + 0 = -2$$

5. Find all solutions to the system of equations or matrix equation. For systems, first put them in the matrix form $Ax = b$.

   a.
$$\begin{cases} 2x_1 = 10 \\ 3x_1 - x_2 = 14 \end{cases}$$

   b.
$$\begin{pmatrix} 1 & -1 \\ 1/2 & -1/2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

   a. We can put it into matrix form:

$$\begin{pmatrix} 2 & 0 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 10 \\ 14 \end{pmatrix}$$

   then calculate the inverse:

$$A^{-1} = -\frac{1}{2} \begin{pmatrix} -1 & 0 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} 1/2 & 0 \\ 3/2 & -1 \end{pmatrix}$$

   and then we can just multiply both sides of the equation on the left to get:

$$x = \begin{pmatrix} 1/2 & 0 \\ 3/2 & -1 \end{pmatrix} \begin{pmatrix} 10 \\ 14 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

   b. This one is already in matrix form, so we just have to try to invert the matrix. But since $\det A = 0$, this is not invertible. The corresponding equations are just multiples of one another, i.e. these are parallel lines, so there are infinitely many solutions.