Worksheet 9

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1. Derive the Taylor polynomial/series at $a = 0$ for the following functions:

   a.
   $$f(x) = x^2$$

   b.
   $$f(x) = e^x$$

   c.
   $$f(x) = \ln(x+1)$$

   d.
   $$f(x) = \cos x + \sin x$$

   e.
   $$f(x) = \frac{1}{1-x}$$

   The definition:
   $$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)\, x^n}{n!}$$
   will give all of the following to you. However there are many tricks which can get them for you with far less work.

   a. This one is just x^2. It is good to check for yourself, but as it turns out, polynomials are their own Taylor polynomial.

   b. This was in lecture in the "hall of fame" but you can always rederive it without much trouble since e^x is such a simple function:
   $$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

   c. This was also in class, but is a bit more difficult to rederive.
   $$\ln(x+1) = \sum_{n=0}^{\infty} (-1)^{n+1} \frac{x^n}{n}$$

d. This is tricky, since it is the sum of two functions which we have seen the expansion of before. If you remember those, then the Taylor series of the sum is the sum of the Taylor series so we just get

$$\cos x + \sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} + (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

You can also just take derivatives manually, but every time you take the derivative, it will just split up by DL+.

e. We can take the derivatives of this one, and we start to see the following pattern:

$$\frac{d^n}{dx^n}\left(\frac{1}{1-x}\right) = \frac{n!}{(1-x)^{-n-1}}$$

But if we plug $x = 0$ in we just get $f^{(n)}(0) = n!$, so the series becomes:

$$\frac{1}{1-x} = \sum_{x=0}^{\infty} x^n$$

This one sure is easy to remember.

2. Find the following anti-derivatives:

a.
$$f(x) = 5x^{1/2} - 7x$$

b.
$$f(x) = (x+1)(2x-1)$$

c.
$$f(x) = \frac{20}{x^3}$$

d.
$$f(x) = \frac{10}{x}$$

e.
$$f(x) = \cos x$$

f.
$$f(x) = e^{x/2}$$

g.
$$f(x) = \frac{1}{\sqrt{1-x^2}}$$

Most anti-derivatives will be derivative problems you have seen some version of before, so it is a good ideal to keep a mental log of derivatives you have seen before. Throughout this problem c is a constant.

a. By the power rule, this is

$$F(x) = \frac{10}{3}x^{3/2} - \frac{7}{2}x^2 + c$$

b. Distribute, and work out the inverse power laws term by term to get:

$$F(x) = \frac{2}{3}x^3 + \frac{1}{2}x^2 - x + c$$

c. Again this is the power rule only it is negative this time:

$$F(x) = \frac{-5}{x^4} + c$$

d. Recall that $(\ln x)' = 1/x$, which means

$$F(x) = 10 \ln x + c$$

e. Recalling our trig derivatives, we get:

$$F(x) = \sin x + c$$

f. Recalling the chain rule, we get

$$F(x) = 2e^{x/2} + c$$

g. Finally, if we recall the derivatives of inverse trig functions we get:

$$F(x) = \arcsin(x) + c$$