Worksheet 10

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1. Calculate the following integrals:

a.
$$\int_1^9 \frac{x-1}{\sqrt{x}}\, dx$$

b.
$$\int_0^1 x\left(\sqrt[3]{x}+\sqrt[4]{x}\right)\, dx$$

c.
$$\int_0^{\pi/4} \frac{1}{\cos^2 x}\, dx$$

d.
$$\int_1^9 \frac{1}{2x}\, dx$$

e.
$$\int_{-1}^1 e^{x+1}\, dx$$

f.
$$\int_1^2 \frac{x^3+3x^6}{x^4}\, dx$$

g.
$$\int_{-1}^0 x^2\,(x+1)^3\, dx$$

a. Compute the anti-derivative and use the FTC:

$$\begin{aligned}\int_1^9 \frac{x-1}{\sqrt{x}}\, dx &= \int_1^9 x^{1/2} - x^{-1/2} = \left[\frac{2x^{3/2}}{3} - 2\sqrt{x}\right]_1^9 \\ &= \frac{27\cdot 2}{3} - 6 - \left(\frac{2}{3} - 2\right) = \frac{26\cdot 2}{3} - 4 = \boxed{\frac{40}{3}}\end{aligned}$$

b. Compute the anti-derivative and use the FTC:

$$\int_0^1 x\left(\sqrt[3]{x}+\sqrt[4]{x}\right) = \int_0^1 x^{4/3}+x^{5/4} = \left[\frac{3}{7}x^{7/3}+\frac{4}{9}x^{9/4}\right]_0^1$$
$$= \frac{3}{7}+\frac{4}{9} = \boxed{\frac{55}{63}}$$

c. Compute the anti-derivative and use the FTC:

$$\int_0^{\pi/4} \frac{1}{\cos^2 x}\,dx = [\tan x]_0^{\pi/4} = \boxed{1}$$

d. Compute the anti-derivative and use the FTC:

$$\int_1^9 \frac{1}{2x}\,dx = \frac{1}{2}[\ln x]_1^9 = \frac{\ln 9}{2} = \boxed{\ln 3}$$

e. Compute the anti-derivative and use the FTC:

$$\int_{-1}^1 e^{x+1}\,dx = \left[e^{x+1}\right]_{-1}^1 = e^2 - e^0 = \boxed{e^2-1}$$

f. Compute the anti-derivative and use the FTC:

$$\int_1^2 \frac{x^3+3x^6}{x^4}\,dx = \int_1^2 x^{-1}+3x^2 = \left[\ln x + x^3\right]_1^2$$
$$= \ln 2 + 8 - (\ln 1 + 1) = \boxed{\ln 2 + 7}$$

g. Compute the anti-derivative and use the FTC:

$$\begin{aligned}\int_{-1}^0 x^2(x+1)^3\,dx &= \int_{-1}^0 x^2\left(x^3+3x^2+3x+1\right)\\ &= \int_{-1}^0 x^5+3x^4+3x^3+x^2\\ &= \left[\frac{x^6}{6}+\frac{3x^5}{5}+\frac{3x^4}{4}+\frac{x^3}{3}\right]_{-1}^0\\ &= 0-\frac{1}{6}-\frac{3}{5}+\frac{3}{4}-\frac{1}{3}\\ &= -\frac{1}{60}(10-12\cdot 3+15\cdot 3-20)\\ &= \frac{1}{60}(10-9) = \boxed{\frac{1}{60}}\end{aligned}$$