Worksheet 24

1. find the eigenvalues of the following matrices:

   a.
   $$\begin{pmatrix} 2 & 0 \\ 3 & 0 \end{pmatrix}$$

   b.
   $$\begin{pmatrix} 3 & -1 \\ 0 & 2 \end{pmatrix}$$

2. In general, the eigenvalues of an upper triangular matrix are given by the entries on the diagonal. The same is true for a lower triangular matrix. Verify this for the 2×2 case. I.e. find the eigenvalues of a generic upper and lower triangular matrix and verify they are the diagonal entries.

3. Supposed that an eigenvalue of a matrix A is zero. Prove that A must be singular.

4. Find the eigenvalues and eigenvectors of the following matrices.

   a.
   $$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

   b.
   $$\begin{pmatrix} 1 & 2 \\ 3 & -3 \end{pmatrix}$$

5. Given the vector $[x_{n+1}, x_n]$ and $[x_n, x_{n-1}]$ find the matrix for the Fibonacci sequence.

6. Find the eigenvalues of the matrix from the previous question.

7. Suppose that λ is an eigenvalue of any matrix A. Show that λ^2 is an eigenvalue of A^2.

8. Find the eigenvalues for the generalized Fibonacci sequences' matrix.