Worksheet 8

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1. Evaluate the integrals using the "area under the curve" interpretation:

   a.
   $$\int_0^4 (3x - 3)\, dx$$

   b.
   $$\int_{-\pi}^{\pi} \sin x\, dx$$

   c.
   $$\int_{-1}^{1} x^3$$

   a. The function inside this integral is just a line of slope 3 which intersects the y axis at -3. So $f(0) = -3$, $f(1) = 0$, and $f(4) = 9$. This tells us that for $0 \leq x \leq 1$ we have a triangle below the x axis, and for $1 \leq x \leq 4$ we have a triangle above the x-axis. The first triangle contributes an area of $-3/2$. This is negative since it is below the axis. Then The rest of the graph, between $x = 1$ and $x = 4$, forms a triangle with area 18, so the total integral is $18 - 3/2 = 33/2$.

   b. First consider the function $\sin x$ between 0 and π. It is above the axis for this entire range, so the area under the curve in this range is positive, but it's hard to tell exactly what, so we will write this amount as A. Now in the range between $-\pi$ and 0 it is below the axis, so this will contribute some negative area, but again we can't tell exactly what it is. However, we can notice that these areas are the same, so the total integral is $A - A = 0$.

   c. By the exact same logic as the previous problem this is also 0.