Worksheet 17

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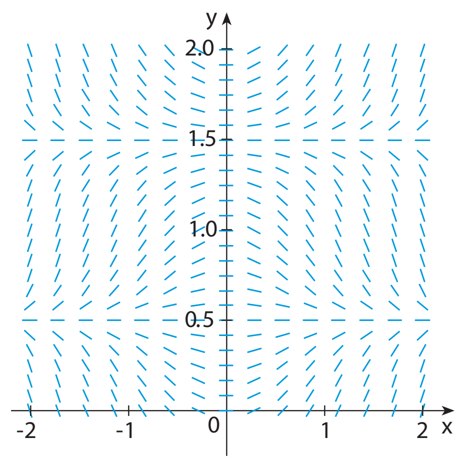
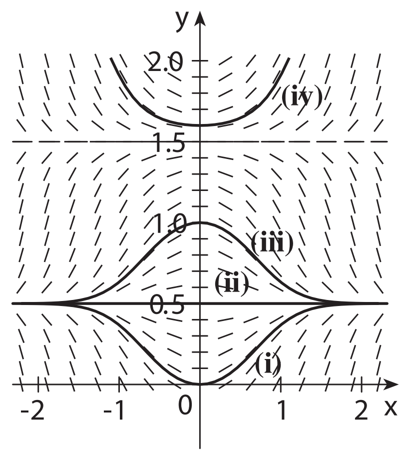
1. Below is the "direction field" for the differential equation: $y' = x \cos \pi y$.



   a. Sketch the graphs of the solutions that satisfy the given initial conditions.
      1. $y(0) = 0$
      2. $y(0) = 0.5$
      3. $y(0) = 1$
      4. $y(0) = 1.6$

   b. Find all of the equilibrium solutions.

   a. This is them:

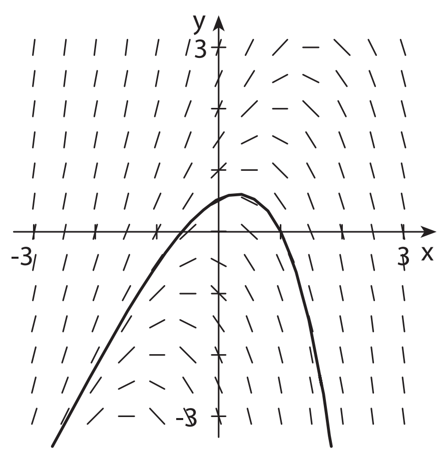


b. Recall this just means where the "flow lines" are flat, so the solutions are $y = 0.5$ and $y = 1.5$.

2. a. Sketch the direction field corresponding to the differential equation $y' = y - 2x$.

   b. Sketch the solution which passes through $(1, 0)$.

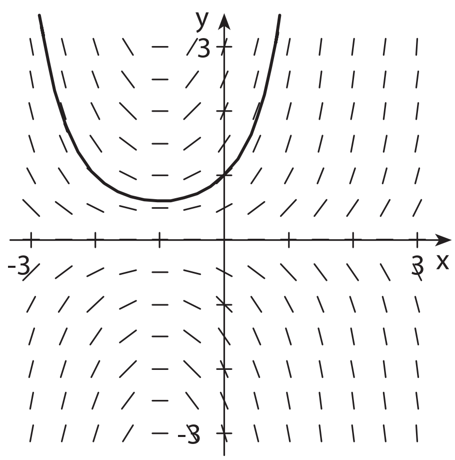
This is it:



3. a. Sketch the direction field corresponding to the differential equation $y' = y + xy$.

   b. Sketch the solution which passes through $(0, 1)$.

This is it:



4. **True or False:** Direction fields are only useful for analysing homogeneous differential equations.

   This is **false**.

5. Use Eulers method with step size 0.5 to compute the approximate y-values y_1, y_2, y_3, and y_4 of the solution of the initial-value problem $y' = y - 2x$ and $y(1) = 0$.

   Start at $x_0 = 1$ and $y(1) = 0$. Then $y'(x_0) = y'(1) = y - 2 \cdot 1 = 0 - 2 = -2$. Therefore if we move by 0.5 in the x direction, we must move -1 in the y direction, so so $y_1 = 0 - 1 = -1$.

   Now we repeat the same process. $y'(x_1) = y'(1.5) = -1 - 2 \cdot 1.5 = -4$. Therefore if we move 0.5 in the x-direction, we should move -2 in the y direction, so $y_2 = -3$.

   Now we repeat the same process. $y'(x_2) = y'(2) = y(2) - 2 \cdot 2 = -3 - 4 = -7$. Therefore if we move 0.5 in the x-direction we should move -3.5 in the y direction, so $y_3 = -6.5$.

   Now we repeat the same process. $y'(x_3) = y'(2.5) = y(2.5) - 2 \cdot 2.5 = -6.5 - 5 = -11.5$. Therefore if we move 0.5 in the x-direction we should move -5.75 in the y-direction, so $y_4 = -12.25$.

   So all together:

n	x_n	y_n
1	1.5	-1
2	2	-3
3	2.5	-6.5
4	3	-12.25

6. (Extra challenge) Solve the following second-order ODE: $t^2y''+3ty'-3y=0$.

   Guess solutions of the form $y=t^r$. Then if this is to satisfy the equation we must have:

$$\begin{aligned}
0 &= t^2\left(t^r\right)''+3t\left(t^r\right)'-3t^r \\
&= t^2\left(r\right)\left(r-1\right)t^{r-2}+3rtt^{r-1}-3t^r \\
&= r\left(r-1\right)t^r+3rt^r-3t^r \\
&= \left(r^2+2r-3\right)t^r
\end{aligned}$$

   so $y=0$ is a solution, and so is $y=t^r$ such that r satisfies the polynomial:

$$0=r^2+2r-3=\left(r-1\right)\left(r+3\right)$$

   which has solutions $r_1=1$ and $r_2=-3$. This means all of the solutions are

$$\boxed{y=C_1t+C_2t^{-3}}$$

   for any C_1 and C_2. Note we did not state $y=0$ separately, since for $C_1=C_2=0$ this is $y=0$!