Worksheet 25

1. Show that the two given vector functions are solutions to the linear system of ODE's $\vec{y}\,' = A\vec{y}$ where A is the given matrix.

   a. $\vec{y}(t) = \begin{pmatrix} 2e^{2t} \\ 5e^{2t} \end{pmatrix}$ and $\vec{z}(t) = \begin{pmatrix} e^{t} \\ 3e^{t} \end{pmatrix}$ where $A = \begin{pmatrix} 7 & -2 \\ 15 & -4 \end{pmatrix}$.

   b. $\vec{y}(t) = e^{-3t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\vec{z}(t) = e^{t}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ where $A = \begin{pmatrix} -1 & -2 \\ -2 & -1 \end{pmatrix}$.

   c. (Challenge) $\vec{y}(t) = e^{-t}\begin{pmatrix} \cos 2t \\ \sin 2t \end{pmatrix}$ and $\vec{z}(t) = e^{-t}\begin{pmatrix} -\sin 2t \\ \cos 2t \end{pmatrix}$ where $A = \begin{pmatrix} -1 & -2 \\ 2 & -1 \end{pmatrix}$.

2. In each case of Problem 1:

   - Write down the eigenvalues and eigenvectors of A **without** reapplying the algorithm for finding them but just using what is already given. (Part (c) is still "Challenge" here.)
   - Write down the **general** solution to the given system, **without** resolving the problem but just using what is already given.

3. Find two different solutions of the system, and then write the general solution:

   a. $\vec{y}\,' = A\vec{y}$ where $\vec{y}(t) = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$.

   b. $\vec{y}\,' = A\vec{y}$ where $\vec{y}(t) = \begin{pmatrix} -1 & 3 \\ -3 & -1 \end{pmatrix}$.

   c. (Challenge) $\vec{y}\,' = A\vec{y}$ where $\vec{y}(t) = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$.

4. Solve the initial value problem:

   a. $\vec{y}\,'(t) = \begin{pmatrix} 1 & 2 \\ 6 & -3 \end{pmatrix}\vec{y}(t),\ \vec{y}(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

   b. (Challenge) $\vec{y}\,'(t) = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}\vec{y}(t),\ \vec{y}(0) = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$.