Worksheet 29

1. Let

$$\mathbf{A} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 4 & 1 \\ 2 & 3 \\ 1 & 0 \end{pmatrix} \qquad \mathbf{C} = \begin{pmatrix} -4 & 1 \\ -2 & 3 \\ 1 & 0 \end{pmatrix} .$$

   (If possible) find $A^T B$, BA^T, and $B - 2C$.

2. Find the determinants of the following:

   $\mathbf{D} = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$, $\mathbf{E} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 6 & 4 \\ 1 & 3 & 3 \end{pmatrix}$

   Is either matrix singular?

3. True/False: A^T is the inverse of matrix A

   Find the inverse of matrix $\mathbf{F} = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$

4. (a) Write the following as a system of equations, then solve for x using inverse matrices:

$$\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} \mathbf{x} = \begin{pmatrix} c \\ 4 \end{pmatrix}$$

   (b) Solve the same system of equations using Gaussian elimination.

   (c) Challenge: Suppose that the voltage vector $\mathbf{v}_t$ of the heart changes from one beat to the next according to the equation:

$$\mathbf{v}_{t+1} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \mathbf{v}_t$$

   An equilibrium is a value of the vector for which no change occurs (that is $\mathbf{v}_{t+1} = \mathbf{v}_t$). Find all possible equilibrium values $\mathbf{v}$.

5. Find the inverse of the following matrix using gaussian elimination:

$$\begin{pmatrix} 1 & 0 & 3 \\ 2 & 7 & 9 \\ 0 & 2 & 1 \end{pmatrix}$$

6. Three-Dimensional Geometry

   (a) Find an equation of the sphere with center $(2, -6, 4)$ and radius 5. Describe its intersection with each of the coordinate planes.

   (b) Write an inequality to describe the region between the yz-plane and the plane $x = 5$

7. The Dot Product and Vector Projections Review:
   Vectors are matrices of what dimension?
   Why can't we multiply vectors using our techniques for matrix algebra?

   $a = [1, 2]$, $b = [-4, 1]$

   (a) Find the dot product of **a** and **b**

   (b) Find the angle between **a** and **b**

   (c) For what values of b are the vectors $[-6b, b, 2]$ and $[b, b^2, b]$ orthogonal?

8. Identifying Eigenvectors Are the following vectors eigenvectors of the matrix? If so what are their corresponding eigenvalues?
   $\mathbf{A} = \begin{pmatrix} 1 & -1 \\ -2 & 0 \end{pmatrix}$, $\mathbf{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

9. Review: Do we need to use the characteristic equation to find eigenvalues of upper and lower triangular matrices? Why or why not?

   Find the eigenvalues of the matrix:

$$A = \begin{pmatrix} -1 & 2 & 3 \\ 0 & 1 & 7 \\ 0 & 2 & 1 \end{pmatrix}$$

10. Sequences Find a direct formula for $a_{n+1} = 2a_n + 3a_{n1}$ for $n1$, $a_0 = 1$, $a_1 = 1$, by encoding the recurrence sequence with linear algebra

11. Systems of Differential Equations Solve the IVP $y'' - 2y' - 3y = 0$, $y(0) = 1$, $y'(0) = 1$ by encoding the DE with linear algebra.