Récoltes et Semailles

Alexandre Grothendieck

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Contents

Presentation of the themes or PRELUDE IN FOUR PARTS

Chapter 1

By way of a foreward

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All that was left to write was the foreward, in order for Récoltes et Semailles to be given to the publisher. And I swear that I went into it with all the will in the world to write something which would be suitable. Something **reasonable** this time. No more than three or four pages, but carefully phrased in order to introduce this huge tome of more than one thousand pages. Something which "grabs" the attention of the jaded reader, which gives him a glimpse that in these frightening "more than a thousand pages", there could be things of interest to him (or things which concern him, who knows?). It is not really my style to pander. But I was ready to make an exception for once! The "publisher crazy enough to give it a shot" (to publish this visibly unpublishable monster) had to make ends meet one way or another.

But then, it didn't come. And yet I tried my best. And not only for an afternoon, as I originally planned. Tomorrow will mark three weeks since I started, since the sheets began accumulating. What came, for sure, isn't what one could decently call a "forward". It is yet another miss! Blame it on my old age - I have never been a salesman. Even when it comes to pleasing (oneself or friends...).

What came is a sort of long "Walk" with commentary, through my work as a mathematician. A Walk intended mostly for the "layman" - he who "never understood anything about math". And for myself as well, having never indulged in such a Walk. Step by step, I found myself unearthing and saying things that had previously remained unspoken. As if by chance, these are also things which I feel are most essential, both in my practice and its outcome. They are things which are not technical in nature. It will be up to you to decide whether or not I succeeded in my naive enterprise to "get the message through" - an exterprise which surely is also a bit mad. My satisfaction and my pleasure will come from making you feel these things. Things that many of my wise colleagues do not feel anymore. Maybe they have become too wise and too prestigious. This often leads to losing touch with the simplest and most essential things.

During this "Walk through a body of work" I also speak of my life. As

well as, here and there, what Récoltes et Semailles is about. I mention this
p. A2 in more detail in the "Letter" (dated from May of last year) which follows the "Walk". This Letter was directed towards my previous students and to my "old friends" in the mathematical community. But even the Letter is not technical in nature. It can be read by any reader interested in learning through a "heartfelt" narrative, the odds and ends that led me to writing Récoltes et Semailles. Even more than the Walk, the Letter will provide a preview to a particular atmosphere in the "prestigious mathematical world". And also (just as in the Walk) of my writing style, as peculiar as it may seem, and of the spirit that is expressed through this style - a spirit which is not universally appreciated.

In the Walk and throughout Récoltes et Semailles, I speak of the activity of **doing mathematics**. It is an activity for which I have first-hand experience and know very well. Most of the things I say anbout it can surely be said of any kind of creative work, or work involving discovery. In any case it is true of all "intellectual" work, that which is done using the "brain" and in writing. All such work proceeds through the the outbreak and development of an **understanding** of the things which are being probed. But to take an example at the opposite extreme, romantic passion is also an activity of discovery. It opens us to understanding of a "physical" nature which also renews itself, develops, and deepends over time. Both of these impulses - that which, say, livens the mathematician at work and that of the lover - are much close in nature than we generally assume or we readily admit. I hope that the pages of Récoltes et Semailles will make you feel this impulse in your work and in your daily life.

Most of the Walk focuses on mathematical work itself. I remain mostly silent concerning the **context** in which this work takes place, and concerning the **motivations** at play outside of mathematical work itself. This risks giving me, or the mathematician, or the "scientist" in general a flattering but deformed image. In the style of "grand and noble passion" without any form of rectification. In accordance with the great "Myth" of Science (with a capital S, if you will!). The heroic myth, "promethean", to which writers and thinkers have succumbed (and continue to succumb). Only historians, maybe, manage to sometimes resist this tantalizing myth. The truth is, within the motivation of these "scientists", which sometimes lead them to devote themselves entirely to their work, ambition and vanity play a role just as important and universal
p. A3 as they do in any other profession. This phenomenon appears in blunt or subtle ways depending on the person - and I am no exception to this pattern. The reading of my testimony will hopefully leave no doubt about this fact.

It is true also that even the most intense ambitions are powerless at discovering or proving a novel mathematical statement - just as they are powerless (for instance) to "make one hard" (in the proper sense of the term). Whether man or woman, what "makes one hard" is not ambition, nor the desire to shine, to exhibit power, of a sexual nature in this case - quite the contrary! It is the acute perception of something strong, at once very real and very delicate. One could call it "beauty", thought this is one of a thousand faces of this thing. Being ambitious doesn't prevent one from sensing the beauty of a being or a thing. But it is **not** ambition which makes us feel it...

The person that first discovered and mastered fire was somebody just like you and me. Not at all what we refer to as "hero", or "demi-god", and so on. Surely, just like you and me, he has encountered the grip of anxiety as well as the time-worn remedy of vanity which alleviates the grip. At the instant at which he "knew" fire, there was no fear nor vanity. Such is the truth in the heroic myth. The myth becomes insipid when it is used to to disguise another aspect of things which is just as real and essential.

My aim in Récoltes et Semailles has been to address both aspects of the myth - that of the impulse towards understanding, and that of fear and its vain antidotes. I believe I "understand", or at least **know** this impulse and its origin (or perhaps one day I will discover to what extent I was deluded). But concerning fear and vanity, as well as the resulting insidious creativity blocks, I know thta I have yet to thoroughly uncover this great enigma. And who knows if I will ever reach the conclusion of this mystery in the year I have left...

As I was writing Récoltes et Semailles two images emerged in order to represent the two aspects of the human journey: that of the **child** (aka the **worker**), and that of the **boss**. In the Work which we are about to undertake, we will be dealing mostly with the "child". It is also him that is featured in the subtitle "**The Child and the Mother**". The motivation for this name will hopefully become clear over the course of this work. p. A4

In the remainder of this reflection, however, it is the Boss who takes the lead. He is living up to his name! It would be more accurate to speak of multiple bosses of competing enterprises rather than of a **singular** boss. But it is also true that all bosses essentially resemble one another. And once we mention bosses it is implied that we will also have to deal with "villains". In part I of the reflection (named "Fatuity and Renewal", which follows the present introductory section). I mostly take on the role of the "villain". In the following three parts it is mostly the "others". Chacun son Tour! That is to say that, in addition to philosophical reflections and "confessions" (not contrite), there will be "vitriolic portraits" (to use the expression of one of my colleagues who found himself tormented). Not to mention large-scale well-oiled "operations". Robert Jaulin[1] assured me (half jokingly) that in Récoltes et Semailles I was making the "ethnology of the mathematical community" (or maybe the sociology I do not quite remember). It is flattering of course to learn that one has been (unknowingly) doing scholarly things! It is true that during the "investigation" segment of the reflection, I saw in passing, in the pages I was writing, a good chunk of the mathematical establishment without counting a number of my colleagues and friends of more modest status. Over the past few months, since I have been sending preliminary versions of Récoltes et Semailles this has been "brought up" again. My testimony arrived like a tome landing in a pond. There were responses of every kind (except for boredom...). Yet almost every time the response was far from what I expected. There was also a lot of silence, which speaks volumes. Visibly, I had (and still have) a lot to learn about what

[1]Rober Jaulin is an old friend. From what I understand, his position with respect to the establishment of the ethnological milieu mirrors mine with respect to the "high society" of mathematics (as white wolves).

happens in people's minds, among my previous students and other colleagues - excuse me I meant about "the sociology of the mathematical milieu"! To all those that contributed to the great sociological work of my old days, I would like to express sincere recognition.

Of course, I was particularly sensitive to warm responses. There were also some rare colleagues who conveyed a sentiment (thus far unexpressed) of crisis,
p. A5 or of degradation of the inner workings of the mathematical milieu with which they identify themselves.

Outside of this milieu, among the very first to respond positively to my testimony, I would like to recognize Sylvie et Catherine Chevalley[2] Robert Jaulin, Stéphane Deligeorge, Christian Bourgois. If Récoltes et Semailles achieves a wider diffusion than that of the initial printing (addressed to a very limited social circle of people), it is mostly thanks to them. Thanks mostly to them communicating their conviction that what I strived to seize and say had to be said. And that it could have an audience outside of my colleagues (who are often sullen, sometimes even belligerent, and strictly opposed to question their position...). Indeed Christian Bourgois did not hesitate to risk publishing the unpublishable, and Stéphane Deligeorge did not hesitate to place my indigestible testimony alongside works of Newton, Cuivier, and Arago (I could not ask for better company). To each of them, for their repeated expressions of sympathy and trust, intervening at an especially sensitive moment, I happily extend all my gratitude.

And here we are at the beginning of a Walk through a life's work, serving as a prelude to a journey through a lifetime. A long journey, over a thousand pages long, each of which is densely packed. I spent a lifetime undergoing this journey without ever exhausting it, and it then took me more than a year to rediscover it, one page at a time. Words were sometimes hard to come by, as they were intended to convey an experience which evaded comprehension - just as ripe grapes stacked in a press occasionally seem to evade the force upon them... But even in those moments when words come flooding, it is not by happenstance. Each word has been carefully weighed in passing, or after the fact. Thus this reflection - testimony - journey is not meant to be read hastily, in a day or a
p. A6 month, by a reader rushed to read the final word. There is **no** "final word", no "conclusion" in Récoltes et Semailles, no more than there are any in my life or yours. There is only a wine, aged over the course of a lifetime, in the core of my being. The last glass which you will be drinking will be no better or worse than the first or the hundredth. They are all "the same", and they are all different. And if the first glass is spoiled, so is the rest of the barrel; it is better to drink fresh water (if such can be found), than to drink bad wine.

But a good wine ought not to be drunk in haste, or expeditiously.

[2]Sylvie et Catherine Chevalley are the widow and daughter of Claude Chevalley, the colleague and friend to whom the central part of Récoltes et Semailles is devoted (ReS III, the key of the Yin and the Yang). At multiple times in the reflection I speak of him and of the role he played in my journey.

Chapter 2

A walk through a life's work, or the child and the Mother

January 1986

2.1 The magic of things

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When I was little I liked going to school. The same teacher taught us reading, writing, arithmetic, singing (he accompanied us with a small violin), and even about prehistoric men and the discovery of fire. I do not ever recall being bored at school, during those days. There was the magic of numbers, that of words, of signs, and of sounds. That of **rhymes** as well, through songs and small poems. There seemed to be, within rhymes, a mystery which extended beyond words. I believed this until the day somebody told me that there was a simple "trick" to it; that rhyme was simply when one ends two consecutive spoken movements by the same syllable, so that, as if by magic, these phrases became **verses**. It was a revelation! At home, where I found a good audience, for weeks or months on end, I amused myself by making verses. At one point, I even started exclusively speaking in rhymes. That period has passed, fortunately. Yet even to this day, I still sometimes write poems - but without trying to force the rhyme, if it doesn't seem to come by itself.

On another occasion an older friend, who was already in high school taught me about negative numbers. It was also a fun game, although I rapidly exhausted it. And then there were crosswords - I spent days and weeks constructing them, evermore interwoven. Within that game the magic of form, that of signs, and that of words found themselves combined. But even that passion subsided without leaving a trace.

During my high school years, which began in Germany during my first year,

then later in France, I was a good student without quite being the "star student". I devoted myself without restraint to the courses which I cared most about, and tended to neglect the others, without really caring for the appreciation of my "prof". During my first year of high school in France, in 1940, I was interned at a concentration camp with my mother, at Rieucros near Mende. It was wartime, and we were foreigners - "undesirables", as they said. But the administration of the camp turned a blind eye towards the kids, however undesirable they may be. We came and went as we pleased. I was the oldest, and the only one to go to high school, which was four or five kilometers away, in the rain and the wind,
p. P2 in makeshift shoes that always got wet.

I still remember my first "math examination", in which the teacher gave me a bad grade, for my proof of one of the "three cases of equality of triangles". My proof wasn't exactly that of the book, which he followed religiously. Yet, I knew very well that my proof was no less convincing than that of the book which I followed min spirit, through repeated invocation of the traditional "we slide this figure in such and such a way onto that figure". Visibly, this teacher did not feel capable of judging things on his own (namely the validity of the reasoning). He had to report to a higher authority, that of a book in this case. I must have been stricken by such dispositions, for me to still remember this incident. Ever since then, I have been presented with more than enough evidence to realize that such dispositions are far from exceptional, but rather they are the quasi-universal norm. There is a lot to be said on this subject - one which I approach more than once in one way or another in Récoltes et Semailles. Yet to this day, I find myself invariably taken aback whenever I am confronted with such behavior...

During the last few years of the war, while my mother was still interned at the camp, I lived in a youth refugee house called "Secours Suisse", in Chambon sur Lignon. Most of us were jewish, and when we were told (by the local police) that there would be raids by the Gestapo, we went to hide in the woods for a night or two, in small groups of no less than 3, without quite realizing that our life was on the line. The region was filled with jews hiding in Cévenol country, and many of us survived thanks to the solidarity of the lcoal population.

What struck me most about "Collège Cévenol" (where I was raised), was the extent to which my peers were disinterested in learning. As for myself, I devoured our textbooks at the beginning of the school year, thinking that this time around, we would finally learn **truly** interesting things; and for the rest of the year I utlized my time as best as I could while classes dragged along inexorably one trimester at a time. Yet we had some wonderful professors. The natural history professor teacher, mister Friedel, was a person with remarkable intellectual and social qualities. However, as he lacked authority, the class was acting out of control, to the extent that it became impossible to hear what he had to say, as his voice was lost in the hurly-burly. This may be the reason I
p. P3 haven't become a biologist!

I spent a fair amount of time, including class time (shh...), solving math problems. The ones in the book soon became insufficient. Perhaps it was because they tended to resemble one another after a while; but mostly, I believe,

because they seemed to come out of the blue à la queue-leue-leue, with no indication as to where they came from or where they're going. These were the books problems, not mine. And yet, natural questions were plentiful. For instance, once the three side lengths a, b, and c of a triangle are known, so that the triangle itself is known (up to its position), there has to be an explicit "formula" that expresses the area of the triangle as a function of a, b, and c. Likewise, for a tetrahedron of which the six side lengths are known - what is the volume? I struggled through that one for a bit, but I must have gotten there eventually. In any case, when a problem "grabbed me", I did not count the hours or days that I spent working on it, even if it meant losing track of everything else! (And such remains the case to this day...)

What I found least satisfying, in our math textbooks, was the absence of a serious definition of the notion of length (of a curve), of area (of a surface), or of volume (of a solid). I promised myself to make up for this omission as soon as I could. This is what I devoted most of energy to between the years of 1945-1948, while I was a student at the University of Montpellier. University lectures weren't for me. Without ever quite realizing it, I must have been under the impression that all my professor did was recite the contents of the textbooks, just like my first math teacher at the lycée de Mende. I barely ever set foot on university grounds, just enough to keep up to date with the perennial "program". Books sufficed to cover said program, but it was clear that they offered no answers to the questions I was asking myself. Truly, they did not even **see** them, no more than my high-school textbooks did. As long as we were provided with recipes for all sorts of calculations, such as lengths, areas, volumes, through single, double, triple integrals (dimensions higher than 3 were carefully avoided...), the problem of providing an intrinsic definition was omitted by both my professors and textbook authors.

From my then limited experience, it seemed that I was the only person in the world to be gifted with a curiosity for mathematical questions. Such was, in any case, my unexpressed conviction, during those years spent in complete intellectual solitude (which did not bother me).[1] To be fair, it never occurred to me at that time to investigate whether or not I was the only person in the p. P4
world to take interest in what I was doing. My energy was sufficiently absorbed by the task I set for myself: to develop a fully satisfactory theory.

I never doubted that I would succeed in reaching the end of the story, as long as I was committed to scrutinizing these structures, spelling out on paper

[1]Between 1945-1948, I lived with my mother in a small hamlet about 10 kilometers away from Montpellier, Mairargues (near Vendargues), lost in the middle of vineyards. (My father disappeared in Auschwitz in 1942.) We scraped by on my meager student funding. To make ends meet, I took part in the harvest every year, and after the harvest season I would sell wine under the table (in contravention of the legislation, or so I hear...). On top of that, there was a self-regulating garden which supplied us with an abundance of figs, spinach, and even (towards the end) tomatoes planted by a complacent neighbor, amidst a sea of splendid poppies. It was the good life - although occasionally a bit rough along the edges, when we had to replace a pair of glasses, or a pair of worn-out shoes. Luckily, because my mother was weak and sick due to her long stay in the camps, we received free medical assistance. We would never have been able to afford a doctor otherwise...

what they were telling me. The intuition behind **volume**, say, was irrecusable. It could only be the reflection of a **reality**, momentarily elusive, but perfectly reliable. What had to be done was simply to seize this reality - a bit, perhaps, the way the magic reality of the "rhyme" had been seized, "understood" one day.

When I began this pursuit, at age 17, freshly out of high-school, I thought it would only take a few weeks. I spent three years on the project. It even caused me to fail an exam at the end of my second year of university - that of spherical trigonometry (in the "further astronomy" module), because of a stupid computational mistake. (I was never very good at computations, I must say, ever since I left high school...). That is why I had to spend a third year in Montpellier to complete my bachelor's instead of going to Paris right away - the only place, I was told, where I would be able to find people aware of what was considered important in Mathematics. My informant, Mister Soula, assured me that the last problem left in mathematics had been resolved twenty or thirty years ago by a so-called Lebesgue. He had apparently developed (funny coincidence!) a theory of measure and integration which brought *point final* to mathematics.

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Mister Soula, my "diff calc" teacher, was a benevolent man who took a liking to me. I was still not convinced by his claim. There must have already been, within me, the prescience that mathematics is a thing which is infinite in scope and depth. Does the sea have a "*point final*"? Yet I never thought of looking for that book by Lebesgue which Mister Soula had told me about, and he probably never held it either. In my mind there was nothing in common between anything a book contained and the work that **I** had been doing, in my own way, in order to answer questions which intrigued me.

2.2 The importance of being alone

When I finally made contact with the mathematical world in Paris, one or two years later, I ended up learning, among many other things, that the work which I had been doing independently, and with the means at hand, was (essentially) what "everybody" knew as the "Lebesgue theory of measure and integration". According to the two or three experts to whom I mentioned my work (or even showed a manuscript), I had just wasted my time redoing something "already known". I actually do not recall being disappointed. At that moment, the idea of receiving "credit", or even simply receiving approbation for the work that I was doing, must have still been foreign to my mind. Furthermore, my energy was completely taken by the process of familiarizing myself with an entirely different milieu and mostly learning what was considered in Paris to be the basic toolkit of the mathematician.[2]

Yet, thinking back to those three years, I realized that they were not in any way wasted. Unknowingly, I learned in solitude what is essential to the work of

[2] I briefly narrate this rough transition period in the first part of Récoltes et Semailles (ReS I), in the section "The Welcome Stranger" (nb. 9).

a mathematician - something no master could truly teach. Without ever having been told, without ever having to encounter someone with whom I could share my quest for understanding, I knew "in my gut" that I was a mathematician: somebody who "does" math, in its fullest sense - the way one makes "love". Mathematics had become, for me, a mistress always accommodating my desires. These years of solitude laid the foundation for a trust that has never been shaken - not by the discovery (upon arrival in Paris at age 20) of the scope of my ignorance and the vastness of what I had to learn; nor (more than 20 years later) by the eventful episode of my permanent departure from the mathematical world; nor, in these last few years, by the often crazy episodes of a "Funeral" (anticipated and cleanly executed) of my person and life's work, orchestrated p. P6
by those who used to be my closest companions...

To phrase it differently: I learned in those crucial years to "be alone".[3] That is, I learned to approach the things which I want to know with my own eyes, rather than rely on the expressed or implicit ideas that eminate from the group with which I identify, or a group to which I attribute authority. An unspoken consensus told me, both in high school and in university, that there was no need to question the notion of "volume", which was presented as "well-known", "self-evident", "unproblematic". Naturally I turned a blind eye to this consensus - just as Lebesgue, a few decades earlier, had to **turn a blind eye**. It is in this act of "turning a blind eye", of being oneself rather than the mere expression of the reigning consensus, of not to remain inscribed within the imperative circle to which they assign us - it is within this solitary act, above all else, that "**creation**" lies. Everything else comes after.

In the following years, within the mathematical world which welcomed me, I had the opportunity to meet multiple people, both older and younger, which were clearly more brilliant, "gifted" than I was. I admired the facility with which they learned new notions, as if at play, juggling them as if they had known them their whole life - while I felt heavy-handed and clumsy, laboriously making my way, akin to a mole, through an amorphous mountain of important things (or so I was told) which I had to learn, despite having no sense of their ins and out. Actually, I was far from the brilliant student who aced every prestigious *concours* and assimilating at once the most prohibitive courses.

Many of my more brilliant peers went on to become competent famous mathematicians. In hindsight, after 30-35 years, it does not seem to me that they left a deep imprint upon the mathematics of today. They did things, often times p. P7
beautiful things, in a pre-existing context which they would never have considered altering. They unknowingly remained prisoners in their imperious circles, which delimitate the Universe of a given time and milieu. In order to overcome

[3] This formulation is somewhat clumsy. I never had to "learn to be alone", for the simple reason that I never **unlearned** during the course my childhood, this innate skill which I had since birth, just as we all do. Yet these three years of solitary work, during which I could walk to my own beat, following my own exigence criteria, confirmed within me a degree of trust and tranquil confidence in my relationship with mathematics which owed nothing to the reining trends and consensus. I make allusion to these again in the note "Roots and Solitude" (Res IV, n°171_3) notably page .

them, they would have had to rediscover within them the ability which they had since birth, just as I did: the capacity to be alone.

The small child has no difficulty being alone. He is solitary by nature, even though he enjoys the occasional company, and knows when to ask for mom's permission teat. And he knows, without having ever been told, that the teat is his, and that he **knows** how to drink. Yet often times we lose touch with out inner child. And thus we constantly miss out on the best without even seeing it...

If in Récoltes et Semailles I address somebody other than myself, it is not a "public". I address myself to you, reader, as I would a **person**, and a person **alone**. It is to the person inside of you that knows how to be alone, the child, with whom I would like to speak, and nobody else. I am aware that the child is often far away. He has gone through all sorts of things for quite some time. He went hiding god knows where, and it can be hard, often times, to get to him. One could swear that he has been dead forever, or rather that he has never existed - and yet I am sure that he is there somewhere, well alive.

I know too what the **sign** is that I am being heard. It is when, beyond all cultural and experiential differences, what I share about my personal experiences echos within you and finds resonance; when you find within it **your own life**, your own self-experience, in a new light which you may never have considered before that. It is not about an "identification" with something or someone far from you. Rather, perhaps, you will rediscover a bit of your own life, or of what is **closest** to you, as you follow my own rediscovery of myself throughout Récoltes et Semailles, including within these very pages which I am currently writing.

2.3 The inner journey - or myth and testimony

Before all else, Récoltes et Semailles is a **reflection** upon myself and my life. Because of this, it is also a **testimony**, in two distinct ways. It is a testimony about my **past**, upon which the principle component of the reflection is concerned with. But it is also, at the same time, a testimony about the immediate present, about the very moment at which I am writing, and during which
p. P8 Récoltes et Semailles is born, in the course of hours, nights, and days. These pages serve as faithful witnesses to a long meditation upon my life, such as it was really carried out (and continues to be carried out at this very moment...).

These pages have no literary pretense They only constitute a document about myself. I only allowed myself to modify them within very narrow bounds[4] (notably for occasional stylistic edits). If there is pretense, it is only that of faithfulness. And that is saying a lot.

This document is also far from an "autobiography". You will learn neither my date of birth (which would be of little relevance unless one is making astrological predictions), nor the names of my mother and father or what they

[4]Thus, the occasional rectification of mistakes (material and of viewpoint) does not appear in the first pass but rather in footnotes or in later reconsideration.

did in life, nor the names of the person who was my spouse and other women who have been very important in my life, nor those of the children that were born from these unions, nor what these children have made of their lives. This does not mean that these things were not important in my life. Rather, the way this reflection on myself was engaged and developed never incited me to give a description of those things, which I lightly touch on here and there, but never take the time to consciously flesh them out with names and numbers. It never seemed to me that doing so would add anything whatsoever to the point which I was making at any given time. (Whereas in the few pages above I was brought, almost inadvertently, to include perhaps more material details on my life than you will find in the thousand pages to come...)

And if you were to ask me what "point" I have attempted to make over the course of these thousand pages, I would answer: it is to narrate, and thereby **discover**, the **inner journey** that my life has been and still is. This narrative/testimony of a journey is happening simultaneously at the two level which I have mentioned above. There is the exploration of a journey past, of its roots and of its origin, tracing all the way back to my childhood. And there is the continuation and renewal of this "very" journey, over the course of the days during which I am writing Récoltes et Semailles in spontaneous response to a violent stimulus coming from the outside world.[5] p. P9

External facts come to nourish the reflection, only to the extent they they induce and provoke new developments in my inner journey, or help clarify it. The burial and the plunder of my mathematical work, of which I will speak at length, has been such a provocation. It awoke in me a host of powerful reactions, and at the same time revealed to me the profound, and hitherto unknown links that continue to tie me to the work I have created.

It is true that my being "good at math" is not necessarily a reason (and even less so a good reason) for you to be interested in my particular journey - nor is the fact that I have had trouble with my colleagues, after shifting milieu and lifestyle. Colleagues, or even friends abound, who find it ridiculous to publicly spread one's "inner moods" - as they say. To them, what matters are "results". The "soul", meaning that within us which **witnesses** the production of these result, as well as apprehends them in various ways (as much in the life of the "producer" as in those of his peers), is looked down upon, sometimes even targeted with open derision. This attitude aims to display some form of modesty, but what I see is a sign of withdrawal or asynchrony promoted by the very air which we breath. I do not write for he who is stricken by this latent self disgust, which makes him reject the best I have to offer. A disdain for what truly makes his **own life**, and for what makes mine: the superficial or profound, course or subtle motions which animate the psyche, that "soul" which lives the experience and reacts to it, which freezes or blossoms, which retreats or learns...

The narrative of an inner journey can only be told by the person living it and

[5] For more details about this "violent stimulus", see "The Letter", notably sections 3 through 8.

no one else. Even though the narrative is only aimed towards oneself, it often times inserts itself within the construction of a **myth**, of which the narrator is the hero. Such a myth is born, not in the creative imagination of a people and a culture, but rather from vanity of he who dared not assume a humble reality, but instead substitutes a construction for it. But a **true** narrative (if such a thing exists), of a journey such as it was truly lived, is to be prized. And this is not because of renown which is (rightly or wrongly) attributed to the narrator, but simply by virtue of its **existence**, and of its truthfulness. Such a testimony is precious, whether it comes from an illustrious person, a small clerk with no future and with family responsibilities, or from a common criminal.

If there is value for one in such a narrative, it is first and foremost that of self confrontation, through this unvarnished testimony of the experience of an other. But also (to phrase it differently) to erase within oneself (be it only for the span of a reading) this disdain by which one holds one's **own journey**, and that "soul" of which one is both the passenger and the captain...

p. P10

2.4 The painting of mores

In speaking of my past as a mathematician, and later in discovering (almost against my will) the twists and turns of the intricacies of the gigantic Burial of my work, I was brought, inadvertently, to paint the picture of a particular milieu and era - an era affected by the decay of certain values which provided meaning to the work of individuals. That is what I mean by "painting of mores", centered around a "fait divers" which is thoughtlessly unique in the annals of Science as I have said rather clearly earlier, I believe, you will not find in Récoltes et Semailles a "folder" concerning a certain unordinary "case", quickly bringing you up to date. And yet a friend of mine, looking for such a folder, blindly passed by nearly everything constituting the substance and flesh of Récoltes et Semailles.

As I explain in much more detail in the Letter, the "investigation" (or the "painting of mores") carries on in parts II and "The Funeral (1) - or the robe of the Emperor of China" and "The Funeral (3) - or the Four Operation". Page after page I persistently extract one after another, a number of juicy facts (to say the least) which I attempt to "classify" bit by bit. Slowly, these fact assemble into a global painting which progressively emerges from the fog, taking on brighter colors and sharper contours. In these daily notes, the raw facts "which just appeared" are inextricably mixed with personal reminiscing, as well as with commentaries and reflections of a psychological, philosophical, or even (occasionally) mathematical nature. That's how it is, and there is nothing I can do about it!

Starting with work I had already done, which occupied me for over a year, producing a sort of "investigation proceedings" folder should not have taken longer than a few hours or days worth of work, depending on the curiosity or demands of the interested reader. I tried at one point to produce such a folder. That is how I started writing a note which was to be called "the four

operations".[6] But in the end I could not bring myself to do it! That is decidedly not my style of expression, and in my old age less so than ever. I now consider, having written Récoltes et Semailles, that I have done enough for the benefit of the "mathematical community", and therefore can leave, without remorse, the task of producing the necessary "folder" to others (in particular to any of my colleagues who would feel concerned). p. P11

2.5 The heirs and the builders

It is now time for me to say a few words about my mathematical work, which has played an important role in my life, and continues to do so (to my own surprise). I come back to this work more than once in Récoltes et Semailles - sometimes in a way that should be understandable by all, and other times in slightly more technical terms.[7] The latter will mostly go "above the heads", not only of the "profane", but also of the mathematical colleague who may not be completely "in" the field in question. One can of course feel free to skip the sections which seem too "involved". Just as one can try to go through them, glimpsing as one goes, a shadow of the "mysterious beauty" (in the words of a non-mathematician friend of mine) of the universe of mathematical things, appearing as a multitude of "strange inaccessible islands" in the vast moving waters of reflection...

Most mathematicians, as I mentioned earlier, are inclined to constrain themselves to a conceptual framework, a "**universe**" fixed once and for all - the one, essentially, which they have found "ready made" at the time of their studies. They are like the heirs of a large and beautiful fully-furnished house, with its lounges, kitchens, workshops, and its kitchenware and tools left and right, with which there is, I trust, plenty to cook and tinker. How this house was built, progressively over the course of multiple generations, and how and why these tools (and not others...) were conceived and built, why the pieces are disposed and organized in such a way - these are all questions the heirs would never think of asking themselves. This is the "universe", the "given", in which we must live, and that is that! Something which appears massive (and which most of the time we have only been able to partially explore), yet at the same time **familiar**, and mostly: **immutable**. They mostly busy themselves with maintaining or embellishing a patrimony: fixing a faulty piece of furniture, restoring a facade, sharpening a tool, or even sometimes, for the most enterprising, building an entire workshop, or a whole new piece of furniture. It even happens, when they fully commit to the task, that the piece of furniture is truly beautiful, so that the whole house appears embellished by its addition. p. P12

[6] The note eventually exploded into part (also named "The four operations") of Récoltes et Semailles, comprising about seventy notes running over more than four-hundred pages.

[7] One will also find here and there, in addition to mathematical notes concerning my previous work, sections containing new mathematical developments. The longest of these is "the five pictures (crystals and $\mathcal{D}$-modules)" in ReS IV, note n°171 (ix).

Even more rarely, one of them will consider modifying one of the main tools, or even, under repeated and insistent pressure or need, to imagine and build a whole new tool. And in so doing, he often feels on the brink of profusely apologizing for what he feels is infringing on the piety owed to the familial tradition, which he has disturbed through his brazen innovation.

In most of the rooms of the house, the windows and shutters are carefully closed, probably on account of a fear that a foreign wind would blow in. And when the pretty new furnishings, here and there, together with their progeny, begin to clutter the rooms and invade the corridors, none of these heirs will agree to face the fact that his familiar and cozy Universe is beginning to feel cramped. Rather than come to terms with such a fact, most will prefer to awkwardly slither, and try not to get trapped, some between a Louis XV buffet and a rocking chair in rattan, others between a boisterous toddler and an Egyptian sarcophagus, while others, as a last resort, will try to climb over a heteroclite and crumbling pile of chairs and benches...

The picture I have just sketched is not unique to the world of mathematicians. It illustrates the deeply engrained and immemorial conditioning which one encounters in every milieu and sphere of human activity, regardless, as far as I can tell, of the society or era in question. I have mentioned such a phenomenon already, and I do not in any way pretend to fall outside of its influence. As will be clear from my testimony, the contrary is true. It only happens to be the case that at the relatively limited level of the act of intellectual creation, I was barely affected[8] by this conditioning which may be called "cultural blindness" - the incapacity to see (and to evolve) outside of the "Universe" fixed by the surrounding culture.

As for myself, I feel that I belong to the lineage of mathematicians whose spontaneous vocation and joy was to continuously construct new houses[9] In
p. P13 so doing, they cannot help but invent all of the required tools, utensils, and furnishings for both the construction of the house from its foundation, and to fill the kitchens and workshops of the house in abundance, so that one may live in it comfortably. Yet, once everything down to the last sapling and stool has been taken care of, the builder rarely lingers on the premises, of which every stone and every piece of wood carries a trace of the hand which shaped and placed it. The builder's place lies not in the quietude of fully finished universes, however welcoming and harmonious they may be, whether they are a product of his own hands or those of his predecessors. His place is in the open air. He is friends with the wind, and does not fear solitude at work for weeks, years or, if need be, for an entire lifetime if no welcome succession presents itself. Just like everybody else, the builder only has two hands - but two hands which at each moment know what they need to do, which refuse neither the largest nor the most delicate tasks, and which never tire of comprehending, again and again, the multitude of things which become them. Two hands might be few, given

[8] I believe the main reason for such immunity is a certain favorable climate which surrounded me until age 5, the note "The innocence" (ReS III,n°107).

[9] This archetypal picture of a "house" to be built, surfaces and is formulated for the first time in "Yin, the Servant, and the new masters" (ReS III , n°135).

that the World is infinite. They will never exhaust it! And yet, two hands can be a lot...

History is not my strong suit, but if I had to give a list of mathematicians inscribed in this lineage, names that spontaneously come to my mind are those of Galois, Riemann (from the past century), and Hilbert (at the beginning of the current century). If I were to name a candidate among the elders which welcomed me into the mathematical world,[10] the name of Jean Leray comes to my mind before any other, even though my contact with him has always been episodic.[11]

p. P14

I have just roughly sketched two pictures: that of the "homebody" mathematician, who is content with maintaining and embellishing a heritage, and that of the builder-pioneer,[12] who is drawn to repeatedly crossing these "invisible and imperious circles" which delimitate a given Universe.[13] These two groups may also be called, somewhat bluntly but also suggestively, "conservatives" and "innovators". Both have their raison d'être, in one collective adventure that is carried out through the generations, through centuries and millennia. During the fruitful periods of a science or art there is neither opposition, nor is there antagonism among these two temperament.[14] They are different and mutually complementary, just as dough and yeast.

Between these two extremes (not at all opposed by nature), one can find a plethora of intermediary temperament. There is the "homebody" that would never think of leaving a familiar dwelling, and would be even less willing to take on the task of building another, god knows where, yet will not hesitate, when the house gets cramped to build a basement, raise the ceiling, or even, if need be, to build a dependency of modest proportions.[15] Without being a

[10] I speak of these beginnings in the section "The welcome stranger" (ReS I , n°9).

[11] I was nonetheless (following H. Cartan and G. Serre) one of the first users and promoters of one of the great innovative notions introduced by Leray, that of a sheaf, which has been an essential tool throughout my work as a geometer. It is also the notion which has provided me with the key to enlarge the notion of a topological space into that of topos, about which I will be speaking later.

Leray differs from the portrayal I have given of the "builder", I believe, in that he does not seem drawn to "construct houses from their foundations to their completion". Rather be was compelled to lay out vast foundations, in places where nobody would have thought to look while leaving to others the care of carrying the construction to its completion, and once the house is built, to settle into the premises (be it only for a short time)...

[12] I have just surreptitiously attached herein two qualifiers with male connotation (that of "builder" and that of "pioneer"), which express very different aspects of the impulse of discovery, one which is of a nature more delicate than what these qualifiers might evoke. Such a discussion will be carried out later in this walk-reflection, in the step "The discovery of the Mother - or the two versants" (n°17).

[13] At the same time, and without really meaning to do so, the builder-pioneer assigns to the old Universe (if not for himself, at the very least for his more sedentary colleagues) new boundaries, thereby inscribing circled which may be larger, but are just as invisible and imperious as those which they have come to replace.

[14] Such has been the case, notably in the mathematical world, during the period (1948-1969) of which I was a direct witness, as I myself belonged to that world. Following my departure in 1970, there seems to have been a large scale reaction, a sort of "consensus of disdain" for the "ideas" in general, and more specifically the great innovative ideas that I have introduced.

[15] Most of my "elders" (about whom I speak for instance in "a welcome debt" (Introduc-

builder at heart, he will often view with a sympathetic eye, or at the very least without concern nor secret reprobation towards another who had shared the
p. P15 same dwelling, and who is already out and about assembling beams and stones in some impossible boonies, with the confidence of somebody who already sees a castle...

2.6 Viewpoint and vision

Allow me to return to myself and my work.

If I excelled in the art of mathematics, it was not through the ability and perseverance to solve problems left by my predecessors, but rather through a natural tendency within me to discover **questions**, evidently crucial, yet that nobody had yet seen, or to excavate the **"right notions"** that were missing (often without anyone realizing until the new notion appeared), as well as the **"right statements"** of which nobody had thought. Often, notions and statements mesh in such a perfect way, that there can be no doubt in my mind as to their validity (give or take small adjustments at most) - so that often, when it boils down to "travail sur pièces" destined for publication, I refrain from going further, and from taking the time to flesh out a proof that often, once the statement and its context are well-understood, consists of no more than a matter of "trade", not to say routine. Things which solicit ones attention are countless, and it is impossible to follow them all to their end! The fact remains that carefully proved propositions and theorems in my written and published work appear in the thousands, and almost all of them have entered the patrimony of things commonly accepted as "known" and frequently used all over mathematics.

I am led more towards the discovery of fertile **viewpoints** than towards the discovery of questions, notions, and statements, by my particular type of genius, which is constantly leading me to introduce, and more or less develop, entirely novel **themes**. It is this, I reckon, which is my most essential contribution to the mathematics of my time. In fact, these innumerable questions, notions, and statements which I just mentioned, only truly make sense for me once they are subjected to such a "viewpoint" - or more precisely they **arise** spontaneously from it; in the same way that a light (even a dim one) appearing in a pitch black night seems to invoke from the shadows contours which it suddenly reveals to us. Without this light uniting them in a common sheaf, the ten, or one-hundred, or one thousand questions, notions, statements would appear as a heterogeneous
p. P16 amorphous pile of "mental widgets" all isolated from one another - rather than as the many parts of a **Whole** which, while perhaps remaining invisible, escaping within the folds of the night, is nonetheless clearly felt.

The fertile viewpoint is that which reveals to us, organized as the many

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tion n°10)) conform to this intermediary temperament. I was thinking notably about Henri Cartan, Claude Chevalley, André Weil, Jean-Pierre Serre, Laurent Schwartz. With the exception maybe of Weil, they have all turned a "sympathetic eye", without "concern nor secret reprobation" towards the solitary adventures into which they saw me embark.

living parts of a common Whole, enveloping them and giving them meaning, these pressing questions which no one had asked, and (as if in response, perhaps, to these questions) these extremely natural notions which nobody had thought of expressing, and these statements finally which seem to immediately follow, and which nobody had dared to conjecture, for as long as the questions which brought them about, and the notions that allowed us to formulate them had remained hidden. Even more than what we call "key theorems" in mathematics, it is the fertile viewpoints which, in our art,[16] constitute the most powerful tools of discovery - or rather, they are not tools, but they are the very eyes of the researcher who passionately strives to understand the nature of mathematical things.

Thus, the fertile viewpoint provides us with an "eye" which at once helps us **discover**, and helps us **recognize the unity** of the multiplicity of what is discovered. And such unity is truly the very life and breath which connects and animates these discoveries.

But just as the word itself suggests, a "viewpoint" by itself remains fragmentary. It reveals to us **one of the aspects** of a scenery or panorama, among a multiplicity of others which are equally valuable, equally "real". It is when complementary viewpoints of a common reality are conjugated, that is, when our "eyes" are multiplied, that the gaze is able to penetrate further ahead in the reckoning of things. The richer and more complex the reality which we desire to know, the more important it is to be equipped with several "eyes"[17] in order to apprehend it in all its ampleness and subtlety.

By virtue of our innate ability to grasp the "multiple" as the **One**, it also happens, sometimes, that a sheaf of viewpoints converging to a unique and vast scenery, gives rise to a novel thing; a thing which transcends each of the partial perspectives, in the same way that a living being transcends each of its limbs and p. P17
organs. This new thing could be called a **vision**. The vision unites the known viewpoints which constitute it, while also revealing to us other viewpoints that were previously ignored, just as the fertile viewpoint makes us discover and apprehend, as part of the same Whole, a multiplicity of new questions, notions, statements.

To say this in another way: the vision is to the viewpoints, from which it seems to arise and which it unites, as the clear and warm daylight is to the various components of the solar spectrum. A vast and profound vision is like an inexhaustible source, made to inspire and guide the work, not only of the one within whom the vision was once conceived and who made himself its servant, but that of generations, fascinated perhaps (as he first was) by these distant horizons which it lets us glimpse...

[16] Such a phenomenon is not exclusive to "our art", but (it seems to me) it appears in every act of discovery, at the very least when such an act happens at the level of intellectual reckoning.

[17] Every viewpoint leads to the development of a language which is best suited to expressing it. Having several "eyes" or several "viewpoints" to apprehend a situation, also means (at least in mathematics) having several different languages to tackle the situation.

2.7 The "great idea" - or the trees and the forest

The so-called "productive" period of my mathematical life, meaning it was marked by proper publications, ran between 1950 and 1969, twenty years that is. And for twenty-five years, between 1945 (when I was seventeen) and 1969 (when I was in my forty-second year), I invested nearly the totality of my energy in mathematical research. An excessive investment certainly. This cost me a long period of spiritual stagnation, an incremental "thickening", to which I will be coming back multiple times in the pages of Récoltes et Semailles. Yet, within the limited scope of a purely intellectual activity, and through the burgeoning and maturation of a vision restricted to the world of mathematical things, these were years of intense creativity.

During this long period of my life, the near totality of my time and energy were devoted to what might be called "**travail sur pièces**": a minute process of shaping, assembling, and honing, required for the construction from start to finish of houses that an inner voice (or an inner demon...) called for me to build, following a blueprint that it whispered to me at every step of the way. Occupied by the tasks of the "trade": such as those of a sculptor, bricklayer, carpenter, even plumber, woodworker, cabinet-maker - I rarely stopped to write down, even in rough sketches, the master-plan, which was invisible to all (as it only appeared later...) except to me, who over the course of the days, months, and years, guided my hand with the certainty of a sleepwalker. [18] I have to say

p. P18

[18] The metaphor of the "sleepwalker" was inspired by the title of the wonderful book "the sleepwalkers" by Koestler (Calman Lévy), presenting an "Essay on the history of the conceptions of the universe", starting from the origins of scientific thought, all the way to Newton. One of the facets of this history which struck Koestler, and which he highlights is the extent to which, often, the path from a given state of our understanding of the world, to some other state which (logically and with hindsight) seems very close, sometimes takes the most astounding detours, which appear to defy reason; and to think that yet, despite those thousand detours that could conceivably have lost them forever, and with the "certainty of sleepwalkers", men who have gone on the quest for the "keys" of the Universe find, as if unintentionally and without even realizing it, other "keys" that they would never have thought of, and which nonetheless appear to be "the right ones".

From what I have observed all around me, at the level of mathematical discovery, these extraordinary detours in the path towards discovery are the actuality of some high-caliber researchers, but not of all. This could be due to the fact that for the past two or three centuries, research in the natural sciences, and even more so in mathematics, has freed itself from the religious presupposition or metaphysical imperatives, pertaining to a given culture or era, which have been strong barriers to the deployment (for better or for worse) of a "scientific" understanding of the Universe. It is nonetheless true that some of the most fundamental and evident ideas and notions in mathematics (such as the notions of displacement, group, the number 0, symbolic arithmetic, the coordinates of a point in space, the notion of a set, or that of topological "shape" without even mentioning negative numbers and complex numbers) took millennia without making an appearance. These oversights are signs of this ingrained "block", deeply embedded in the psyche, against the conceptualization of entirely novel ideas, even in the cases where these are of a childlike simplicity and seem to impose themselves with the strength of evidence, over the course of generations or even millennia...

Returning to my own work, I have the impression that within it, the "mess-ups" (perhaps more frequent than in the work of most of my colleagues) pertain exclusively to matters of detail, generally spotted quickly by my own hand. These are simple "potholes", of purely "local" nature, and with no serious implications concerning the validity of the examined sit-

that the "travail sur pièces" to which I like to devote a loving care, was not at all displeasing to me. Moreover, the mode of mathematical expression which was professed and practiced by my elders, gave preeminence (to say the least) to the technical aspect of one's work, and in no way encouraged the "digressions" that would have idled on the "motivations"; or even those which appeared to bring out of the mist some vision which perhaps was inspiring, but which, because it failed to be presented in the form of tangible constructions in wood, stone, or hard cement, was likened more to dream fragments, than to the craft of a p. P19
conscientious and diligent artisan.

At the quantitative level, my work during these intense years of productivity manifested itself mostly in the form of about twelve-thousand pages of publications, in the form of articles, monographs, or seminaries,[19] as well as hundreds, if not thousands, of new notions which have entered into the mathematical patrimony, with the very names which I had given them upon first discovering them.[20] In the history of mathematics, I think I may have been the person who introduced the largest number of new notions into our science, and at the same time, the person who was brought, as a consequence, to invent the largest number of new names, with the intention of expressing these notions with delicacy, and in as suggestive a way as I could.

These indications give no more than a very rough feel for my work, ignoring what truly lies at its heart, its life and vigor. As I wrote about earlier, what I have brought best to mathematics are the new "**viewpoints**" that I have been able to first **glimpse**, and later patiently **excavate** and to some extent develop. These new viewpoints, like the notions I just mentioned, inserting themselves in a vast multiplicity of different situations, are also nearly enumerable.

The fact remains that certain viewpoints are more vast than others, those which by themselves encompass a multitude of partial viewpoints, within a multitude of particularly different situations. Such a viewpoint can be called, rightly, a "**great idea**". Through its internal fecundity, such an idea gives rise to a vast progenitor of ideas which themselves inherit its fecundity, although most (if not all) are of a lesser scope than the mother idea.

The task of **expressing** a great idea, of "saying" it, can be as delicate as its conception and slow gestation within the person who conceived of it. To better put it, the laborious process of patiently excavating the idea, day after day,

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uation. At the same time, at the level of ideas and larger guiding intuitions, my work is free of any "mistakes" as incredible as that might seem. It is this always reliable certainty in apprehending each moment, if not the eventual conclusions of an argument (which often remain hidden from sight), at the very least the most fertile directions which present themselves to take me straight to what is essential - it is that certainty which has brought to my mind Koestler's metaphor of the "sleepwalker".

[19] Starting from the 1960's, part of these publications were written in collaboration with colleagues (mostly J. Dieudonné) and students.

[20] The most important of these notions are reviewed in the Thematic Sketch, as well as in the Historical Commentary which accompanies it - both of these are included in Volume IV of the Reflections. Some of these names were suggested by friends or students, such as the term "smooth morphism" (J. Dieudonné) or the panoply "site, stack, gerbe, and band" in the thesis of Jean Giraud.

p. P20

from the veils of mist which surround it at birth, to slowly succeed in giving it a tangible form, as a painting which grows richer, firmer, and finer over the course of weeks, months, years. To simply **name** the idea by some striking formula, or by more or less technical keywords, can fit in the span of a few lines, or a few pages - but rare are those who, without knowing it beforehand, will know how to listen to this "name" and recognize its face. And when the idea reaches full maturity, a hundred pages might suffice to express it to the full satisfaction of the worker within whom it was born - just as it can happen that ten-thousand pages, crafted and weighted at length, may not suffice.[21]

In this case, as in others, among those who gained awareness of the project, which presented the idea in full bloom, many people could see the vigorous trees, and used them (some to climb, others used them as lumber, some as firewood, ...) yet few could see the forest from the trees...

2.8 The vision - or twelve themes for a harmony

Perhaps one might say that the "great idea" is the viewpoint which, not only itself reveals new and fertile ideas, but that which introduces a novel and vast **theme** which embodies it. Every science, when understood not as a tool to gain power or domination, but rather as a journey towards the understanding of our species through the ages, is nothing but the harmony, more or less vast and more or less rich from one era to the next, which deploys itself through generations and centuries, through the delicate counterbalancing of all the themes which appeared one by one, as if summoned from the void, to integrate into the science and be interlaced within it.

p. P21

Among the multiple viewpoints, which I have unearthed in mathematics, there are **twelve**, in hindsight, which I would call "great ideas".[22] To see my

[21] Upon leaving the mathematical world in 1970, the totality of my publications (including several works in collaboration) on the central theme of schemes amounted to some $10,000$ pages. Yet this only represented a modest fragment of the vast program which I glimpsed ahead, concerning schemes. This program was abandoned sine die dès my departure despite the fact that almost everything which had already been published for the use of everyone had already entered the common patrimony of results and notions considered "well known".

The part of my program concerned with schemes together with its extensions and ramifications, which I had completed at the time of my departure, alone represented the most vast foundational work ever accomplished in the history of mathematics, and surely one of the most vast in the history of Science as well.

[22] Here are, for the mathematically inclined reader, the twelve key ideas, or "maître thèmes" (in chronological order of appearance):

1. Topological tensor products and nuclear spaces.
2. "Continuous" and "discrete" duality (derived categories, "six operations").
3. Riemann-Roch-Grothendieck yoga (K-theory, relationship with intersection theory).
4. Schemes.
5. Topos.
6. Étale and ℓ-adic cohomology.
7. Motives, Motivic Galois Groups (Grothendieck $\otimes$-categories).

mathematical work, to "feel" it, is to see and "feel" at least some of these ideas, and the great themes that they introduce and which lie at the heart of my work.

By the nature of things, some of these ideas are "greater" than others (which in turn are "smaller"!). In other words, among these novel themes, some are vaster than others, and some plunge deeper into the heart of the mysteries of mathematical things.[23] There are three (and not the least of them) which, having appeared only after I had left the mathematical world, remain at the embryonic stage; "officially" did not exist as no proper publication can be pointed to as a birth certificate.[24] p. P22

8. Crystals, crystalline cohomology, yoga of "de Rham coefficients", "Hodge coefficients", ...
9. "Topological algebra": ∞-stacks, derivators; cohomological formalism of topoi, serving as inspiration for a new conception of homotopical algebra.
10. Tame topology.
11. The yoga of nonabelian algebraic geometry, Galois-Teichmüller Theory.
12. "Schematic" or "arithmetic" point of view for regular polyhedra and regular configurations of all kinds.

Apart from the first theme, a large portion of which appeared in my thesis of 1953 and was further developed in the period in which I worked in functional analysis between 1950 and 1955, these themes were discovered and developed during my time working as a geometer, starting in 1955.

[23]Among these themes, the most **vast** in its **reach** seems to me to be that of **topoi**, in that it provides the idea of a synthesis of algebraic geometry, topology, and arithmetic. The most vast in terms of the **extent of the developments** to which it has given birth thus far, is the theme of schemes. (See the note ref.) It is that theme which provides the framework "par excellence" in which eight of the other listed themes are developed (namely all but themes 1,5,10), and it also provides the central notion for a fundamental renewal of algebraic geometry, and of the algebraic geometric language.

On the other extreme, the first and last of these twelve themes seem to me to be of more modest dimensions than the others. Yet, concerning the last, introducing a new viewpoint on the ancient theme of regular polyhedra and regular configurations, I doubt that the life of a mathematician who would devote themselves exclusively to this would suffice to exhaust it. As for the first of all of these themes, topological tensor products, it played a role of a new tool ready for use, rather than a source of inspiration for later developments. The fact remains that I still, to this day, receive sporadic echoes of more or less recent solutions (twenty or thirty years later) to some of the questions which I had left open.

The deepest (to my eyes) of these twelve themes, are the notion of **motives**, and the closely related **yoga of nonabelian algebraic geometry, and Galois-Teichmüller theory**.

From the viewpoint of **powerful tools**, perfectly polished under my care, and commonly used in various "cutting edge fields" in research during the past two decades, the themes of "**schemes**" and "**étale and ℓ-adic cohomology"** have been the most noteworthy. For a well-informed mathematician, I believe there is no doubt that the schematic tools, as well as the theory of ℓ-adic cohomology which arises from it, are among some of the most important contemporary acquisitions, having come to nourish and renew our science over the course of the previous generations.

[24]The only "semi-official" text where these themes are briefly sketched, is the Esquisse d'un Programme written in January 1984, in the context of a detachment request to the CNRS. This text (which is also mentioned in the Introduction 3, "compass and luggage") will be included in principle in volume 4.

Part I

I

Part II

II

Part III

III

Part IV

IV