Récoltes et Semailles

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Contents

Presentation of the themes or PRELUDE IN FOUR PARTS

Chapter 1

By way of a foreward

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All that was left to write was the foreward, in order for Récoltes et Semailles to be given to the publisher. And I swear that I went into it with all the will in the world to write something which would be suitable. Something **reasonable** this time. No more than three or four pages, but carefully phrased in order to introduce this huge tome of more than one thousand pages. Something which "grabs" the attention of the jaded reader, which gives him a glimpse that in these frightening "more than a thousand pages", there could be things of interest to him (or things which concern him, who knows?). It is not really my style to pander. But I was ready to make an exception for once! The "publisher crazy enough to give it a shot" (to publish this visibly unpublishable monster) had to make ends meet one way or another.

But then, it didn't come. And yet I tried my best. And not only for an afternoon, as I originally planned. Tomorrow will mark three weeks since I started, since the sheets began accumulating. What came, for sure, isn't what one could decently call a "forward". It is yet another miss! Blame it on my old age - I have never been a salesman. Even when it comes to pleasing (oneself or friends...).

What came is a sort of long "Walk" with commentary, through my work as a mathematician. A Walk intended mostly for the "layman" - he who "never understood anything about math". And for myself as well, having never indulged in such a Walk. Step by step, I found myself unearthing and saying things that had previously remained unspoken. As if by chance, these are also things which I feel are most essential, both in my practice and its outcome. They are things which are not technical in nature. It will be up to you to decide whether or not I succeeded in my naive enterprise to "get the message through" - an exterprise which surely is also a bit mad. My satisfaction and my pleasure will come from making you feel these things. Things that many of my wise colleagues do not feel anymore. Maybe they have become too wise and too prestigious. This often leads to losing touch with the simplest and most essential things.

During this "Walk through a body of work" I also speak of my life. As

well as, here and there, what Récoltes et Semailles is about. I mention this
p. A2 in more detail in the "Letter" (dated from May of last year) which follows the "Walk". This Letter was directed towards my previous students and to my "old friends" in the mathematical community. But even the Letter is not technical in nature. It can be read by any reader interested in learning through a "heartfelt" narrative, the odds and ends that led me to writing Récoltes et Semailles. Even more than the Walk, the Letter will provide a preview to a particular atmosphere in the "prestigious mathematical world". And also (just as in the Walk) of my writing style, as peculiar as it may seem, and of the spirit that is expressed through this style - a spirit which is not universally appreciated.

In the Walk and throughout Récoltes et Semailles, I speak of the activity of **doing mathematics**. It is an activity for which I have first-hand experience and know very well. Most of the things I say anbout it can surely be said of any kind of creative work, or work involving discovery. In any case it is true of all "intellectual" work, that which is done using the "brain" and in writing. All such work proceeds through the the outbreak and development of an **understanding** of the things which are being probed. But to take an example at the opposite extreme, romantic passion is also an activity of discovery. It opens us to understanding of a "physical" nature which also renews itself, develops, and deepends over time. Both of these impulses - that which, say, livens the mathematician at work and that of the lover - are much close in nature than we generally assume or we readily admit. I hope that the pages of Récoltes et Semailles will make you feel this impulse in your work and in your daily life.

Most of the Walk focuses on mathematical work itself. I remain mostly silent concerning the **context** in which this work takes place, and concerning the **motivations** at play outside of mathematical work itself. This risks giving me, or the mathematician, or the "scientist" in general a flattering but deformed image. In the style of "grand and noble passion" without any form of rectification. In accordance with the great "Myth" of Science (with a capital S, if you will!). The heroic myth, "promethean", to which writers and thinkers have succumbed (and continue to succumb). Only historians, maybe, manage to sometimes resist this tantalizing myth. The truth is, within the motivation of these "scientists", which sometimes lead them to devote themselves entirely to their work, ambition and vanity play a role just as important and universal
p. A3 as they do in any other profession. This phenomenon appears in blunt or subtle ways depending on the person - and I am no exception to this pattern. The reading of my testimony will hopefully leave no doubt about this fact.

It is true also that even the most intense ambitions are powerless at discovering or proving a novel mathematical statement - just as they are powerless (for instance) to "make one hard" (in the proper sense of the term). Whether man or woman, what "makes one hard" is not ambition, nor the desire to shine, to exhibit power, of a sexual nature in this case - quite the contrary! It is the acute perception of something strong, at once very real and very delicate. One could call it "beauty", thought this is one of a thousand faces of this thing. Being ambitious doesn't prevent one from sensing the beauty of a being or a thing. But it is **not** ambition which makes us feel it...

The person that first discovered and mastered fire was somebody just like you and me. Not at all what we refer to as "hero", or "demi-god", and so on. Surely, just like you and me, he has encountered the grip of anxiety as well as the time-worn remedy of vanity which alleviates the grip. At the instant at which he "knew" fire, there was no fear nor vanity. Such is the truth in the heroic myth. The myth becomes insipid when it is used to to disguise another aspect of things which is just as real and essential.

My aim in Récoltes et Semailles has been to address both aspects of the myth - that of the impulse towards understanding, and that of fear and its vain antidotes. I believe I "understand", or at least **know** this impulse and its origin (or perhaps one day I will discover to what extent I was deluded). But concerning fear and vanity, as well as the resulting insidious creativity blocks, I know thta I have yet to thoroughly uncover this great enigma. And who knows if I will ever reach the conclusion of this mystery in the year I have left...

As I was writing Récoltes et Semailles two images emerged in order to represent the two aspects of the human journey: that of the **child** (aka the **worker**), and that of the **boss**. In the Work which we are about to undertake, we will be dealing mostly with the "child". It is also him that is featured in the subtitle "**The Child and the Mother**". The motivation for this name will hopefully become clear over the course of this work. p. A4

In the remainder of this reflection, however, it is the Boss who takes the lead. He is living up to his name! It would be more accurate to speak of multiple bosses of competing enterprises rather than of a **singular** boss. But it is also true that all bosses essentially resemble one another. And once we mention bosses it is implied that we will also have to deal with "villains". In part I of the reflection (named "Fatuity and Renewal", which follows the present introductory section). I mostly take on the role of the "villain". In the following three parts it is mostly the "others". Chacun son Tour! That is to say that, in addition to philosophical reflections and "confessions" (not contrite), there will be "vitriolic portraits" (to use the expression of one of my colleagues who found himself tormented). Not to mention large-scale well-oiled "operations". Robert Jaulin[1] assured me (half jokingly) that in Récoltes et Semailles I was making the "ethnology of the mathematical community" (or maybe the sociology I do not quite remember). It is flattering of course to learn that one has been (unknowingly) doing scholarly things! It is true that during the "investigation" segment of the reflection, I saw in passing, in the pages I was writing, a good chunk of the mathematical establishment without counting a number of my colleagues and friends of more modest status. Over the past few months, since I have been sending preliminary versions of Récoltes et Semailles this has been "brought up" again. My testimony arrived like a tome landing in a pond. There were responses of every kind (except for boredom...). Yet almost every time the response was far from what I expected. There was also a lot of silence, which speaks volumes. Visibly, I had (and still have) a lot to learn about what

[1]Rober Jaulin is an old friend. From what I understand, his position with respect to the establishment of the ethnological milieu mirrors mine with respect to the "high society" of mathematics (as white wolves).

happens in people's minds, among my previous students and other colleagues - excuse me I meant about "the sociology of the mathematical milieu"! To all those that contributed to the great sociological work of my old days, I would like to express sincere recognition.

Of course, I was particularly sensitive to warm responses. There were also some rare colleagues who conveyed a sentiment (thus far unexpressed) of crisis,
p. A5 or of degradation of the inner workings of the mathematical milieu with which they identify themselves.

Outside of this milieu, among the very first to respond positively to my testimony, I would like to recognize Sylvie et Catherine Chevalley[2] Robert Jaulin, Stéphane Deligeorge, Christian Bourgois. If Récoltes et Semailles achieves a wider diffusion than that of the initial printing (addressed to a very limited social circle of people), it is mostly thanks to them. Thanks mostly to them communicating their conviction that what I strived to seize and say had to be said. And that it could have an audience outside of my colleagues (who are often sullen, sometimes even belligerent, and strictly opposed to question their position...). Indeed Christian Bourgois did not hesitate to risk publishing the unpublishable, and Stéphane Deligeorge did not hesitate to place my indigestible testimony alongside works of Newton, Cuivier, and Arago (I could not ask for better company). To each of them, for their repeated expressions of sympathy and trust, intervening at an especially sensitive moment, I happily extend all my gratitude.

And here we are at the beginning of a Walk through a life's work, serving as a prelude to a journey through a lifetime. A long journey, over a thousand pages long, each of which is densely packed. I spent a lifetime undergoing this journey without ever exhausting it, and it then took me more than a year to rediscover it, one page at a time. Words were sometimes hard to come by, as they were intended to convey an experience which evaded comprehension - just as ripe grapes stacked in a press occasionally seem to evade the force upon them... But even in those moments when words come flooding, it is not by happenstance. Each word has been carefully weighed in passing, or after the fact. Thus this reflection - testimony - journey is not meant to be read hastily, in a day or a
p. A6 month, by a reader rushed to read the final word. There is **no** "final word", no "conclusion" in Récoltes et Semailles, no more than there are any in my life or yours. There is only a wine, aged over the course of a lifetime, in the core of my being. The last glass which you will be drinking will be no better or worse than the first or the hundredth. They are all "the same", and they are all different. And if the first glass is spoiled, so is the rest of the barrel; it is better to drink fresh water (if such can be found), than to drink bad wine.

But a good wine ought not to be drunk in haste, or expeditiously.

[2]Sylvie et Catherine Chevalley are the widow and daughter of Claude Chevalley, the colleague and friend to whom the central part of Récoltes et Semailles is devoted (ReS III, the key of the Yin and the Yang). At multiple times in the reflection I speak of him and of the role he played in my journey.

Chapter 2

A walk through a life's work, or the child and the mother

January 1986

2.1 The magic of things

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When I was little I liked going to school. The same teacher taught us reading, writing, arithmetic, singing (he accompanied us with a small violin), and even about prehistoric men and the discovery of fire. I do not ever recall being bored at school, during those days. There was the magic of numbers, that of words, of signs, and of sounds. That of **rhymes** as well, through songs and small poems. There seemed to be, within rhymes, a mystery which extended beyond words. I believed this until the day somebody told me that there was a simple "trick" to it; that rhyme was simply when one ends two consecutive spoken movements by the same syllable, so that, as if by magic, these phrases became **verses**. It was a revelation! At home, where I found a good audience, for weeks or months on end, I amused myself by making verses. At one point, I even started exclusively speaking in rhymes. That period has passed, fortunately. Yet even to this day, I still sometimes write poems - but without trying to force the rhyme, if it doesn't seem to come by itself.

On another occasion an older friend, who was already in high school taught me about negative numbers. It was also a fun game, although I rapidly exhausted it. And then there were crosswords - I spent days and weeks constructing them, evermore interwoven. Within that game the magic of form, that of signs, and that of words found themselves combined. But even that passion subsided without leaving a trace.

During my high school years, which began in Germany during my first year,

then later in France, I was a good student without quite being the "star student". I devoted myself without restraint to the courses which I cared most about, and tended to neglect the others, without really caring for the appreciation of my "prof". During my first year of high school in France, in 1940, I was interned at a concentration camp with my mother, at Rieucros near Mende. It was wartime, and we were foreigners - "undesirables", as they said. But the administration of the camp turned a blind eye towards the kids, however undesirable they may be. We came and went as we pleased. I was the oldest, and the only one to go to high school, which was four or five kilometers away, in the rain and the wind,
p. P2 in makeshift shoes that always got wet.

I still remember my first "math examination", in which the teacher gave me a bad grade, for my proof of one of the "three cases of equality of triangles". My proof wasn't exactly that of the book, which he followed religiously. Yet, I knew very well that my proof was no less convincing than that of the book which I followed min spirit, through repeated invocation of the traditional "we slide this figure in such and such a way onto that figure". Visibly, this teacher did not feel capable of judging things on his own (namely the validity of the reasoning). He had to report to a higher authority, that of a book in this case. I must have been stricken by such dispositions, for me to still remember this incident. Ever since then, I have been presented with more than enough evidence to realize that such dispositions are far from exceptional, but rather they are the quasi-universal norm. There is a lot to be said on this subject - one which I approach more than once in one way or another in Récsoltes et Semailles. Yet to this day, I find myself invariably taken aback whenever I am confronted with such behavior...

During the last few years of the war, while my mother was still interned at the camp, I lived in a youth refugee house called "Secours Suisse", in Chambon sur Lignon. Most of us were jewish, and when we were told (by the local police) that there would be raids by the Gestapo, we went to hide in the woods for a night or two, in small groups of no less than 3, without quite realizing that our life was on the line. The region was filled with jews hiding in cévenol country, and many of us survived thanks to the solidarity of the lcoal population.

What struck me most about "Collége Cévenol" (where I was raised), was the extent to which my peers were disinterested in learning. As for myself, I devoured our textbooks at the beginning of the school year, thinking that this time around, we would finally learn **truly** interesting things; and for the rest of the year I utlized my time as best as I could while classes dragged along inexorably one trimester at a time. Yet we had some wonderful professors. The natural history professor teacher, mister Friedel, was a person with remarkable intellectual and social qualities. However, as he lacked authority, the class was acting out of control, to the extent that it became impossible to hear what he had to say, as his voice was lost in the hurly-burly. This may be the reason I
p. P3 haven't become a biologist!

I spent a fair amount of time, including class time (shh...), solving math problems. The ones in the book soon became insufficient. Perhaps it was because they tended to resemble one another after a while; but mostly, I believe,

because they seemed to come out of the blue á la queue-leue-leue, with no indication as to where they came from or where they're going. These were the books problems, not mine. And yet, natural questions were plentiful. For instance, once the three side lengths a, b, and c of a triangle are known, so that the triangle itself is known (up to its position), there has to be an explicit "formula" that expresses the area of the triangle as a function of a, b, and c. Likewise, for a tetrahedron of which the six side lengths are known - what is the volume? I struggled through that one for a bit, but I must have gotten there eventually. In any case, when a problem "grabbed me", I did not count the hours or days that I spent working on it, even if it meant losing track of everything else! (And such remains the case to this day...)

What I found least satisfying, in our math textbooks, was the absence of a serious definition of the notion of length (of a curve), of area (of a surface), or of volume (of a solid). I promised myself to make up for this omission as soon as I could. This is what I devoted most of energy to between the years of 1945-1948, while I was a student at the University of Montpellier. University lectures weren't for me. Without ever quite realizing it, I must have been under the impression that all my professor did was recite the contents of the textbooks, just like my first math teacher at the lycée de Mende. I barely ever set foot on university grounds, just enough to keep up to date with the perennial "program". Books sufficed to cover said program, but it was clear that they offered no answers to the questions I was asking myself. Truly, they did not even **see** them, no more than my high-school textbooks did. As long as we were provided with recipes for all sorts of calculations, such as lengths, areas, volumes, through single, double, triple integrals (dimensions higher than 3 were carefully avoided...), the problem of providing an intrinsic definition was omitted by both my professors and textbook authors.

From my then limited experience, it seemed that I was the only person in the world to be gifted with a curiosity for mathematical questions. Such was, in any case, my unexpressed conviction, during those years spent in complete intellectual solitude (which did not bother me).[1] To be fair, it never occurred to me at that time to investigate whether or not I was the only person in the p. P4
world to take interest in what I was doing. My energy was sufficiently absorbed by the task I set for myself: to develop a fully satisfactory theory.

I never doubted that I would succeed in reaching the end of the story, as long as I was committed to scrutinizing these structures, spelling out on paper

[1]Between 1945-1948, I lived with my mother in a small hamlet about 10 kilometers away from Montpellier, Mairargues (near Vendargues), lost in the middle of vineyards. (My father disappeared in Auschwitz in 1942.) We scraped by on my meager student funding. To make ends meet, I took part in the harvest every year, and after the harvest season I would sell wine under the table (in contravention of the legislation, or so I hear...). On top of that, there was a self-regulating garden which supplied us with an abundance of figs, spinach, and even (towards the end) tomatoes planted by a complacent neighbor, amidst a sea of splendid poppies. It was the good life - although occasionally a bit rough along the edges, when we had to replace a pair of glasses, or a pair of worn-out shoes. Luckily, because my mother was weak and sick due to her long stay in the camps, we received free medical assistance. We would never have been able to afford a doctor otherwise...

what they were telling me. The intuition behind **volume**, say, was irrecusable. It could only be the reflection of a **reality**, momentarily elusive, but perfectly reliable. What had to be done was simply to seize this reality - a bit, perhaps, the way the magic reality of the "rhyme" had been seized, "understood" one day.

When I began this pursuit, at age 17, freshly out of high-school, I thought it would only take a few weeks. I spent three years on the project. It even caused me to fail an exam at the end of my second year of university - that of spherical trigonometry (in the "further astronomy" module), because of a stupid computational mistake. (I was never very good at computations, I must say, ever since I left high school...). That is why I had to spend a third year in Montpellier to complete my bachelor's instead of going to Paris right away - the only place, I was told, where I would be able to find people aware of what was considered important in Mathematics. My informant, Mister Soula, assured me that the last problem left in mathematics had been resolved twenty or thirty years ago by a so-called Lebesgue. He had apparently developed (funny coincidence!) a theory of measure and integration which brought *point final* to mathematics.

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Mister Soula, my "diff calc" teacher, was a benevolent man who took a liking to me. I was still not convinced by his claim. There must have already been, within me, the prescience that mathematics is a thing which is infinite in scope and depth. Does the sea have a "*point final*"? Yet I never thought of looking for that book by Lebesgue which Mister Soula had told me about, and he probably never held it either. In my mind there was nothing in common between anything a book contained and the work that **I** had been doing, in my own way, in order to answer questions which intrigued me.

2.2 The importance of being alone

When I finally made contact with the mathematical world in Paris, one or two years later, I ended up learning, among many other things, that the work which I had been doing independently, and with the means at hand, was (essentially) what "everybody" knew as the "Lebesgue theory of measure and integration". According to the two or three experts to whom I mentioned my work (or even showed a manuscript), I had just wasted my time redoing something "already known". I actually do not recall being disappointed. At that moment, the idea of receiving "credit", or even simply receiving approbation for the work that I was doing, must have still been foreign to my mind. Furthermore, my energy was completely taken by the process of familiarizing myself with an entirely different milieu and mostly learning what was considered in Paris to be the basic toolkit of the mathematician.[2]

Yet, thinking back to those three years, I realized that they were not in any way wasted. Unknowingly, I learned in solitude what is essential to the work of

[2] I briefly narrate this rough transition period in the first part of Récoltes et Semailles (ReS I), in the section "The Welcome Stranger" (nb. 9).

a mathematician - something no master could truly teach. Without ever having been told, without ever having to encounter someone with whom I could share my quest for understanding, I knew "in my gut" that I was a mathematician: somebody who "does" math, in its fullest sense - the way one makes "love". Mathematics had become, for me, a mistress always accommodating my desires. These years of solitude laid the foundation for a trust that has never been shaken - not by the discovery (upon arrival in Paris at age 20) of the scope of my ignorance and the vastness of what I had to learn; nor (more than 20 years later) by the eventful episode of my permanent departure from the mathematical world; nor, in these last few years, by the often crazy episodes of a "Funeral" (anticipated and cleanly executed) of my person and life's work, orchestrated p. P6
by those who used to be my closest companions...

To phrase it differently: I learned in those crucial years to "be alone".[3] That is, I learned to approach the things which I want to know with my own eyes, rather than rely on the expressed or implicit ideas that eminate from the group with which I identify, or a group to which I attribute authority. An unspoken consensus told me, both in high school and in university, that there was no need to question the notion of "volume", which was presented as "well-known", "self-evident", "unproblematic". Naturally I turned a blind eye to this consensus - just as Lebesgue, a few decades earlier, had to **turn a blind eye**. It is in this act of "turning a blind eye", of being oneself rather than the mere expression of the reigning consensus, of not to remain inscribed within the imperative circle to which they assign us - it is within this solitary act, above all else, that "**creation**" lies. Everything else comes after.

In the following years, within the mathematical world which welcomed me, I had the opportunity to meet multiple people, both older and younger, which were clearly more brilliant, "gifted" than I was. I admired the facility with which they learned new notions, as if at play, juggling them as if they had known them their whole life - while I felt heavy-handed and clumsy, laboriously making my way, akin to a mole, through an amorphous mountain of important things (or so I was told) which I had to learn, despite having no sense of their ins and out. Actually, I was far from the brilliant student who aced every prestigious *concours* and assimilating at once the most prohibitive courses.

Many of my more brilliant peers went on to become competent famous mathematicians. In hindsight, after 30-35 years, it does not seem yo me that they left a deep imprint upon the mathematics of today. p. P7

[3] This formulation is somewhat clumsy. I never had to "learn to be alone", for the simple reason that I never **unlearned** during the course my childhood, this innate skill which I had since birth, just as we all do. Yet these three years of solitary work, during which I could walk to my own beat, following my own exigence criteria, confirmed within me a degree of trust and tranquil confidence in my relationship with mathematics which owed nothing to the reining trends and consensus. I make allusion to these again in the notes "Roots and Solitude" (Res IC, nb. 171) notably .