DRP WEEK 4: ONE BIG EXAMPLE

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Let G and H be groups and let X be a set. Consider the set of functions $f : X \to H$. Write this set as F.

Assume we have a group action of G on X, written $G \circlearrowright X$. This means we have a function:

$$a : G \times X \to X \ .$$

This means for every $g \in G$, we get a function $\varphi_g : X \to X$ given by

$$\varphi_g(x) = a(g, x) \ . \tag{1}$$

Remark 1. The idea of restricting to an element of G to get a function on X is a very subtle, confusing, and important one. So take some time to meditate on it.

Exercise 1. Show that G acts on F too. [Hint: we need a rule for eating a group element $g \in G$ and a function $f : X \to H$, and producing a new function $X \to H$. From the action of G on X, g defines a function $\varphi_g : X \to X$. So now we can rephrase the question: how do we turn f and φ_g into a new function $X \to H$?]

Exercise 2. Let H have n elements and H have m elements. How many elements does F have? Your answer should be in terms of m and n.

Exercise 3. Define a group homomorphism from G to a symmetric group (of some order). [Hint: We know G has an action on F. Does a symmetric group act on F?] [Another hint: The order of the correct symmetric group is the size of F...]

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