HOMEWORK 8

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Exercise 1 (Hatcher Chapter 0: 23)**.** Show that a CW complex is contractible if it is the union of two contractible subcomplexes whose intersection is also contractible.

Exercise 2 (Hatcher §1.A 4)**.** If X is a finite graph and Y is a subgraph homeomorphic to S^1 and containing the basepoint x_0, show that $\pi_1(X, x_0)$ has a basis in which one element is represented by the loop Y.

Exercise 3 (Hatcher §1.A 6)**.** Let F be the free group on two generators and let F' be its commutator subgroup. Find a set of free generators for F' by considering the covering space of the graph $S^1 \vee S^1$ corresponding to F'.

Exercise 4. Let T_g denote the surface $\#_g T^2$, $g \geq 0$. Show that T_h is a covering space of T_g iff there exists $n \geq 1$ such that $h = n(g-1) + 1$.

Exercise 5. Using covering spaces, show that a finite index subgroup of a finitely generated group is finitely generated.

Exercise 6. Using covering spaces, find the commutator subgroups of $\mathbb{Z}2\mathbb{Z} * \mathbb{Z}/2\mathbb{Z}$ and $\mathbb{Z}/2\mathbb{Z} * \mathbb{Z}/3\mathbb{Z}$.