HOMEWORK 9

JACKSON VAN DYKE

Exercise 1 (Hatcher §2.1.11)**.** Show that if A is a retract of X then the map $H_n(A) \to H_n(X)$ induced by the inclusion $A \subset X$ is injective.

Exercise 2 (Hatcher §2.1.15)**.** For an exact sequence $A \to B \to C \to D \to E$ show that $C = 0$ iff the map $A \to B$ is surjective and $D \to E$ is injective. Hence for a pair of spaces (X, A), the inclusion $A \hookrightarrow X$ induces isomorphisms on all homology groups iff $H_n(X, A) = 0$ for all n.

Exercise 3 (Hatcher §2.1.16)**.**
(a) Show that $H_0(X, A) = 0$ iff A meets each path-component of X.
(b) Show that $H_1(X, A) = 0$ iff $H_1(A) \to H_1(X)$ is surjective and each path-component of X contains at most one path-component of A.

Exercise 4. Recall that a SES of chain complexes $0 \to C \xrightarrow{\varphi} D \xrightarrow{\psi} E \to 0$ induces a long exact sequence in homology

$$\cdots \to H_q(C) \xrightarrow{\varphi_*} H_q(D) \xrightarrow{\psi_*} H_q(E) \xrightarrow{\Delta} H_{q-1}(C) \to \cdots \quad . \tag{1}$$

Prove

(i) Δ is well-defined,
(ii) Δ is a homomorphism, and
(iii) exactness at $H_q(E)$.

Exercise 5. Let $0 \to A_0 \to A_1 \to \cdots \to A_{n-1} \to A_n \to 0$ be an exact sequence of finite-dimensional vector spaces. Show that $\sum_{i=0}^{n} (-1)^i \dim A_i = 0$.

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