HOMEWORK 1

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Exercise 1 (Chapter 0, 2; Hatcher)**.** Construct an explicit retraction of $\mathbb{R}^{n+1} \setminus \{0\}$ to S^n.

Exercise 2 (Chapter 0, 11; Hatcher)**.** Show that $f : X \to Y$ is a homotopy equivalence if there exist maps $g, h : Y \to X$ $fg \simeq \mathrm{id}_Y$ and $hf = \mathrm{id}_X$. More generally, show that f is a homotopy equivalence if fg and hf are homotopy equivalences.

Exercise 3. Let $f : S^1 \to S^1$ be a map that is not homotopic to id_{S^1}. Show that there exists $x \in S^1$ such that $f(x) = -x$.

Exercise 4. Let X, Y be closed subsets of $X \cup Y$. Let $f : X \to Z$ and $g : Y \to Z$ be maps such that $f|_{X \cap Y} = g|_{X \cap Y}$. Show that $f \cup g : X \cup Y \to Z$ is continuous.

Exercise 5. Let $f : X \to Y$ be a map and W a space. Define

$$f_* : [W, X] \to [W, Y]$$

by $f_*([h]) = [fh]$. Show

(i) f_8 is well-define
(ii) if $f : X \to Y$, $g : Y \to Z$ are maps and W a space then

$$(fg)_* = f_* g_* : [W, X] \to [W, Z]$$

(iii) $(\mathrm{id}_X)_* = \mathrm{id}_{[W,X]}$.
(iv) if $f : X \to Y$ is a homotopy equivalence then f_* is a bijection.

(Corresponding dual properties hold for

$$f^* [Y, W] \to [X, W]$$

defined by $f^*([h]) = [hf]$.)

Exercise 6. Recall that a space X has the fixed point property (FPP) if for every map $f : X \to X$ there exists $x \in X$ such that $f(x) = x$.

(i) Suppose $X \simeq Y$ and X has the FPP. Does Y have the FPP?
(ii) If A is a retract of X and X has the FPP does A have the FPP?
(iii) If A is a retract of X and A has the FPP does X have the FPP?

Exercise 7. Use path-connectedness to show that there is no continuous injection from $\mathbb{R}^n$ to $\mathbb{R}^1$ for $n > 1$.

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