HOMEWORK 2

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Exercise 1 (Chapter 0, 9; Hatcher)**.** Show that a retract of a contractible space is contractible.

Exercise 2. Let $X = \left\{ (x, y) \in S^1 \times S^1 \,\middle|\, x \neq -y \right\}$. Show that the map $f : S^n \to X$ given by $f(x) = (x, x)$ is a homotopy equivalence.

Exercise 3. A *topological group* is a topological space which is also a group, such that the multiplication and inverse functions:

$$G \times G \longrightarrow G$$

$$(x, y) \longmapsto xy$$

$$G \longrightarrow G$$

$$x \longmapsto x^{-1}$$

are continuous. (Examples: $\mathbb{R}$, S^1, $\mathrm{GL}(n, \mathbb{R})$, $\mathrm{SO}(n)$, ...)

Let G be a topological group and $e \in G$ be the identity. If σ, τ are loops in G based at e let $\sigma \bullet \tau$ be the loop in G based at e defined by

$$(\sigma \bullet \tau)(s) = \sigma(s)\,\tau(s)$$

for all $s \in I$.

Show that

$$\sigma \bullet \tau \simeq \sigma * \tau \,(\mathrm{rel}\,\partial I)\, \sigma \bullet \tau \simeq \tau * \sigma \,(\mathrm{rel}\,\partial I) \ .$$

Deduce that $\pi_1(G, e)$ is abelian.

Exercise 4. Show that a contractible space is path-connected.

Exercise 5. Show that there is no retraction from I^2 to the comb space C.