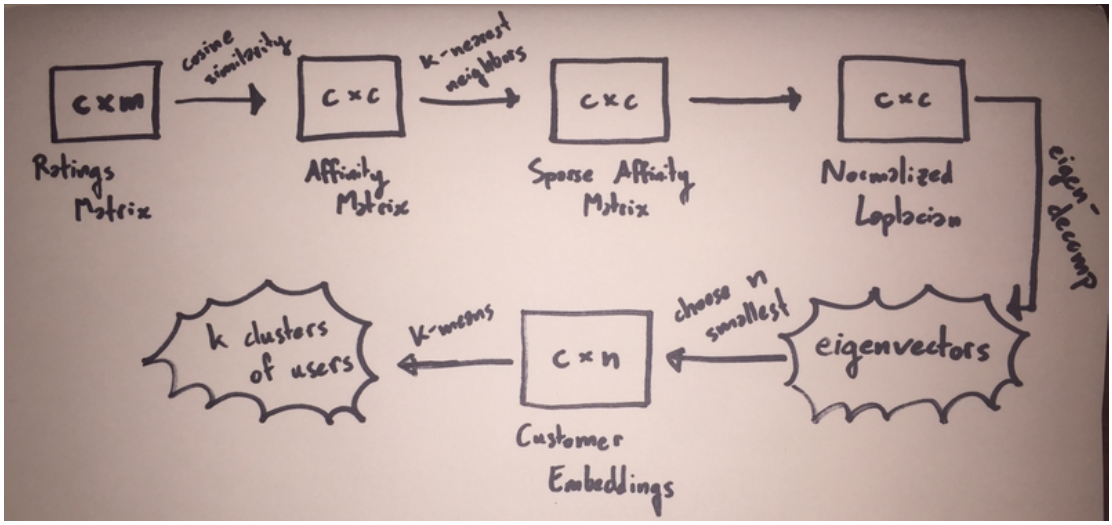
Report: ECS 271, Programming Assignment 1

Spectral Clustering

Overview:

With a spectral-clustering based recommendation system, the approach I took was the one that most closely mirrored my intuition of how to solve the problem: if we want to know how a particular user might rate a particular movie, identify other users with similar tastes and find out what they thought of that movie. To this end, I used spectral-clustering to identify communities of Netflix users with similar taste in movies. When answering the query, "*What would user x rate movie y?*", the predicted rating is the arithmetic mean of the ratings given for movie y by users in the same cluster as x. Ratings of zero indicate members who have not seen movie y or not rated it and are dropped before calculating the mean.



Technical Approach:

Splitting the Training Set. Though `sklearn` provides a `train_test_split` function, I did not find it to be suitable for this project based on a fundamental assumption: that approximately all movies and customers represented in the testing data would also be represented in the training data. The `train_test_split` function provides no such guarantee. The given training data had enough customers and movies with single digit representation with respect to ratings that simple random sampling proved to be insufficient in practice. Instead, I wrote my own splitting function.

This function calculates 10% of the count of each movie-id and customer-id in the training set, uses a floor function on the result, and mandates that no movie-id or customer-id can be present more than that number of times in the test data. Using random sampling combined with this check guarantees that all movies and customers in the testing data are present in the training data while also yielding approximately a 90-10 split between testing and training data.

Constructing Customer Nodes: The Ratings Matrix. Each customer-id is a node associated with a vector of movie ratings representing their taste in film. Given c customer-ids and m movies, a $c \times m$ sparse matrix R of movie-ratings is constructed that contains this information. Therefore, the components of the ith row vector of R are the m movie ratings associated with customer c_i and the components of the jth column vector of R are the c movie ratings associated with movie m_j.

Calculating Edge Weights: The Affinity Matrix. Each edge weight between two customer nodes is intended to represent the level of similarity between the movie tastes of those two customers. To this end, the cosine-similarity between movie-rating vectors is used. The resulting similarities are kept in a $c \times c$ affinity matrix W. Therefore, if $R[i]$ represents the ith row of R, we have:

$$W_{i,j} = \frac{R_i \cdot R_j}{\|R_i\| \|R_j\|}$$

In which $W_{i,j}$ represents the similarity between the movie tastes of the two customers represented in rows i and j of matrix R.

Note that because the domain of ratings is strictly positive, all vectors of customer ratings are in the same direction and, as a result, cosine-similarity will give us similarity values strictly between 0 and 1. Cosine-similarity is often used with positive high-dimensional spaces like this one and also has the advantage of being low-complexity for sparse vectors.

To yield the final affinity matrix, a k-nearest neighbors algorithm is applied to W, thus removing noise and reducing the complexity of calculating our clusters.

Finding Clusters: Hyper-parameter Tuning for Spectral Clustering. In order to identify communities of individuals with similar movie tastes, k-means clustering is applied to a projection of the graph of customer nodes using the eigen-decomposition of the normalized graph Laplacian. This equates to finding minimal normalized cuts in the graph of customer nodes. Hyper-parameters include the number of clusters ($n\_clusters$), the number of eigenvectors used ($n\_components$), the number of iterations to use in the k-means clustering (n_iter), and the number of nearest neighbors ($n\_neighbors$).

To get a general idea of the effect of hyper-parameter values, tuning started with two rounds of very-coarse and naively-initiated searches over different configurations of values for $n\_clusters$, $n\_components$, and $n\_neighbors$ (n_iter was held constant at 10) with the goal of minimizing mean-squared error.

In the first round, $n\_clusters \in \{15, 20, 25, 30, 35\}$, $n\_components \in \{5, 7, 10\}$, and $n\_neighbors \in \{50, 250, 500, 1000\}$, making for 60 total configurations. Of these, the configuration yielding the lowest mean-squared error used 15 clusters, 5 eigenvectors, and 1000 nearest neighbors. In general, it appeared that fewer clusters, fewer components, and more nearest neighbors was optimal.

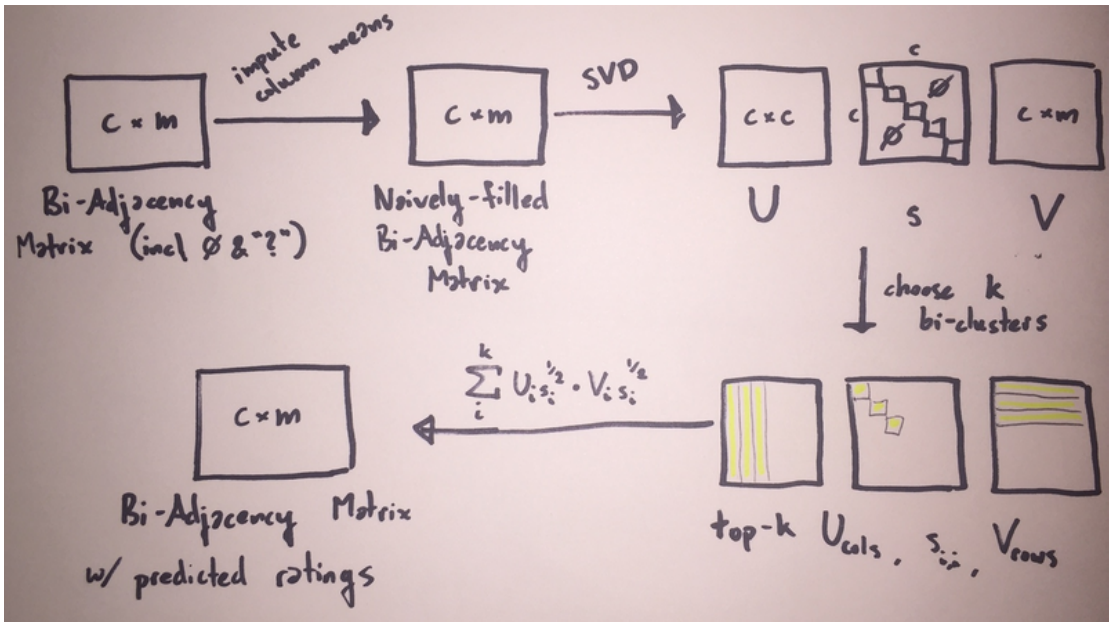
In the second round, $n\_clusters \in \{10, 11, 12, 13, 14\}$, $n\_components \in \{3, 5, 7, 10\}$, and $n\_neighbors \in \{250, 500, 1000\}$, again making for 60 total configurations. The conclusion from the first round appeared to hold true: the configuration with the lowest mena-squared error used 10 clusters, 3 eigenvectors, and 500 nearest neighbors.

These two rounds of systematic testing gave a general direction within the search-space; Further trial-and-error tuning eventually led to the final choice of 5 clusters, 3 eigenvectors, and 750 nearest neighbors.

Matrix Completion

Overview:

With a matrix completion-based recommendation system, we start by viewing the data as a bipartite graph $G = (C, M, E)$ in which C is the set of nodes representing customers, M is the set of nodes representing movies, and E is the set of weighted edges $\{(c, m, w) | c \in C, m \in M, w \in \{0, 1, 2, 3, 4, 5\}\}$ where the weight w is the rating user u gave to movie v. We then consider a $|C| \times |M|$ bi-adjacency matrix A in which each row represents one user's ratings, each column represents one movie's ratings, and $A_{i,j}$ represents a single user's rating of a single movie. In this context, the problem becomes one of completing the missing entries in A. Solutions to this problem are predicated on the assumption that A is of low-rank. In this context, the assumption of low-rank equates to the assumption that compared to the total number of movies and users, the number of *latent factors* that influence a user's opinion of a movie is small. In other words, the ratings of all users can be expressed as a linear combination of the ratings of a small number of prototypical users. In this assignment, I make use of the *singular value decomposition* strategy for matrix completion.



Technical Approach:

Splitting the Training Set. See discussion above.

Constructing the Bipartite Graph: The Bi-Adjacency Matrix. This process looks just like the one described above for the Customer Nodes Ratings Matrix for Spectral Clustering with one addition; the test set of blank ratings is included as well. This ultimately results in a Bi-Adjacency Matrix in which $A_{i,j} \in \{0, 1, 2, 3, 4, 5, ?\}$.

Finding Bi-clusters: Hyper-parameter Tuning for Matrix Completion Before performing the singular value decomposition, we fill in each element in A that is a zero or a question mark with the arithmetic mean of the other values in the column. This equates to filling in missing ratings with the average of the existing ratings for that film.

A singular value decomposition is performed on this matrix, resulting in a $|C| \times |C|$ matrix U, a $|C| \times |C|$ diagonal matrix S, and a $|C| \times |M|$ matrix V. These matrices are then used to construct a matrix of predicted ratings. For a given choice of k bi-clusters, we then take the top k columns of U, the top k rows of V, and the top k values of the diagonal matrix. Each of the column and row vectors is scaled by the square root of its respective value from s before ultimately taking a series of k outer-products and summing them up! In the source code, this can be done cleverly with a series of dot products.

With only a single value to tune with this strategy, the search space was much easier to manage than with the spectral clustering solution. I began by using the optimal number of clusters discovered above as a base line, $k = 5$. Variations in this quantity did not show any appreciable difference; MSE trended slightly higher with a higher value of k, but in order to avoid significantly overfitting the model, I settled on $k = 6$.