

# FUNDAMENTALS OF QUANTITATIVE MODELING

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*Module 1: Introduction and core modeling math*



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# Course goals

- Goals
  - Exposure to the language of modeling
  - See a variety of quantitative business models and applications
  - Learn the process of modeling and how to critique models
  - Associate business process characteristics with appropriate models
  - Understand the value and limitations of quantitative models
  - Provide the foundational material for the other three courses in the Specialization

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# Resources

- Software used in this Specialization
  - Excel (<https://office.live.com/start/Excel.aspx>)
  - Google sheets (<https://www.google.com/sheets/about/>)
  - R – an open source modeling platform  
(<https://www.r-project.org/>)
- Math review
  - E-book: Business Math for MBAs  
(<https://books.google.com/books?vid=ISBN97809990497912&hl=en>) - essential mathematics for business modeling

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# Module 1 content

- Examples and uses of models
- Keys steps in the modeling process
- A vocabulary for modeling
- Mathematical functions
  - Linear
  - Power
  - Exponential
  - Log

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# What is a model?

- A formal **description** of a business process
- It typically involves **mathematical** equations and/or random variables
- It is almost always a **simplification** of a more complex structure
- It typically relies upon a set of **assumptions**
- It is usually implemented in a computer program or using a spreadsheet

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# Examples of models

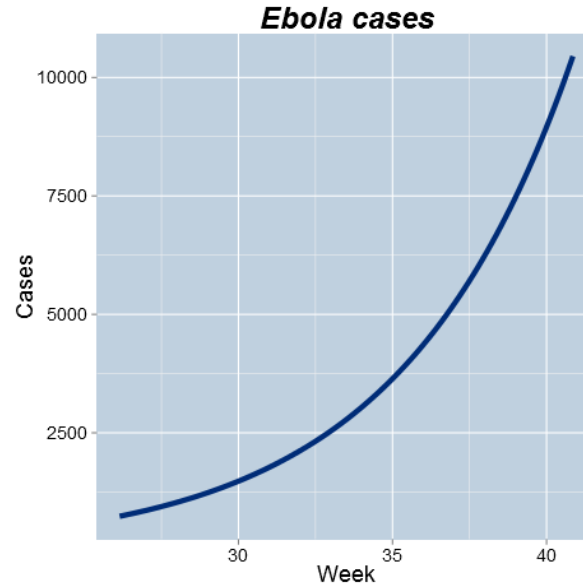
- The price of a diamond as a function of its weight
- The spread of an epidemic over time
- The relationship between demand for, and price of, a product
- The uptake of a new product in a market

# Diamonds and weight



Model: Expected price =  $-260 + 3721 \text{ Weight}$

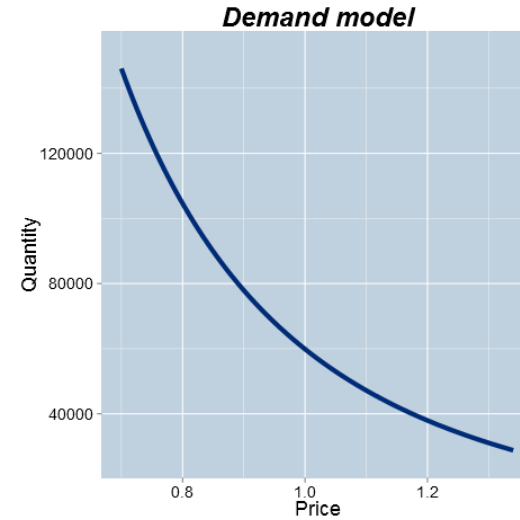
# Spread of an epidemic



$$\text{Model: Cases} = 6.69 e^{0.18 \text{ Weeks}}$$

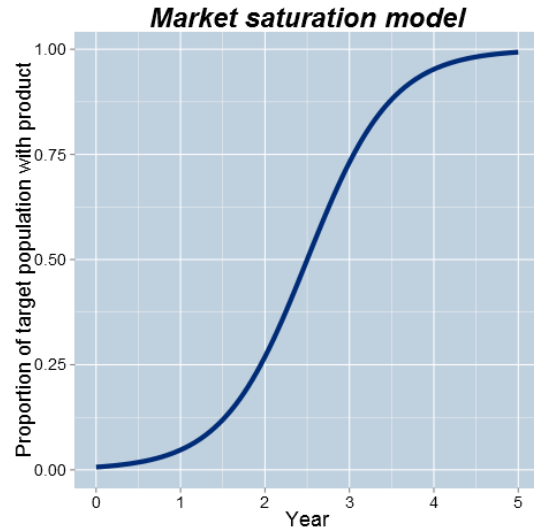


# Demand models

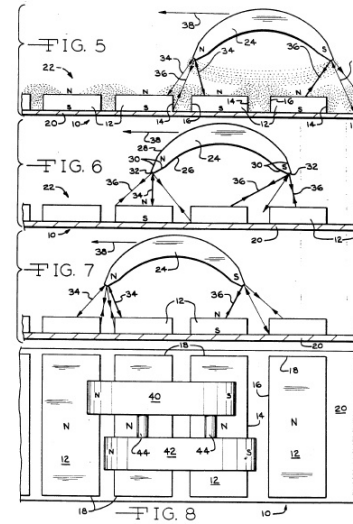


$$\text{Model: Quantity} = 60,000 \text{ Price}^{-2.5}$$

# The uptake of a new product



U.S. Patent Apr. 24, 1979 Sheet 2 of 2 4,151,431



$$\text{Model: Prop} = \frac{e^{2(\text{Year} - 2.5)}}{1 + e^{2(\text{Year} - 2.5)}}$$

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# How models are used in practice

- Prediction: calculating a single output
  - What's the expected price of a diamond ring that weighs 0.2 carats?
- Forecasting
  - How many people are expected to be infected in 6 weeks?
  - Scheduling – who is likely to turn up for their outpatient appointment?
- Optimization
  - What price maximizes profit?
- Ranking and targeting
  - Given limited resources, which potential diamonds for sale should be targeted first for potential purchase?

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# How models are used in practice

- Exploring what-if scenarios
  - If the growth rate of the epidemic increased to 20% each week, then how many infections would we expect in the next 10 weeks?
- Interpreting coefficients in model
  - What do we learn from the coefficient -2.5 in the price/demand model?
- Assessing how sensitive the model is to key assumptions

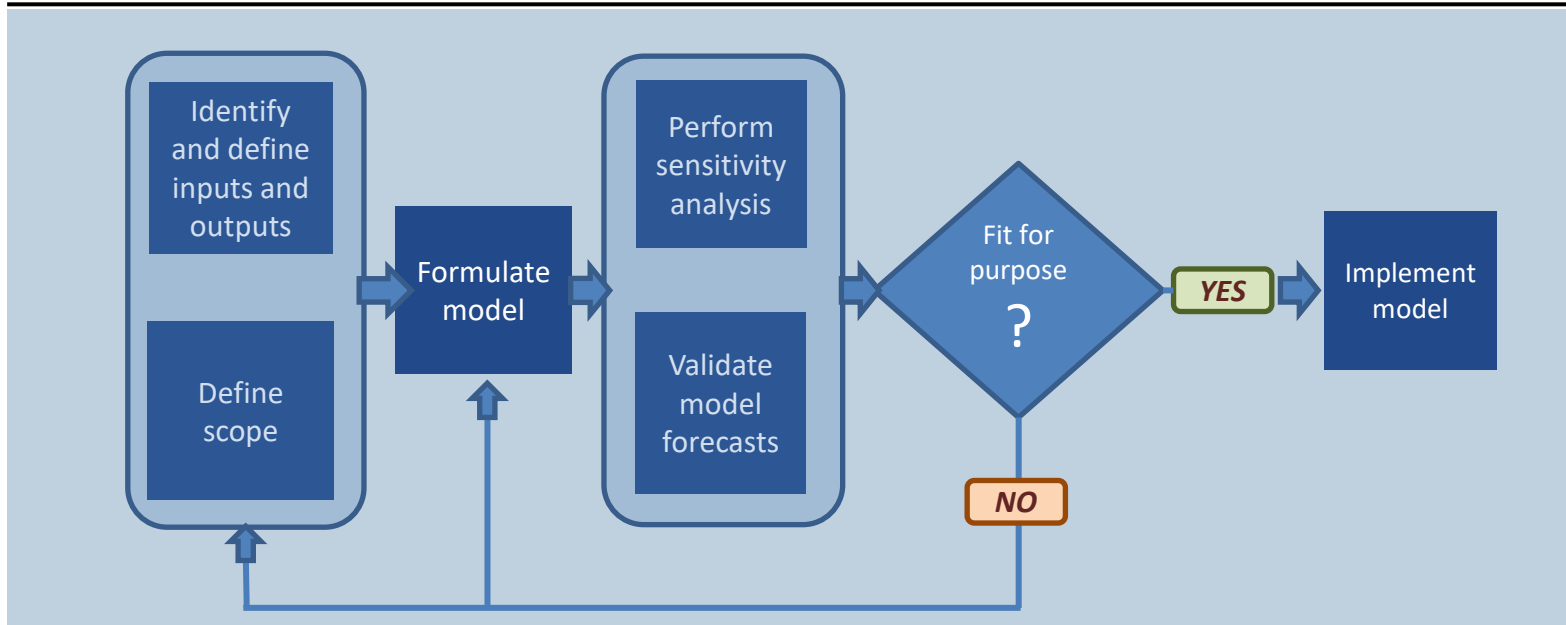
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# Benefits of modeling

- Identify gaps in current understanding
- Make assumptions explicit
- Have a well-defined description of the business process
- Create an institutional memory
- Used as a decision ***support*** tool
- Serendipitous insight generator

# Key steps in the modeling process

## *Modeling Process Workflow*



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# What if the model doesn't always work?

- When the observed outcome differs greatly from the model's prediction, then there is the possibility of learning from this event if we can understand why the difference occurs
- Modeling is a continuous and evolutionary process
- We identify the weaknesses and limitations and ***iterate*** the modeling process to overcome them

# A modeling lexicon





## Data driven v. theory driven



- Theory: given a set of assumptions and relationships, then what are the logical consequences?
  - Example: if we assume that markets are efficient then what should the price of a stock option be?
- Data: given a set of observations, how can we approximate the underlying process that generated them?
  - Example: I've separated out my profitable customers from the unprofitable ones. Now, what features are able to differentiate them?

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## Deterministic v. probabilistic/stochastic

- Deterministic: given a fixed set of inputs, the model always gives the same output
  - Example: Invest \$1000 at 4% annual compound interest for 2 years. After 2 years the initial \$1000 will **always** be worth \$1081.60.
- Probabilistic: Even with identical inputs, the model output can vary from instance to instance
  - Example: A person spends \$1000 on lottery tickets. After the lottery is drawn how much they are worth depends on a random variable, whether or not they won the lottery.

# Discrete v. continuous variables

- Watches can be digital or analog



- Likewise models can involve discrete or continuous variables
  - Discrete: characterized by jumps and distinct values
  - Continuous: a smooth process with an infinite number of potential values in any fixed interval

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# Static v. dynamic

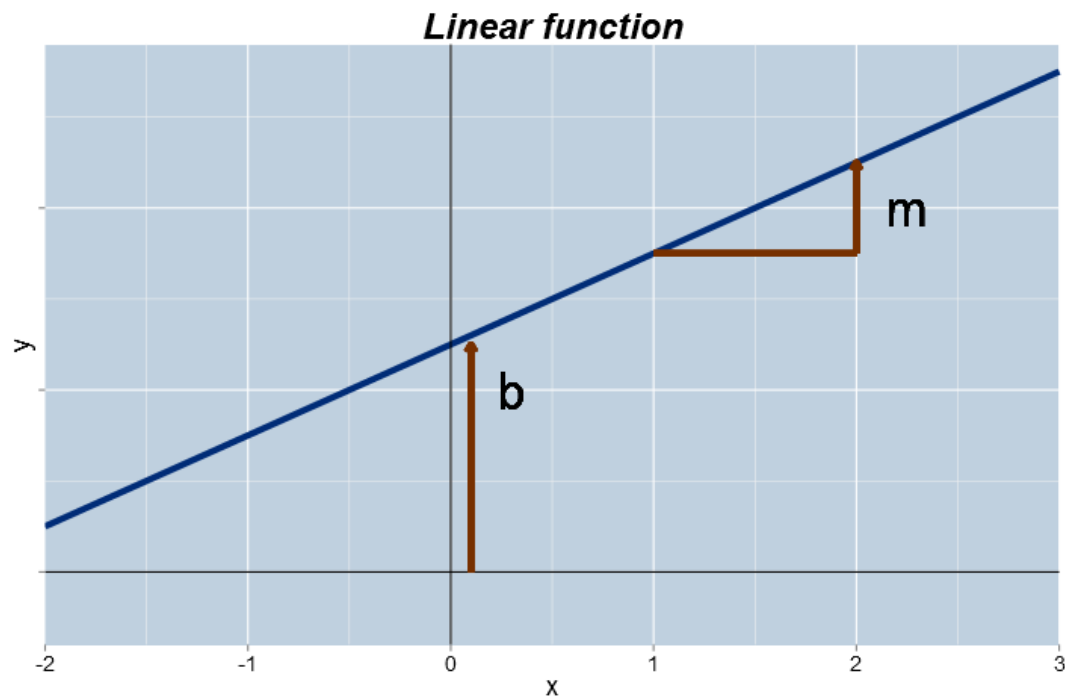
- Static: the model captures a single snapshot of the business process
  - Given a website's installed software base, what are the chances that it is compromised today?
- Dynamic: the evolution of the process itself is of interest. The model describes the movement from state to state
  - Given a person's participation in a job training program, how long will it take until he/she finds a job and then, if they find one, for how long will they keep it?

# Key mathematical functions

- Math: the language of modeling
  - Four key mathematical functions provide the foundations for quantitative modeling
  - 1. Linear
  - 2. Power
  - 3. Exponential
  - 4. Log



# The linear function

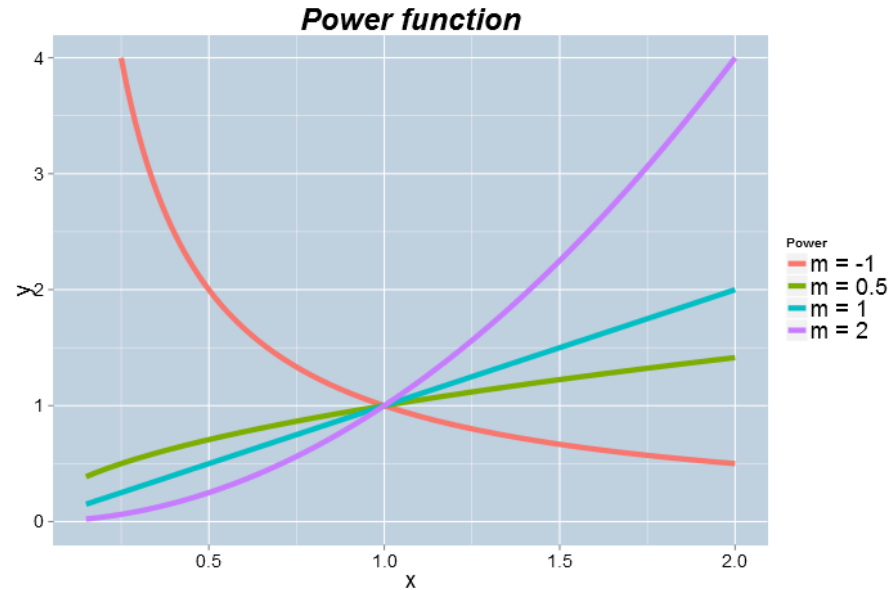


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# The linear function

- $y = mx + b$
- $x$  is the input,  $y$  is the output
- $b$  is the intercept
- $m$  is the slope
- Essential characteristic: the slope is constant
  - A one-unit change in  $x$  corresponds to an  $m$ -unit change in  $y$ .

# The power function for various powers of $x$

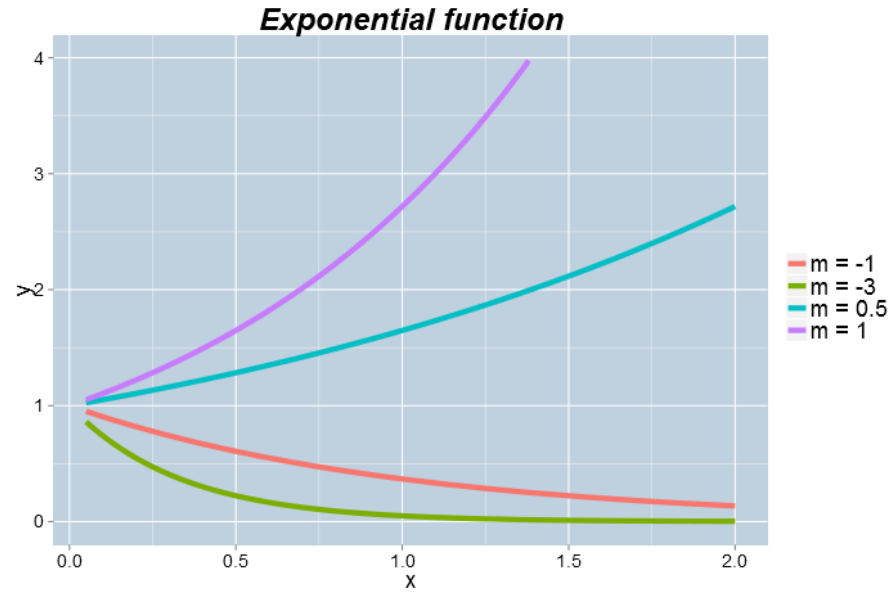




# The power function

- $y = x^m$
- $x$  is the **base**
- $m$  is the **exponent**
- Essential characteristic:
  - A one **percent** (proportionate) change in  $x$  corresponds to an approximate  $m$  **percent** (proportionate) change in  $y$ .
- Facts
  1.  $x^m x^n = x^{m+n}$
  2.  $x^{-m} = 1/x^m$

# The exponential function for various values of $m$



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# The exponential function

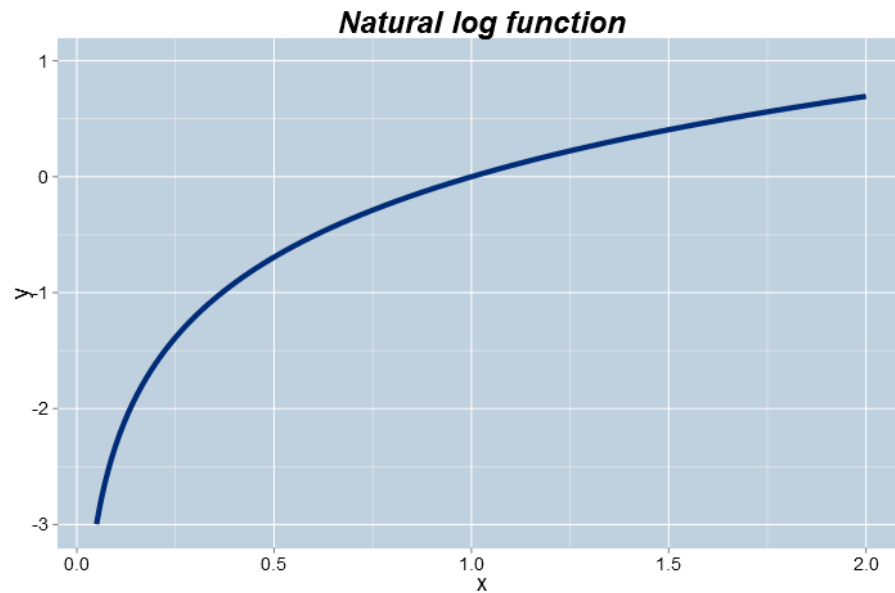
- $y = e^{mx}$
- $e$  is the mathematical constant: 2.71828...
- Notice that as compared to a power function,  $x$  is in the exponent of the function and not the base

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# The exponential function

- Essential characteristic:
  - the rate of change of  $y$  is proportional to  $y$  itself
- Interpretation of  $m$  for small values of  $m$  (say  $-0.2 \leq m \leq 0.2$ ):
  - For every one-unit change in  $x$ , there is an approximate  $100m\%$  (proportionate) change in  $y$
  - Example: if  $m = 0.05$ , then a one-**unit** increase in  $x$  is associated with an approximate 5% increase in  $y$

# The log function

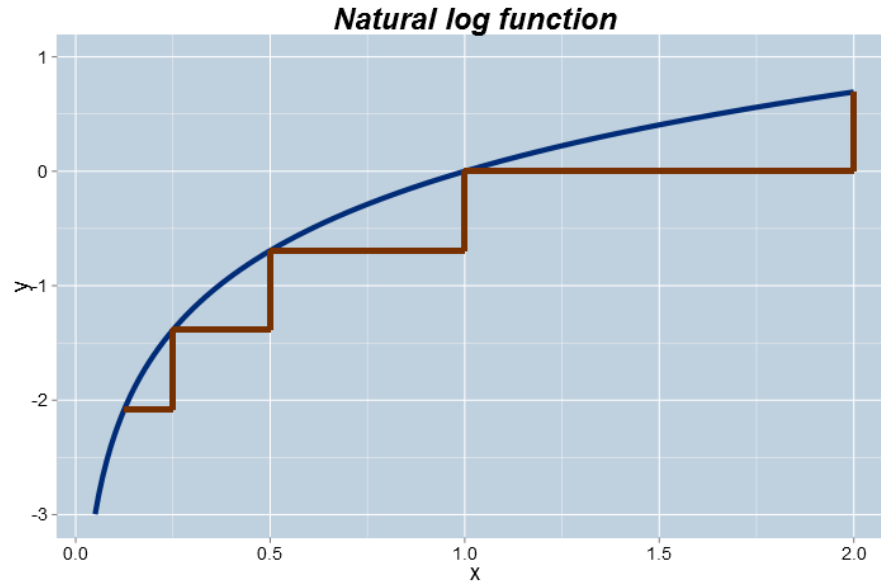


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# The log function

- The log function is very useful for modeling processes that exhibit ***diminishing returns to scale***
- These are processes that increase but at a decreasing rate
- Essential characteristic:
  - A constant proportionate change in  $x$  is associated with the same absolute change in  $y$

# The log function



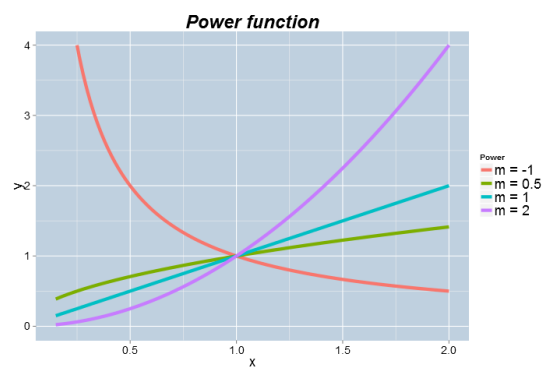
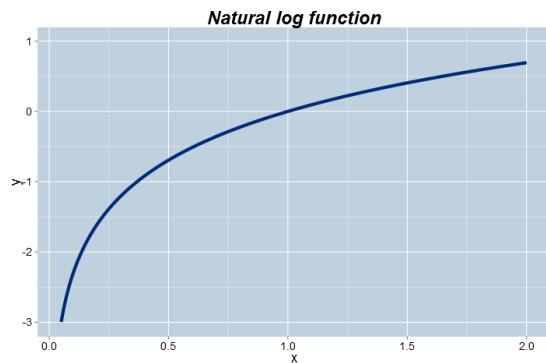
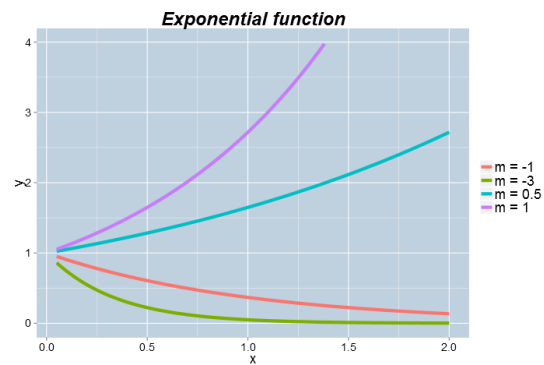
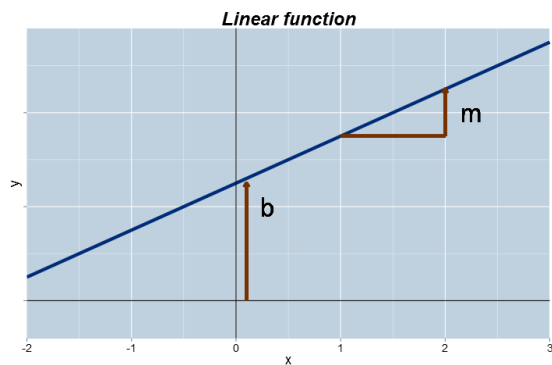
- Proportionate change in  $x$  is associated with constant change in  $y$

# The log function

- $y = \log_b(x)$
- $b$  is called the base of the logarithm
- The most frequently used base is the number “ $e$ ” and the logarithm is called the “natural log”
- The log undoes (is the inverse of) the exponential function:
  - $\log_e e^x = x$
  - $e^{\log_e x} = x$
- $\log(xy) = \log(x) + \log(y)$
- In this course we will always use the natural log and write it simply as  $\log(x)$



# The four functions



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# Module summary

- Uses for models
- Steps in the modeling process
  - It is an iterative process and model validation is key
- Discussed various types of models, discrete v. continuous etc.
- Reviewed essential mathematical functions that form the foundation of quantitative models



