# Discrete–Time Controller Design For a Two–DOF Helicopter


Ishaan Sharma
ID: 40053363
i_harma@encs.concordia.ca

Nidhi Patel
ID: 40043592
patelnidhi114@gmail.com

**Concordia University**
**Department of Electrical and Computer Engineering**


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# 1 Introduction

The system simulates two–dimensional flight that is equipped with two propellers driven by DC motors. The front propeller (pitch propeller) is used to control the pitch angle $\theta$ and the back propeller (yaw propeller) is used to control the yaw angle $\psi$.

# 2 Discrete Design of Pitch Controller

The transfer function for pitch angle $\theta$(s) is:

$$\theta(s) = \frac{37.2021}{s^2 + 0.2830s + 2.7452} V_p(s) + \frac{3.5306}{s^2 + 0.2830s + 2.7452} V_y(s) \tag{1}$$

The dynamics of pitch and yaw channels are coupled therefore yaw propeller affects the pitch angle and the pitch propeller affects the yaw angle. The objective is to design a (single–input–single–output) discrete–time controller for the pitch angle. For the design of pitch channnel, we ignore the coupling and treat the yaw voltage as disturbance.

The block diagram of the controller is as follows:

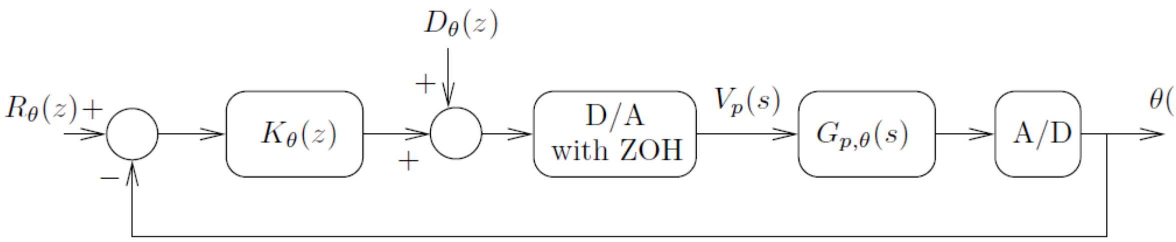


Figure 1: Pitch Channel

The transfer function is reduced to:

$$\theta(s) = \frac{37.2021}{s^2 + 0.2830s + 2.7452} V_p(s) \tag{2}$$

The requirements for the controller design are:
(DS1): Percentage of overshoot for step reference input $\leq 20\%$
(DS2): Settling time of step response $\leq$ 18s
(DS3): Rise time of step response $\leq$ 3s
(DS4): Steady–state error for step reference input = 0
(DS5): Steady–state output in response to step disturbance = 0

Based on the requirements of designing (Calculations have been attached), we find that:

$$\zeta \geq 0.45$$
$$\omega_n \zeta \geq 0.25$$
$$\omega_n \geq 0.6$$

Chosen Values of $\zeta = 0.7$, $\omega_n = 1$ and $T_s = 0.3$ because it has to 8 to 10 times of rise time.
Using Matlab the discrete equivalent of the plant in this case is:

$$G_0 = \frac{1.595z + 1.55}{z^2 - 1.687 + 0.9186} \tag{3}$$

Sample time: 0.3 seconds
Discrete-time transfer function.
The poles of the plant are $0.8433 + 0.4555i$ and $0.8433 - 0.4555i$
The zero of the plant is $-0.9719$

- Since we need the steady state error to unit step input to be zero therefore $K_p$ must be equal to $\infty$. **Therefore the Controller must have one pole at z=1.**
- Since the steady state output must also be zero for unit step disturbance, **therefore Controller must have one pole at z=1.**

Root locus design method is used for controller design. The rootlocus of the plant is as follows:

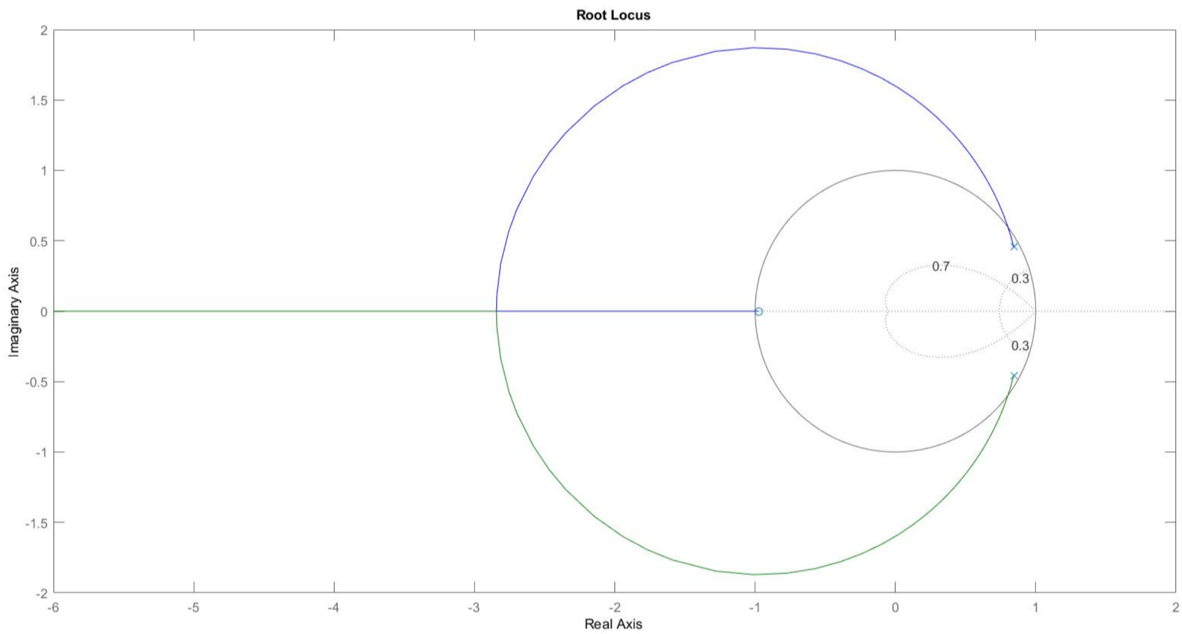


Figure 2: Root locus of $G_0$ for $\theta$

## 2.1 Controller design methodology

Previously a Lead compensator was designed to bend the root locus of the plant in the desired area. Then a lag was added to meet the steady state requirements. All the requirements were met with the design but the problem was the system was too fast with Rise time = 0.3 sec. Which would result in a costly actuator.

Therefore, the pole zero cancellation approach is used now. The poles of the plant are taken as the zeros of the controller. One pole of the controller is 1 and the other pole is found by the angle criterion of the root locus. The gain can be found analytically be the magnitude criterion of root locus and also graphically from the root locus of the open loop transfer function of the plant and controller.

The suggested controller is:

$$K_\theta = K\frac{z^2 - 1.687 + 0.9186}{(z-1)(z-a)} \tag{4}$$

After the calculations for 'a' and K. 'a' is found to be 0.6208, and K= 0.0227. Then the root locus of open loop $G_0K_\theta$ is plotted as follows:

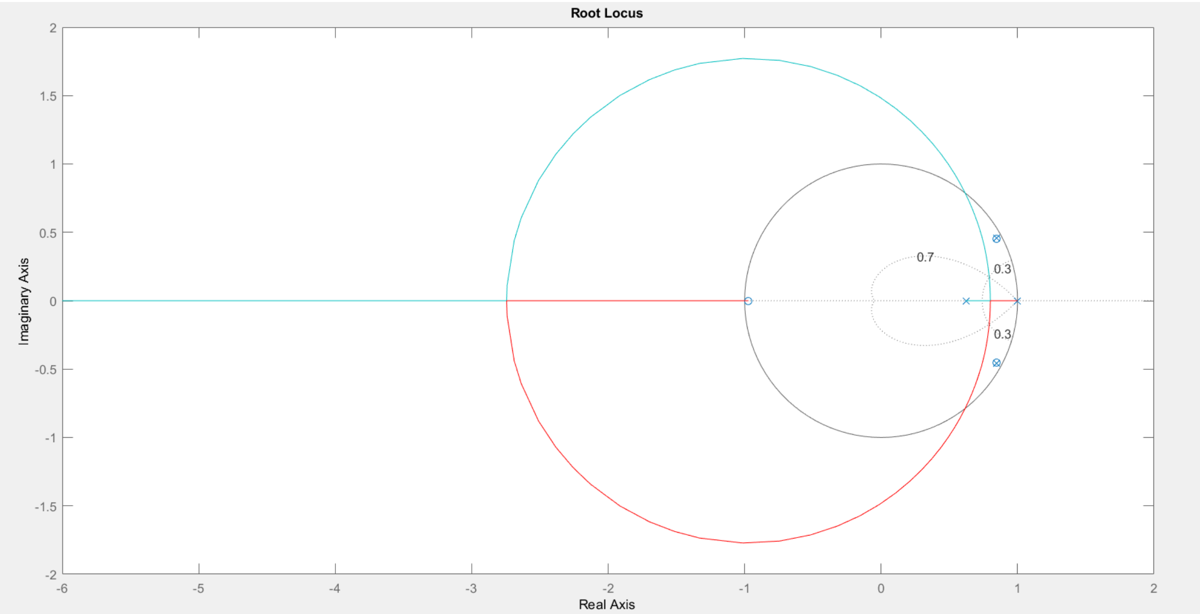


Figure 3: Root locus of $G_0K_\theta$

After iterating for few times and comparing the step responses, finally the controller is:

$$K_\theta = 0.022\frac{z^2 - 1.687 + 0.9186}{(z-1)(z-0.6208)} \tag{5}$$

## 2.2 Step response of the closed loop system $G_0K_\theta$

```
    RiseTime: 2.4000
SettlingTime: 6.3000
 SettlingMin: 0.9349
 SettlingMax: 1.0369
   Overshoot: 3.6895
  Undershoot: 0
        Peak: 1.0369
    PeakTime: 4.8000
```

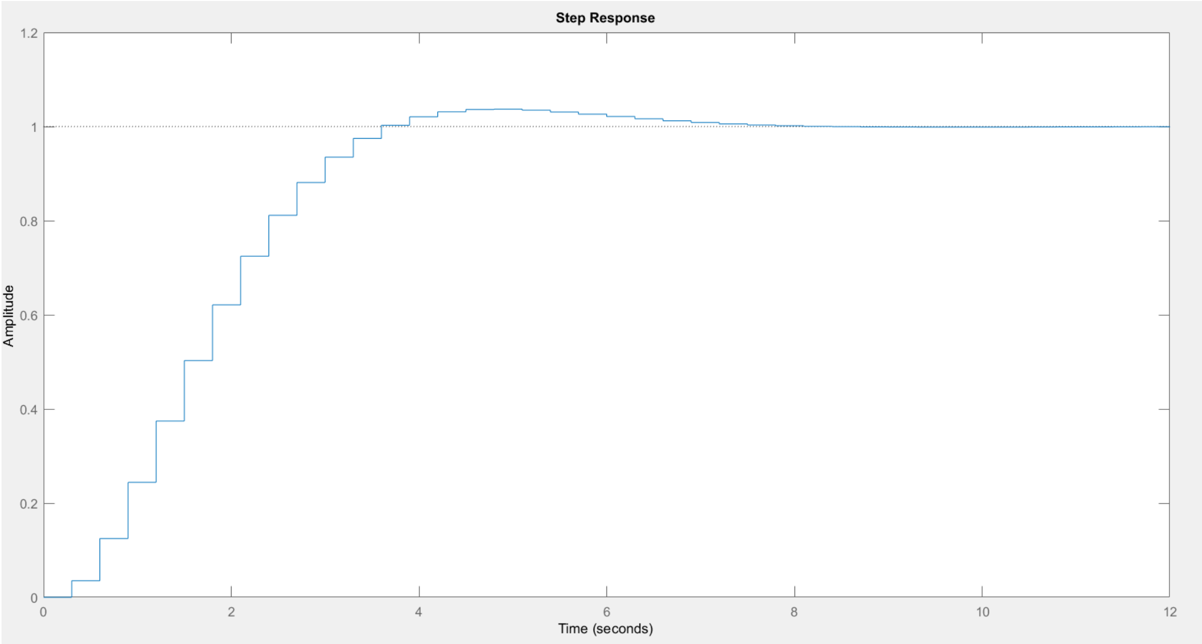


Figure 4: Step Response of Closed loop $G_0 K_\theta$

### 2.3 Disturbance to output response

```
     1.595 z^3 - 1.035 z^2 - 1.522 z + 0.9621
  ----------------------------------------------
  z^4 - 3.272 z^3 + 4.248 z^2 - 2.561 z + 0.6016

Sample time: 0.3 seconds
Discrete-time transfer function.
```

**The unit step disturbance response is:**

```
    RiseTime: 0
SettlingTime: 27
 SettlingMin: -10.7090
 SettlingMax: 19.1120
   Overshoot: 1.3790e+17
  Undershoot: 7.7271e+16
        Peak: 19.1120
    PeakTime: 1.8000
```

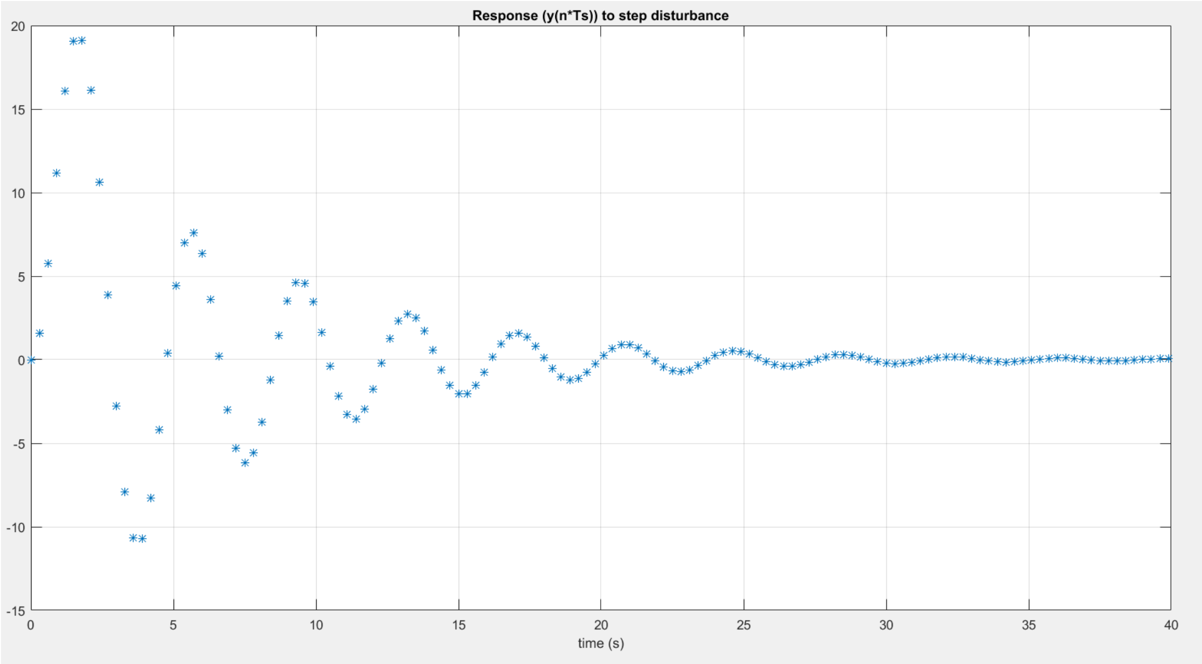


Figure 5: Step Response of Disturbance

# 3 Discrete Design of Yaw Controller

The Transfer function for the yaw angle $\psi$ is

$$\psi(s) = \frac{2.3892}{s(s+0.2701)}V_p(s) + \frac{7.461}{s(s+0.2701)}V_y(s) \tag{6}$$

The yaw angle $\psi$ is coupled with the pitch voltage $V_p$, but it is treated as disturbance. The block diagram of the yaw channel is:

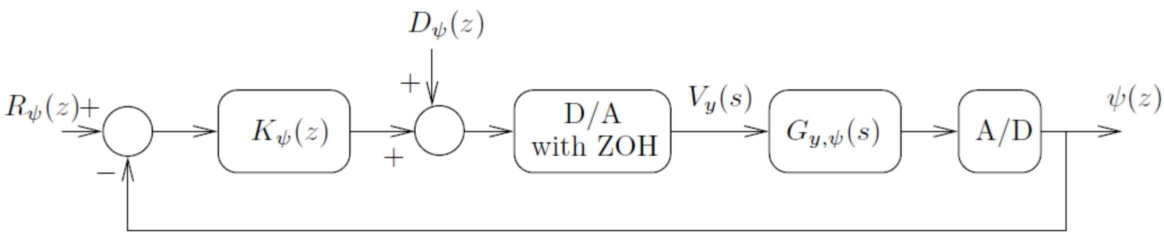


Figure 6: Yaw Channel

The transfer function is reduced to:

$$\psi(s) = \frac{7.461}{s(s+0.2701)}V_y(s) \tag{7}$$

Using Matlab the discrete equivalent of the plant in this case is:

$$G_0 = \frac{0.3269z + 0.3181}{z^2 - 1.922z + 0.9222} \tag{8}$$

Sample time: 0.3 seconds
Discrete-time transfer function.
The poles of the plant are 1 and 0.9222
The zero of the plant is $-0.9734$

- Since we need the steady state error to unit step input to be zero therefore $K_p$ must be equal to $\infty$. The plant has a pole at z=1 therefore Type 1 system already exists.

Root locus design method is used for controller design. The rootlocus of the plant is as follows:

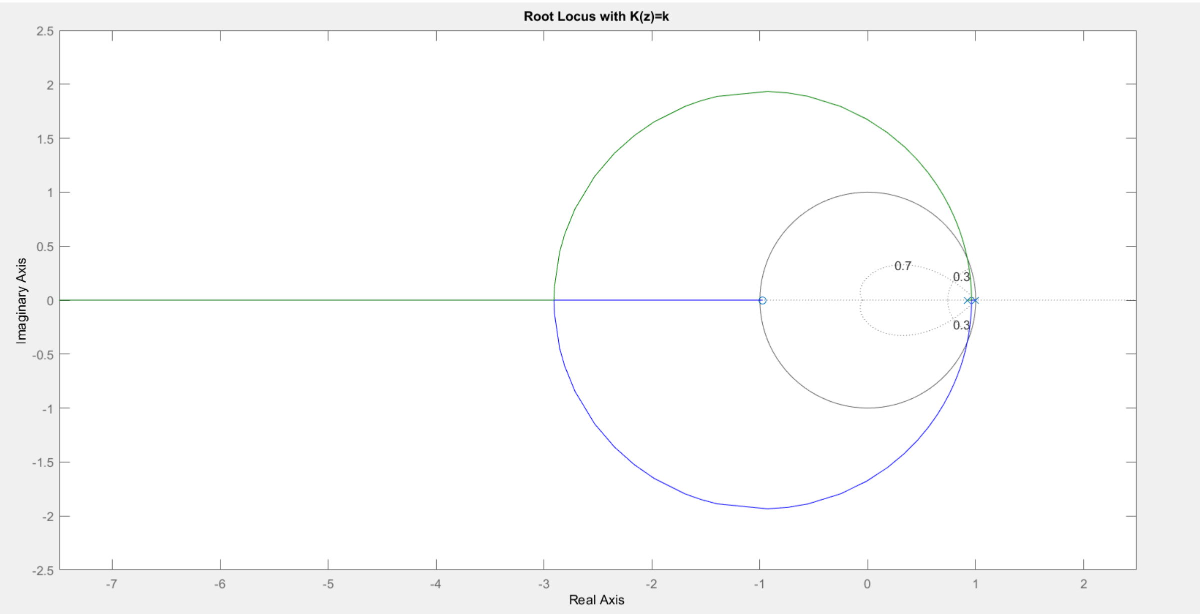


Figure 7: Root locus of $G_0$ for $\psi$

## 3.1 Controller design methodology

A Lead compensator is designed to bend the root locus of the plant in the desired area. The zero of the Lead is placed near the pole of the plant i.e 0.9. The pole of the Lead is placed such that the root locus bends towards the desired area.

With only a lead compensator the required transient characteristics are achieved. But, on finding the unit step disturbance response, its found that it does not go to zero.

Therefore, a lag is added to meet the disturbance requirements. The pole and zero of the lag compensator is kept very close to each other such that the transient response is affected to the minimum. All the requirements were met with the **lead-lag compensator design**. The angle and magnitude criterion is utilized as well to verify the location of pole and value of gain.

The suggested controller is:

$$K_\psi = 0.113 \frac{(z-0.89)}{(z-0.6)} \frac{(z-0.991)}{(z-0.999)} \tag{9}$$

Then the root locus of open loop $G_0 K_\psi$ is plotted as follows:

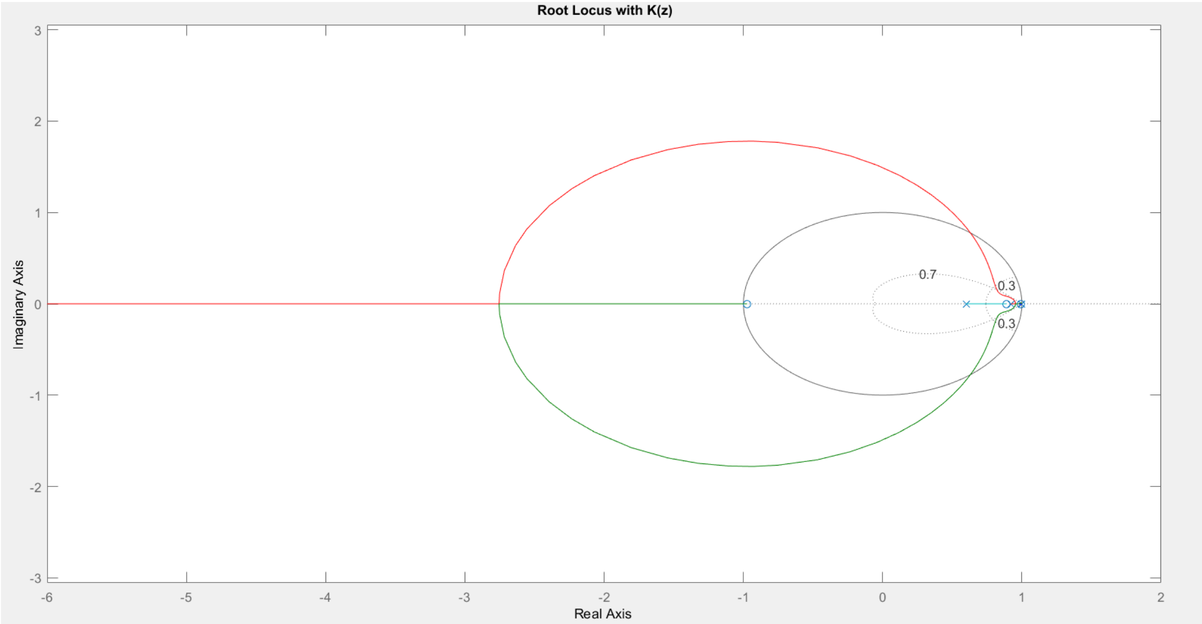


Figure 8: Root locus of $G_0 K_\psi$

### 3.2 Step response of the closed loop system $G_0 K_\psi$

```
     RiseTime: 1.8000
 SettlingTime: 15.9000
  SettlingMin: 0.9013
  SettlingMax: 1.1737
    Overshoot: 17.3724
   Undershoot: 0
         Peak: 1.1737
     PeakTime: 4.5000
```

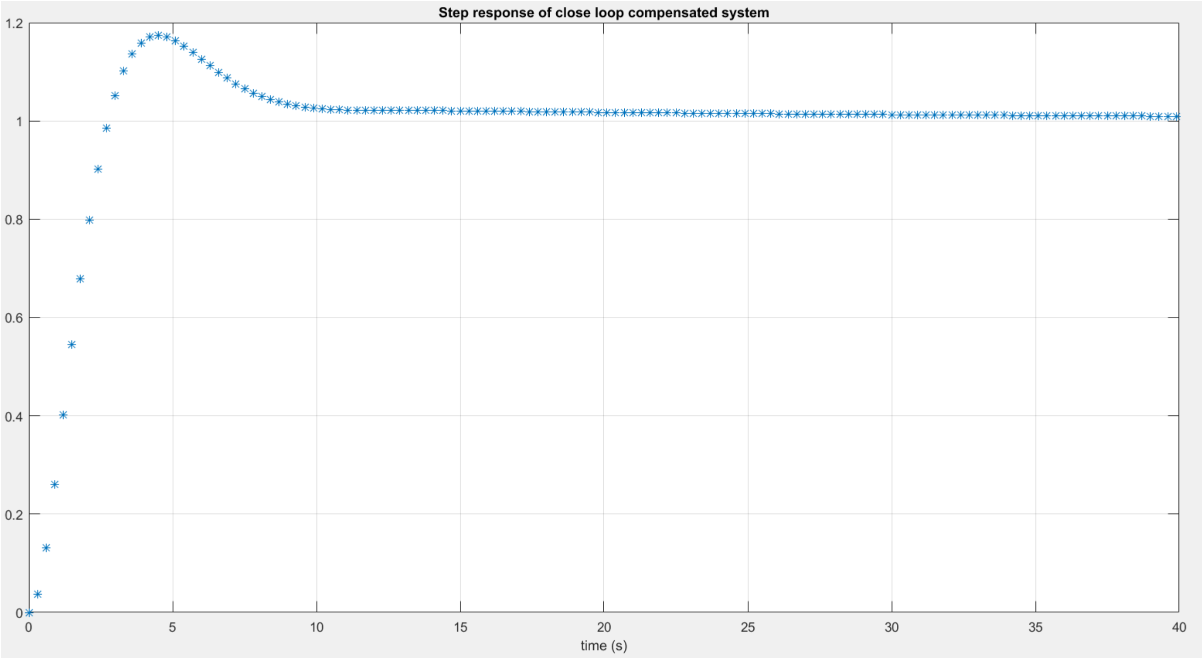


Figure 9: Step Response of Closed loop $G_0K_\psi$

### 3.3 Disturbance to output response

```
   0.3269 z^3 - 0.2045 z^2 - 0.3128 z + 0.1907
  ----------------------------------------------
  z^4 - 3.484 z^3 + 4.562 z^2 - 2.662 z + 0.5845

Sample time: 0.3 seconds
Discrete-time transfer function.
```

**The unit step disturbance response is:**

```
        RiseTime: 0.6000
    SettlingTime: 133.8000
     SettlingMin: 3.6088
     SettlingMax: 29.1678
       Overshoot: 715.7493
      Undershoot: 0
            Peak: 29.1678
        PeakTime: 6.9000
```

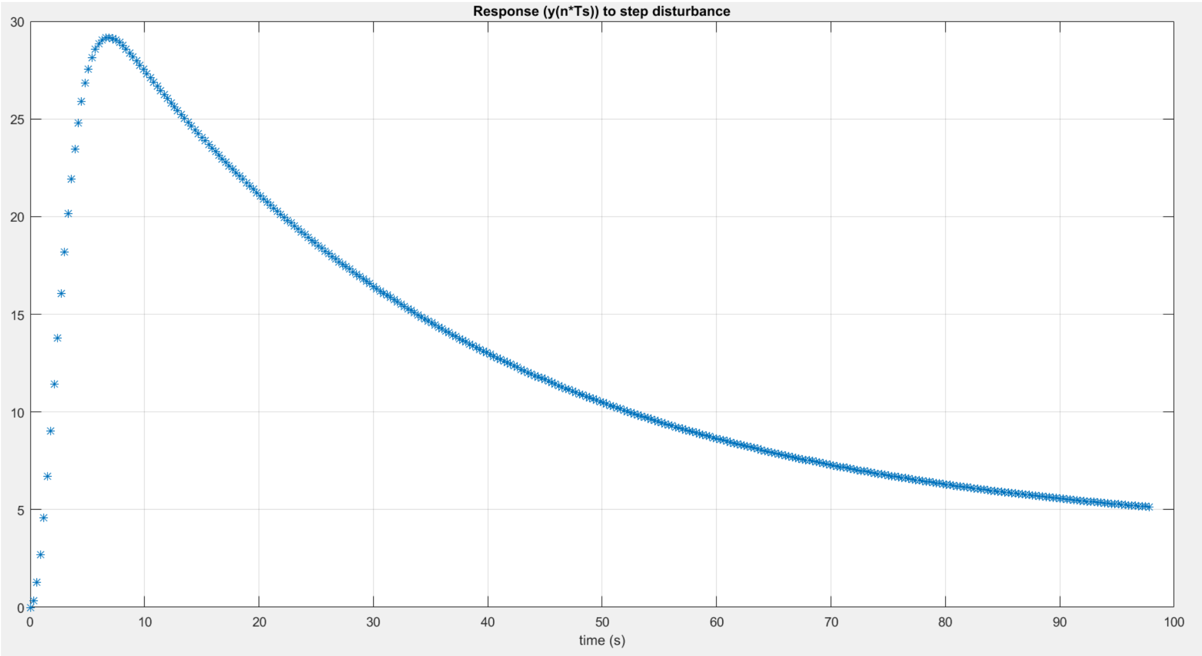


Figure 10: Step Response of Disturbance

# 4 Simulating the Complete System

The block diagram of the complete system is:

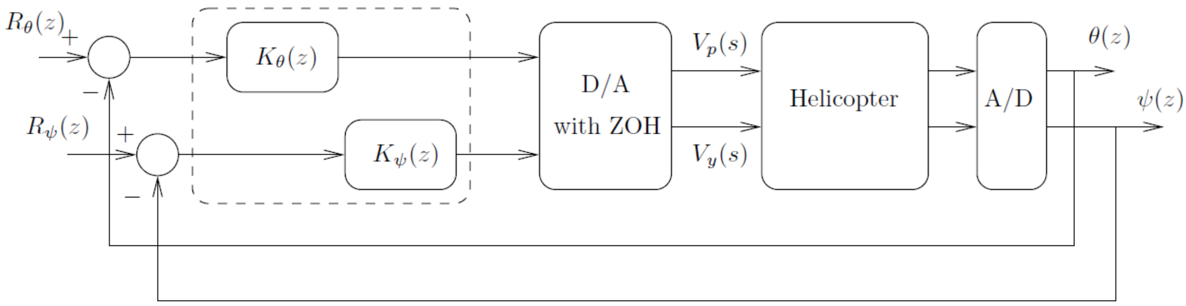


Figure 11: Complete Closed Loop System

## 4.1 Step Response of Complete Closed Loop System

After appending the individual Pitch and Yaw controllers the step response is simulated as follows

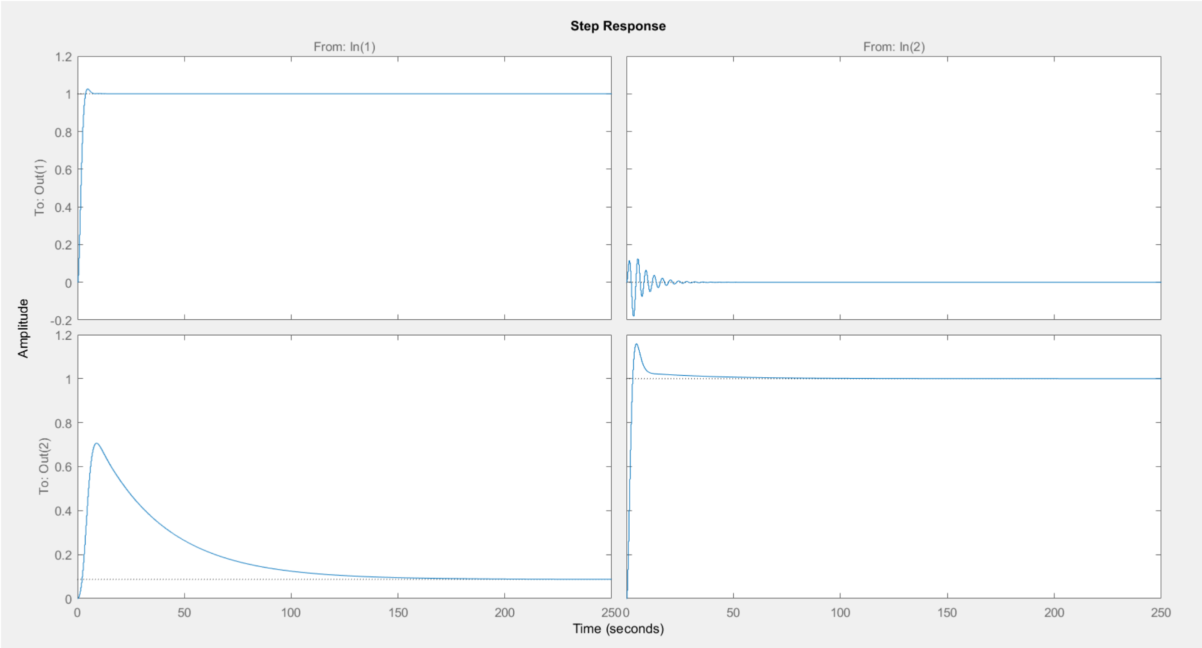


Figure 12: Step Response of Complete Closed Loop System

## 4.2 Step Response of pitch reference to $\theta$ and $\psi$

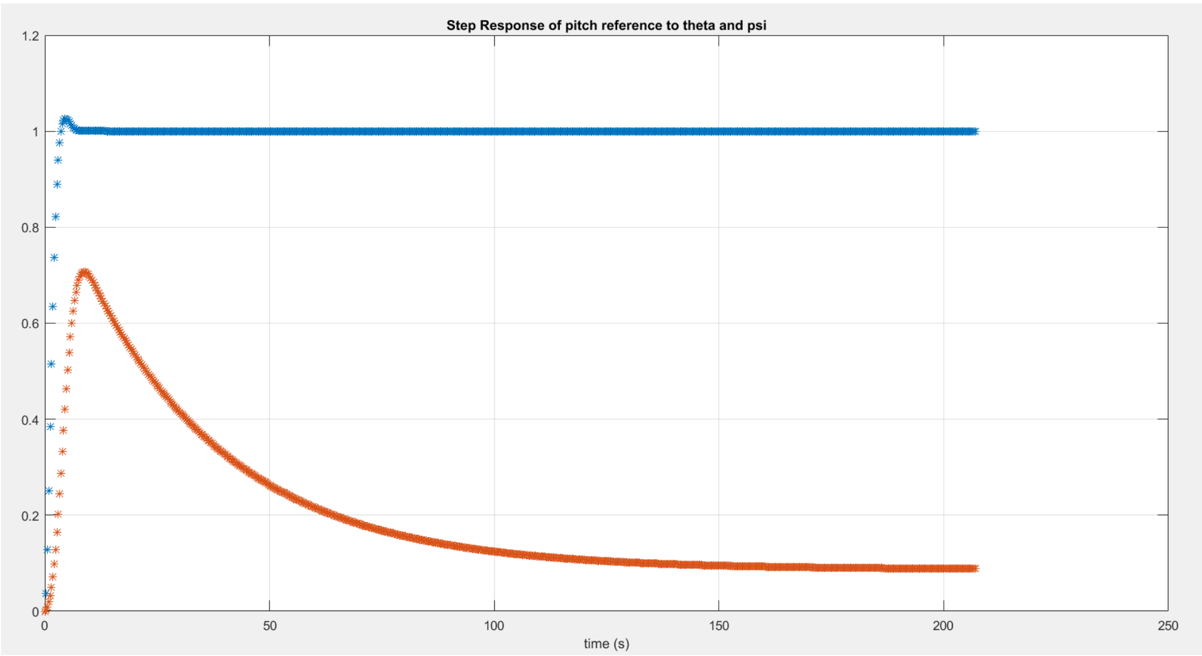


Figure 13: Step Response of pitch reference to $\theta$ and $\psi$

| Blue Color(\theta) | Orange Color(\psi) |
|---|---|
| RiseTime: 2.4000 | RiseTime: 1.5000 |
| SettlingTime: 5.4000 | SettlingTime: 135.3000 |
| SettlingMin: 0.9394 | SettlingMin: 0.0876 |
| SettlingMax: 1.0257 | SettlingMax: 0.7065 |
| Overshoot: 2.5731 | Overshoot: 710.8492 |
| Undershoot: 0 | Undershoot: 0 |
| Peak: 1.0257 | Peak: 0.7065 |
| PeakTime: 4.5000 | PeakTime: 8.7000 |

Figure 14: Step Response info of pitch reference to $\theta$ and $\psi$

Note that the Orange color is for $\psi$ and Blue color is for $\theta$

## 4.3 Step Response of yaw reference to $\theta$ and $\psi$

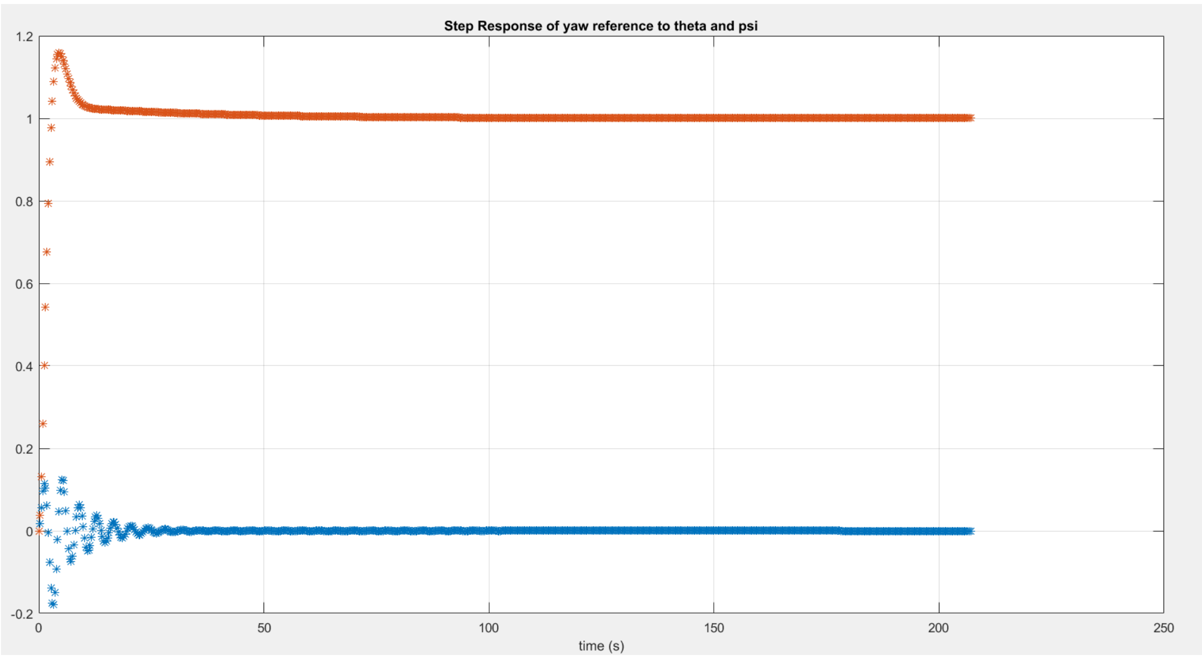


Figure 15: Step Response of yaw reference to $\theta$ and $\psi$

| Blue Color(\theta) | Orange Color(\psi) |
|---|---|
| RiseTime: 0 | RiseTime: 2.1000 |
| SettlingTime: 28.5000 | SettlingTime: 16.8000 |
| SettlingMin: -0.1792 | SettlingMin: 0.9762 |
| SettlingMax: 0.1240 | SettlingMax: 1.1585 |
| Overshoot: 1.2751e+18 | Overshoot: 15.8453 |
| Undershoot: 8.8230e+17 | Undershoot: 0 |
| Peak: 0.1792 | Peak: 1.1585 |
| PeakTime: 3.3000 | PeakTime: 4.5000 |

Figure 16: Step Response info of yaw reference to $\theta$ and $\psi$

Note that the Orange color is for $\psi$ and Blue color is for $\theta$

## 4.4 Comparison of Step responses With and Without Coupling

Consider the tables below:

| Step Response Info | Output(Θ/ψ) | Without Coupling | With Coupling (Complete Closed Loop System) |
|---|---|---|---|
| Pitch Controller | Θ | RiseTime: 2.4000 | RiseTime: 2.4000 |
| | | SettlingTime: 6.3000 | SettlingTime: 5.4000 |
| | | SettlingMin: 0.9510 | SettlingMin: 0.9394 |
| | | SettlingMax: 1.0420 | SettlingMax: 1.0257 |
| | | Overshoot: 4.1961 | Overshoot: 2.5731 |
| | | Undershoot: 0 | Undershoot: 0 |
| | | Peak: 1.0420 | Peak: 1.0257 |
| | | PeakTime: 4.500 | PeakTime: 4.5000 |
| | | | |
| | ψ | RiseTime: 0.6000 | RiseTime: 1.5000 |
| | | SettlingTime: 133.8000 | SettlingTime: 135.3000 |
| | | SettlingMin: 3.6088 | SettlingMin: 0.0876 |
| | | SettlingMax: 29.1678 | SettlingMax: 0.7065 |
| | | Overshoot: 715.7493 | Overshoot: 710.8492 |
| | | Undershoot: 0 | Undershoot: 0 |
| | | Peak: 29.1678 | Peak: 0.7065 |
| | | PeakTime: 6.9000 | PeakTime: 8.7000 |

Figure 17: Comparison of Step responses of Pitch Controller With and Without Coupling

**Effects of Cross Coupling on the Pitch Controller are Acceptable:**

- Since there is no drastic change in achieving $\theta$. The step response remains almost the same, actually a little better. The rise time remains same, overshoot and settling time reduce a little after coupling.
- The effect of pitch controller on $\psi$ has been improved after coupling because the peak value has been reduced a lot from 29.16 to only 0.7.

| Step Response Info | Output(Θ/ψ) | Without Coupling | With Coupling (Complete Closed Loop System) |
|---|---|---|---|
| Yaw Controller | ψ | RiseTime: 1.8000 | RiseTime: 2.1000 |
| | | SettlingTime: 15.9000 | SettlingTime: 16.8000 |
| | | SettlingMin: 0.9013 | SettlingMin: 0.9762 |
| | | SettlingMax: 1.1737 | SettlingMax: 1.1585 |
| | | Overshoot: 17.3724 | Overshoot: 15.8453 |
| | | Undershoot: 0 | Undershoot: 0 |
| | | Peak: 1.1737 | Peak: 1.1585 |
| | | PeakTime: 4.5000 | PeakTime: 4.5000 |
| | | | |
| | Θ | RiseTime: 0 | RiseTime: 0 |
| | | SettlingTime: 27 | SettlingTime: 28.5000 |
| | | SettlingMin: -11.0050 | SettlingMin: -0.1792 |
| | | SettlingMax: 18.9749 | SettlingMax: 0.1240 |
| | | Overshoot: 1.4127e+17 | Overshoot: 1.2751e+18 |
| | | Undershoot: 8.1933e+16 | Undershoot: 8.8230e+17 |
| | | Peak: 18.9749 | Peak: 0.1792 |
| | | PeakTime: 1.5000 | PeakTime: 3.3000 |

Figure 18: Comparison of Step responses of Yaw Controller With and Without Coupling

**Effects of Cross Coupling on the Yaw Controller are Acceptable:**

- Since there is no drastic change in achieving $\psi$. The step response remains almost the same, actually a little better. The rise time gets closer to the required value, overshoot and settling time change a little after coupling.
- The effect of yaw controller on $\theta$ has been improved after coupling because the peak value has been reduced a lot from 18.97 to only 0.17.

# 5 Appendix- Matlab Code