Homework 2 - CSE 291 - Math for Robotics

Due: Thursday, 8 February 2018

1. Prove that the first derivative $p_2'(x)$ of the parabola interpolating $f(x)$ at $x_0 < x_1 < x_2$ is equal to the straight line which takes on the value $f[x_{i-1}, x_i]$ at the point $(x_{i-1} + x_i)/2$, for $i = 1, 2$.

2. (a) Implement Müller's method.

   (b) Use Müller's method to find all the roots of the polynomial $p(x) = x^3 - 4x^2 + 6x - 4$.

3. Suppose you wish to build an interpolation table with entries of the form $(x, f(x))$ for the function $f(x) = \sin x$ over the interval $[0, \pi]$. Please use uniform spacing between points.

   - How fine must the table spacing be in order to ensure 6 decimal digit accuracy, assuming that you will use linear interpolation between adjacent points in the table?
   - How fine must it be if you will use quadratic interpolation?
   - In each case, how many entries do you need in the table?

4. Implement Newton's Method. Consider the following equation:

   $$x = \tan x.$$

   There are an infinite number of solutions x to this equation. Use Newton's method (and any techniques you need to start Newton in regions of convergence) to find the two solutions that are closest to 5.