Homework 1 - CSE 291 - Math for Robotics

Due: Thursday, 25 January 2018

1. Implement the $PA = LDU$ decomposition algorithm by yourself (i.e. do not just call a built-in function in Matlab or Python. You may assume the matrix A is square and of full rank. Show that your implementation is functional.

2. Compute the $PA = LDU$ decomposition and the SVD decomposition for each of the following matrices:

   (you can use your own LDU implementation and it is OK to use a pre-defined implementation for SVD).

   a.
   $$A_1 = \begin{bmatrix} 4 & 7 & 0 \\ 2 & 2 & -6 \\ 3 & 2 & 1 \end{bmatrix}$$

   b.
   $$A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

   c.
   $$A_3 = \begin{bmatrix} 2 & 2 & 5 \\ 3 & 2 & 5 \\ 1 & 1 & 5 \end{bmatrix}$$

3. Solve the following system of equations $Ax = b$ given the below values for A and b. For each system specify if it has zero, one or more solutions. For the systems with zero solutions give the SVD solution. Relate your answers to the SVD decomposition.

   a.
   $$A = \begin{bmatrix} 2 & 1 & 3 \\ 2 & 1 & 2 \\ 5 & 5 & 5 \end{bmatrix} \quad b = \begin{bmatrix} 5 \\ -5 \\ 0 \end{bmatrix}$$

   b.

$$A = \begin{bmatrix} 4 & 7 & 0 \\ 2 & 2 & -6 \\ 1 & 2 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix}$$

c.

$$A = \begin{bmatrix} 4 & 7 & 0 \\ 2 & 2 & -6 \\ 1 & 2 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 18 \\ -12 \\ 8 \end{bmatrix}$$

4. A frequent problem in perception is that you see an object from multiple positions. Consider that you have recorded two datasets $\{a_i\}$ and $\{b_i\}$ with $i = 1 \ldots n$ and we assume we have established correspondance between the data points. How could you use SVD or similar techniques to compute the relatively transformation between the datasets **a** and **b** assuming that the datasets have many data points and you are expected to use all the data-points.