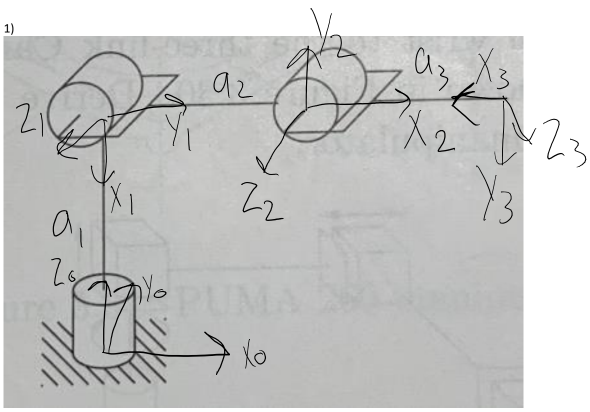
Assignment 3

Tuesday, October 1, 2019 9:48 PM

1)



$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$O_1 = \begin{bmatrix} 0 \\ 0 \\ a_1 \end{bmatrix}$$

$$Z_1 = Z_2 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

$$O_2 = \begin{bmatrix} a_2 \\ 0 \\ a_1 \end{bmatrix}$$

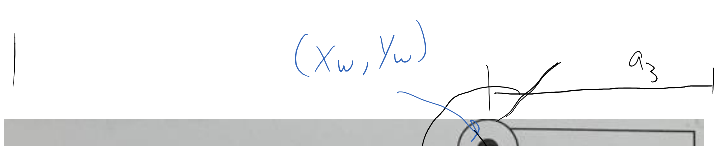
$$Z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$O_3 = \begin{bmatrix} a_2 + a_3 \\ 0 \\ a_1 \end{bmatrix}$$

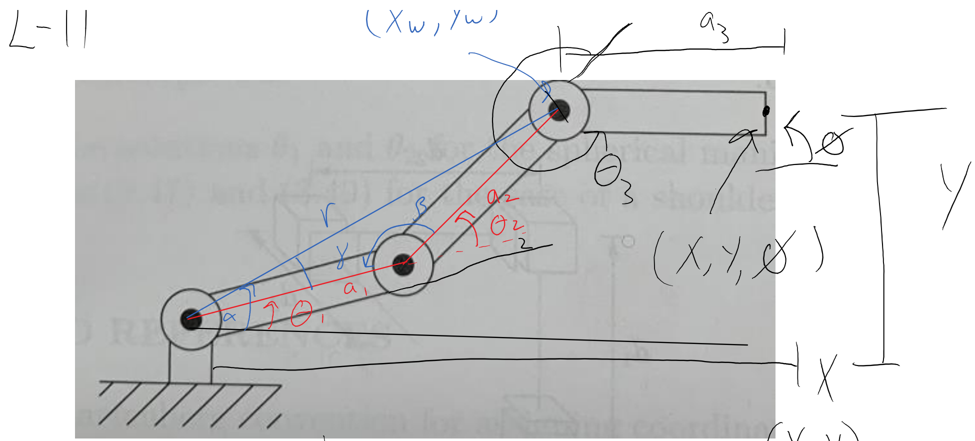
$$J(q) = \begin{bmatrix} Z_0 \times (O_3 - O_0) & Z_1 \times (O_3 - O_1) & Z_2 \times (O_3 - O_2) \\ Z_0 & Z_1 & Z_2 \end{bmatrix}$$

$$J(q) = \begin{bmatrix} 0 & 0 & 0 \\ (a_2 + a_3) & a_2 + a_3 & a_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

2-11



L-11



End effector has its position given (X, Y) & orientation given by ϕ

Geometrically, we can see

$$X = a_1 \cos\theta_1 + a_2 \cos(\theta_1 + \theta_2) + a_3 \cos(\theta_1 + \theta_2 + \theta_3)$$

$$Y = a_1 \sin\theta_1 + a_2 \sin(\theta_1 + \theta_2) + a_3 \sin(\theta_1 + \theta_2 + \theta_3)$$

$$\phi = \theta_1 + \theta_2 + \theta_3$$

Before doing any calculations,

It is clear there are infinite solutions when only looking for positions. This is b/c there are 3 variables $(\theta_1, \theta_2, \theta_3)$ & 2 equations.

For the exact inverse kinematics we do

$$X_w = a_1 \cos\theta_1 + a_2 \cos\theta_2$$

$$Y_w = a_1 \sin\theta_1 + a_2 \sin\theta_2$$

$$\alpha = \operatorname{atan}\left(\frac{X_w}{Y_w}\right)$$

$$r^2 = a_1^2 + a_2^2 - 2 a_1 a_2 \cos\beta$$

$$\theta_2 = 180 - \operatorname{acos}\left(\frac{a_1^2 + a_2^2 - X_w^2 + Y_w^2}{2 a_1 a_2}\right)$$

$$r^2 + a_1^2 - 2 r a_1 \cos\gamma = a_2^2$$

$$\gamma = \operatorname{acos}\left(\frac{r^2 + a_1^2 - a_2^2}{2 r a_1}\right)$$

$$\theta_1 = \alpha - \gamma = \operatorname{atan}\left(\frac{X_w}{Y_w}\right) - \operatorname{acos}\left(\frac{X_w^2 + Y_w^2 + a_1^2 - a_2^2}{2 a_1 \sqrt{X_w^2 + Y_w^2}}\right)$$

$$\theta_3 = \phi - \theta_1 - \theta_2$$

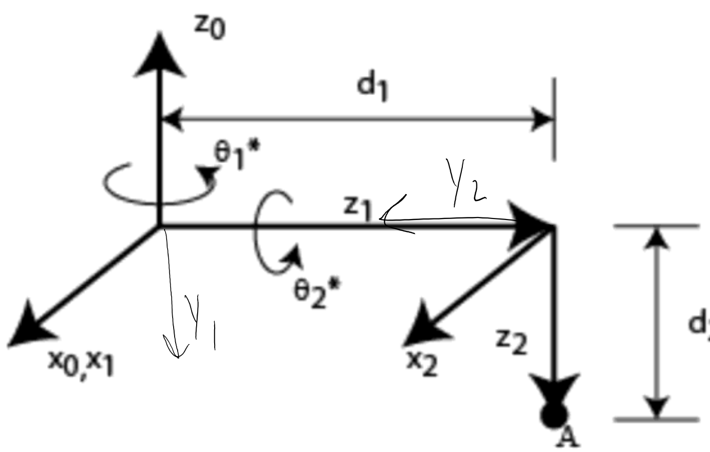
So, for position & orientation there will either be 1 or 0 solutions.

3)

$$V_1^1 = \begin{bmatrix} 3 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$$V_0^1 = H_1^0 \, V_1^1 = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

4)



a) With respect to frame 0

And assuming that in the current arm orientation $\theta_1^* = 90^\circ$, $\theta_2^* = 180^\circ$

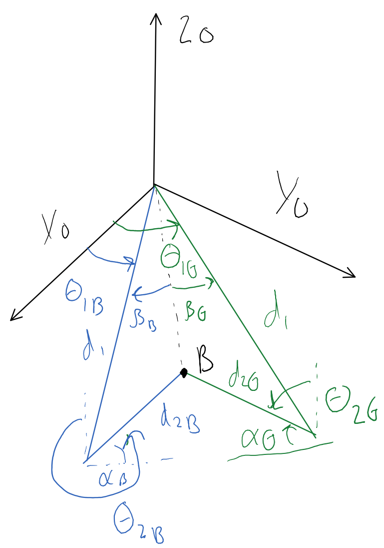
$$Z = d_2^* \cos(\theta_2^*)$$

$$X = d_1 \cos(\theta_1^*) + d_2^* \sin(\theta_2^*) \sin(\theta_1^*)$$

$$Y = d_1 \sin(\theta_1^*) - d_2^* \sin(\theta_2^*) \cos(\theta_1^*)$$

so, any value for x, y, z within the above constraints are reachable by the arm.

b) Because these equations have 3 unknowns for 3 variables, it would be easy to say each position has one unique solution. However, because of the nature of sinusoids, many positions have two solutions. Namely, in a situation like the one outlined below.



$d_{2B} = d_{2G}$
$\beta_B = \beta_G$
$\alpha_B = \alpha_G$

Essentially, any position that satisfies $d_2^* \neq 0$ & $\theta_2 \neq 0, 180$ will have a θ_1 s.t. $\theta_2' = -\theta_2$ gives the same position

By '$-$' I don't mean negative, but that it will have an α as depicted so that the arm is pointed 'backward' to its first solution.

c) $O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $O_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $O_2 = \begin{bmatrix} 0 \\ d_1 \\ 0 \end{bmatrix}$ $O_3 = \begin{bmatrix} 0 \\ d_1 \\ -d_2 \end{bmatrix}$

$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $z_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $z_2 = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$

$O_3 - O_0 = \begin{bmatrix} 0 \\ -d_1 \\ d_2 \end{bmatrix} = O_3 - O_1$

$O_3 - O_2 = \begin{bmatrix} 0 \\ 0 \\ d_2 \end{bmatrix}$

$J = \begin{bmatrix} d_1 & d_2 & 0 \\ \ldots \end{bmatrix}$

$$J(a) = \begin{bmatrix} d_1 & d_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ d_2 \end{bmatrix}$$