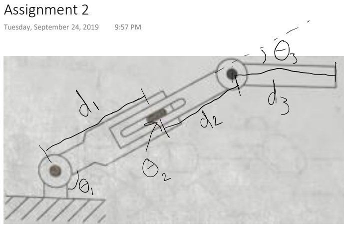
Assignment 2

Tuesday, September 24, 2019 9:57 PM



	θi	di	αi	ai
1	θ1*	0	0	a1
2	0	0	0	a2*
3	θ3*	0	0	a3

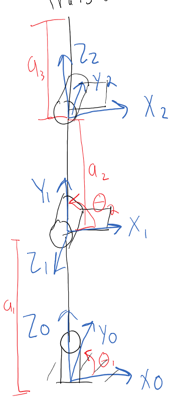
$$A_1(\theta_1) = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & a_1\cos(\theta_1) \\ \sin(\theta_1) & \cos(\theta_1) & 0 & a_1\sin(\theta_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = T_1^0$$

$$A_2(a_2) = \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow T_2^0 = A_1 A_2 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & \cos\theta_1(a_1+a_2) \\ \sin\theta_1 & \cos\theta_1 & 0 & \sin\theta_1(a_1+a_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3(a_3) = \begin{bmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & a_3\cos(\theta_3) \\ \sin(\theta_3) & \cos(\theta_3) & 0 & a_3\sin(\theta_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow T_3^0 = T_2^0 A_3 = \begin{bmatrix} \cos(\theta_1+\theta_3) & -\sin(\theta_1+\theta_3) & 0 & a_3\cos(\theta_1+\theta_3) + \cos\theta_1(a_1+a_2) \\ \sin(\theta_1+\theta_3) & \cos(\theta_1+\theta_3) & 0 & a_3\sin(\theta_1+\theta_3) + \sin\theta_1(a_1+a_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3-6)

Transform to be



	θ	d	α	a
1	θ_1^*	0	90	a_1
2	θ_2^*	0	0	a_2
3	θ_3^*	0	0	a_3

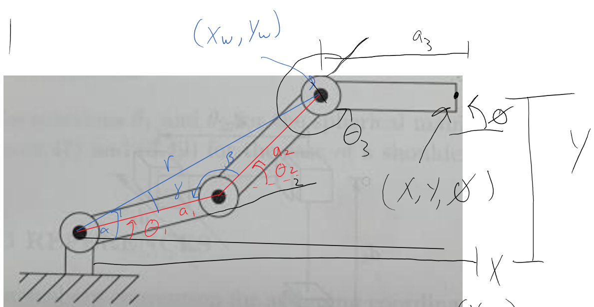
$$A_1(\theta_1) = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & a_1\cos\theta_1 \\ \sin\theta_1 & 0 & -\cos\theta_1 & a_1\sin\theta_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2(\theta_2) = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & a_2\cos\theta_2 \\ \sin(\theta_2) & \cos(\theta_2) & 0 & a_2\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3(\theta_3) = \begin{bmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & a_3\cos\theta_3 \\ \sin(\theta_3) & \cos(\theta_2) & 0 & a_3\sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$
T_3^0 = \begin{bmatrix}
\cos(\theta_1)(\cos(\theta_2+\theta_3)) & -\cos(\theta_1)(\sin(\theta_2+\theta_3)) & \sin(\theta_1) & a_1\cos(\theta_1) + a_2\cos(\theta_1)\cos(\theta_2) + a_3\cos(\theta_1)\cos(\theta_2+\theta_3) \\
\sin(\theta_1)(\cos(\theta_2+\theta_3)) & -\sin(\theta_1)(\sin(\theta_2+\theta_3)) & -\cos(\theta_1) & a_1\sin(\theta_1) + a_2\sin(\theta_1)\sin\theta_2 + a_3\sin(\theta_1)\sin(\theta_2+\theta_3) \\
\sin(\theta_2+\theta_3) & \cos(\theta_2+\theta_3) & 0 & a_2\sin(\theta_2) + a_3\sin(\theta_2+\theta_3) \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

2-11



End effector has its position given (X, Y)
& orientation given by $\emptyset$

Geometrically, we can see

$$X = a_1\cos\theta_1 + a_2\cos(\theta_1+\theta_2) + a_3\cos(\theta_1+\theta_2+\theta_3)$$

$$Y = a_1\sin\theta_1 + a_2\sin(\theta_1+\theta_2) + a_3\sin(\theta_1+\theta_2+\theta_3)$$

$$\emptyset = \theta_1 + \theta_2 + \theta_3$$

Before doing any calculations,

It is clear there are infinite solutions when only looking for positions. This is b/c there are 3 variables $(\theta_1, \theta_2, \theta_3)$ & 2 equations.

For the exact inverse kinematics we do

$$X_w = a_1\cos\theta_1 + a_2\cos\theta_2$$

$$Y_w = a_1\sin\theta_1 + a_2\sin\theta_2$$

$$\alpha = a\tan\left(\frac{X_w}{Y_w}\right)$$

$$r^2 = a_1^2 + a_2^2 - 2a_1a_2\cos\beta$$

$$\theta_2 = 180 - a\cos\left(\frac{a_1^2 + a_2^2 - X_w^2 + Y_w^2}{2a_1a_2}\right)$$

$$r^2 + a_1^2 - 2ra_1\cos\gamma = a_2^2$$

$$\gamma = a\cos\left(\frac{r^2 + a_1^2 - a_2^2}{2ra_1}\right)$$

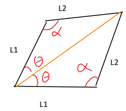
$X_w^2 + Y_w^2 + a_1^2 - a_2^2$

$$\theta_1 = \alpha - \gamma = \operatorname{atan}\left(\frac{x_w}{y_w}\right) - a\cos\left(\frac{x_w^2 + y_w^2 + a_1^2 - a_2^2}{2 a_1 \sqrt{x_w^2 + y_w^2}}\right)$$

$$\theta_3 = \phi - \theta_1 - \theta_2$$

So, any point can be reached either 2, 1, or 0 ways. This is done as follows:

- Any point that is not reachable by the first two arms completely extended, or the second arm oriented 180 degrees of the first is reachable in two orientations, show to the right. So, the third joint will have a specified orientation, and it is simply attached to one of these two orientations
- If it is on the extremes of the reachability of the first two arms mentioned earlier, then there is only one solution because the third orientation is predefined
- 0 solutions if the spot is too close to the origin, or outside of the reachability of all three arms

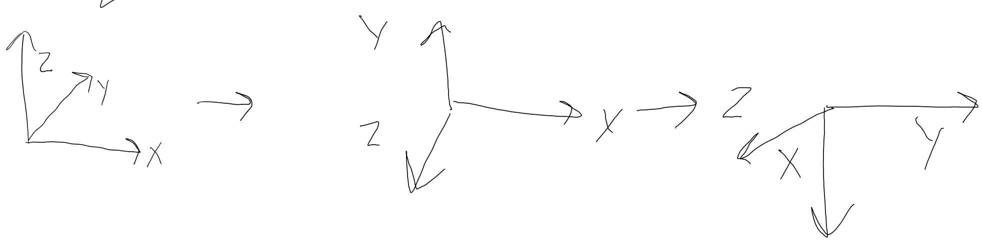


2-14)

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$R_y = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$R = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$$



2-15)

$$R^1_2 R^2_3 = R^1_3$$

$${R^1_2}^{-1} R^1_2 R^2_3 = {R^1_2}^{-1} R^1_3$$

$$R^2_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \end{bmatrix}$$