Atomic and nuclear physics

X-ray physics
Compton effect on X-rays

Compton effect: verifying the energy loss of the scattered x-ray quantum

Objects of the experiment

■ Measuring the transmission values T_1 and T_2 of a Cu foil for unscattered x-rays and x-rays scattered at an aluminum body.

■ Determining the wavelength shift for scattered x-rays from the change in the transmission.

■ Comparing the measured wavelength shift with that calculated on the basis of the Compton scattering.

Principles

Compton effect:

In 1923, the American physicist *A. H. Compton* observed a wavelength shift in x-rays scattered at a scattering body, which he explained on the basis of the quantum nature of x-rays. He interpreted this effect as a collision between an x-ray quantum and an electron of the scattering material, in which the energy

$$E = \frac{h \cdot c}{\lambda} \tag{I}$$

h: Planck's constant
c: velocity of light
λ: wavelength

of the x-ray quantum changes, as the kinetic energy is transferred to the electron.

In the collision, energy and momentum are conserved. Before the collision, the electron can be considered as being at rest (see Fig. 1). When *v* is the velocity of the electron after the

collision and λ_1 and λ_2 are the wavelengths of the x-ray quantum before and after collision, a relativistic formulation of the conservation of energy gives us

$$\frac{h \cdot c}{\lambda_1} + m_0 \cdot c^2 = \frac{h \cdot c}{\lambda_2} + \frac{m_0 \cdot c^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \tag{II}$$

m_0: mass of electron

The momentum of an x-ray quantum is

$$p = \frac{h}{\lambda} \tag{III}$$

The conservation of momentum is thus

$$\frac{h}{\lambda_1} = \frac{h}{\lambda_2} \cdot \cos\vartheta + \frac{m_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \cdot v \cdot \cos\varphi$$

and

$$0 = \frac{h}{\lambda_2} \cdot \sin\vartheta + \frac{m_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \cdot v \cdot \sin\varphi \tag{IV}.$$

ϑ, φ: collision angles (see Fig. 1).

We can apply some transformations to (II) and (IV) to arrive at the following relation for the change in the wavelength:
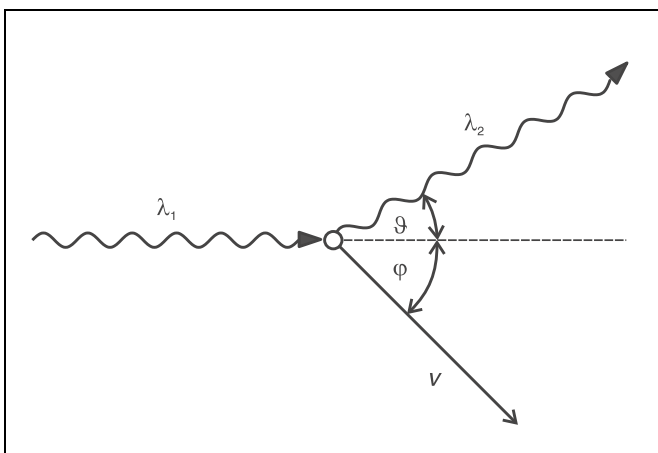
$$\lambda_2 - \lambda_1 = \frac{h}{m_0 \cdot c}(1 - \cos\vartheta) \tag{V}$$

The quantity

$$\frac{h}{m_0 \cdot c} = 2.43 \text{ pm} \tag{VI}$$

is termed the Compton wavelength λ_C. It is identical to the wavelength of a photon having an energy equal to the resting energy of the electron.

Fig. 1 Compton scattering of an x-ray quantum at an electron at rest.

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Apparatus

1 X-ray apparatus 554 811

1 End-window counter
for α, β, γ and x-ray radiation 559 01

1 Compton accessory x-ray 554 836

Verifying the wavelength shift:

Verification of the wavelength shift is based on an arrangement developed by R. W. Pohl, in which the attenuation of the unscattered x-radiation in a copper foil is compared with that of radiation scattered using an aluminum scattering body. The transmission T_{Cu} of the copper foil depends in the wavelength of the x-rays (see Fig. 2). Therefore, a shift in the wavelength of the x-radiation due to Compton scattering becomes apparent as a change in the transmission or the counting rate.

The evaluation is facilitated by the fact that the wavelength-dependency of the transmission of the copper foil can be formulated as

$$T_{Cu} = e^{-a \cdot \left(\frac{\lambda}{100pm}\right)^n} \tag{VII}$$

with $a = 7.6$ and $n = 2.75$.

This experiment initially measures the counting rate R_0 for x-rays scattered at an aluminum body without attenuation, and then the counting rates R_1 and R_2 with the copper foil placed in front of and behind the aluminum body, respectively (see Fig. 3). As the counting rates are low, the counting rate R of background radiation is also taken into consideration. Using equation (VII), the transmission values

$$T_1 = \frac{R_1 - R}{R_0 - R} \tag{VIII}$$

and

$$T_2 = \frac{R_2 - R}{R_0 - R} \tag{IX}$$

allow us to calculate the "mean" wavelengths λ_1 of the unscattered x-rays and λ_2 of the unscattered x-rays. In accordance with (VII) to (IX), the wavelength shift

$$\Delta\lambda = \lambda_2 - \lambda_1 \tag{X}$$

under study here is

$$\Delta\lambda = 100pm \cdot \left(\left(\frac{\ln(R_0 - R) - \ln(R_2 - R)}{a} \right)^{\frac{1}{n}} - \left(\frac{\ln(R_0 - R) - \ln(R_1 - R)}{a} \right)^{\frac{1}{n}} \right) \tag{XI}.$$

Safety notes

The x-ray apparatus fulfills all regulations governing an x-ray apparatus and fully protected device for instructional use and is type approved for school use in Germany (NW 807/97 Rö).

The built-in protection and screening measures reduce the local dose rate outside of the x-ray apparatus to less than 1 μSv/h, a value which is on the order of magnitude of the natural background radiation.

■ Before putting the x-ray apparatus into operation inspect it for damage and to make sure that the high voltage is shut off when the sliding doors are opened (see Instruction Sheet for x-ray apparatus).
■ Keep the x-ray apparatus secure from access by unauthorized persons.

Do not allow the anode of the x-ray tube Mo to overheat.

■ When switching on the x-ray apparatus, check to make sure that the ventilator in the tube chamber is turning.

The goniometer is positioned solely by electric stepper motors.

■ Do not block the target arm and sensor arm of the goniometer and do not use force to move them.

Fig. 2   Transmission of copper foil (d = 0.07 mm) in the wavelength range from 50 to 80 pm, measured at the LD DIDACTIC laboratories
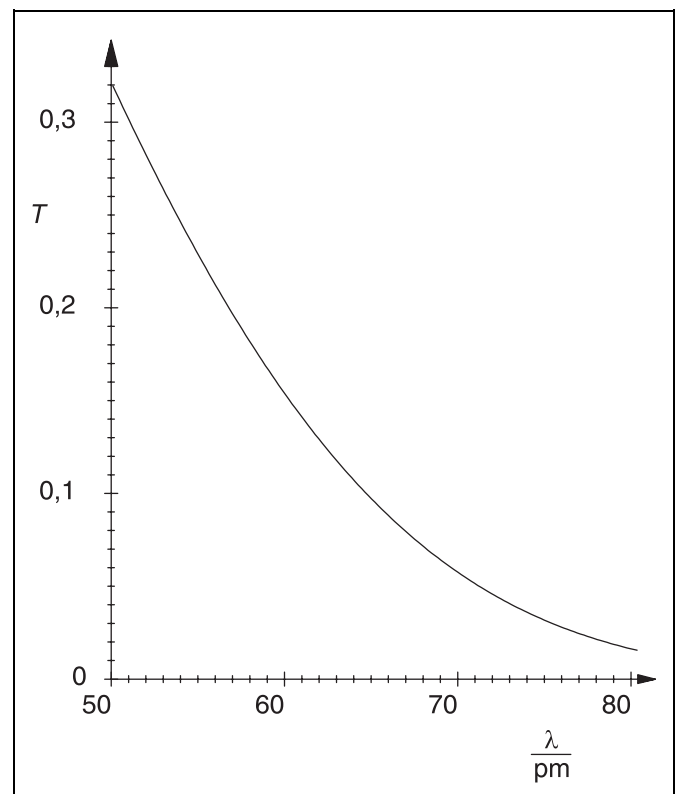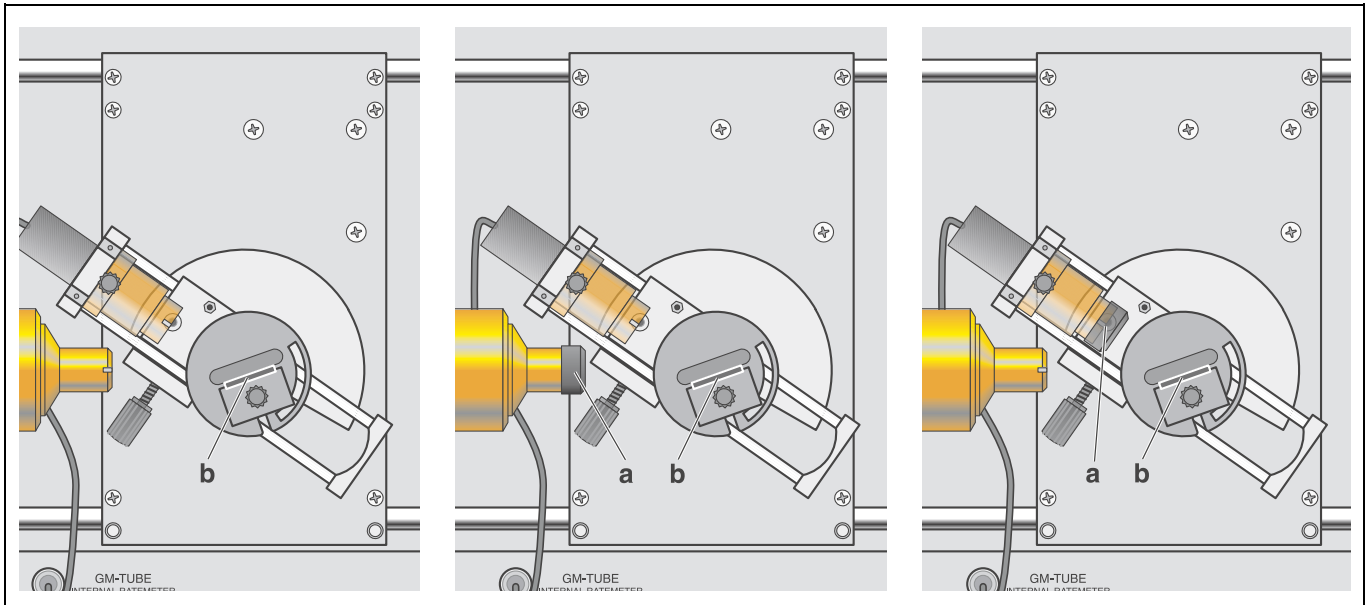
Fig. 3   Experiment setup for verifying the wavelength shift due to
        Compton scattering
        Left: without copper filter
        Center: with copper filter **(a)** in front of aluminum scatter-
        ing body (**b**)
        Right: with copper filter **(a)** behind aluminum scattering
        body **(b)**

Setup

Set up the experiment as shown in Fig. 3. Carry out the
following steps:

–   Remove the collimator and mount the zirconium filter sup-
    plied with the x-ray apparatus on the radiation inlet aperture
    side of the collimator.

–   Mount the collimator together with the zirconium filter.

–   If necessary, mount the goniometer and end window
    counter (see the Instruction Sheet of the x-ray apparatus).

–   Set a distance between the collimator and target of approx.
    5 cm and a distance between the target and the detector
    inlet diaphragm of approx. 4 cm.

–   Mount the aluminum scattering body from the Compton
    accessory x-ray as the target.

–   Press the TARGET key and manually set the target angle to
    20° using the ADJUST knob.

–   Press the SENSOR key and manually set the sensor angle
    to 145° using the ADJUST knob.

Carrying out the experiment

–   Set the tube high voltage U = 30 kV and the emission
    current I = 1.00 mA.

–   Set the angular step width $\Delta\beta$ = 0.0°.

a) Without copper filter:

–   Set the measuring time per angular step to Δt = 60 s.

–   Start the measurement with the SCAN key and display the
    mean counting rate R after the measuring time elapses by
    pressing REPLAY. Record the result as counting rate R_0.

b) Copper filter in front of aluminum scattering body:

–   Place the copper filter on the collimator.

–   Increase the measuring time per angular step to Δt = 600 s.

–   Start the measurement with the SCAN key and display the
    mean counting rate R after the measuring time elapses by
    pressing REPLAY. Record the result as counting rate R_1.

c) Copper filter behind aluminum body:

–   Mount the copper filter on the sensor seat.

–   Start the measurement with the SCAN key and display the
    mean counting rate R after the measuring time elapses by
    pressing REPLAY. Record the result as counting rate R_2.

d) Background effect:

–   Set the emission current I = 0.

–   Start the measurement with the SCAN key and display the
    mean counting rate R after the measuring time elapses by
    pressing REPLAY. Record the result as counting rate R.

Measuring example

U = 30 kV, I = 1.00 mA

Target angle = 20°, sensor angle = 145°

a) R_0 = 3.88 s^{-1} Δt = 60 s

b) R_1 = 0.669 s^{-1} Δt = 600 s

c) R_2 = 0.496 s^{-1} Δt = 600 s

d) R = 0.218 s^{-1} Δt = 600 s

Evaluation and results

From (VIII) and (IX), we can calculate the respective transmis-
sion values as

T_1 = 0.119 and T_2 = 0.071

Using (VII), we can then calculate the wavelengths of the unscattered and scattered x-radiation as

$\lambda_1 = 62.6$ pm and $\lambda_2 = 67.5$ pm

and the wavelength shift as

$\Delta\lambda = (4.9 \pm 1.3)$ pm.

(See below for calculation of the statistical error $\sigma_{\Delta\lambda}$).

Comparison with theory of Compton scattering:

The value to be expected according to (V) at a scattering angle $\vartheta = 145°$ is $\Delta\lambda mda = 4.42$ pm.

Additional information

Statistical error:

When measuring a counting rate R at a time interval Δt,

$N = R \cdot \Delta t$

events are counted each time. The statistical error of the counting rate R here is thus

$$\sigma_R = \frac{\sqrt{N}}{\Delta t} = \sqrt{\frac{R}{\Delta t}}.$$

For the counting rates given in the measuring example, it follows that

$\sigma_{R_0} = 0.3$ s^{-1}, $\sigma_{R_1} = 0.03$ s^{-1}, $\sigma_{R_2} = 0.03$ s^{-1} and $\sigma_R = 0.02$ s^{-1}

These errors carry over into the determination of the wavelength shift $\Delta\lambda$. For this statistical error $\sigma_{\Delta\lambda}$, we can say

$$\left(\sigma_{\Delta\lambda}\right)^2 = \left(\frac{\partial\Delta\lambda}{\partial R_1} \cdot \sigma_{R_1}\right)^2 + \left(\frac{\partial\Delta\lambda}{\partial R_2} \cdot \sigma_{R_2}\right)^2$$
$$+ \left(\frac{\partial\Delta\lambda}{\partial R_0} \cdot \sigma_{R_0}\right)^2 + \left(\frac{\partial\Delta\lambda}{\partial R} \cdot \sigma_R\right)^2$$

We can use the abbreviations

$$A_{01} = \frac{1}{R_0 - R} \cdot \frac{(100\ \text{pm})^n}{n \cdot a \cdot \lambda_1^{n-1}} = 3.0\ \text{pm} \cdot \text{s},$$

$$A_{02} = \frac{1}{R_0 - R} \cdot \frac{(100\ \text{pm})^n}{n \cdot a \cdot \lambda_2^{n-1}} = 2.6\ \text{pm} \cdot \text{s}$$

$$A_{11} = \frac{1}{R_1 - R} \cdot \frac{(100\ \text{pm})^n}{n \cdot a \cdot \lambda_1^{n-1}} = 24\ \text{pm} \cdot \text{s and}$$

$$A_{22} = \frac{1}{R_2 - R} \cdot \frac{(100\ \text{pm})^n}{n \cdot a \cdot \lambda_2^{n-1}} = 34\ \text{pm} \cdot \text{s}$$

to reformulate the partial derivations appearing here as:

$$\frac{\partial\Delta\lambda}{\partial R_0} = -A_{01} + A_{02} = -0.4\ \text{pm} \cdot \text{s},$$

$$\frac{\partial\Delta\lambda}{\partial R_1} = A_{11} = 24\ \text{pm} \cdot \text{s},$$

$$\frac{\partial\Delta\lambda}{\partial R_2} = -A_{22} = -34\ \text{pm} \cdot \text{s and}$$

$$\frac{\partial\Delta\lambda}{\partial R} = -A_{11} + A_{22} - A_{01} + A_{02} = +10\ \text{pm} \cdot \text{s}$$

As expected, the errors of the two counting rates R_1 and R_2 make up the largest part of this statistical error, while the effect of the counting rate R_0 can be largely ignored.

From the numerical values of the measuring example, we obtain

$\sigma_{\Delta\lambda} = 1.3$ pm

This value can be halved by increasing the measuring times for R_1 and R_2 by a factor of four.

Corrections:

In addition to inelastic Compton scattering, it would also be appropriate to take the elastic scattering of x-rays into consideration, in which by definition no wavelength shift occurs. The measured transmission T_2 is reduced less with respect to T_1 than the transmission T_2' for purely inelastic scattered x-rays which we are actually trying to detect. The evaluation thus estimates the wavelength shift $\Delta\lambda$ due to Compton scattering too low.