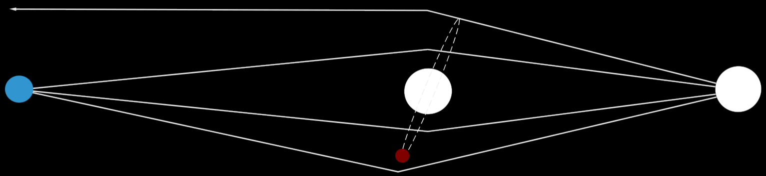
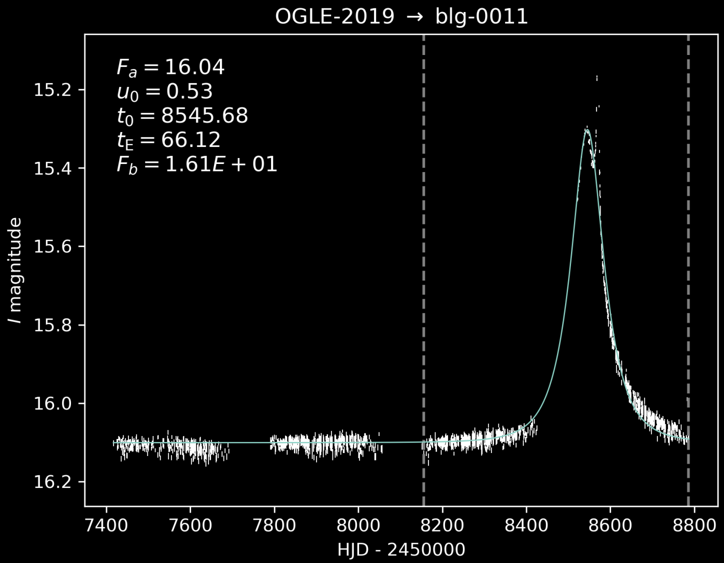
The Statistics of Gravitational Microlensing Photometry

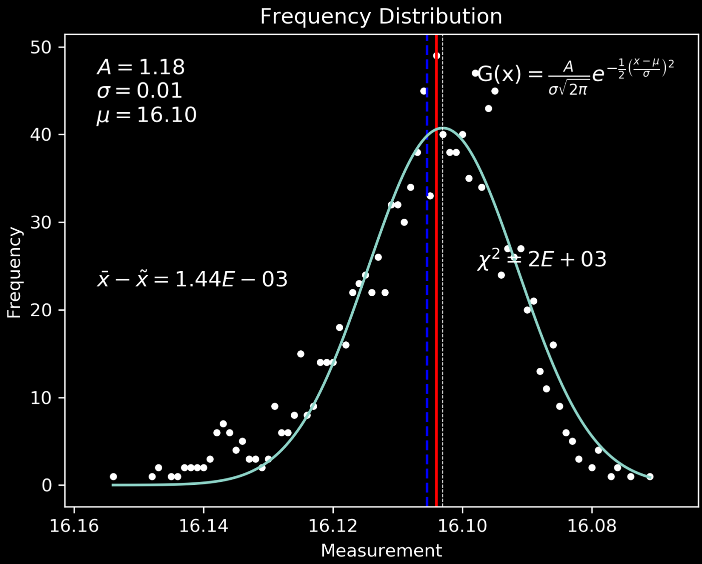
Jack Symonds





OGLE-2019 → blg-0011
$F_a = 16.04$
$u_0 = 0.53$
$t_0 = 8545.68$
$t_{\mathrm{E}} = 66.12$
$F_b = 1.61E+01$
15.2
15.4
15.6
15.8
16.0
16.2
I magnitude
7400
7600
7800
8000
8200
8400
8600
8800
HJD - 2450000

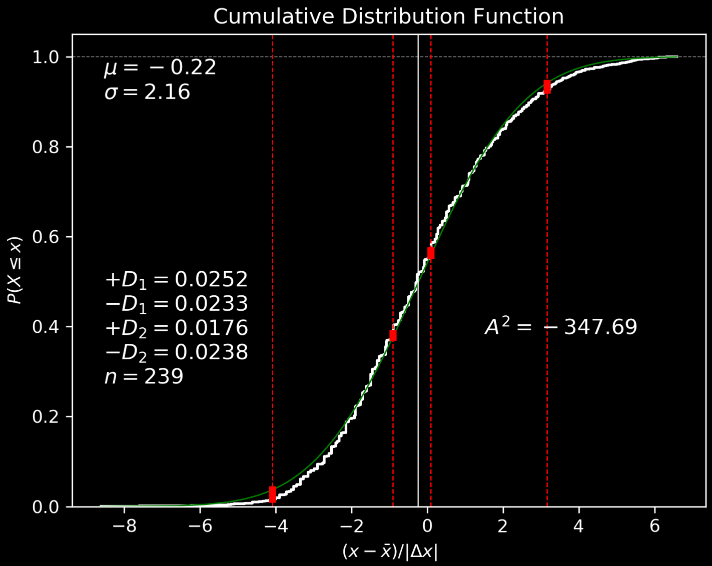
Constant Magnitude Distribution



$$\mathcal{R} = \frac{x - \bar{x}}{|\Delta x|} \qquad x \text{ is the magnitude measuremnet}$$

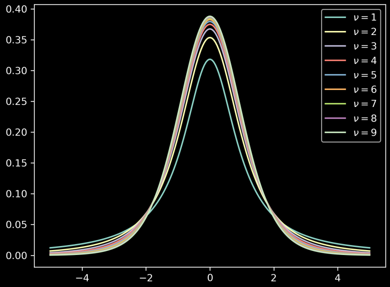
$$\text{PDF } f(x) \longrightarrow \text{CDF } F(x)$$

$$F(x) = P(X \leq x) = \frac{1}{n} \int_{\infty}^{x} f_X(t)\ \mathrm{d}t$$


Cumulative Distribution Function
$\mu = -0.22$
$\sigma = 2.16$
$+D_1 = 0.0252$
$-D_1 = 0.0233$
$+D_2 = 0.0176$
$-D_2 = 0.0238$
$n = 239$
$A^2 = -347.69$
$P(X \leq x)$
$(x - \bar{x})/|\Delta x|$
0.0
0.2
0.4
0.6
0.8
1.0
−8
−6
−4
−2
0
2
4
6

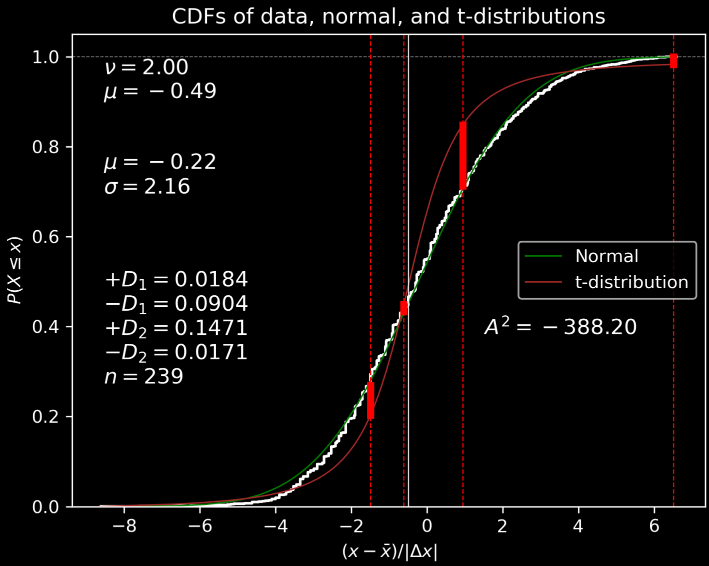
student's t-distribution

$$f(t; t_0, \nu) = \frac{1}{\sqrt{\nu}\ \mathrm{B}(\frac{1}{2}, \frac{\nu}{2})} \left(1 + \frac{(t - t_0)^2}{\nu}\right)^{-\frac{\nu+1}{2}} \qquad \mathrm{B}(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1}\, \mathrm{d}t$$



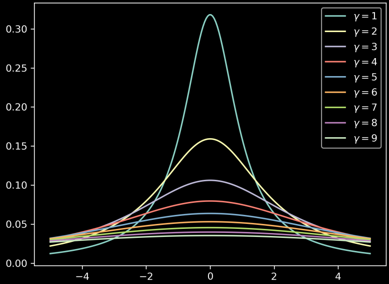
$$F(t; u_0, \nu) = \int_{-\infty}^{t} f(u)\ \mathrm{d}u$$

$$= \int_{-\infty}^{t} \frac{1}{\sqrt{\nu}\ \mathrm{B}(\frac{1}{2}, \frac{\nu}{2})} \left(1 + \frac{(u - u_0)^2}{\nu}\right)^{-\frac{\nu+1}{2}} \mathrm{d}u$$

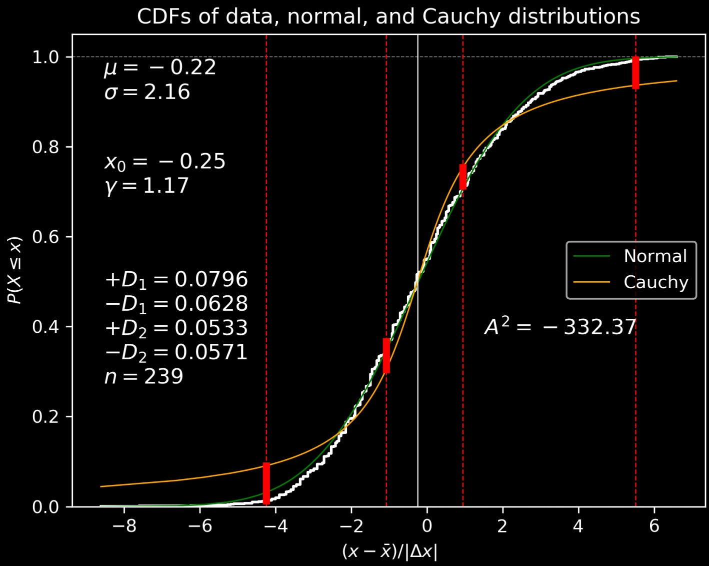

CDFs of data, normal, and t-distributions
$\nu = 2.00$
$\mu = -0.49$
$\mu = -0.22$
$\sigma = 2.16$
$+D_1 = 0.0184$
$-D_1 = 0.0904$
$+D_2 = 0.1471$
$-D_2 = 0.0171$
$n = 239$
$A^2 = -388.20$
Normal
t-distribution
$P(X \leq x)$
$(x - \bar{x})/|\Delta x|$
1.0
0.8
0.6
0.4
0.2
0.0
−8
−6
−4
−2
0
2
4
6

Cauchy / Lorentzian Distribution

$$f(x;x_0,\gamma) = \frac{1}{\pi\gamma}\left[1+\left(\frac{x-x_0}{\gamma}\right)^2\right] = \frac{1}{\pi\gamma}\left[\frac{\gamma^2}{(x-x_0)^2+\gamma^2}\right]$$



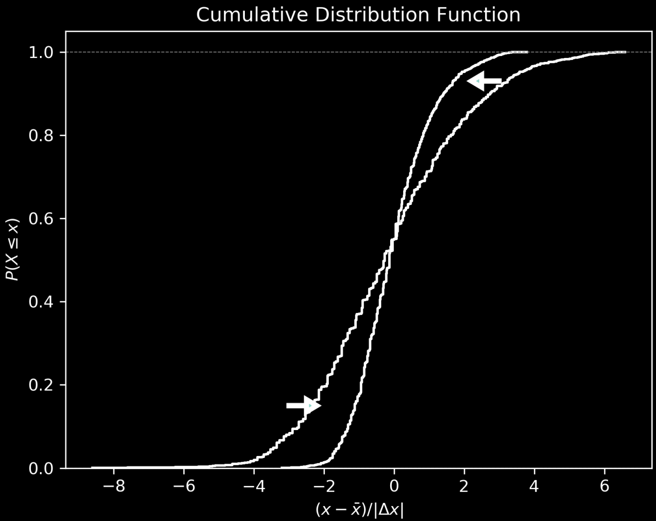
$$F(x;x_0,\gamma) = \int_{-\infty}^{x} f(t)\ \mathrm{d}t$$
$$= \frac{1}{\pi}\arctan\left(\frac{x-x_0}{\gamma}\right)+\frac{1}{2}$$



CDFs of data, normal, and Cauchy distributions
$\mu = -0.22$
$\sigma = 2.16$
$x_0 = -0.25$
$\gamma = 1.17$
$+D_1 = 0.0796$
$-D_1 = 0.0628$
$+D_2 = 0.0533$
$-D_2 = 0.0571$
$n = 239$
$A^2 = -332.37$
Normal
Cauchy
$P(X \leq x)$
$(x - \bar{x})/|\Delta x|$
1.0
0.8
0.6
0.4
0.2
0.0
−8
−6
−4
−2
0
2
4
6

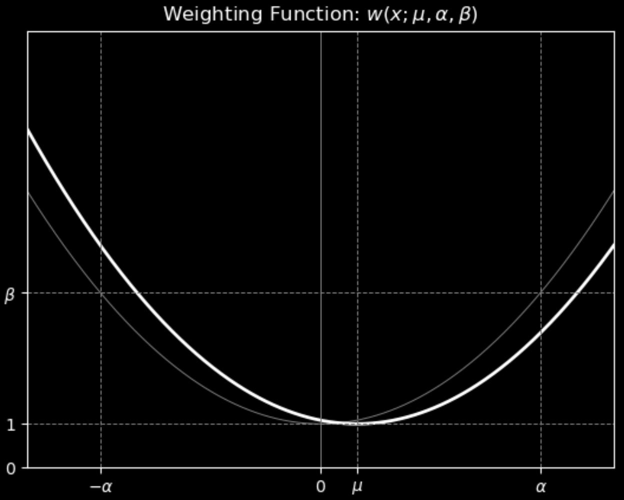
Error Correcting

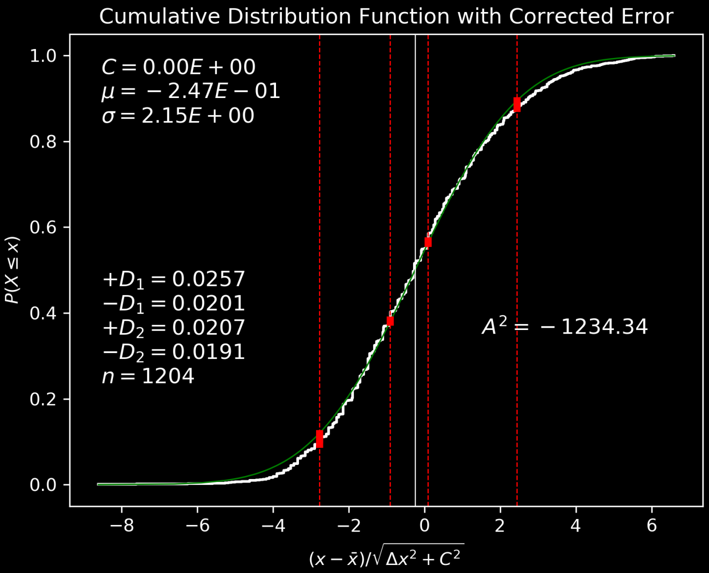
$$\Delta x \to \sqrt{\Delta x^2 + C^2} \quad \text{so that} \quad \mathcal{R} = \frac{x - \bar{x}}{\sqrt{\Delta x^2 + C^2}}$$



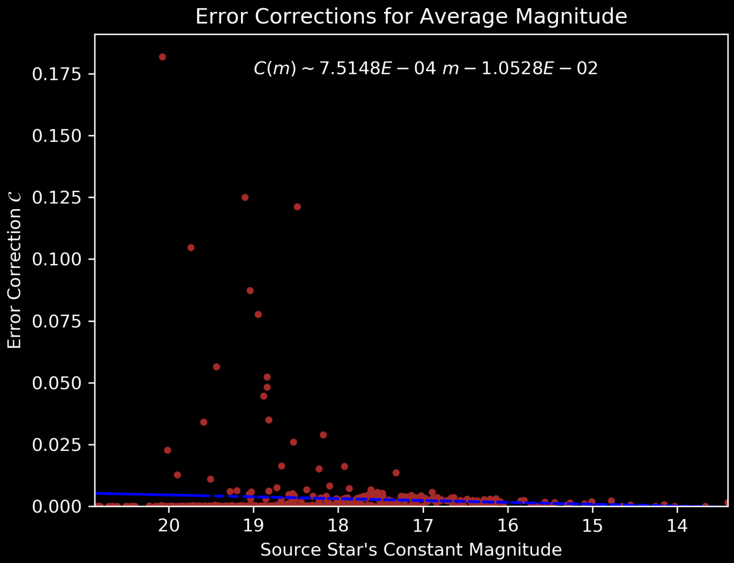
$$\mathcal{S}(C,\mu,\sigma)=\sum_{i=1}^{n}\left[F_{\mathrm{G}}(\mathcal{R}_i(C);\mu,\sigma)-F(\mathcal{R}_i(C))\right]^2\cdot w(\mathcal{R}_i(C);\mu)$$

$$w(x;\mu,\alpha,\beta)=\frac{\beta-1}{\alpha^2}(x-\mu)^2+1$$





Cumulative Distribution Function with Corrected Error
$C = 0.00E + 00$
$\mu = -2.47E - 01$
$\sigma = 2.15E + 00$
$+D_1 = 0.0257$
$-D_1 = 0.0201$
$+D_2 = 0.0207$
$-D_2 = 0.0191$
$n = 1204$
$A^2 = -1234.34$
$P(X \leq x)$
$(x - \bar{x})/\sqrt{\Delta x^2 + C^2}$
1.0
0.8
0.6
0.4
0.2
0.0
−8
−6
−4
−2
0
2
4
6



Error Corrections for Average Magnitude
$C(m) \sim 7.5148E-04\ m - 1.0528E-02$
0.175
0.150
0.125
0.100
0.075
0.050
0.025
0.000
Error Correction $\mathcal{C}$
20
19
18
17
16
15
14
Source Star's Constant Magnitude