CAL POLY

MEMORANDUM

February 7, 2026

TO: Charlie Refvem
FROM: Jack Toyama

SUBJECT: Homework 0x03 – System Modeling

The following kinematic model can be applied to the Romi robot and its position, orientation, and movement. In the motor differential equation: $\tau_m \dot{\Omega}_m = K_m V_m$, K_m is the speed constant, determining speed given voltage input, V_m, and τ_m is the motor time constant accounting for the response rate of the motor. Using the physical characteristics of the robot and the state and input vectors, the system dynamics can be determined. In addition, the output state can be calculated using the output equations and the output vectors: values from the encoders, IMU, and GPS. Equation 1 defines the angular velocity of the robot with respect to its environment. Equation 2 represents the velocity of the robot in reference to the robot's coordinates, while equations 3 and 4 represent the velocity in referent to the environment. Equation 5 is the set of differential equations for the system dynamics from the state variables and the system input voltages. Equation 6 is the set of output variables given the measurable parameters of the system; from the encoders, IMU, and GPS. The derivations of the equations can be found in Attachment A.

$$\Omega = \frac{v_R - v_L}{w} \tag{1}$$

$$\text{v} = \frac{v_R + v_L}{2} \tag{2}$$

$$\dot{X}_R = v\cos(\psi_R) \tag{3}$$

$$\dot{Y}_R = v\sin(\psi_R) \tag{4}$$

$$\dot{x} = \begin{bmatrix} \dot{\psi}_R \\ \dot{S} \\ \dot{\Omega}_R \\ \dot{\Omega}_L \\ \dot{X}_R \\ \dot{Y}_R \end{bmatrix} = \begin{bmatrix} \frac{r}{w}(\Omega_r - \Omega_l) \\ \frac{r}{2}(\Omega_r + \Omega_l) \\ \frac{1}{\tau_m}(K_m u_R - \Omega_R) \\ \frac{1}{\tau_m}(K_m u_L - \Omega_L) \\ \frac{r}{2}(\Omega_R + \Omega_L)\cos(\psi_R) \\ \frac{r}{2}(\Omega_R + \Omega_L)\sin(\psi_R) \end{bmatrix} \tag{5}$$

$$y = \begin{bmatrix} S_L \\ S_R \\ \psi_R \\ \Omega \\ X_R \\ Y_R \end{bmatrix} = \begin{bmatrix} S_L = S - \frac{w}{2}\psi_R \\ S_R = S + \frac{w}{2}\psi_R \\ \psi_R = \psi_R \\ \Omega = \dot{\psi}_R \\ X_R = X_R \\ Y_R = Y_R \end{bmatrix} \tag{6}$$

Discussion Questions

1. The kinematic model for the Romi assumes no slippage by the wheels. This is a safe assumption for the level of precision we need for our functionality when: speeds are low, acceleration is slow, and the tire material can grab the surface well. When these factors are not met, small slippages can add up and result in differences in the expected and actual location. The full dynamic model is much more accurate, but requires a much more complex system to implement, which is not necessary for our implementation.

2. The motor parameters are not the same when driving straight vs turning. While driving straight, both wheels experience nearly identical loads, the chassis does not rotate, and the inertia is in a translational direction. On the other hand, when the wheels are rotating at different speeds, causing the robot to turn; rotational inertia, lateral friction, and differences in wheel loads become factors. While not exactly the same, if we account for it in the design of our pathing and moving algorithms, the effects will be negligible.

3. To correct for drift from small real-world errors, additional information besides the wheel encoders is needed. An IMU directly measures the angular velocity and linear acceleration to ensure it matches what is expected. By combining the information from the encoders and the IMU, both translational and rotational data can be input to a feedback system to account for inconsistencies in motion. GPS would potentially be helpful over long periods of time where drift adds up. However, for the Romi-scale, GPS is likely not accurate enough to correct the miniscule errors caused by wheel slippage. Orientation is also impossible to determine using traditional GPS.

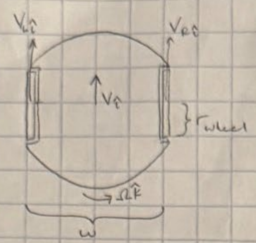
4. Utilizing proportional and integral control feedback, the robot can maintain a stable steady state with minimal overshoot and oscillation. The proportional component gets the measurements close to the setpoint, and the integral component closes the distance to the setpoint and holds it there by rejecting disturbances to the system. Feedback from motor encoders, line sensors, and an IMU in combination would allow for accurate feedback inputs, and by splitting up a complex path into uniform components, such as straight lines and arcs, feedback can be applied more accurately because the motion is simpler.

5. It is not possible to know the exact orientation and position of the robot just using the net angles swept by each wheel. Different paths can result in the same rotations. For example, 1 wheel making one full rotation and then the other making one full rotation will result in the robot being in the same position as it began, while both wheels making a full rotation at the same time will result in a change in position. With time history, and assuming no wheel slippage, the robot's path can be reconstructed by combining

each arc created by the wheel motions. Conversely, given the robot's orientation and location, you can determine many possible sets of angles swept to reach the position, but it is impossible to know the exact set without the time history of the motor rotation.

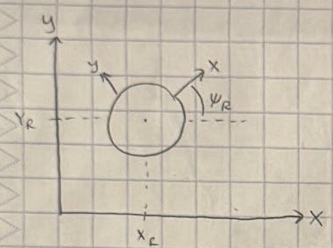
To track absolute position, state variables must be kept and updated throughout the motion of the robot. This requires the encoder counts, which can be used to continuously updating the position state. For arcs, the initial orientation, change in orientation, and the wheel distances can be tracked to determine the arc specifications.

Attachment A: System schematic and Romi's dynamics.



$$V_R\hat{\imath} = V_L\hat{\imath} + \Omega\hat{k} \times (-w)\hat{\jmath}$$
$$(V_R - V_L)\hat{\imath} = w\Omega\hat{\imath}$$
$$\boxed{\Omega = \frac{V_R - V_L}{w}}$$

$$V\hat{\imath} = V_L\hat{\imath} + \Omega\hat{k} \times \left(-\frac{w}{2}\right)\hat{\jmath}$$
$$V\hat{\imath} = \left(V_L + \frac{w}{2}\Omega\right)\hat{\imath} \longrightarrow \text{integrate for } S_L\ S_R$$
$$\boxed{V = \frac{V_L + V_R}{2}}$$



$$\hat{\imath} = \cos\psi_R\,\hat{I} + \sin\psi_R\,\hat{J}$$
$$\hat{\jmath} = -\sin\psi_R\,\hat{I} + \cos\psi_R\,\hat{J}$$
$$\dot{X}_R\hat{I} + \dot{Y}_R\hat{J} = V\hat{\imath}$$
$$\dot{X}_R\hat{I} + \dot{Y}_R\hat{J} = V(\cos\psi_R\hat{I} + \sin\psi_R\hat{J})$$
$$\boxed{\begin{aligned}\dot{X}_R &= V\cos\psi_R \\ \dot{Y}_R &= V\sin\psi_R\end{aligned}}$$

$$T_m\dot{\Omega}_m + \Omega_m = K_m V_m$$

T_m is the motor time constant accounting for the response rate

K_m is the speed constant, determining speed given Voltage input (V_m)

$$\underline{x} = \begin{bmatrix} \psi_R \\ S \\ \Omega_L \\ \Omega_R \\ X_{R0} \\ Y_{R0} \end{bmatrix} \qquad \underline{u} = \begin{bmatrix} u_L \\ u_R \end{bmatrix}$$

$$V_R = r\Omega_R$$
$$V_L = r\Omega_L$$
$$V_R - V_L = \dot{\psi}_R w$$
$$r\Omega_R - r\Omega_L = \dot{\psi}_R w$$
$$\dot{\psi}_R = \frac{r\Omega_R - r\Omega_L}{w} = \frac{r}{w}(\Omega_R - \Omega_L)$$

$$\dot{S}_L = V_L = r\Omega_L$$
$$\dot{S}_R = V_R = r\Omega_R$$
$$\dot{S} = \frac{r\Omega_R + r\Omega_L}{2} = \frac{r}{2}(\Omega_R + \Omega_L)$$

$$V = \frac{V_L + V_R}{2} = \frac{r}{2}(\Omega_R + \Omega_L)$$
$$\dot{X}_R = \frac{r}{2}(\Omega_R + \Omega_L)\cos(\psi_R)$$
$$\dot{Y}_R = \frac{r}{2}(\Omega_R + \Omega_L)\sin(\psi_R)$$

$$\dot{x} = \begin{bmatrix} \dot{\psi}_R \\ \dot{S} \\ \dot{\Omega}_L \\ \dot{\Omega}_R \\ \dot{X}_R \\ \dot{Y}_R \end{bmatrix} = \begin{bmatrix} \frac{r}{w}(\Omega_R - \Omega_L) \\ \frac{r}{2}(\Omega_R + \Omega_L) \\ \frac{1}{T_m}(K_m U_R - \Omega_R) \\ \frac{1}{T_m}(K_m U_L - \Omega_L) \\ \frac{r}{2}(\Omega_R + \Omega_L)\cos(\psi_R) \\ \frac{r}{2}(\Omega_R + \Omega_L)\sin(\psi_R) \end{bmatrix}$$

$$T_m\dot{\Omega}_m + \Omega_m = K_m V_m$$
$$T_m\dot{\Omega}_m = K_m V_m - \Omega_m$$
$$\dot{\Omega}_m = \frac{1}{T_m}(K_m V_m - \Omega_m)$$
$$\dot{\Omega}_R = \frac{1}{T_m}(K_m V_R - \Omega_R) = \cancel{\frac{1}{T_m}(K_m r\Omega_R - \Omega_R)}$$
$$\dot{\Omega}_L = \frac{1}{T_m}(K_m V_L - \Omega_L) = \cancel{\frac{1}{T_m}(K_m r\Omega_L - \Omega_L)}$$

$$y = \begin{matrix} \psi_R = \psi_R \\ x_R = X_R \\ y_R = y_R \\ \Omega = \dot{\psi}_R \end{matrix}$$

$$V\hat{\imath} = \left(V_L + \frac{w}{2}\Omega\right)\hat{\imath} \rightarrow \int V\hat{\imath}\,dt = \int \left(V_L + \frac{w}{2}\Omega\right)\hat{\imath}\,dt$$

$$S = S_L + \frac{w}{2}\psi_R$$

$$S_L = S - \frac{w}{2}\psi_R$$

$$y = \begin{bmatrix} S_L \\ S_R \\ \psi_R \\ \Omega \\ X_R \\ Y_R \end{bmatrix} = \begin{bmatrix} S - \frac{w}{2}\psi_R \\ S + \frac{w}{2}\psi_R \\ \psi_R \\ \dot{\psi}_R \\ X_R \\ Y_R \end{bmatrix}$$

$$V = \frac{V_R + V_L}{2}$$

$$2V = V_R + V_L$$

$$V_L = 2V - V_R$$

$$V\hat{\imath} = \left(2V - V_R + \frac{w}{2}\Omega\right)\hat{\imath}$$

$$\int V\,dt = \int 2V - V_R + \frac{w}{2}\Omega\,dt$$

$$S = 2S - S_R + \frac{w}{2}\psi_R$$

$$S_R = S + \frac{w}{2}\psi_R$$