Problem 2

Group 4

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$$\int \frac{x^3}{\sqrt{x^2-1}}$$

First we can see that for $g(x) = \frac{x^3}{\sqrt{x^2-1}}$, the value of $g(x)$ is undefined at $x = 1$ or $x = -1$, as the denominator is 0. Outside of $(-1, 1)$, the value under the square root will be negative, and so the denominator will be imaginary. We can therefore evaluate this integral on $(-1, 1)$.

$$\lim_{\alpha \to 1^-} \int_{-\alpha}^{\alpha} \frac{x^3}{\sqrt{x^2-1}}$$

Now, however, we can see that the this integral is symmetric around the origin. That is, $g(-x) = -g(x)$. We can see that is true by plugging in x and $-x$:

$$-\frac{x^3}{\sqrt{x^2-1}} = \frac{(-x)^3}{\sqrt{(-x)^2-1}}$$

$$\frac{-x^3}{\sqrt{x^2-1}} = \frac{-x^3}{\sqrt{x^2-1}}$$

This means that the positive and negative values will cancel out, and so the value of the integral will be 0.

$$\int \frac{x^3}{\sqrt{x^2-1}} = 0$$