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~ 1 hr
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1a) $u_n = 4 + 3n$ Arithmetic starting at $n=0$
4, 7, 10

b) $u_n = 4^n$ Geometric - Find nterm by
1, 4, 16, 64 $x_n = ar^{(n-1)}$
$\times 4$ $\times$

c) $u_n = n \cdot 3^n$ Neither since not same multiplier increase
0, 3, 18, 27
$\times 4$ $\times$ not 4 so neither

Find the limit

a) $u_n = 1 + \frac{1}{2}n$ as $n \to \infty$ $u_n \to \infty$ $\lim_{n\to\infty}(1+\frac{1}{2}n) = \infty$

b) $u_n = \left(\frac{1}{2}\right)^n$ $\lim_{n\to\infty}\left(\frac{1}{2}\right)^n = 0$

c) $\lim_{x\to -4} \frac{x^2+5x+4}{x^2+3x-4} = \frac{\cancel{(x+4)}(x+1)}{\cancel{(x+4)}(x-1)} = \frac{-4+1}{-4-1} = \frac{-3}{-5}$

$= \frac{3}{5}$

sec 8/8

1a) $a_n = \dfrac{3+5n^2}{n+n^2}$ $\quad \lim_{n\to\infty} \left(\dfrac{3+5n^2}{n+n^2}\right)$

$$= \frac{\lim_{n\to\infty}(3+5n^2)}{\lim_{n\to\infty}(n+n^2)} = \frac{\lim_{n\to\infty}\left(\frac{3}{n^2}+5\right)}{\lim_{n\to\infty}\left(\frac{1}{x}+1\right)} = \frac{5}{1} = 5$$

b) $a_n = \dfrac{(-1)^{n-1}n}{n^2+1}$

I'll use squeeze theorem to do this

$$-1 \le (-1)^{n-1} \le 1$$

$$-n \le (-1)^{n-1}n \le n$$

$$\frac{-n}{n^2+1} \le \frac{(-1)^{n-1}n}{n^2+1} \le \frac{n}{n^2+1}$$

$\lim_{n\to\infty} \dfrac{-n}{n^2+1} = 0 \qquad \lim_{n\to\infty} \dfrac{n}{n^2+1} = 0$ so

by Squeeze theorm

$$\lim_{n\to\infty} a_n = 0$$

Find More Limits

$\lim_{x \to a} f(x) = -3$ $\lim_{x \to a} g(x) = 0$ $\lim_{x \to a} h(x) = 8$

a) $\lim_{x \to a} [f(x) + h(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} h(x) = 5$

b) $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} =$ DNE → Divide by 0

c) $\lim_{x \to a} \frac{2f(x)}{h(x) - f(x)} = \frac{2 \lim_{x \to a} f(x)}{\lim_{x \to a} h(x) - \lim_{x \to a} f(x)} = \frac{-6}{3} = -2$

Check for discontinues

a) $f(x) = \frac{9x^3 - x}{(x-1)(x+1)}$ Discontinus at $x=1$ and $x=-1$

b) e^{-x^2} - continuous

Finite limits

a) $\lim_{x \to 1} \left[\frac{x^4 - 1}{x - 1}\right] = \frac{(x^2+1)(x^2-1)}{(x-1)} = \frac{(x^2+1)(x+1)(x-1)}{(x-1)}$

$= (x^2+1)(x+1) = 4$

b) $\lim_{x \to -4} \left[\frac{x^2+5x+4}{x^2+3x-4} \right] = \frac{\cancel{(x+4)}(x+1)}{\cancel{(x+4)}(x-1)}$

$\frac{-3}{-5} = \frac{3}{5}$

Infinite Limits

a) $\lim_{x \to \infty} \left[\frac{9x^2}{x^2+3} \right] = \frac{\infty}{\infty}$ so yes to L'Hopitals

$\lim_{x \to \infty} \left(\frac{18x}{2x} \right) = 9$

b) $\lim_{x \to \infty} \left[\frac{3^x}{x^3} \right]$ = yes to $= \frac{\ln(3)\,3^x}{3x^2}$

$= \frac{\ln(3)\,\ln(3)\,3^x}{6x} = \frac{\ln(3)^3\,3x}{6}$

$= \infty \quad \infty$

Accessing Continuity, Differentiating

a) $f(x) = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$

Continuous $\quad 0^2 = -0^2$

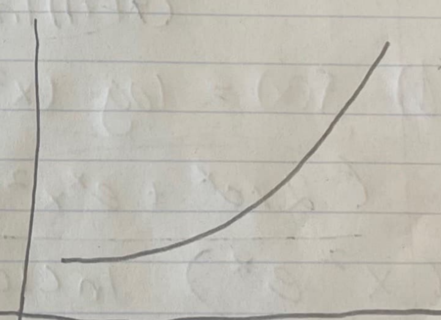
Differentiable $\quad 2x = -2x \qquad 2(0) = -2(0),$

b) $x^3 = x \qquad 1^3 = 1 \quad$ continuous

$3x^2 = 1 \qquad 3 \neq 1 \quad$ not differentiable

Possible Derivative

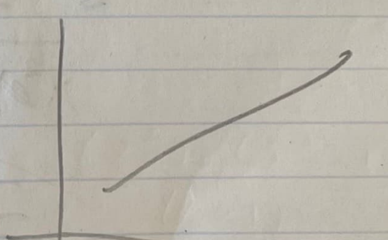
Assuming function is
Somewhat like x^2+1
(for simplicity)



then

$f'(x) = 2x$

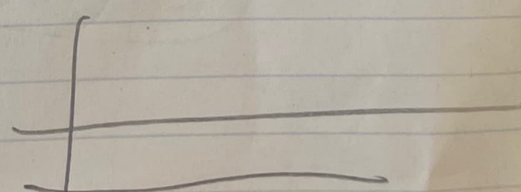
C and D



then

$f''(x) = 2$

A and B



Calculate Derivs

a $f(x) = 4x^3 + 2x^2 + 5x + 11$

$f'(x) = 12x^2 + 4x + 5$

b) $y = \sqrt{30} = 0$

$y' = 0$

$\frac{d}{dx} \log_a x = \frac{1}{x \ln(a)}$

c) $h(x) = \log(9t+1)$

$h'(x) = \frac{9}{(9t+1)\ln(10)}$

d) $f(x) = \log(x^2 e^x)$

$$\frac{(2xe^x + e^x x^2)}{(x^2 e^x) \ln(10)}$$

$$= \frac{2x\cancel{e^x}}{x^2 \cancel{e^x} \ln(10)} + \frac{\cancel{e^x} \cancel{x^2}}{\cancel{x^2} \cancel{e^x} \ln(10)}$$

$$\frac{2}{x \ln(10)} + \frac{1}{\ln(10)}$$