
Question1 (2 points)

There has been a rumour that your instructor loves senior students more than junior students. And it is easier for seniors to successfully enrol in VE414. Of course, it is just a rumour, but where there is smoke; there is fire.

	Unsuccessful	Successful
Junior	71	16
Senior	10	34

Let us look for evidence in the above data to see whether he is guilty! He claims JI's course enrolment system employs a random mechanism when the course initial cap is smaller than the total number of students tries to register for the course in the first round. This random mechanism decides whether each of the students is successfully enrolled or not according to a single unknown probability p by the end of the first round. Suppose the random mechanism decides each student's enrolment independently. And the actual course cap is the number of students that are successfully enrolled after the first round. Assume no student has dropped VE414 since the first round, who would in their right mind, and the initial course cap for VE414 is 70, and $\text{Beta}(3, 7)$ is assumed to be the prior for p. Determine whether the instructor is guilty using Bayesian analysis.

Question2 (3 points)

Suppose we have n i.i.d observations $x_1, x_2, \ldots, x_n$, which we will model using a Poisson distribution with an unknown mean y,

$$X_i \mid Y \sim \text{Poisson}\,(y)$$

Assume a gamma distribution with mean of 15 and variance of 25 is used as the prior.

(a) (2 points) Find the posterior of Y given those n observations.

(b) (1 point) Find the posterior predictive distribution of a new data point X^*.

Question3 (5 points)

Let us consider Bayes' original problem again,

$$X_k \mid P \sim \text{Binomial}\,(k, p)$$

(a) (2 points) Suppose $x_3 = 2$, plot and compare the two posteriors obtained by using the uniform prior and the Jeffreys prior.

(b) (2 points) Suppose $x_{300} = 200$, plot and compare the two posteriors obtained by using the uniform prior and the Jeffreys prior.

(c) (1 point) Suppose $x_2 = 1$, and apply the Bayes theorem using the following

$$f_P \propto p^{-1}(1-p)^{-1}$$

which is an improper prior and should be understood as the limit of Beta distribution as $\alpha, \beta \to 0$. Give a suggestion to why the above improper prior is also considered to reflect our ignorance in a certain sense.