UM–SJTU Joint Institute

Bayesian Analysis
(VE414)

Class Notes

Class 2 Inference

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1 Basic Information

Thomas Bayesian's origin problem: Given the number of times in which an unknown event has happened and failed: Required the chance that the probability of its happening in a single trial lies somewhere between any two degrees of probability that can be named."

It is about statistical inference on binomial proportion p:

$$X \sim \text{Binominal}(k, p)$$

A Ball Game Example[H]
Suppose we have a black ball and a red ball. We first throw the black ball on the table, but we don't know where the black ball locates. Then we throw the red ball on the table, and we can know whether the red ball is farther or closer. Our aim is to find out where exactly the black ball locates.

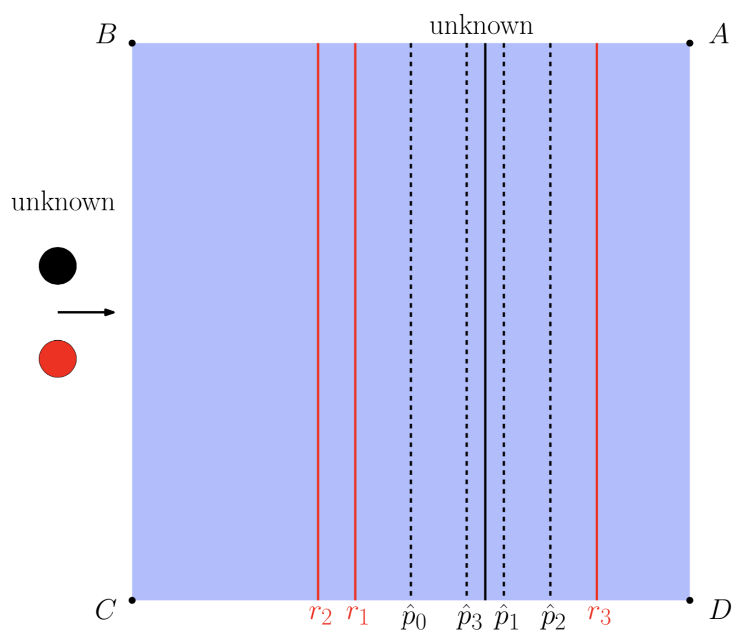

Figure 1: Ball Game Example

2 Maximum Likelihood Estimator

Since each throw of red ball is independent, we can assume that the result should follows the binominal distribution.

$$X \sim \text{Binominal}(k, p)$$

Then the likelihood of this should be

$$L(p;x) = f_{X|p}(x \mid p) = \frac{k!}{x!(k-x)!} p^x (1-p)^{k-x}$$

A very arrogant way of solving this problem is named Maximum Likelihood Estimate, where we try to maximize the likelihood. It is one of typical frequentist solutions.

$$\tilde{\theta}_{MLE} = \underset{\theta \in \Theta}{\operatorname{argmax}} \, L(\theta; x)$$

And for this ball game, we can conclude that the Estimator should be

$$\tilde{p} = \frac{x}{k}, \quad x \text{ for number of success, } k \text{ for number of trials}$$

As we mentioned above, the maximum likelihood estimate is an arrogant approach, where one can get some results far from the real value when the trial is small. Consider the case where one throw the black ball to the middle, and only throw one trial of read ball. In this case, the maximum likelihood estimate will have an estimator either to be 0, or be 1, which are both far from the actual result.

3 Probability Mass Function and Probability Density Function

Recall that for a discrete random variable, its PMF(probability mass function) is

$$f_{Y|X}(y \mid x) = \frac{f_{X,Y}(x,y)}{f_X(x)}, \qquad f_X(x) \neq 0$$

It is meaningful because PMF gives probability directly by

$$f_{Y|X}(y \mid x) = \Pr(Y = y \mid X = x) = \frac{Pr(X = x, Y = y)}{Pr(X = x)} = \frac{f_{X,Y}(x,y)}{f_X(x)}$$

However, for a continuous random variable x, PDF does not link to probability directly

$$f_{Y|X}(y \mid x) \neq \Pr(Y = y \mid X = x)$$

What's more, for any continuous random variable x_c here, we always have

$$\Pr(c) = 0$$

4 Some Generally Used Distributions

Binominal Distribution

Distribution:

$$X \sim \text{Binominal}(k, p)$$

Probability Mass Function:

$$f_X(x) = \frac{k!}{x!(k-x)!} p^x (1-p)^{k-x}$$

Support[1]:

$$x \in \{0, 1, 2, \cdots, k\}$$

Mean and Variance:

$$\mathbb{E}(x) = kp \qquad \text{Var}(x) = kp(1-p)$$

Geometric Distribution

Distribution:

$$X \sim \text{Geometric}(p)$$

Probability Mass Function:

$$f_X(x) = p(1-p)^{x-1}$$

Support:

$$x \in \{1, 2, \cdots, k\}$$

Mean:

$$\mathbb{E}(x) = \frac{1}{p}$$

Variance:

$$\text{Var}(x) = \frac{1-p}{p^2}$$

5 Quiz

[1]Support: The possible values for random variable x