Question1 (5 points)

Harvey Dent possesses five coins, two of which are double-headed, one is double-tailed, and two are normal. Assume each face is equally likely to turn up.

(a) (1 point) He shuts his eyes, picks a coin at random, and tosses it; what is the probability that the lower face of the coin is a head?

(b) (2 points) He opens his eyes and sees that the coin is showing heads; what is the probability that the lower face is a head?

(c) (2 points) He shuts his eyes and tosses the coin again, and then he opens his eyes and sees that the coin is showing heads again; what is the probability that the lower face is a head?

Question2 (3 points)

Suppose you have Beta $(4, 4)$ as your prior distribution on the probability p that a coin will yield a head when spun in a specific manner. The coin is independently spun ten times, and heads appear fewer than 3 times. You are not told how many heads were seen, only that the number is less than 3. Find the posterior density (up to a proportionality constant) for p and produce a graph of it in R.

Question3 (2 points)

Suppose you train people in a simple learning experiment, as follows. When people see the two words "radio" and "ocean" on the computer screen, they should press the **F** key on the computer keyboard. They see several repetitions and learn the response well. Then you introduce another correspondence for them to learn: Whenever the words "radio" and "mountain" appear, they should press the **J** key on the computer keyboard. You keep training them until they know both correspondences well.

Now you probe what they've learned by asking them about two novel test items. For the first test, you show them the word "radio" by itself and instruct them to make the best response (**F** or **J**) based on what they learned before.

For the second test, you show them the two words "ocean" and "mountain" and ask them to make the best response. You do this procedure with 50 people. Your data show that for "radio" by itself, 40 people chose **F** and 10 chose **J**. For the word combination "ocean" and "mountain" 15 chose **F** and 35 chose **J**. Are people biased toward **F** or toward **J** for either of the two probe types? Justify your answer using Bayesian analysis.

Question4 (0 points)

Let $X_1, X_2, \ldots, X_n$ denote a random sample from a geometric distribution with PDF

$$f_{X_i|p}(x_i \mid p) = p(1-p)^{x_i - 1}$$

(a) (1 point (bonus)) Find maximum likelihood estimate $\tilde{p}$ of p.

(b) (1 point (bonus)) The invariance property of maximum likelihood estimates tells that for any smooth monotone function $q = h(p)$ of p, $\tilde{q} = g(\tilde{p})$ is the maximum likelihood estimate of q. Find the maximum likelihood estimate of $q = p(1-p)$.

(c) (1 point (bonus)) Assuming a uniform prior. Find the posterior mean $\hat{p}$.