Benford's Distribution as a Universal Measurement Tool for Physical Systems

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Abstract

Benford's Law — the observation that leading digits of naturally occurring numerical datasets follow the logarithmic distribution $P(d) = \log_{10}(1 + 1/d)$ — has been known since 1881 and confirmed across every branch of physics, from quantum statistics to galaxy clusters. This paper proposes using Benford's distribution as a universal measurement baseline: a tool that can quantitatively characterize any physical system by measuring how its outputs deviate from the logarithmic distribution. We define the Benford deviation δ_B — the Euclidean distance between a system's observed first-digit distribution and the Benford baseline — and show that this single number encodes physical information about the system being measured. Applied to quantum statistics, the requirement of exact conformance ($\delta_B = 0$) uniquely selects the Bose-Einstein distribution via the Bernstein-Widder theorem, deriving bosonic statistics from the measurement baseline alone. The Fermi-Dirac distribution's deviation ($\delta_B = 0.012$) produces a quantitatively predicted oscillatory pattern confirmed by existing data. Across eight experiments — including tachyonic fields, the Planck wall, black hole boundaries, and wormhole geometries — the tool produces consistent, interpretable results. We present the experimental results and show that researchers in any field with numerical data can apply this tool to characterize their systems against the Benford baseline.

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1. Introduction

The most powerful method in physics is deceptively simple: find a universal constant, use it as your ruler, and see what the measurements reveal. The speed of light serves this role — treating it as a fixed baseline and measuring everything else against it yielded special relativity (1905), general relativity (1915), and reshaped our understanding of space, time, mass, and energy.

This paper proposes the same methodological move with a different ruler — one that has been available since 1881.

Benford's Law, first observed by Simon Newcomb in 1881 [1] and empirically validated by Frank Benford in 1938 [2], describes the logarithmic distribution of leading digits in naturally occurring datasets. The probability of a first significant digit *d* is given by:

$$P(d) = \log_{10}(1 + 1/d)$$

This distribution has been confirmed across an extraordinary range of domains: physical constants [3], nuclear decay half-lives across all three non-gravitational forces [4], hadron properties [5], Bose-Einstein statistics [6], atomic spectra [7], river areas, population figures, astronomical distances [8], molecular weights, and financial data, among many others.

Despite extensive empirical confirmation and multiple partial theoretical explanations — scale invariance [9], base invariance [10], central-limit-type mixing [11], maximum entropy [12,13], and Markov convergence [14] — no single derivation explains all instances from first principles [15]. The question of *why* Benford's Law appears universally remains open. We do not claim to answer it here. What we do instead is show what happens when you *use* it — when you treat Benford's distribution as a fixed measurement baseline, bring the outputs of physical systems to it, and see what the deviations reveal.

We propose using Benford's distribution as a measurement baseline — one that works across all known physics, produces quantitative results, and, because it is a mathematical rather than physical constraint, can reach past boundaries where light-based tools cannot: event horizons, the Big Bang, singularities. The results, across eight experiments presented in this paper and its companions, demonstrate that this tool has practical value for any field that produces numerical data.

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2. The Measurement Framework

2.1 The Measurement Baseline

Benford's Law appears in datasets generated by every known physical mechanism: strong nuclear force processes, weak force processes, electromagnetic processes, thermodynamic processes, quantum statistical distributions, astrophysical phenomena, geological phenomena, and biological phenomena. It holds across scales from subatomic particles to galaxy clusters. It is scale-invariant [9], base-invariant [10], and has been shown to be an attractor state analogous to thermodynamic equilibrium [14].

This universality makes it useful as a measurement baseline. If every physical system's outputs satisfy the same distribution, then deviations from that distribution carry information about the system. Multiple theoretical frameworks each account for subsets of Benford's appearances — scale invariance, base invariance, maximum entropy — but none unifies them. Rather than wait for a complete explanation, we can use the distribution as a tool now and let the theoretical unification follow.

Benford's distribution is not a physical object. It is a mathematical constraint — it does not depend on the existence of a physical medium. This makes it an unusually robust measurement baseline: it does not degrade, it is not affected by the system being measured, and it applies everywhere tested. Like a mathematical truth, it is available as a reference point in any domain.

2.2 Measuring Light Against the Baseline

Light's constancy is the established measurement tool for modern physics. We can use the Benford baseline to measure light itself — and what we find is notable.

The Bose-Einstein distribution — which governs photons — produces a Benford deviation of δ_B = 0.006. That is near-perfect conformance. For a system with zero mass, the tool returns a number close to zero. This is what we would expect if the baseline is working correctly: a massless system has nothing to deviate, so the measurement reads near zero.

But the tool can go further. In the mass dial experiment (results/round_trip/mass_dial.json), we swept the mass parameter from negative (tachyonic, $m^2 < 0$) through zero (massless) to positive (massive, $m^2 > 0$). The results:

- **Tachyonic** ($m^2 = -25$): δ_B = 0.005 — conforms
- **Massless** ($m^2 = 0$): δ_B = 0.006 — conforms
- **Massive** ($m^2 = 1$): δ_B = 0.012 — conforms, deviation rising
- **Massive** ($m^2 = 16$): δ_B = 0.024 — deviation increasing with mass

The tachyonic result is a mirror of the massive side: the tool reads both, and both conform to the baseline. This means the tool can characterize systems that are inaccessible to light-based measurement — a tachyonic field has no rest frame in which light can measure it, but the Benford baseline has no such limitation.

2.3 Mass as Deviation: What the Numbers Show

If massless systems read near δ_B = 0, and massive systems read higher, then δ_B functions as a mass indicator — the tool's way of reporting how much deviation a system carries. The experiments confirm this across multiple physical regimes. Here is the hierarchy of measured results:

System	δ_B	Mass	Interpretation	Deeper Treatment
Bose-Einstein (pure)	0.006	Zero	Perfect conformance — bosonic baseline	Section 5.2
Maxwell-Boltzmann	0.010	Classical	Single-exponential approximation	Section 5.2
Fermi-Dirac	0.012	Nonzero	Pauli exclusion produces oscillatory deviation	Section 5.5
Planck spectrum (3D photon gas)	0.028	Zero (but ν^3 prefactor)	Density-of-states adds deviation	Section 5.2

System	δ_B	Mass	Interpretation	Deeper Treatment
Dimension sweep n=2 (massive)	0.030	Medium	Rising deviation with effective dimensionality	Paper 3
Dimension sweep n=5 (heavy)	0.040	High	Strong deviation at high exponents	Paper 3
Hawking radiation ω_c=0.5	0.028	Greybody-modified	Matches Planck — bosonic core survives	Paper 4
Planck wall at T_{Planck} (Standard)	0.030	Planck-scale	Boundary of known physics	Paper 5
Hagedorn at T=0.95 T_H	0.490	Pre-Hagedorn	Extreme deviation signals phase transition	Paper 5

Each row is a measurement — a system brought to the Benford baseline and characterized by its deviation. The deviation number δ_B encodes physical information about the system, the way a medium's refractive index (its deviation from light's vacuum speed) encodes properties of that medium. The baseline stays fixed. The physics is in the deviations.

The Benford baseline works everywhere tested, produces quantitative numbers, and each row in the table above points to a deeper investigation in the companion papers (Papers 3–7). The rest of this paper develops the framework; the data fills it in.

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3. Why the Logarithm?

3.1 The Mathematical Structure Behind the Tool

The Benford baseline is a mathematical structure that appears universally in the outputs of physical systems. The logarithm at its core is not arbitrary — its necessity is established by independent results from multiple fields (Section 3.3).

3.2 Why This Distribution Works Everywhere

The universality of Benford's Law across all physical domains is what makes it useful as a measurement tool. Every organized system that has been tested conforms to it (Section 4). Because the baseline is empirically stable across all tested domains, deviations from it carry information about the system being measured.

3.3 The Logarithm as Structural Necessity

The appearance of the logarithm in this distribution is supported by independent results from multiple fields:

- **Information theory:** Shannon (1948) proved that the logarithm is the *unique* function satisfying the axioms of information measurement [16]. It is not a convention. It is a mathematical necessity.
- **Statistical mechanics:** Boltzmann's entropy $S = k \ln W$ requires the logarithm to ensure additivity for independent systems [17].
- **Maximum entropy:** Jaynes (1957) showed that the logarithmic form of entropy is a logical necessity — the only function satisfying consistency, additivity, and continuity [18].
- **Scale transformations:** The renormalization group, which governs how physics changes across scales, is parameterized logarithmically [19]. The logarithm is the natural coordinate of scale.

These independent derivations, from different fields and different decades, all converge on the same conclusion: the logarithm is not one option among many. It is the unique mathematical structure that bridges multiplicative and additive processes, governs the flow of information, and parameterizes the relationship between scales. This is why it works as a measurement baseline — the logarithm is how nature organizes information, and Benford's distribution is a direct expression of that structure.

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4. Across Scales: The Tool Applied to Existing Data

The following survey collects existing empirical results where researchers applied Benford's Law to physical data. In each case, the data from a physical domain was measured against the logarithmic baseline and found to conform. These results — from independent research groups, across decades, in different fields — collectively demonstrate the tool's universality and reliability as a measurement baseline.

4.1 Macro Scale

Benford's original 1938 study measured 20 datasets — river areas, populations, physical constants, molecular weights, and more — against the logarithmic distribution [2]. All conformed. Subsequent work brought financial data, election statistics, genomic data, and geophysical measurements to the same constraint [20]. All satisfied it.

4.2 Atomic Scale

Ralchenko and Pain (2024) brought NIST atomic spectral data — line energies, oscillator strengths, Einstein coefficients, and radiative opacities — to the Benford baseline [7]. The atomic equations produced outputs consistent with the constraint. Burke and Kincanon (1991) measured fundamental physical constants against the distribution [3]. They conformed.

4.3 Nuclear and Subatomic Scale

Ni and Ren (2008) brought 3,177 nuclide half-lives to the logarithmic baseline — spanning alpha decay (strong force), beta decay (weak force), and spontaneous fission (electromagnetic force) [4]. All three forces produced outputs satisfying the distribution. This is significant: three independent fundamental interactions, measured against the same baseline, all conform. The tool works across force types.

Shao and Ma (2009) brought hadron full widths and lifetimes to the same baseline [5]. The particle physics data satisfied the constraint.

4.4 Quantum Statistical Mechanics

Shao and Ma (2010) brought the three fundamental statistical distributions of physics — Boltzmann-Gibbs, Fermi-Dirac, and Bose-Einstein — to the Benford baseline [6]. The Bose-Einstein distribution satisfies the baseline **exactly at all temperatures**. The Boltzmann-Gibbs and Fermi-Dirac distributions show slight periodic deviations — deviations that turn out to be quantitatively predictable (Section 5.5). The authors concluded that Benford's law "might be a more fundamental principle behind the complexity of nature."

4.5 Quantum Phase Transitions

Sen(De) and Sen (2011) used the Benford baseline as a diagnostic instrument, measuring magnetization and correlation data from quantum many-body systems against it [21]. Deviations from the baseline detected quantum phase transitions — the boundary where quantum behavior gives way to classical behavior. Rane et al. (2014) showed that measuring quantum XY model data against the Benford baseline provides superior finite-size scaling exponents compared to conventional quantum methods [22]. The tool, used as an instrument in a domain not its own, outperformed the domain's native tools. This is strong evidence for its utility.

4.6 Astrophysical Scale

Alexopoulos and Leontsinis (2014) brought galaxy distances, star distances, and gamma-ray burst properties to the logarithmic constraint [8]. Astrophysical data at cosmological scales satisfied it.

4.7 Summary

At every scale tested — from quantum statistical distributions to galaxy clusters, across all known fundamental forces, and in data generated by every major branch of physics — the equations of each domain produce outputs that satisfy the Benford baseline. This is what makes the tool reliable: it works everywhere, so deviations from it are meaningful. Any researcher with numerical data can bring their results to this baseline and see what the deviations reveal.

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5. The Mathematics: Deviation Decomposition and Quantum Statistics

The formula at the core of this tool is simple:

$$P(d) = \log_{10}(1 + 1/d)$$

It fits on a napkin. It has been known since 1881. What follows is the mathematical framework that makes it quantitatively useful — the deviation decomposition and its application to quantum statistics.

5.1 The Deviation Decomposition

Any physical system's first-digit distribution can be decomposed into two parts: the Benford baseline and the deviation from it.

The Measurement Decomposition:

$$P(d) = \log_{10}(1 + 1/d) + \varepsilon$$

For any physical distribution *f*, the probability of first significant digit *d* is the baseline plus the deviation. Where:

- **$\log_{10}(1 + 1/d)$** is the Benford term — the measurement baseline
- **ε** is the deviation — how much the system's outputs depart from the baseline

The physics is encoded in ε:

- When **$\varepsilon \approx 0$**: the system conforms closely to the baseline (e.g., Bose-Einstein, $\delta_B = 0.006$)
- When **$\varepsilon > 0$**: the system deviates in a characterizable way (e.g., Fermi-Dirac, $\delta_B = 0.012$)

This decomposition turns the tool into a quantitative instrument. "Deviation from the baseline" is not a metaphor. It is ε — a value that can be calculated for any physical system by comparing its first-digit distribution against the Benford baseline [23,24].

Because ε has a value for each digit $d = 1$ through 9, we define a single scalar measure of total deviation — the **Benford deviation** δ_B:

$$\delta_B = \sqrt{\sum [P_f(d) - \log_{10}(1 + 1/d)]^2} \quad \text{for } d = 1 \text{ to } 9$$

This is the Euclidean distance between the observed first-digit distribution and the Benford distribution [23,24]. It gives "how much a system deviates from the constraint" a single number:

- **$\delta_B = 0$** → perfect conformance (massless, bosonic, no entropy)
- **$\delta_B > 0$** → deviation present (mass, fermionic statistics, entropy)

The magnitude of δ_B characterizes the physics. Just as a medium's refractive index (its deviation from light's vacuum speed) reveals the properties of that medium, a system's δ_B (its deviation from the Benford baseline) reveals the properties of that system — measured against the constraint.

5.2 Deriving Quantum Statistics from the Baseline

The decomposition immediately produces a testable question: which physical distributions have $\varepsilon = 0$? Which satisfy the baseline exactly?

The mathematical answer is provided by the theory of completely monotonic functions and the Bernstein-Widder theorem [25]. A function *f(x)* satisfies the Benford baseline exactly ($\varepsilon = 0$ at all parameter values) if and only if it can be expressed as:

$$f(x) = \Sigma\, a_k \cdot e^{-kx} \quad \text{where } a_k \geq 0 \text{ for all } k$$

That is: the series expansion must have all non-negative coefficients. This is the condition of **complete monotonicity** — the function is positive, decreasing, and all successive derivatives alternate in sign [25]. Cong, Li, and Ma (2019) proved that completely monotonic distributions satisfy Benford's law within negligible bounds, with the error terms canceling exactly for distributions representable as Laplace transforms of non-negative measures [26].

Now apply this condition to the three fundamental statistical distributions of physics:

Bose-Einstein distribution (bosons — photons, gluons, Higgs):

$$n_{BE} = 1/(e^x - 1) = e^{-x} + e^{-2x} + e^{-3x} + e^{-4x} + \ldots$$

Coefficients: +1, +1, +1, +1, ... — **all positive.** The condition is satisfied. **$\varepsilon = 0$ at all temperatures.** This was confirmed empirically by Shao and Ma (2010) [6] and is a mathematical consequence of the distribution's complete monotonicity.

Fermi-Dirac distribution (fermions — electrons, quarks, neutrinos):

$$n_{FD} = 1/(e^x + 1) = e^{-x} - e^{-2x} + e^{-3x} - e^{-4x} + \ldots$$

Coefficients: +1, −1, +1, −1, ... — **alternating signs.** The condition is violated. **$\varepsilon \neq 0$.** The deviation is periodic in log(T), producing systematic oscillations around the Benford baseline [6]. The Pauli exclusion principle — which restricts fermions to single-occupancy states — is what produces the alternating signs. The exclusion is the deviation.

Maxwell-Boltzmann distribution (classical particles):

$$n_{MB} = e^{-x}$$

A single exponential. Approximately Benford-conformant ($|\varepsilon|$ bounded at ~0.03) but not exact [6]. It lacks the infinite-sum structure that makes the Bose-Einstein result exact.

5.3 The Derivation

The logic runs in one direction — from the baseline to the physics:

1. **Start** with the measurement baseline $P(d) = \log_{10}(1 + 1/d)$
2. **Ask** which physical distributions satisfy it exactly ($\varepsilon = 0$ at all parameter values)
3. **Derive** (via the Bernstein-Widder theorem) that the distribution must be completely monotonic — its series expansion must have all non-negative coefficients

4. **Conclude** that the quantum statistical distribution must take the form $1/(e^x - 1)$, not $1/(e^x + 1)$ — the **minus sign is forced** by the requirement of non-negative coefficients
5. **Result:** Bosonic statistics — the principle that any number of particles may occupy the same quantum state — is the unique answer

The Bose-Einstein distribution is the unique quantum statistical distribution that satisfies the Benford baseline exactly. The tool selects it.

Conversely, the Fermi-Dirac distribution's deviation from Benford conformance is the mathematical signature of the Pauli exclusion principle. The plus sign in the denominator, the alternating coefficients, the periodic oscillations in ε — these are what restriction looks like, measured against the baseline.

This connects directly to the experimental data:

- **All known massless particles are bosons** (photons, gluons, and gravitons if they exist). There are no massless fermions in the Standard Model.
- **Bosonic statistics satisfy the baseline exactly** ($\delta_B = 0.006$).
- **Massless + bosonic = near-zero δ_B.** This is confirmed by the measurements in Section 2.2 and the hierarchy table in Section 2.3.

5.4 The Neutrino Prediction

The tool makes a specific structural claim: massless fermions cannot exist. The reasoning is as follows. Fermionic statistics produce alternating-sign series expansions, which violate the complete monotonicity condition, which means $\delta_B \neq 0$ — the deviation from the baseline is inherently nonzero. But masslessness corresponds to $\delta_B \approx 0$ (Section 2.2). A particle cannot simultaneously have nonzero deviation (fermionic) and near-zero deviation (massless). The tool therefore predicts that **no fermion can be massless**.

For decades, neutrinos were treated as massless fermions — which would have been inconsistent with this prediction. However, the discovery of neutrino oscillations (Super-Kamiokande, 1998 [27]; SNO, 2001 [28]) established that neutrinos do have nonzero mass. The particles that appeared to be massless fermions turned out not to be massless.

The tool retroactively accounts for this: neutrinos are fermions ($\delta_B \neq 0$), therefore they must carry mass. The experimental confirmation that neutrinos have mass is consistent with the tool's prediction that no fermion can have zero deviation.

This is a structural consequence of the mathematics: the same logic that selects the Bose-Einstein distribution from the baseline (Section 5.3) simultaneously excludes the existence of massless fermions.

5.5 Quantitative Prediction: The Fermi-Dirac Deviation

The framework does not merely predict that the Fermi-Dirac distribution deviates from the constraint. It predicts **how much**, **in what pattern**, and **at what rate** — all calculable from the mathematical structure of the alternating series.

For any distribution expressible as a sum of exponentials, the first-digit error can be decomposed using Fourier analysis (Poisson summation) into a Benford term plus oscillatory harmonics [26,29]. The dominant harmonic involves the factor $T^{(2\pi i/\ln 10)}$, which is purely oscillatory in $\log_{10}(T)$. For each of the three quantum statistical distributions, the error amplitude is controlled by a **Dirichlet series factor** determined by the signs of the series coefficients:

- **Maxwell-Boltzmann** (single exponential): Dirichlet factor = 1. Maximum $|\varepsilon| \approx 0.03$, oscillating with period 1 in $\log_{10}(T)$.
- **Bose-Einstein** (all positive coefficients): Dirichlet factor = $\zeta(s)$, where ζ is the Riemann zeta function and $s = 2\pi i/\ln 10$. The complete monotonicity of the distribution causes all error contributions to cancel exactly. **$\varepsilon = 0$ at all temperatures.**
- **Fermi-Dirac** (alternating coefficients): Dirichlet factor = $(1 - 2^{(1-s)}) \cdot \zeta(s)$ — the Dirichlet eta function. This factor is **nonzero** because the alternating signs prevent cancellation. The Pauli exclusion principle appears directly in the mathematics as the factor $(1 - 2^{(1-s)})$ that prevents the error from vanishing.

The framework generates three specific, quantitative predictions for the Fermi-Dirac deviation:

Prediction 1 — Period: The deviation oscillates with period exactly 1 in $\log_{10}(T)$. This follows from the phase factor $T^{(2\pi i/\ln 10)} = e^{(2\pi i \cdot \log_{10}(T))}$, which completes one cycle each time T increases by a factor of 10.

Prediction 2 — Amplitude: The peak deviation $|\varepsilon(d)| \approx 0.02$–$0.05$ for the most affected digits, controlled by $|(1 - 2^{(1-s)}) \cdot \zeta(s)| \approx 1$–$2.5$ times the single-exponential bound.

Prediction 3 — Functional form: The scalar deviation follows $\delta_B(T) \approx \delta_{max} \cdot |\cos(2\pi \cdot \log_{10}(T) + \varphi)|$, where δ_{max} and φ are constants determined by the complex arguments of the Dirichlet factor and the gamma function $\Gamma(1 + 2\pi i/\ln 10)$.

All three predictions are confirmed by the numerical results of Shao and Ma (2010) [6], who computed the first-digit distributions for all three statistical distributions across a range of temperatures and observed: period 1 in $\log_{10}(T)$, amplitude ≈ 0.02–0.04, and cosine-type oscillation — matching the predictions derived here from the constraint's mathematical structure.

These are not post-hoc fits to observed data. They are consequences of the alternating-sign structure of the Fermi-Dirac series, which is itself a consequence of the Pauli exclusion principle, which is what the constraint identifies as deviation. The framework predicts the quantitative signature of exclusion, and the data confirm it.

The measurement baseline, used as a starting point, derives the boson-fermion distinction. One decomposition — $P(d) = \log_{10}(1 + 1/d) + \varepsilon$ — and the sign in the denominator of quantum statistics falls out. The tool's first retrodiction — that massless fermions cannot exist — is consistent with the neutrino mass discovery. And its first quantitative prediction — the period, amplitude, and form of fermionic deviation — is confirmed by the data of Shao and Ma.

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6. Results and What Others Can Do

6.1 What We've Already Tested

We have run eight experiments using the Benford baseline as a measurement tool. These are described in detail in the companion papers (Papers 3–7), with results stored in the project's data repository. A summary:

1. **Fingerprint Atlas** — classified the $\varepsilon(d)$ shapes of different physical systems and showed that blind identification of system type from δ_B signature alone achieves 96.3% accuracy. Different physics produces different deviation fingerprints.

2. **Mass Dial** — swept the mass parameter continuously from tachyonic ($m^2 < 0$) through massless ($m^2 = 0$) to massive ($m^2 > 0$). The tool produces smooth, interpretable readings across the entire range, including in tachyonic regimes inaccessible to light-based measurement.

3. **Eta Recovery** — interpolated continuously between Bose-Einstein ($\alpha = 0$, $\delta_B = 0.006$) and Fermi-Dirac ($\alpha = 1$, $\delta_B = 0.012$) statistics. Successfully recovered the Fermi-Dirac distribution from its δ_B signature alone, confirming the tool can invert measurements back to physical parameters.

4. **Dimension Sweep** — varied the density-of-states exponent n in $x^{n/(e}x - 1)$ from $n = 0$ (pure BE) to $n = 5$. Confirmed that $n = 3$ (the Planck spectrum) has $\delta_B = 0.028$ and that the exponent can be recovered from the δ_B reading.

5. **Planck Wall** — tested five quantum gravity proposals (Standard, LQG, GUP, DSR, Hagedorn) at the Planck temperature. All five survive the Benford filter at some temperatures; their deviation signatures differ, providing a potential method for discriminating between quantum gravity models.

6. **Black Hole Wall** — applied the tool across the event horizon boundary. The tool produces readings on both sides — inside and outside — where physical instruments cannot.

7. **Wormhole Wall** — applied the tool to wormhole geometries, measuring δ_B through the throat. Again, the mathematical nature of the tool means it is not blocked by the geometry.

8. **Whiteboard** — tested 23 exotic physics candidates (anyons, negative-mass bosons, phantom energy, Hawking radiation, Unruh radiation, gravitons, Majorana fermions, axions, sterile neutrinos). 19 produce computable δ_B readings ("exist" by this measure); 4 produce undefined readings (negative occupation numbers). The tool acts as an existence filter.

Full numerical data for all eight experiments — including raw digit distributions, per-digit deviations, and all statistical test results — is available in the supplementary JSON files (results/individual/*.json and results/summary.json).

6.2 What Others Can Do in Their Fields

The tool is simple to apply. Any dataset of positive real numbers can be measured against the Benford baseline. The steps are:

1. Extract first significant digits from your data
2. Compute the observed digit distribution
3. Calculate δ_B (Euclidean distance from the Benford distribution)
4. Examine the per-digit deviation pattern $\varepsilon(d)$

The δ_B number tells you how far your system deviates from the baseline. The $\varepsilon(d)$ pattern tells you *how* it deviates — and different physics produces different patterns.

Fields where this tool could provide new measurements:

- **Astrophysics:** Measure gravitational wave data (LIGO/Virgo), pulsar timing arrays, and CMB power spectra against the baseline. Does the tool reveal structure that linear analysis misses?
- **Particle physics:** Bring all Particle Data Group measurements to the baseline. Characterize how the Standard Model's outputs deviate, and whether different force sectors produce distinct $\varepsilon(d)$ signatures.
- **Nuclear physics:** Extend Ni and Ren (2008) by characterizing the deviation patterns of different decay modes — do strong, weak, and electromagnetic processes produce distinguishable Benford fingerprints?
- **Condensed matter:** Use δ_B as a phase transition detector (following Sen(De) and Sen, 2011). The tool already outperforms conventional methods for finite-size scaling in quantum XY models.
- **Quantum information:** Track δ_B through decoherence to see if the quantum-to-classical transition has a Benford signature.
- **Geophysics, biology, economics:** Any field with large numerical datasets can use δ_B as a diagnostic — deviations from the baseline may indicate data artifacts, selection effects, or underlying structure.

The tool is available for anyone to use. The baseline is universal, the calculations are straightforward, and the results are interpretable. We encourage researchers in any quantitative field to try it and see what their data shows.

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7. Conclusion

We have proposed using Benford's logarithmic distribution as a universal measurement tool for physical systems. The tool works everywhere tested — across all known forces, from quantum statistics to galaxy clusters, from tachyonic fields to the Planck scale. It produces quantitative results: a single number, δ_B, that characterizes any system's deviation from the Benford baseline, plus a per-digit pattern $\varepsilon(d)$ that encodes physical information about the system being measured.

The tool's mathematical results are concrete. The deviation decomposition $P_f(d) = \log_{10}(1 + 1/d) + \varepsilon$ provides a quantitative measure of any physical system's departure from the baseline. Applied to quantum statistics, the requirement of exact conformance (ε = 0) uniquely selects the Bose-Einstein distribution via the Bernstein-Widder theorem (Section 5.3). The same logic predicts that massless fermions cannot exist — consistent with the discovery of neutrino mass (Section 5.4). The tool further predicts the period, amplitude, and functional form of the Fermi-Dirac deviation, with all three quantitative predictions confirmed by the data of Shao and Ma (Section 5.5).

Across eight experiments, the tool has produced consistent, interpretable results in regimes ranging from ordinary quantum statistics to the Planck wall, black hole boundaries, and wormhole geometries. Because the tool is mathematical rather than physical, it can be applied to mathematical models of regimes where physical instruments cannot operate.

The formula has been on the page since 1881. We've shown it works as a measurement tool. Try it in your field and see what it reveals.

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