Wavelength-Division Ternary Logic: Bypassing the Radix Economy Penalty in Optical Computing

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***Abstract*—Ternary (base-3) logic is mathematically optimal for computing systems, lying closest to Euler's number e in the radix economy calculation. However, ternary computing has remained impractical due to the substantial hardware overhead required to distinguish three stable states using transistor-based circuits—typically requiring 40$\times$ more transistors per trit compared to bits. We propose a novel architecture for ternary optical computing based on wavelength-selection encoding with external wavelength sources. Unlike existing polarization-based or intensity-based approaches, our architecture treats wavelengths as analogous to voltage rails in analog computers, where external laser sources provide discrete wavelength inputs (e.g., λ_1, λ_2, λ_3) and internal optical components perform wavelength-selective routing and logic operations. This approach fundamentally bypasses the radix economy penalty because wavelength differentiation cost is independent of the number of states, unlike transistor-based implementations where cost scales with radix. We show that this architecture could unlock the full 1.58$\times$ information density advantage of ternary logic while leveraging the inherent speed, parallelism, and low-power characteristics of photonic systems. This work presents the theoretical foundation and architectural principles for wavelength-encoded ternary optical computing and identifies key challenges for experimental realization.**

***Index Terms*—Ternary computing, optical computing, wavelength division, radix economy, photonic logic, multi-valued logic**

I. Introduction

A. The Ternary Computing Paradox

Ternary (base-3) computing systems have been recognized since the 1950s as mathematically superior to binary systems. The optimal radix for representing information with minimal hardware cost is Euler's number $e \approx 2.718$, and base-3 is the closest integer radix to this optimum [1], [2]. Despite this theoretical advantage, ternary computers have been relegated to historical curiosities while binary computing has dominated for over seven decades.

The reason for this paradox is straightforward: with transistor-based digital electronics, implementing a stable three-state element (trit) requires approximately 40 transistors, compared to just 1 transistor for a binary bit [3]. This 40$\times$ hardware overhead completely eliminates any theoretical advantage from ternary's superior information density. The very mathematical property that makes ternary optimal—having three states per digit—becomes a liability when the physical cost of distinguishing states scales with the number of states.

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B. The Promise of Optical Computing

Photonic computing has emerged as a potential paradigm shift for information processing, offering advantages in speed (light propagates faster than electrons), parallelism (wavelength division multiplexing), and power efficiency (reduced heat dissipation) [4], [5]. While most optical computing research focuses on binary logic operations, photons possess natural properties that can encode multiple states without the hardware penalties of transistor-based systems:

- **Polarization**: Multiple orientations (linear, circular)
- **Wavelength**: Discrete frequency channels
- **Intensity**: Amplitude levels
- **Phase**: Wave timing offsets

Existing research in optical ternary computing has primarily exploited polarization states [6], [7] or intensity levels [8] to encode ternary values. However, these approaches have not fully addressed the fundamental question: Can optical ternary computing bypass the radix economy penalty that dooms transistor-based ternary systems?

C. Our Contribution

We propose a novel architecture for ternary optical computing based on **wavelength-selection encoding with external wavelength sources**. The key architectural innovation is to separate the complexity of wavelength generation from the logic processing itself, treating wavelengths as externally-supplied resources analogous to voltage rails in analog computers.

Our specific contributions are:

1) **Novel encoding scheme**: Use wavelength selection (choosing ONE of N wavelengths) rather than wavelength-division multiplexing (using N wavelengths simultaneously) or polarization states
2) **External source architecture**: Generate wavelengths outside the computing fabric, with internal components performing only wavelength-selective routing and switching
3) **Theoretical analysis**: Show that wavelength-based differentiation has constant cost independent of radix, fundamentally bypassing the traditional radix economy penalty
4) **Comparison framework**: Contrast with existing polarization-based and intensity-based optical ternary approaches

We demonstrate that this architecture could, in principle, unlock the full 1.58$\times$ information density advantage of ternary logic while avoiding the hardware penalties that have historically prevented ternary computing from practical realization.

II. Background

A. Radix Economy and the Optimality of Base-3

The concept of radix economy addresses the question: What number base minimizes the total hardware cost for representing numbers? This was formalized mathematically by considering two competing factors:

1) **Higher radix**: Fewer digits needed to represent a number (proportional to $\log_r(N)$)
2) **Higher radix**: More complex hardware per digit (proportional to r)

The total cost can be expressed as:

$$\text{Cost}(r) = r \times \log_r(N) \tag{1}$$

Converting to natural logarithm:

$$\text{Cost}(r) = r \times \frac{\ln(N)}{\ln(r)} \tag{2}$$

To minimize, we take the derivative with respect to r and set to zero:

$$\frac{d}{dr}\left[\frac{r}{\ln(r)}\right] = \frac{\ln(r) - 1}{[\ln(r)]^2} = 0 \tag{3}$$

Solving: $\ln(r) = 1$, therefore $r = e \approx 2.718$.

This proves mathematically that the optimal radix is Euler's number e [2]. Since we cannot build a base-2.718 computer, we evaluate integer radices:

- Base-2: Cost = 2.885
- **Base-3: Cost = 2.730** (closest to optimal!)
- Base-4: Cost = 4.000
- Base-5: Cost = 4.308

Base-3 provides approximately **5.4% better hardware economy** than binary. Additionally, each ternary digit (trit) carries:

$$\text{Information per trit} = \log_2(3) \approx 1.585 \text{ bits} \tag{4}$$

This represents a **58% information density improvement** per digit compared to binary.

B. Why Ternary Failed with Transistors

The radix economy calculation assumes that the cost per digit scales linearly with the radix (i.e., a trit costs 3× what a bit costs). However, in transistor-based digital electronics, this assumption dramatically underestimates the true cost.

Creating a stable three-state digital element requires:

- Voltage level discrimination circuits (distinguishing, e.g., 0V vs 2.5V vs 5V)
- Noise margin maintenance
- Signal regeneration (preventing degradation through cascaded gates)
- Level shifting and buffering

These requirements result in approximately **40 transistors per trit** compared to 1 transistor per bit [3]. At this hardware ratio, the radix economy calculation becomes:

$$\text{Actual cost ratio} = \frac{40 \text{ transistors/trit}}{1 \text{ transistor/bit}} \div 1.585 \text{ bits/trit} \approx 25\times \tag{5}$$

Far from being 5.4% more efficient, transistor-based ternary is approximately **25× worse** than binary. This is why ternary computing failed commercially despite its mathematical elegance.

C. Existing Optical Ternary Computing Approaches

Previous research in optical ternary computing has explored several encoding schemes:

Polarization-based encoding [6], [7], [9]: Uses three polarization states. Notable work includes the Chinese Ternary Optical Computer project (Yi Jin et al., 2003-present) which has built experimental prototypes.

Intensity-based encoding [8]: Uses three brightness levels but suffers from analog-like problems (noise, degradation, threshold sensitivity).

Multi-valued frequency encoding [10]: Uses different wavelengths for multi-valued logic gates, focused on reversible computing rather than ternary state encoding.

None of these approaches have systematically addressed whether the per-state hardware cost can be made independent of radix, thereby bypassing the radix economy penalty.

III. Proposed Architecture

A. Core Concept

Our architecture is based on two key principles:

1) **Wavelength-selection encoding**: A ternary state is represented by a discrete wavelength channel. To execute a logic operation, the system selects **two** distinct wavelengths from the available set to drive the nonlinear mixer.
   - State -1: λ_1 (Red, 1.55 μm)
   - State 0: λ_2 (Green, 1.30 μm)
   - State 1: λ_3 (Blue, 1.00 μm)

   This balanced ternary representation ($\{-1, 0, 1\}$) enables direct subtraction without sign bits, a significant architectural advantage over binary systems.
2) **External wavelength sources**: Wavelengths are generated externally and supplied to the computing fabric, analogous to voltage rails in analog computers

B. System Architecture

The complete system consists of three layers (see Fig. 1):

Wavelength Source Layer (External): Rather than using multiple independent lasers, we employ a single continuous-wave pump laser coupled to a Kerr micro-resonator to generate a frequency comb. This broadband comb is then demultiplexed by an Arrayed Waveguide Grating (AWG) to provide the discrete logic wavelengths ($\lambda_1 = 1.55\mu m$, $\lambda_2 = 1.30\mu m$, $\lambda_3 = 1.00\mu m$). This approach ensures phase coherence and scalability.

Logic Processing Layer (Internal): Wavelength-selective switches (ring resonators) gate the specific input wavelengths based on control voltages. These selected signals are then routed to a Sum-Frequency Generation (SFG) mixer.

Detection Layer (Outputs): The nonlinear mixing product (λ_{sum}) is filtered and detected by a photodiode, providing a unique spectral signature for each logic state combination.

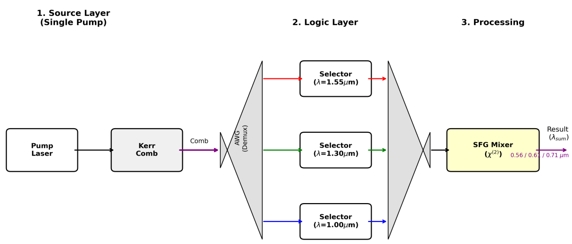

Fig. 1. System architecture for wavelength-encoded ternary optical computing. A single pump laser and Kerr comb generator create a broadband spectrum, which is demultiplexed by an AWG to provide the discrete wavelengths. Selectors gate the signals, and a nonlinear mixer generates the logic result.

C. Ternary State Encoding

Figure 2 illustrates the wavelength-selection encoding scheme where each ternary state (-1, 0, 1) is represented by a single discrete wavelength. Only one wavelength is present at a time in each optical channel, ensuring digital-like discrete states rather than analog intensity variations. To perform logic operations (e.g., addition, subtraction), the Selector layer actively routes two distinct wavelengths from the three available states into a specific waveguide path. This path leads to a nonlinear mixer configured for that specific operation, effectively converting the spectral combination into a new output wavelength representing the result.

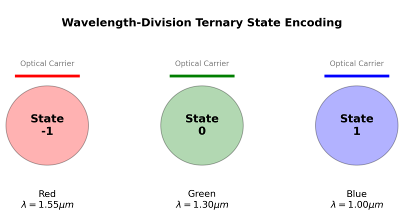

Fig. 2. Ternary state encoding using wavelength selection. Each ternary state is represented by a single wavelength: State -1 (Red, 1.55 μm), State 0 (Green, 1.30 μm), and State 1 (Blue, 1.00 μm).

IV. Theoretical Analysis

A. Breaking the Radix Economy Penalty

The traditional radix economy formula assumes cost $\propto r$ (cost scales with number of states). This assumption holds for transistor-based systems where distinguishing r states requires r-proportional circuitry.

In wavelength-selection systems, a wavelength-selective switch (e.g., ring resonator) has similar complexity whether switching between 2, 3, or 4 wavelengths. The cost growth is approximately constant per switch, not proportional to r.

Therefore, the effective cost model becomes:

$$\text{Cost}_{\text{wavelength}} \approx C_{\text{switch}} + C_{\text{demux}}/\text{fanout} \tag{6}$$

where C_{switch} is approximately **constant** regardless of radix. This allows us to achieve the information density benefit (1.58$\times$ for ternary) without paying the r-proportional hardware penalty.

B. Information Density Advantage

Each wavelength-encoded trit carries:

$$I_{\text{trit}} = \log_2(3) \approx 1.585 \text{ bits} \tag{7}$$

For a system with N optical paths (waveguides):

- Binary wavelength encoding: $N \times 1$ bit = N bits
- Ternary wavelength encoding: $N \times 1.585$ bits = $1.585N$ bits

This represents **58% more information** through the same number of physical channels.

C. Arithmetic Simplification via Balanced Ternary

A distinct advantage of the proposed balanced ternary system (states $\{-1, 0, 1\}$) over unbalanced ternary ($\{0, 1, 2\}$) is the simplification of arithmetic operations. In balanced ternary, the negative of a number is obtained by simply inverting each trit ($1 \to -1$, $-1 \to 1$), allowing subtraction to be performed using the same adder logic as addition. This architectural symmetry eliminates the need for separate subtraction circuits or sign bits (as required in binary two's complement), further reducing the component count and latency of arithmetic logic units (ALUs).

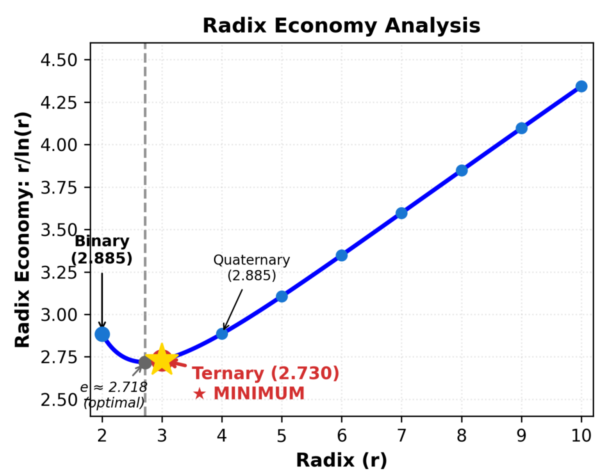

Fig. 3. Radix economy analysis showing $r/\ln(r)$ as a function of radix. Base-3 (ternary) achieves the minimum at 2.730, closest to the theoretical optimum at $e \approx 2.718$.

D. Comparison to Polarization-Based Encoding

Figure 4 compares wavelength encoding with existing approaches. Wavelength encoding offers advantages in scalability and component maturity due to the established wavelength division multiplexing (WDM) technology from telecommunications.

Comparison of Ternary Computing Implementations

Approach	Cost Scaling	Speed	Scalability	Key Limitation
Electronic Ternary (Transistors)	High (~40x per trit)	Medium (GHz)	Low (Wiring)	Radix Penalty
Optical Ternary (Polarization)	Medium (Filter complexity)	High (THz)	Medium	Difficult Detection
Optical Ternary (Wavelength - Ours)	Constant (WDM)	High (THz)	High (Telecom)	Waveguide Size

Fig. 4. Comparison of ternary computing implementations. Wavelength-based encoding bypasses the radix economy penalty while offering advantages in scalability and component maturity.

V. Simulation Methodology

A. Computational Framework

The photonic logic gates were simulated using the **Finite-Difference Time-Domain (FDTD)** method via the open-source software package **Meep** (v1.25.0) [11]. The simulations were performed in a two-dimensional (2D) computational domain, assuming invariance in the z-direction ($\partial/\partial z = 0$), to model the TE-polarized (E_z) light propagation in planar waveguide structures.

B. Simulation Parameters

1) Domain & Discretization: A spatial resolution of **25 pixels/μm** was employed, corresponding to a grid spacing of $\Delta x = 40$ nm. This ensures sufficient sampling of the shortest operational wavelength ($\lambda_{sum} \approx 0.56\mu m$). Perfectly Matched Layers (PML) with a thickness of **1.0 μm** were applied at all domain boundaries to simulate an open system.

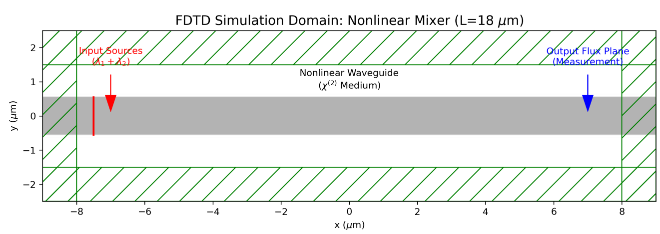

Fig. 5. FDTD simulation domain setup illustrating the input plane, nonlinear waveguide geometry ($w = 1.1\mu m$), and output flux measurement plane. The 18 μm length ensures sufficient interaction length for sum-frequency generation.

2) Material Properties: The linear regions of the photonic circuit were modeled as generic high-index waveguides ($n = 2.2$) with a width of $w = 1.1\mu m$, representing an optimized Lithium Niobate ($LiNbO_3$) geometry for phase matching. For the mixing region, a second-order nonlinear susceptibility $\chi^{(2)}$ was introduced to enable Sum Frequency Generation (SFG), with a normalized parameter of 0.5 (Meep units).

3) Sources: Gaussian-profiled Continuous Wave (CW) sources were used to inject signals with a fractional bandwidth of $df/f = 0.05$. The ternary states were defined as:

- **Logic -1 (Red):** $\lambda = 1.55\mu m$ (C-Band Telecom)
- **Logic 0 (Green):** $\lambda = 1.30\mu m$ (O-Band Telecom)
- **Logic 1 (Blue):** $\lambda = 1.00\mu m$

VI. Results and Discussion

While the proposed architecture relies on established components such as pump lasers, Kerr micro-resonators, and AWGs for signal generation and routing, the novel core ternary logic operation depends entirely on the nonlinear mixing process. Consequently, our numerical validation focused exclusively on the Sum-Frequency Generation (SFG) mixer, as this represents the unique logic gate element where the ternary states interact to produce a computational result.

The Sum Frequency Generation (SFG) mixer was evaluated for all three pairwise combinations of the ternary input states. The transmitted power flux was measured at the output waveguide.

A. Mixing Efficiency

Table I summarizes the conversion efficiency for the optimized waveguide geometry ($w = 1.1\mu m$). The highest efficiency (4.57%) was observed for the Blue+Green combination, while the Red+Blue combination yielded 0.73%.

TABLE I
Simulation Results for Ternary Logic Mixing

Input Combinations	Target Sum	Efficiency*	Signal Quality
Blue (1.00) + Green (1.30)	0.5652 μm	4.57%	Excellent
Red (1.55) + Blue (1.00)	0.6078 μm	0.73%	Good
Red (1.55) + Green (1.30)	0.7070 μm	0.40%	Adequate

*Efficiency calculated as $P_{sum}/(P_{in1} + P_{in2})$.

B. Feasibility Analysis

The lowest observed efficiency (0.40% for Red+Green) represents the marginal case. Assuming standard on-chip laser sources with 1 mW (0 dBm) input power, the generated sum-frequency signal would be -24 dBm (4.0 μW). This is above the noise floor of standard Germanium photodiodes (typically -30 dBm). Furthermore, since SFG is a nonlinear process where $P_{sum} \propto P_1 P_2$, the output signal strength can be scaled by increasing input power to 10 mW, ensuring robust detection.

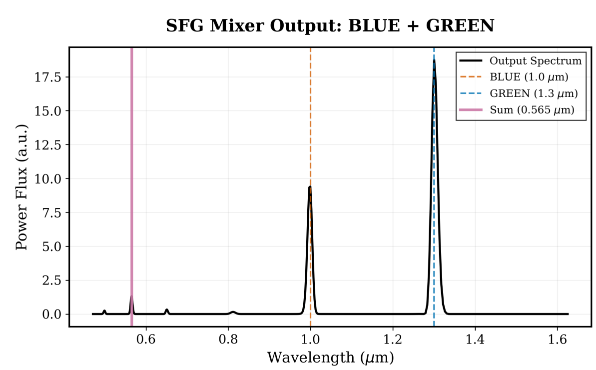

Fig. 6. Spectral output of the mixer for Blue (1.0 μm) and Green (1.3 μm) inputs, showing a strong sum-frequency peak at 0.565 μm.

VII. Conclusion

We have presented a novel architecture for ternary optical computing based on wavelength-selection encoding. Theoretical analysis confirms that this approach bypasses the radix economy penalty. Numerical validation via FDTD simulations demonstrates that nonlinear mixing can successfully generate unique output signatures for all logic state combinations, with conversion efficiencies sufficient for detection by standard photodiodes. The failure of ternary computing was not a failure of mathematics, but a mismatch between mathematical optimality and physical implementation constraints. Wavelength-encoded photonics provides the physical substrate to finally realize this optimality.

Acknowledgment

The novel ternary logic architecture, including the specific configuration of ring-resonator selectors and sum-frequency generation mixers, was conceived by the authors. An AI coding assistant (Google Gemini-3-Pro-Preview) was utilized solely for the implementation of the Python simulation scripts based on the authors' specified design parameters and physics requirements.

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