Simulation and reconstruction of in-line holograms acquired with plane waves

Tatiana Latychevskaia
Physics Department, University of Zurich, Winterthurerstrasse 190, 8057 Zurich, Switzerland
tatiana@physik.uzh.ch

For more details, please read the full article [1].

1 Introduction

The principle of hologram formation with a plane wave is illustrated in Fig. 1.

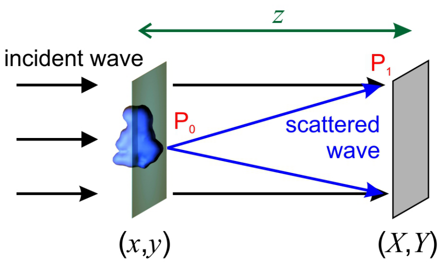

Figure 1: In-line holography schemes realized with a plane wave.

By selecting the optical axis along the propagation of the plane wave we obtain the incident wave:

$$U_{\text{incident}}(x, y) = 1. \tag{1}$$

The exit wave behind the object is given by the object transmission function:

$$U_{\text{exit wave}}(x, y) = t(x, y). \tag{2}$$

1.1 Simulation of hologram

The wave is propagated from the object plane (x, y) towards the detector plane (X, Y), and in the detector plane the hologram is formed as squared amplitude of the wavefront in the detector plane:

$$U_{\text{detector}} H(X, Y) = |U(X, Y)|^2. \tag{3}$$

1.2 Reconstruction of hologram

The reconstruction of a digital hologram recorded with plane waves begins with illumination with the reference wave $R(X, Y) = 1$ followed by propagation backward to the object plane.

2 Angular Spectrum Method

The angular spectrum method was first described by J. A. Ratcliffe [2], and has been explained in detail by J.W. Goodman in his book [3]. The angular spectrum method is based on the notion, that plane wave propagation can be described by the propagation of its spectrum. The components of the scattering vector

$$\vec{k} = \frac{2\pi}{\lambda}(\cos\varphi \sin\theta, \sin\varphi \sin\theta, \cos\theta) \tag{4}$$

are related to the Fourier domain coordinates (u, v) as following:

$$\begin{aligned} \cos\varphi \sin\theta &= \lambda u \\ \sin\varphi \sin\theta &= \lambda v \end{aligned} \tag{5}$$

whereby $(\lambda u, \lambda v)$ are the direction cosines of the vector $\vec{k}$, and therefore the following condition is fulfilled:

$$(\lambda u)^2 + (\lambda v)^2 \leq 1 \tag{6}$$

The complex-valued exit wave $U_{\text{exit wave}}(x, y) = t(x, y)$ is propagated to the detector plane by calculation of the following transformation [3]:

$$U_{\text{detector}}(X, Y) = \text{FT}^{-1}\left[\text{FT}\left(t(x, y)\right)\exp\left(\frac{2\pi i z}{\lambda}\sqrt{1-(\lambda u)^2-(\lambda v)^2}\right)\right], \tag{7}$$

where (u, v) denote the same Fourier domain coordinates as defined above. The reconstruction of the hologram is calculated by using the formula:

$$U(x, y) = \text{FT}^{-1}\left[\text{FT}\left(H(X, Y)\right)\exp\left(-\frac{2\pi i z}{\lambda}\sqrt{1-(\lambda u)^2-(\lambda v)^2}\right)\right]. \tag{8}$$

The term $\exp\left(\pm\frac{2\pi i z}{\lambda}\sqrt{1-(\lambda u)^2-(\lambda v)^2}\right)$ has to be simulated, and it has non-zero values for the range of $(\lambda u, \lambda v)$ constrained by Eq. 6.

References

[1] T. Latychevskaia and H.-W. Fink, "Practical algorithms for simulation and reconstruction of digital in-line holograms," Applied Optics **54,** 2424 – 2434 (2015).

[2] J. A. Ratcliffe, "Some aspects of diffraction theory and their application to the ionosphere," Reports on progress in physics **19,** 188 – 267 (1965).

[3] J. W. Goodman, *Introduction to Fourier optics*, 3 ed. (Roberts & Company Publishers, 2004).