p8131_hw6_xy2395

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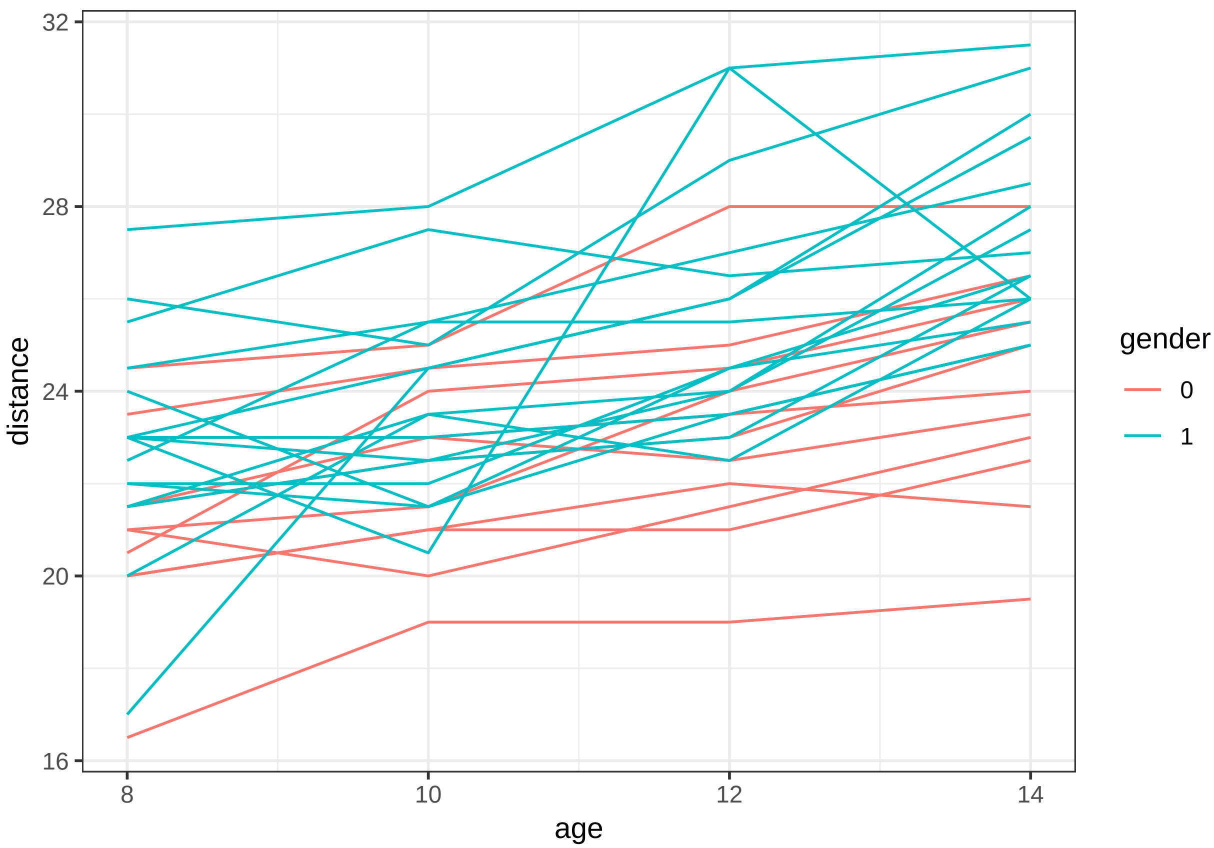
Problem 2

2.1 Spaghetti Plot

```
# Import data
dental <-
  read.table('HW6-dental.txt', header = TRUE) %>%
  as.tibble() %>% janitor::clean_names() %>%
  mutate(gender = as.factor(gender))
```

```
# Spaghetti plot
dental %>%
ggplot(aes(x = age, y = distance, group = child, color = gender)) +
  geom_line() +
  theme_bw()
```



2.2 Marginal Form

$$\begin{aligned}
E\left(Y_{ij}\right) &= E(\beta_0 + a_i + b_0 * I_{(sex_i=0)} + b_1 * I_{(sex_i=1)} + \beta_1 * age_{ij} + e_{ij}) \\
&= \beta_0 + \beta_1 * age_{ij} \\
Var\left(Y_i\right) &= Var\left(a_i + e_{ij} + b_k\right) \\
&= Var\left(a_i\right) + Var\left(e_{ij}\right) + Var\left(b_k\right) \\
&= \sigma_a^2 + \sigma_e^2 + \sigma_b^2
\end{aligned}$$

2.3 Comparing models with different covariance patterns

For the following 3 models, we assume equal variance across measurements at different ages.

```
# Compound Symmetry covariance
compsym = gls(distance ~ gender + age,
              data = dental,
              correlation = corCompSymm(form = ~1 | child),
              method="REML")
# Exponential covariance
expo = gls(distance ~ gender + age,
           data = dental,
           correlation = corExp(form = ~1 | child),
           method = 'REML')
# Autoregressive covariance
auto1 = gls(distance ~ gender + age,
            data = dental,
            correlation = corAR1(form = ~1 | child),
            method = 'REML')
```

```
# Compare coefficient parameter estimates
bind_rows(
  compsym$coefficients,
  expo$coefficients,
  auto1$coefficients,
) %>%
  mutate(CovType = c('CompSym', 'Exp', 'Auto')) %>%
  select(CovType, everything()) %>%
  knitr::kable()
```

CovType	(Intercept)	gender1	age
CompSym	15.38569	2.321023	0.6601852
Exp	15.45999	2.418714	0.6529597
Auto	15.45999	2.418714	0.6529597

The coefficient parameter estimates are similar across the 3 covariance patterns.

```
# Compare covariance estiamtes
# Compound Symmetry
compsym$sigma^2 * corMatrix(compsym$modelStruct$corStruct)[[1]]
```

```
##          [,1]     [,2]     [,3]     [,4]
## [1,] 5.316240 3.266784 3.266784 3.266784
## [2,] 3.266784 5.316240 3.266784 3.266784
## [3,] 3.266784 3.266784 5.316240 3.266784
```

```
## [4,] 3.266784 3.266784 3.266784 5.316240
```

```r
# Exponential covariance
expo$sigma^2 * corMatrix(expo$modelStruct$corStruct)[[1]]
```

```
##          [,1]     [,2]     [,3]     [,4]
## [1,] 5.296881 3.315144 2.074839 1.298574
## [2,] 3.315144 5.296881 3.315144 2.074839
## [3,] 2.074839 3.315144 5.296881 3.315144
## [4,] 1.298574 2.074839 3.315144 5.296881
```

```r
# Autoregressive covariance
auto1$sigma^2 * corMatrix(auto1$modelStruct$corStruct)[[1]]
```

```
##          [,1]     [,2]     [,3]     [,4]
## [1,] 5.296881 3.315144 2.074840 1.298574
## [2,] 3.315144 5.296881 3.315144 2.074840
## [3,] 2.074840 3.315144 5.296881 3.315144
## [4,] 1.298574 2.074840 3.315144 5.296881
```

The covariance estiamtes are similar across the 3 covariance patterns.