代数 Algebra

1 基础运算 Basic Operation

$$a+b=b+a$$
$$a+(b+c)=(a+b)+c$$
$$a\cdot b=b\cdot a$$
$$a\cdot(b\cdot c)=(a\cdot b)\cdot c$$
$$a\cdot(b+c)=a\cdot b+a\cdot c$$

$$a+(b\pm c)=a+b\pm c$$
$$a-(b\pm c)=a-b\mp c$$

2 分数和比 Fraction and Ratio

$$\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}\ (c\neq 0)$$
$$\frac{a}{c}+\frac{b}{d}=\frac{ad+bc}{cd}\ (c\neq 0,d\neq 0)$$
$$a\cdot\frac{b}{c}=\frac{ab}{c}\ (c\neq 0)$$
$$\frac{a}{c}\cdot\frac{b}{d}=\frac{ab}{cd}\ (c\neq 0,d\neq 0)$$
$$\frac{ab}{ac}=\frac{b}{c}\ (a\neq 0,c\neq 0)$$
$$a\frac{b}{c}=\frac{ac+b}{c}\ (c\neq 0)$$
$$\frac{a}{c}\div\frac{b}{d}=\frac{a}{c}\cdot\frac{d}{b}=\frac{ad}{bc}\ (b\neq 0,c\neq 0)$$

$$a:b=a\div b=\frac{a}{b}\ (b\neq 0)$$
$$ac:bc=a:b=\frac{a}{d}:\frac{b}{d}\ (b\neq 0,c\neq 0,d\neq 0)$$
$$a:b=c:d\Leftrightarrow ad=bc\ (b\neq 0,d\neq 0)$$
$$\frac{a}{b}=\frac{c}{d}\Leftrightarrow\frac{a+b}{b}=\frac{c+d}{d}\Leftrightarrow\frac{a-b}{b}=\frac{c-d}{d}$$
$$\frac{a}{b}=\frac{c}{d}\Leftrightarrow\frac{a+b}{a-b}=\frac{c+d}{c-d}$$
$$\frac{a}{100}=a\%$$

3 乘方运算 Exponentiation

$$a^m \cdot a^n = a^{m+n}\ (a \neq 0)$$
$$(a^m)^n = a^{mn}\ (a \neq 0)$$
$$a^m b^m = (a \cdot b)^m\ (a \neq 0)$$
$$\frac{a^m}{b^m} = (\frac{a}{b})^m$$

4 根式运算 Radical operation

$$a^2 = b \Leftrightarrow\ a = \pm\sqrt{b}\ (b \geq 0)$$
$$\sqrt{a^2} = a\ (a \geq 0)$$
$$\sqrt{a^2} = -a\ (a \leq 0)$$
$$(\sqrt{a})^2 = a (a \geq 0)$$
$$\sqrt{a^2 b} = \sqrt{a^2} \cdot \sqrt{b} = a\sqrt{b}\ (a \geq 0, b \geq 0)$$

5 绝对值运算 Absolute value operations

$$|-x| = |x|$$
$$|x - y| = |y - x|$$
$$|xy| = |x| \cdot |y|$$
$$\left|\frac{\mathrm{x}}{\mathrm{y}}\right| = \frac{|x|}{|y|}, y \neq 0$$

6 代数式运算 Algebraic operations

$$x^2 - y^2 = (x + y)(x - y)$$
$$(x \pm y)^2 = x^2 \pm 2xy + y^2$$

7 等差数列 Arithmetic sequence

$$d = a_{n+1} - a_n$$
$$a_n = a_1 + (n-1)d$$
$$S_n = \frac{(a_1 + a_n)n}{2}$$
$$\mathrm{d} = \frac{a_{\mathrm{m}} - a_{\mathrm{n}}}{\mathrm{m} - \mathrm{n}}$$

8 等比数列 Geometric Sequences

$$\mathrm{q} = \frac{a_{n+1}}{a_n}$$
$$a_n = a_1 \times q^{n-1} = a_m \times q^{n-m}$$

9 必背计算结论 Calculation Conclusion

$$2^0 = 1$$
$$2^1 = 2$$
$$2^2 = 4$$
$$2^3 = 8$$
$$2^4 = 16$$
$$2^5 = 32$$
$$2^6 = 64$$
$$2^7 = 128$$
$$2^8 = 256$$
$$2^9 = 512$$
$$2^{10} = 1024$$
$$2^{11} = 2048$$
$$2^{12} = 4096$$
$$2^{13} = 8192$$
$$2^{14} = 16384$$
$$2^{15} = 32768$$
$$2^{16} = 65536$$

$$11^2 = 121$$
$$12^2 = 144$$
$$13^2 = 169$$
$$14^2 = 196$$
$$15^2 = 225$$
$$16^2 = 256$$
$$17^2 = 289$$
$$18^2 = 324$$
$$19^2 = 361$$

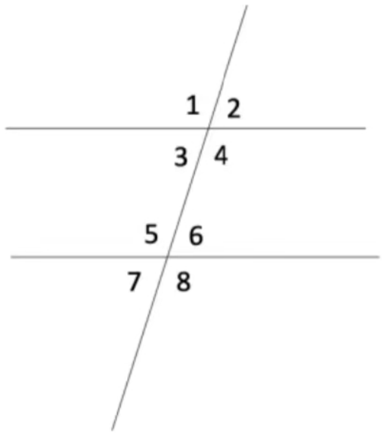
$$15^2 = 225$$
$$25^2 = 625$$
$$35^2 = 1225$$
$$45^2 = 2025$$
$$55^2 = 3025$$
$$65^2 = 4225$$
$$75^2 = 5625$$
$$85^2 = 7225$$
$$95^2 = 9025$$

$$1! = 1$$
$$2! = 2$$
$$3! = 6$$
$$4! = 24$$
$$5! = 120$$
$$6! = 720$$

$$\pi \approx 3.1416$$
$$e \approx 2.7183$$
$$\sqrt{2} \approx 1.4142$$
$$\sqrt{3} \approx 1.7321$$
$$\sqrt{5} \approx 2.2361$$
$$\sqrt{6} \approx 2.4495$$

$$3^2 + 4^2 = 5^2$$
$$5^2 + 12^2 = 13^2$$
$$7^2 + 24^2 = 25^2$$
$$8^2 + 15^2 = 17^2$$

1 角和线 Angles and Lines



- Sum of complementary angles: a+b=90°

  互余的两个角和为90°

- Sum of supplementary angles: a+b=180°

  互补的两个角和为180°

- Relationship of Angles Formed by Parallel Lines:

  平行线间角的关系（“三线八角”）：

  Alternate interior angles: $\angle 3 = \angle 6, \angle 4 = \angle 5$

  内错角相等

  Alternate exterior angles: $\angle 1 = \angle 8, \angle 2 = \angle 7$

  外错角相等

  Corresponding angles: $\angle 1 = \angle 5, \angle 2 = \angle 6, \angle 3 = \angle 7, \angle 4 = \angle 8$

  同位角相等

  Interior angles on the same sides of transversal:

  $\angle 3 + \angle 5 = 180°, \angle 4 + \angle 6 = 180°$

  同旁内角互补

- 三角形的性质 The Property of Triangles

  $$\text{The area of a triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

  三角形的面积 = $\frac{1}{2}$ 底×高

  The sum of any two sides in a triangle is greater than the third side, and the difference between any two sides is less than the third side. 三角形两边之和大于第三边，两边之差小于第三边。

- Pythagoras Theorem: $a^2 + b^2 = c^2$

  勾股定理：直角三角形中，两直角边的平方和等于斜边的平方

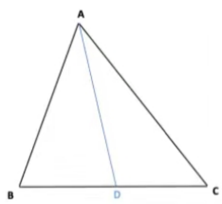
  (*常考勾股数：* $3^2+4^2=5^2, 5^2+12^2=13^2, 7^2+24^2=25^2, 8^2+15^2=17^2$)

- Mid-line of triangle:

  三角形中线性质

  BD = DC,

  $S_{\Delta ABD} = S_{\Delta ADC}$



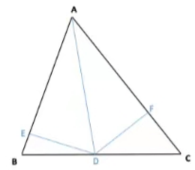
几何 Geometry

- Triangle bisector:

  三角形角平分线性质

  $\angle BAD = \angle CAD$

  DE = DF

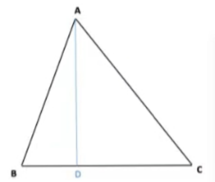


- Altitude of triangle:

  三角形垂线性质

  $AD \perp BC$

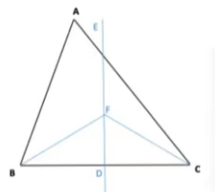
  $$S_{\Delta ABC} = \frac{1}{2} \times AD \times BC$$



- Vertical bisector of a triangle:

  三角形垂直平分线性质

  BD=DC, DE⊥BC, FB=FC

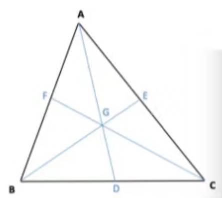


- Center of gravity of triangle:

  三角形重心性质

  BD=DC, AE=EC, AF=FB

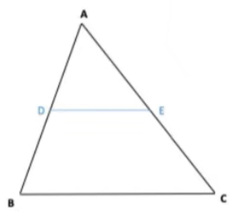
  AG=2GD, BG=2GE, CG=2GF



- Middle line of the triangle:

  三角形中位线性质

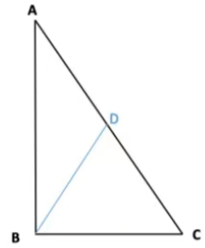
  $$DE = \frac{1}{2} BC, DE // BC$$



- **Isosceles triangle:** Same sides same angles; Three lines in one.
  等腰三角形两腰相等，两底角相等，三线（中线、角平分线、垂线）合一。

  Equilateral triangle: Same side length, same angles; $\mathrm{Area} = \frac{\sqrt{3}}{4}a^2 (a = \text{side length})$
  等边三角形各边、各角都相等，面积= $\frac{\sqrt{3}}{4}$ 边长的平方。

  Right triangle: BD= $\frac{1}{2}$ AC
  直角三角形斜边中线等于斜边的一半

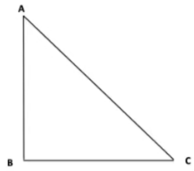


- Isosceles right triangle:

  $AB = BC = \frac{\sqrt{2}}{2}AC$

  $\angle BAC = \angle BCA = 45°$

  等腰直角三角形斜边是两直角边的 $\sqrt{2}$ 倍，两锐角都是45°．



2 相似三角形 Similar Triangles

- Determination of Similar Triangles**相似三角形的判定**

  Principle 1 (SSS)

  方法一：三边对应成比例

  If $\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CA}{C'A'}$, then $\Delta ABC \sim \Delta A'B'C'$.

  Principle 2 (AA)

  方法二：两角对应相等

  If $\angle A = \angle A'$, $\angle B = \angle B'$, then $\Delta ABC \sim \Delta A'B'C'$.

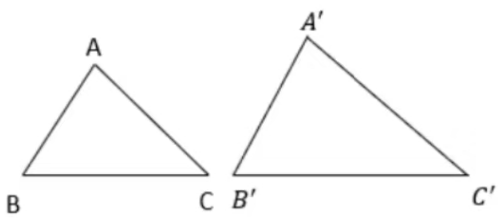
  Principle 3 (SAS)

  方法三：两边对应成比例，它们的夹角对应相等

  If $\frac{AB}{A'B'} = \frac{BC}{B'C'}$, and $\angle B = \angle B'$, then $\Delta ABC \sim \Delta A'B'C'$.

  (AND VICE VERSA)

  （反之，如果两个三角形相似，以上边的比例关系和角的相等关系也成立）



- The properties of similar triangles 相似三角形的性质

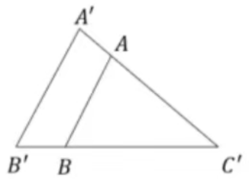
（1） The ratio of corresponding sides of similar triangles is called scale factor. The ratio of perimeters of similar triangles is equal to the scale factor. The ratio of areas of similar triangles is equal to the square of the scale factor.

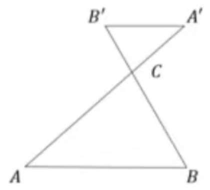
If $\Delta ABC \sim \Delta A'B'C'$, $\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CA}{C'A'} = k$, then $\frac{\text{perimeter of } \Delta ABC}{\text{perimeter of } \Delta A'B'C'} = k$, $\frac{\text{area of } \Delta ABC}{\text{area of } \Delta A'B'C'} = k^2$

相似三角形对应边的比例称为相似比。如果两个三角形相似，那么三边对应成比例，并且周长比等于相似比，面积比等于相似比的平方。

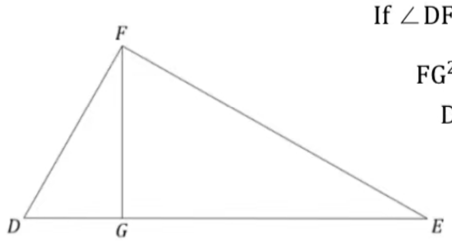
(2) A型和X型相似

If $AB \parallel A'B'$, then $\Delta ABC \sim \Delta A'B'C'$





(3) 相似三角形中的射影定理



If $\angle DFE = \angle FGE = 90°$, then $\Delta DFE \sim \Delta FGE$

$$FG^2 = DG \times GE,\ FG \times DE = DF \times FE$$
$$DF^2 = DE \times DG,\ EF^2 = DE \times EG$$

3 平行四边形 Parallelograms

- **Properties of parallelograms:** opposite sides parallel and equal; opposite angles are equal; diagonals bisect each other.

平行四边形的性质：对边平行且相等，对角相等，对角线互相平分

- **Determinations of parallelograms:** a quadrilateral that has two pairs of opposite sides parallel / two pairs of opposite sides equal / one pair of opposite sides parallel and equal / two pairs of opposite angles equal / two diagonals bisect each other is parallelogram.

平行四边形的判定：一个四边形，如果它的两组对边分别平行/两组对边分别相等/一组对边平行且相等/两组对角分别相等/两条对角线互相平分，那么它是平行四边形。

- **Properties of rectangles:** all angles = 90° ; diagonals are equal.
  矩形的性质：所有内角都是90° ，两条对角线相等

- **Properties of rhombus:** all sides are equal; diagonals bisect each other at 90° ; di-agonals bisect angles
  菱形的性质：所有边相等，对角线互相垂直平分，对角线平分内角

4 多边形 Polygons

- A polygon with all its sides and all its angles equal is called a **regular polygon.**
  所有边和角都相等的多边形叫做**正多边形。**

- **Number of symmetry axis** of a n-side regular polygon: n
  边数为n的正多边形有n条对称轴。

- **order of rotational symmetry** of a n-side regular polygon: n
  边数为n的正多边形是n阶旋转对称图形

- **Sum of interior angles** of a n-side polygon = (n-2) × 180°
  边数为n的多边形内角和为(n-2) × 180°

- **Sum of exterior angles** of a n-side polygon=360°
  边数为n的多边形外角和等于360°

- **Number of diagonals** of a n-side polygon = $\frac{n(n-3)}{2}$
  边数为n的多边形的对角线个数为 $\frac{n(n-3)}{2}$

- **Interior angle** of a n-side regular polygon = $\frac{(n-2)\times 180°}{n}$
  边数为n的正多边形的内角大小为 $\frac{(n-2)\times 180°}{n}$

- **Exterior angle** of a n-side regular polygon = $\frac{360°}{n}$
  边数为n的正多边形的外角大小为 $\frac{360°}{n}$

- **Center angle** of a n-side regular polygon = $\frac{360°}{n}$
  边数为n的正多边形的中心角大小为 $\frac{360°}{n}$

5 圆 Circles

- Circumference of a circle = $\pi d = 2\pi r$

  圆周长= $\pi d = 2\pi r$

- Area of a circle = $\pi r^2 = \frac{1}{4}\pi d^2$

  圆面积= $\pi r^2 = \frac{1}{4}\pi d^2$

- **In the same circle or congruent circles,** if two central angles / circumference angles / arcs / chords / chord lengths on the two chords is equal, then the others are all equal.

  在同一个圆或全等的两个圆中，如果两个圆心角/两个圆周角/两条弧/两条弦/两段弦长相等，满足其中一条，其它的也都成立。

- The degree of the circumference angle is equal to half of the degree of the central angle the arc it is facing.

  圆周角等于它所对的弧对应的圆心角的一半。

- For a sector, $\frac{arc\ length}{circumference} = \frac{central\ angle\ of\ the\ arc}{360°} = \frac{area\ of\ sector}{area\ of\ circle}$.

  扇形中，弧长在整圆周长中所占的比，等于它所对的圆心角在360° 中所占的比，也等于它的面积在整圆面积中所占的比。

6 解析几何 Analytical Geometry

- The distance between two points: $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

  两点间距离= $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

7 立体几何 Solid Geometry

- For a prism, V−E+F=2: V stands for the number of vertices, E stands for the number of edges, F stands for the number of faces.

  在棱柱中，顶点数−棱数+面数=2.

8 立体几何 Solid Geometry

- surface area of cuboid = $6a^2$
  正方体表面积：$6a^2$

- Volume of cuboid = a^3
  正方体体积：a^3

- Surface area of a prism = 2 x area of cross–section + perimeter of cross–section x length
  棱柱表面积 = 2倍横截面面积+横截面周长x棱柱的长

- Volume of a prism = area of cross–section x length
  棱柱体积=横截面面积x棱柱的长

- Surface area of a cylinder = $2\pi rh + 2\pi r^2 = 2\pi r(h + r)$
  圆柱表面积 = $2\pi rh + 2\pi r^2 = 2\pi r(h + r)$

- Volume of a cylinder = $\pi r^2 h$
  圆柱体积 = $\pi r^2 h$

- Volume of a pyramid = $\frac{1}{3}$ × base area × perpendicular height
  正四棱锥体积 = $\frac{1}{3}$ 底面积 × 垂高

- Surface area of a cone = $\pi rl + \pi r^2 = \pi r(l + r)$
  圆锥表面积 = $\pi rl + \pi r^2 = \pi r(l + r)$

- Volume of a cone = $\frac{1}{3}\pi r^2 h$
  圆锥体积 = $\frac{1}{3}\pi r^2 h$

- Surface area of a sphere = $4\pi r^2$
  球表面积 = $4\pi r^2$

- Volume of a sphere = $\frac{4}{3}\pi r^3$
  球体积 = $\frac{4}{3}\pi r^3$

1 加法原理 Sum Rule(Additional Principle)

If an event E_1 can happen in n_1 ways, event E_2 can happen in n_2 ways, event E_k can happen in n_k ways, and if any event E_1, E_2, ... or E_k happens, the job is done, then the total ways to do the job is $n_1 + n_2 + \cdots + n_k$.

加法原理：如果一件事情有k种不同的做法，其中做法E_1有n_1种方式完成，做法E_2有n_2种方式完成，做法E_k有n_k种方式完成，并且做法E_1, E_2, ... or E_k都有可能出现，那么这件事情的总完成方式共有$n_1+n_2+\cdots+n_k$种。

2 乘法原理 Product Rule(Multiplication Principle)

When a task consistes of k separate parts, if the first part can be done in n_1 ways, the second part can be done in n_2 ways, and so on through the k^{th} part, which can be done in n_k ways, then total number of possible results for completing the task is given by the product: $n_1 \times n_2 \times n_3 \times \cdots \times n_k$.

如果一件事情分成k个步骤完成，其中第一步有n_1种方式完成，第二步有n_2种方式完成，直到第k步，有n_k种方式完成，那么完成整件事情的方法数是$n_1 \times n_2 \times n_3 \times \cdots \times n_k$.

3 概率的主要性质 Properties of probability

(1)The probability of an event is between 0 and 1. 任意事件的概率都在0到1之间。

(2)The probability of an impossible event is 0. 不可能事件的概率为0.

(3)The probability of a certain event is 1. 必然事件的概率为1.

The probability that an event will occur is equal to one minus the probability that it will not occur. 一件事情发生的概率等于1减去它不发生的概率。

排列组合 Permutation and Combination

$$n! = n \cdot (n-1) \cdots 3 \cdot 2 \cdot 1$$

正整数n的阶乘n · (n−1) · · ·3·2·1

$$_{n}P_{r} = \frac{n!}{(n-r)!} \ (1 \le r \le n)$$

n个物体中选出r个排序，共有 $\mathbf{{}_nP_r = \frac{n!}{(n-r)!}}$ 种方法

$$ {}_nC_r = \frac{\boldsymbol{n}!}{\boldsymbol{r}!\,(\boldsymbol{n}-\boldsymbol{r})!}\ (0 \le r \le n) $$

n个物体中选出r个，共有 $\mathbf{{}_nC_r = \frac{n!}{r!(n-r)!}}$ 种方法

$$ \frac{{}_nP_n}{p!\ \times q!\ \times r!\ \times \cdots} = \frac{n!}{p!\ \times q!\ \times r!\ \times \cdots}\ (p+q+r+\ldots=n) $$

n个物体排列，其中p个物体是一类，q个物体是另一类，r个物体是第三类，…，

共有 $\frac{{}_nP_n}{p! \times q! \times r! \times \cdots} = \frac{n!}{p! \times q! \times r! \times \cdots}$ 种排列方式

$$ \boldsymbol{{}_nc_0 + {}_nc_1 + \ldots + {}_nc_n = 2^n} $$

$$ Probability = \frac{number\ of\ ways\ th\ at\ a\ certain\ outcome\ can\ occur}{total\ number\ of\ possible\ outcomes} $$

4 概率的基本公式

集合和韦恩图 Sets and Venn Diagrams

$$ n(A \cup B) = n(A) + n(B) - n(A \cap B) $$

两个事件的并集公式：集合A和集合B的并集中的元素数 = 集合A中的元素数 + 集合B中的元素数 − 集合A和集合B交集中的元素数

$$ n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C) $$

三个事件的并集公式：集合A、集合B和集合C的并集中的元素数 = 集合A中的元素数 + 集合B中的元素数 + 集合C中的元素数 − 集合A、集合B和集合C交集中的元素数

1 Odd and Even奇偶性
Odd是奇数，Even是偶数

even ±even = even

odd ±odd = even

even ±odd = odd

even x even = even

odd x odd = odd

even x odd = even

even ÷even = even/odd/fraction

even ÷odd = even/fraction

odd ÷odd = odd/fraction

even ≠odd

- The sum of two consecutive integers is odd: $n+(n+1)=2n+1$
  两个连续整数的和是奇数
- The product of two consecutive integers is even: $n\times(n+1)=n(n+1)$
  两个连续整数的乘积是偶数
- Any two consecutive integers have opposite parity
  两个连续整数有相反的奇偶性（一奇一偶）
- If the product of n positive integers is even, at least one of these n positive integers is even
  如果n个正整数的乘积是偶数，那么这n个正整数中至少有一个是偶数
- If the product of n positive integers is odd, all of these n positive integers are odd
  如果n个正整数的乘积是奇数，那么这n个正整数都是奇数
- If the number of odd integers is even, then the sum of them is even
  偶数个奇数的和是偶数
- If the number of odd integers is odd, then the sum of them is odd
  奇数个奇数的和是奇数

2 2的若干次方的整除性 Divisibility rule for 2, 4, 8, 16, and 2^n

A number is divisible by 2 if the last digit of the number is divisible by 2;

A number is divisible by 4 if the last 2 digits of the number is divisible by 4;

A number is divisible by 8 if the last 3 digits of the number is divisible by 8;

A number is divisible by 16 if the last 4 digits of the number is divisible by 16;

A number is divisible by 2^n if the last n digits of the number is divisible by 2^n.

判断一个数能否被2, 4, 8, 16, … 2^n, 整除只要看这个数的最后1, 2, 3, 4, …, n位组成的数能否被2, 4, 8, 16, …, 2^n 整除

3 3和9的整除性 Divisibility rule for 3 and 9

A number is divisible by 3 if the sum of the digits of the number is divisible by 3;

A number is divisible by 9 if the sum of the digits of the number is divisible by 9.

判断一个数能否被3或者9整除只要看这个数的数位之和能否被3或者9整除

4 5和5的若干次方的整除性 Divisibility rule for 5, 25, 125, and 5^n

A number is divisible by 5 if the last digit of the number is divisible by 5;

A number is divisible by 25 if the last 2 digits of the number is divisible by 25;

A number is divisible by 125 if the last 3 digits of the number is divisible by 125;

A number is divisible by 5^n if the last n digits of the number is divisible by 5^n.

判断一个数能否被5, 25, 125, …, 整除只要看这个数的最后1, 2, 3, 4, …, n位组成的数能否被5, 25, 125, …, 整除

5 11的整除性 Divisibility rule for 11

If the difference between the sum of all digits in odd digit and the sum of all digits in even digit can be divised by 11, then the number is divisible by 11.

判断一个数能否被11整除只要看这个数的所有奇数数位和偶数数位之和的差值能否被11整除

6 有多于等于2种不同质因数的合数的整除性 Divisibility rule for 6, 12, 14, 15, 18, 24

A number is divisible by 6 if the number is divisible by both 2 and 3;

A number is divisible by 12 if the number is divisible by both 3 and 4;

A number is divisible by 14 if the number is divisible by both 2 and 7;

A number is divisible by 15 if the number is divisible by both 3 and 5;

A number is divisible by 18 if the number is divisible by both 2 and 9;

A number is divisible by 24 if the number is divisible by both 3 and 8;

判断一个数能否被一个至少有两种质因数的合数整除只要把这个合数分解质因数后拆分成每个质因数的幂再分别判断，如果都可以整除，那么这个数就能被那个合数整除

7 分解质因数 Prime Factorization

- If a number can be written as $a = p_1{}^{b_1}p_2{}^{b_2}...p_n{}^{b_n}$, then the number of positive factor of is $(b_1+1)(b_2+1)...(b_n+1)$: the sum of its factor is $(p_1^0+p_1^1+...+p_1^{b_1})(p_2^0+p_2^1+...+p_2^{b_2})...(p_n^0+p_n^1+...+p_n^{b_n})$.

  一个整数的正因数数量等于 $(b_1+1)(b_2+1)...(b_n+1)$:这些正因数的和为$(p_1^0+p_1^1+...+p_1^{b_1})(p_2^0+p_2^1+...+p_2^{b_2})...(p_n^0+p_n^1+...+p_n^{b_n})$.

- Define $\lfloor x \rfloor$is the largest integer less than or equal to . Then the power of prime number p does n ! have is $\left\lfloor\frac{n}{p}\right\rfloor+\left\lfloor\frac{n}{p^2}\right\rfloor+\left\lfloor\frac{n}{p^3}\right\rfloor+...$

  定义$\lfloor x \rfloor$是小于等于x的最大整数，则里含有质因数P的数量为$\left\lfloor\frac{n}{p}\right\rfloor+\left\lfloor\frac{n}{p^2}\right\rfloor+\left\lfloor\frac{n}{p^3}\right\rfloor+...$

8 最大公因数和最小公倍数 GCF and LCM

If (a,b) = d, and n is a positive integer, then (na, nb) = nd

(a,1) = 1; (a, a) = a; (a,b) = (b,a); (a,b) = (b, a−b)

$$GCF(a,b) \times LCM(a,b) = a \times b$$