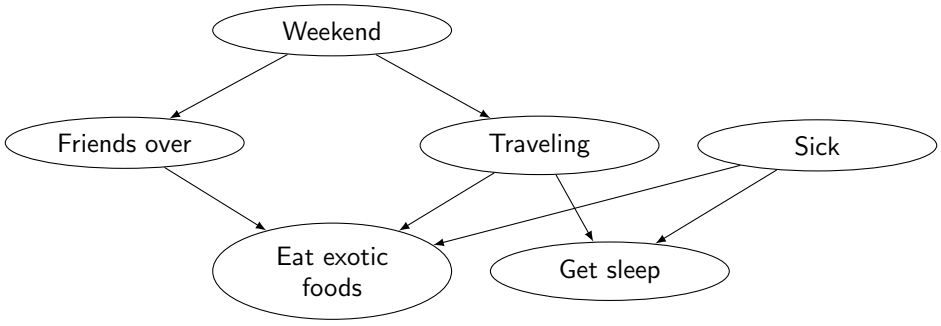
Problem 3 (Bayesian Networks, 10 pts)

In this problem we explore the conditional independence properties of a Bayesian Network. Consider the following Bayesian network representing a fictitious person's activities. Each random variable is binary (true/false).



The random variables are:

- Weekend: Is it the weekend?
- Friends over: Does the person have friends over?
- Traveling: Is the person traveling?
- Sick: Is the person sick?
- Eat exotic foods: Is the person eating exotic foods?
- Get Sleep: Is the person getting sleep?

For the following questions, $A \perp B$ means that events A and B are independent and $A \perp B|C$ means that events A and B are independent conditioned on C.

Use the concept of d-separation to answer the questions and show your work (i.e., state what the blocking path(s) is/are and what nodes block the path; or explain why each path is not blocked).

Example Question: Is Friends over $\perp$ Traveling? If NO, give intuition for why.

Example Answer: NO. The path from Friends over – Weekend – Traveling is not blocked following the d-separation rules. Thus, the two are not independent. Intuitively, this makes sense as if say we knew that the person was traveling, it would make it more likely to be the weekend. This would then make it more likely for the person to have friends over.

Actual Questions:

1. Is Sick $\perp$ Weekend? If NO, give intuition for why.
2. Is Sick $\perp$ Friends over | Eat exotic foods? If NO, give intuition for why.
3. Is Friends over $\perp$ Get Sleep? If NO, give intuition for why.
4. Is Friends over $\perp$ Get Sleep | Traveling? If NO, give intuition for why.
5. Suppose the person stops traveling in ways that affect their sleep patterns (as various famous people have done). Travel still affects whether they eat exotic foods. Draw the modified network. (Feel free to reference the handout file for the commands for displaying the new network in LaTeX).
6. For this modified network, is Friends over $\perp$ Get Sleep? If NO, give an intuition why. If YES, describe what observations (if any) would cause them to no longer be independent.

Solution

1. Yes. We see that there are three paths from Weekend to Sick. These paths are:

   - Weekend-Traveling-Get sleep-Sick
   - Weekend-Traveling-Eat exotic foods-Sick
   - Weekend-Friends Over-Eat exotic foods-Sick

   All three of these paths are blocked. The first path is blocked since Get sleep is unobserved and the Traveling and Sick nodes both point into the Get sleep node. Similarly, since Eat exotic foods is unobserved, the second and third paths are blocked since the Traveling, Friends over, and Sick nodes all point into the Eat exotic foods node.

2. No. Now that Eat exotic foods is observed, the path Friends Over-Eat exotic foods-Sick is no longer blocked. Observing the Eat exotic foods node unblocks the path because the Friends over and Sick nodes both point into the Eat exotic foods node. An intuitive explanation is that if someone has friends and is eating exotic foods, it is less likely that the person is sick, otherwise they would not be engaging in these activities. Since that person is more likely to have friends over and not be sick if they are eating exotic foods, then being sick and having friends over covary.

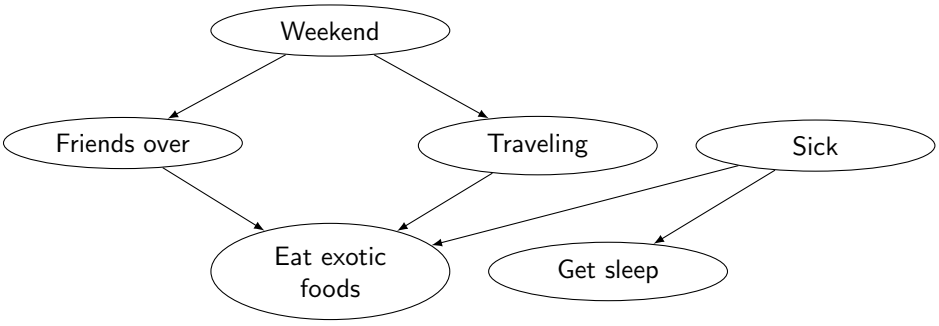
3. No. Since Weekend and Traveling are not observed, the path Friends over-Weekend-Traveling-Get sleep is not blocked. An intuitive explanation is that if someone has Friends over, then it is more likely to be the weekend and they are less likely to be traveling (in order to have friends over, it would be strange if they were also traveling at the same time), which would imply that person has free time to be hanging out with friends and is likely to be getting sleep. Therefore Friends over and Get sleep covary.

4. Yes. There three paths from Friends over to Get sleep:

   - Friends over-Weekend-Traveling-Get sleep
   - Friends over-Eat exotic foods-Traveling-Get sleep
   - Friends over-Eat exotic foods-Sick-Get sleep

   Since Traveling is observed, the first path is blocked, since the Traveling node points into the Get sleep node. Since Eat exotic foods is unobserved, the second and third paths are blocked since the Friends over, Traveling, and Sick nodes all point into the Eat exotic foods node.

5. The only modification that is needed to be done on this network is to remove the connection between Traveling and Get sleep node. This modified network is shown below:



6. Yes. There are two paths from Friends over to Get sleep:

   - Friends over-Eat exotic foods-Sick-Get sleep

- Friends over-Weekend-Traveling-Eat-exotic-foods-Sick-Get sleep

Both of these paths are blocked since Eat exotic foods is unobserved, and because the Friends over, Traveling, and Sick nodes all point into Eat exotic foods. Observing Eat exotic foods would unblock the path and cause Friends over and Get sleep to no longer be independent.