

Matching Planar Objects In New Viewpoints

What Kind of Transformation Happened To My DVD?



$T?$

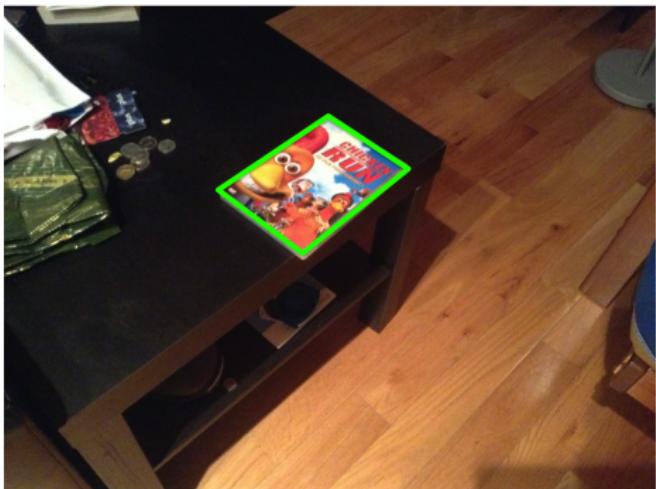


What Kind of Transformation Happened To My DVD?

- Rectangle goes to a parallelogram (almost but not really, but let's believe that for now)



$T?$



All 2D Linear Transformations

Linear transformations are combinations of

- Scale,
- Rotation
- Shear
- Mirror

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

[Source: N. Snavely]

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Affine Transformations

Affine transformations are combinations of

- Linear transformations, and
- Translations

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same as:

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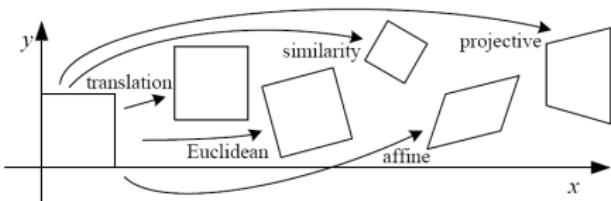
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Properties of affine transformations:

- Origin does not necessarily map to origin
- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition
- Rectangles go to parallelograms

[Source: N. Snavely]

2D Image Transformations

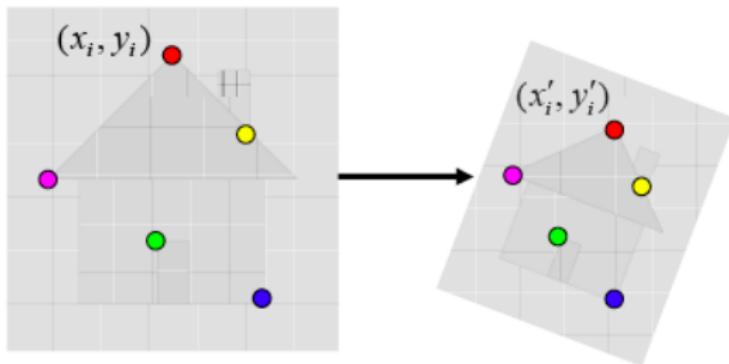


Transformation	Matrix	# DoF	Preserves	Icon
translation	$\begin{bmatrix} I & t \end{bmatrix}_{2 \times 3}$	2	orientation	
rigid (Euclidean)	$\begin{bmatrix} R & t \end{bmatrix}_{2 \times 3}$	3	lengths	
similarity	$\begin{bmatrix} sR & t \end{bmatrix}_{2 \times 3}$	4	angles	
affine	$\begin{bmatrix} A \end{bmatrix}_{2 \times 3}$	6	parallelism	
projective	$\begin{bmatrix} H \end{bmatrix}_{3 \times 3}$	8	straight lines	

- These transformations are a nested set of groups
- Closed under composition and inverse is a member

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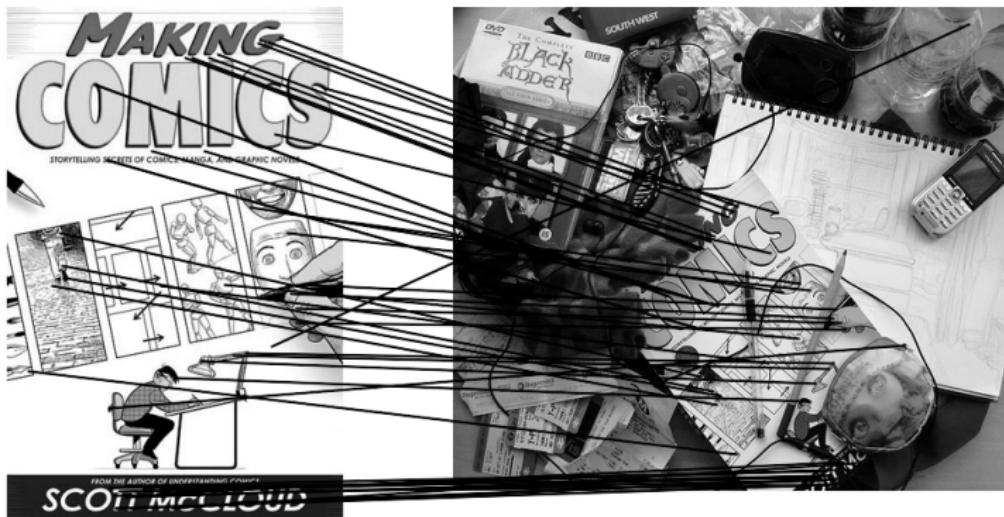
- Affine transformation approximates viewpoint changes for roughly **planar objects** and roughly **orthographic cameras** (more about these later in class)
- DVD went affine!



Computing the (Affine) Transformation

Given a set of matches between images I and J

- How can we compute the affine transformation A from I to J?
- Find transform A that best agrees with the matches

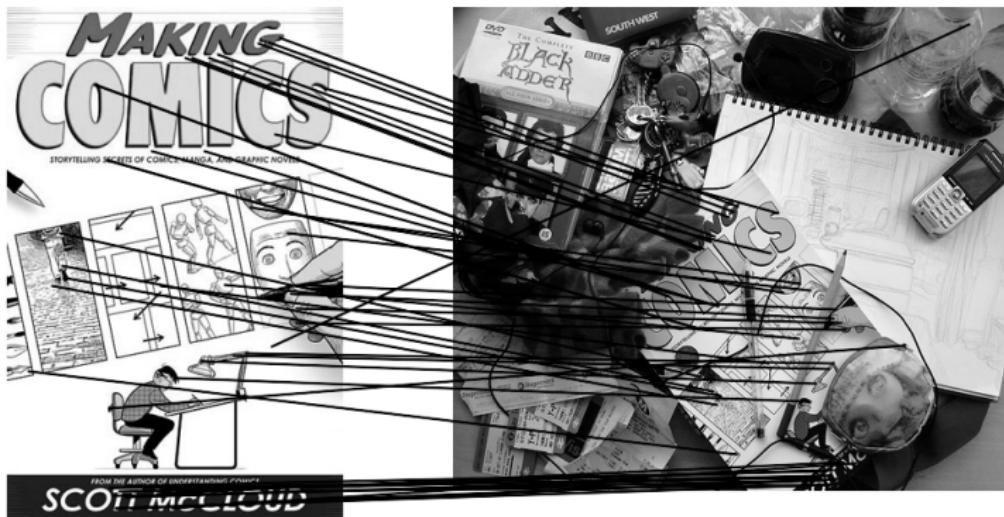


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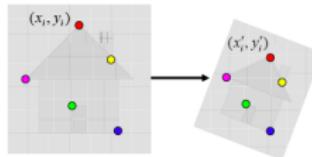
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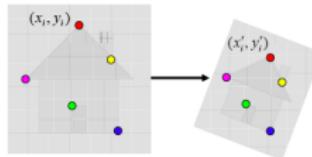
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- Let (x_i, y_i) be a point on the reference (model) image, and (x'_i, y'_i) its match in the test image
- An affine transformation A maps (x_i, y_i) to (x'_i, y'_i) :

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- We can rewrite this into a simple linear system:

$$\begin{bmatrix} x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = \begin{bmatrix} x'_i \\ y'_i \end{bmatrix}$$

Computing the Affine Transformation

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$$\underbrace{\begin{bmatrix} & & \vdots & & & \\ x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \\ & & \vdots & & & \end{bmatrix}}_P \underbrace{\begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix}}_a = \underbrace{\begin{bmatrix} \vdots \\ x'_i \\ y'_i \\ \vdots \end{bmatrix}}_{P'}$$

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- 6 parameters \rightarrow 3 matches
- But the more, the better (more reliable)
- How do we compute A?

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$$\min_{a,b,\dots,f} \|\mathbf{Pa} - \mathbf{P}'\|_2^2$$

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- Which has a closed form solution:

$$\mathbf{a} = (\mathbf{P}^T \mathbf{P})^{-1} \mathbf{P}^T \mathbf{P}'$$

Image Alignment Algorithm: Affine Case

Given images I and J

- ① Compute image features for I and J
- ② Match features between I and J
- ③ Compute affine transformation A between I and J using least squares on the set of matches

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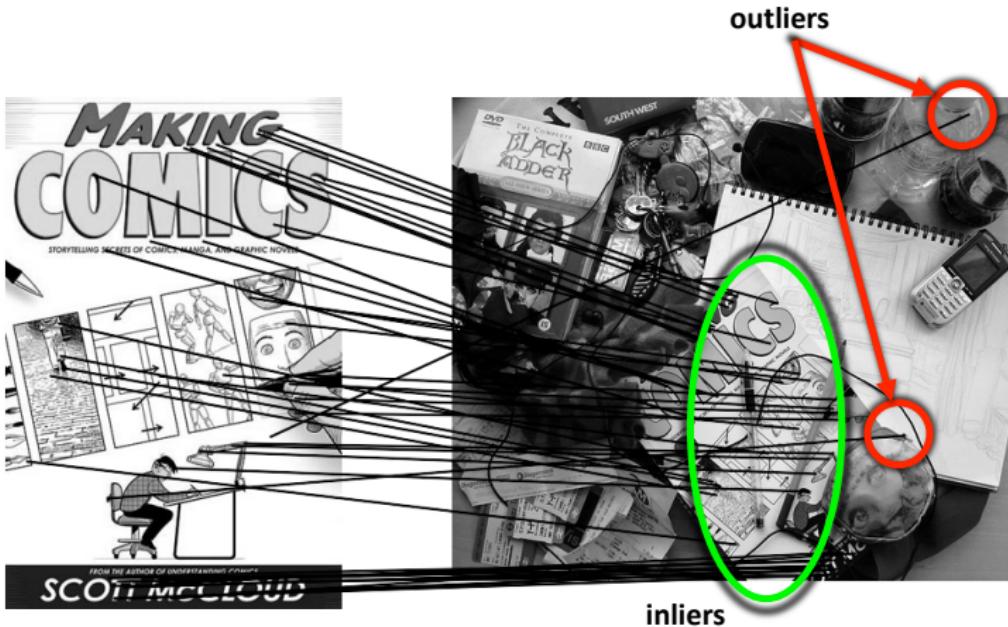
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Is there a problem with this?

[Source: N. Snavely]

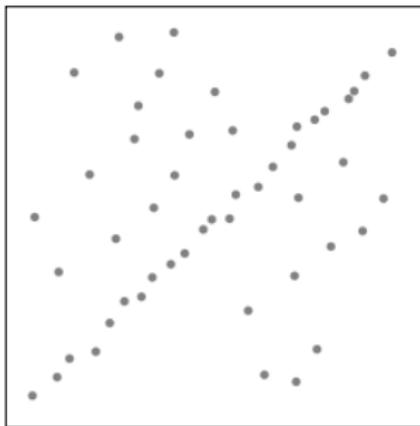
Robustness



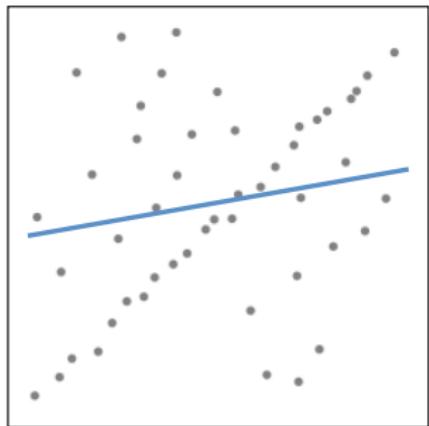
[Source: N. Snavely]

Simple Case

- Lets consider a simpler example ... Fit a line to the points below!



Problem: Fit a line to these datapoints



Least squares fit

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- Take the minimal number of points to compute what we want. In the line example, two points (in our affine example, three matches)

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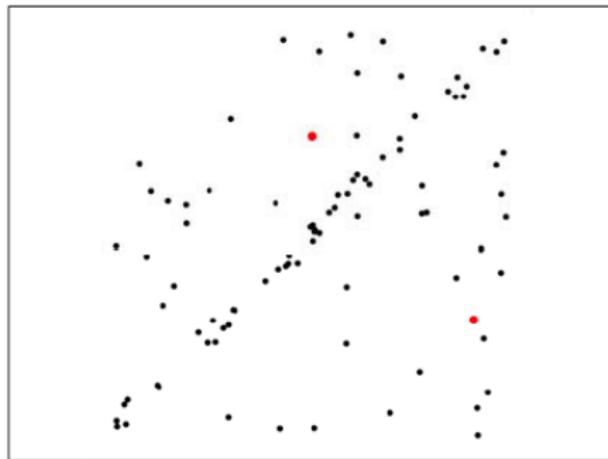
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- “Agree” = within a small distance of the line
- Repeat this many times, remember the number of inliers for each trial
- Among several trials, select the one with the largest number of inliers

This procedure is called **R**Andom **S**Amples **C**onsensus

RANSAC for Line Fitting Example

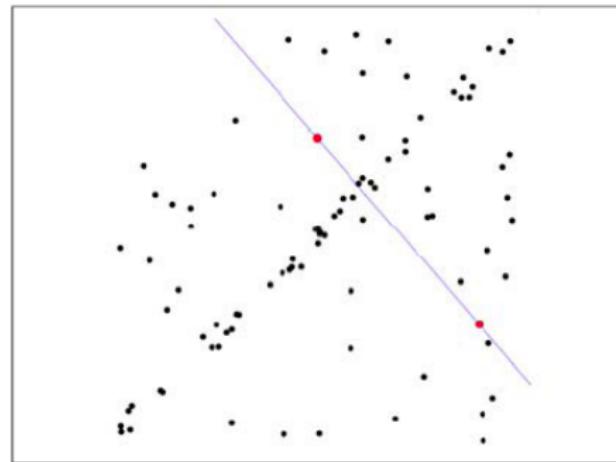
- ① Randomly select minimal subset of points



[Source: R. Raguram]

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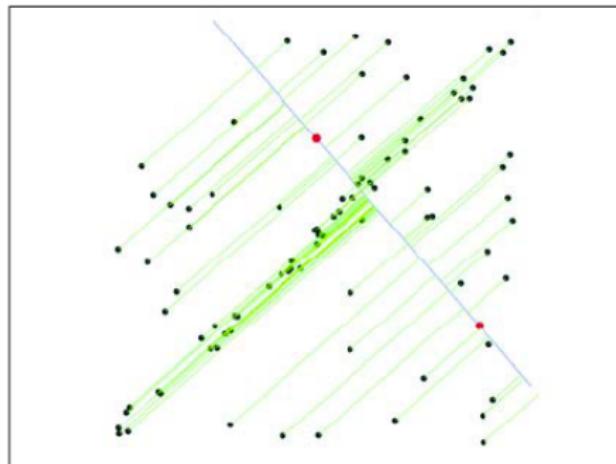
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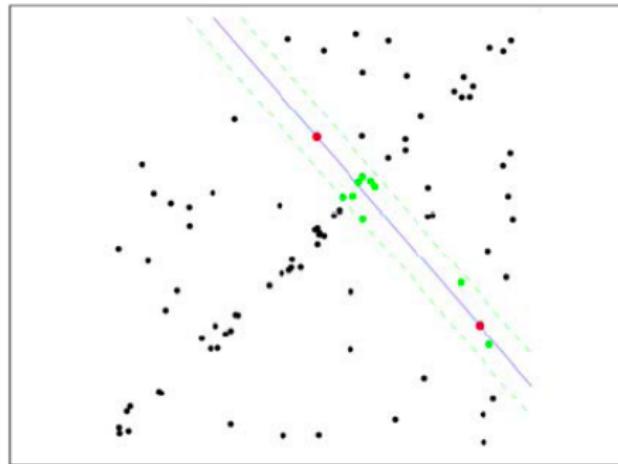
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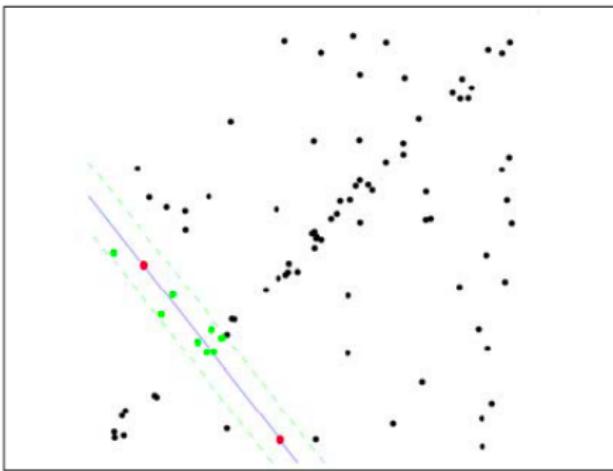
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- ④ Select points consistent with model



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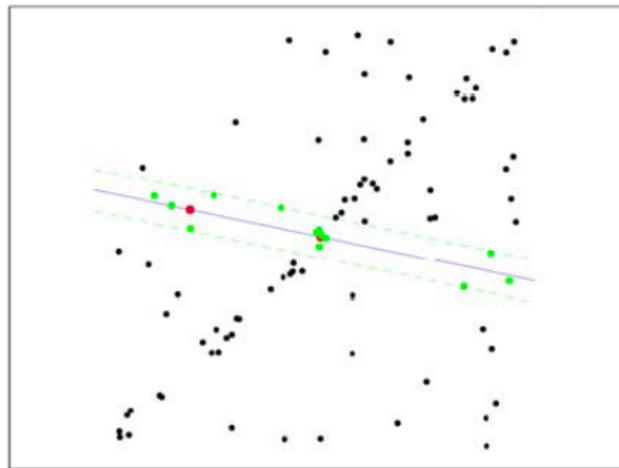
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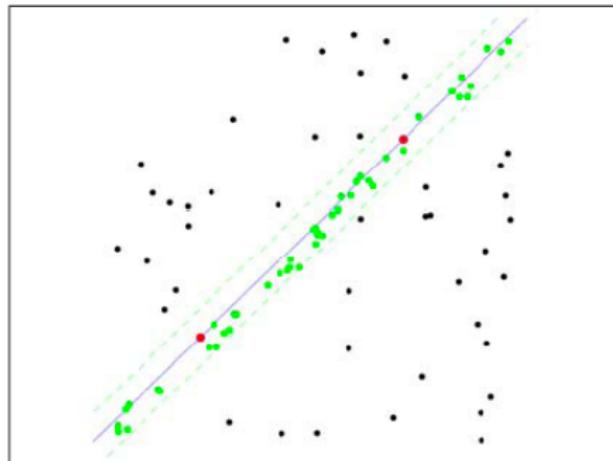
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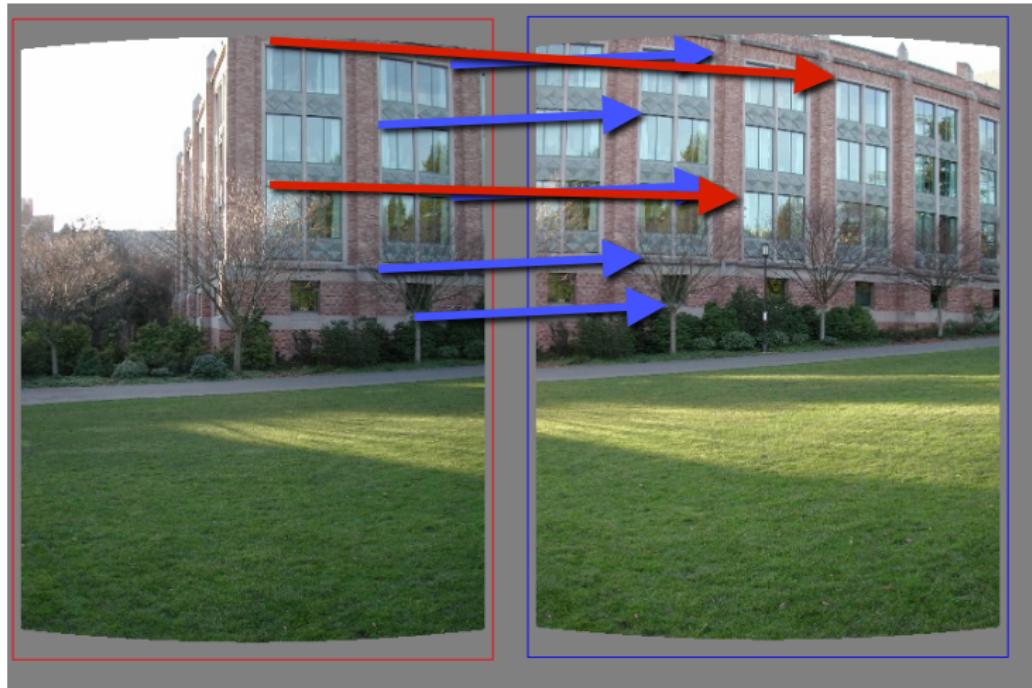
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- ⑥ Choose model with largest set of inliers



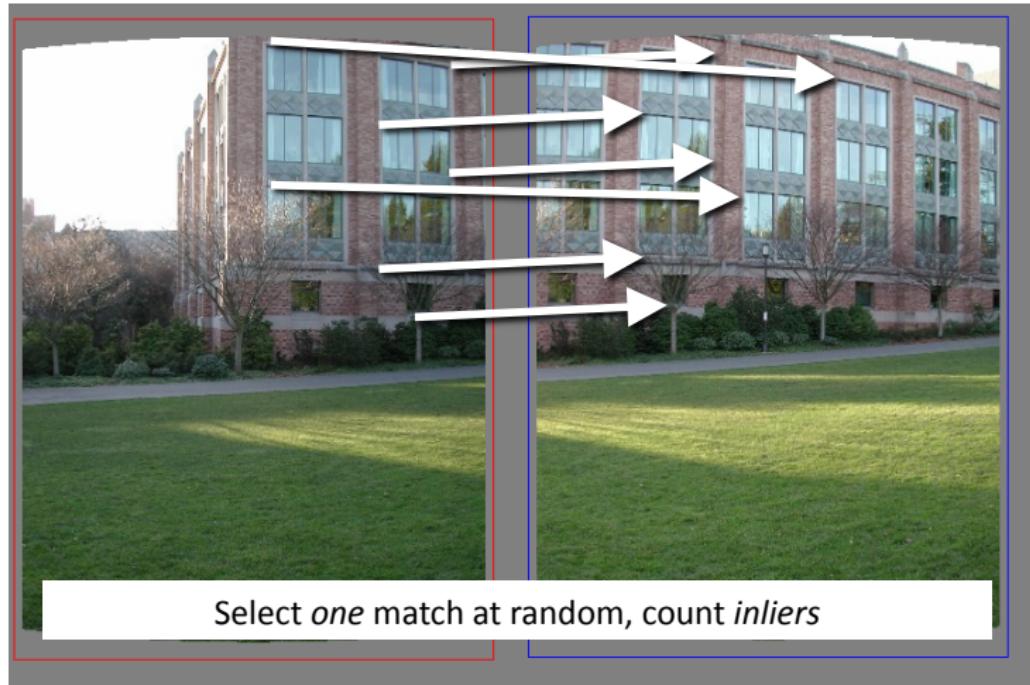
[Source: R. Raguram]

Translations



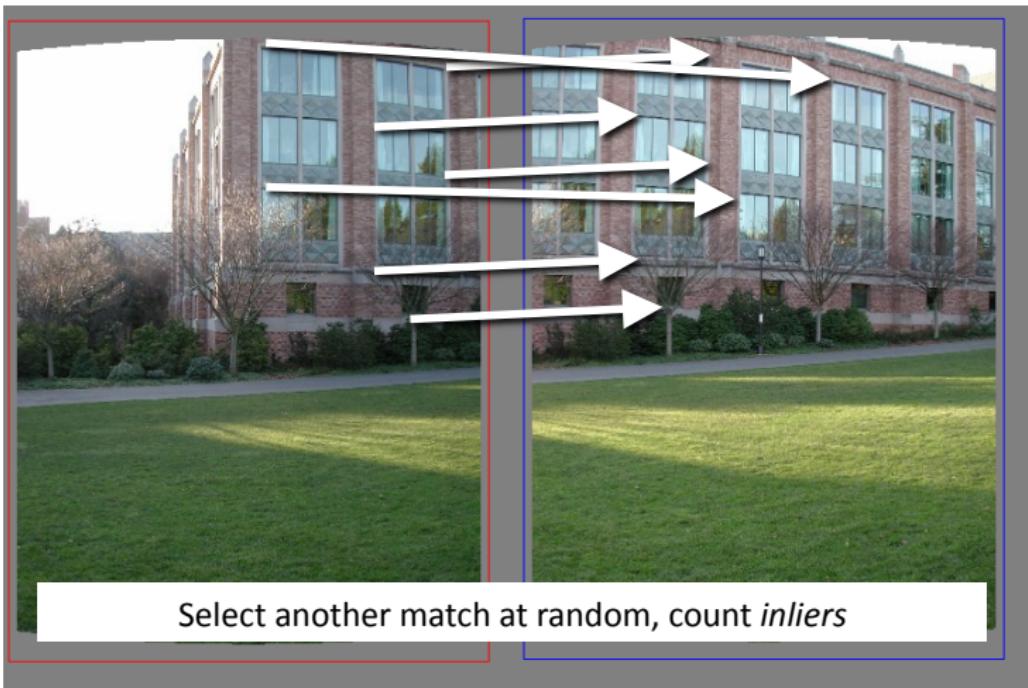
[Source: N. Snavely]

RAndom SAmple Consensus



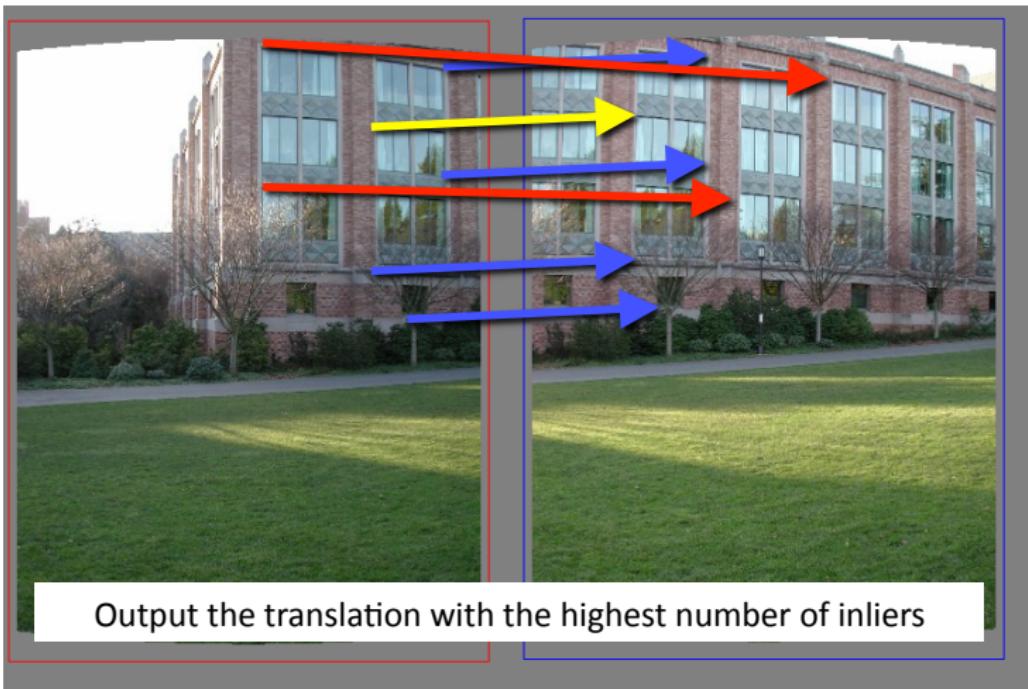
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- RANSAC only has guarantees if there are $< 50\%$ outliers
- "All good matches are alike; every bad match is bad in its own way." – [Tolstoy via Alyosha Efros]

[Source: N. Snavely]

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[Source: R. Urtasun]

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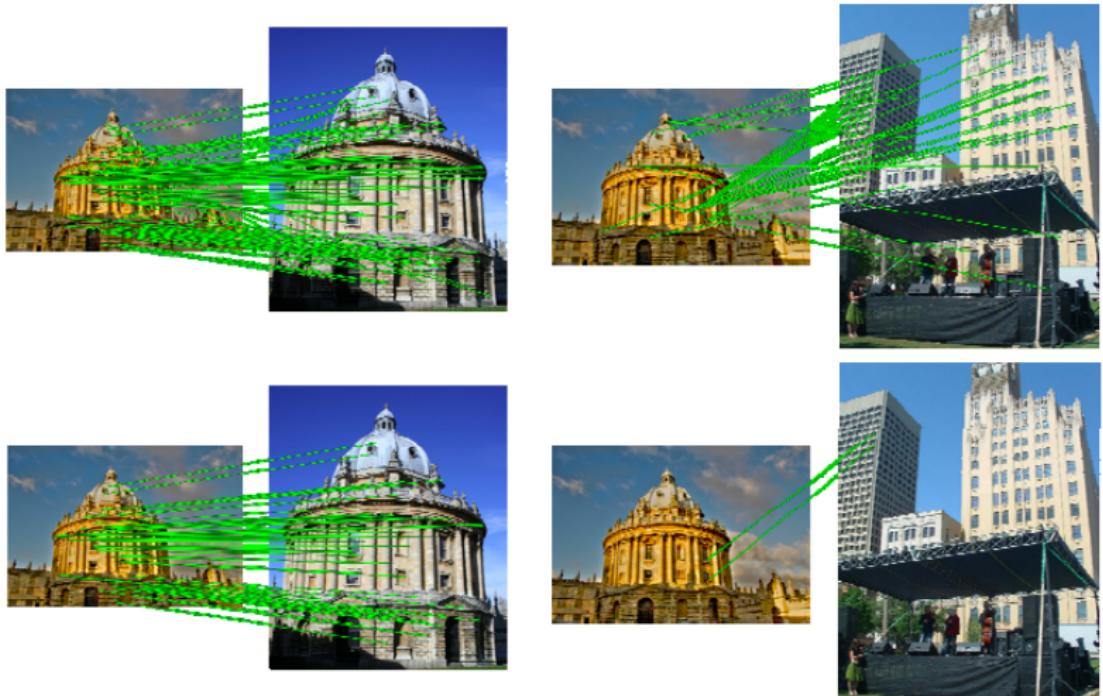
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Cons

- Parameters to tune
- Sometimes too many iterations are required
- Can fail for extremely low inlier ratios
- We can often do better than brute-force sampling

[Source: N. Snavely, slide credit: R. Urtasun]

Ransac Verification



[Source: K. Grauman, slide credit: R. Urtasun]

Summary – Stuff You Need To Know

To match image I and J under affine transformation:

- Compute scale and rotation invariant keypoints in both images
- Compute a (rotation invariant) feature vector in each keypoint (e.g., SIFT)
- Match all features in I to all features in J
- For each feature in reference image I find closest match in J
- If ratio between closest and second closest match is < 0.8 , keep match
- Do RANSAC to compute affine transformation A :
 - Select 3 matches at random
 - Compute A
 - Compute the number of inliers
 - Repeat
 - Find A that gave the most inliers