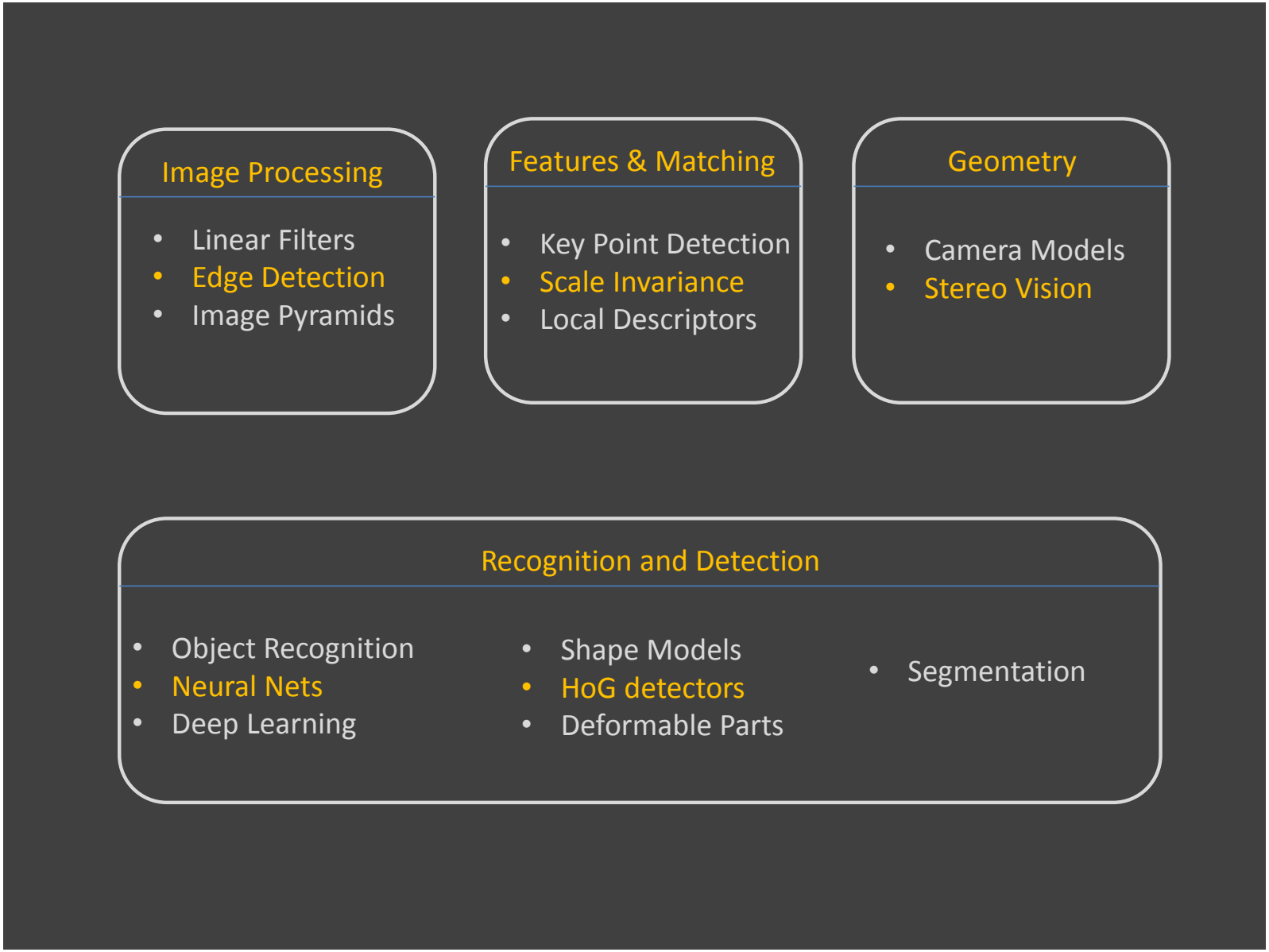
Announcements

- **Class Website**:

  Quercus: https://q.utoronto.ca
- Everything is / will be on Quercus! (assignments on MarkUs)
- **Office Hours (confirmed)**:

  Tuesdays 11:00-12:00 (BA2283), Thursdays 4:00-5:00 pm (BA3289)
- **Waitlist ...**

Syllabus Overview



Images

Digital Image

- Image is a matrix with integer values
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255	255	255	255	255	255	255	255	255	254	211	219	180	160	184	194	191	170	153	172	187	188	179	140	155	210	214	250	255	255	255	255
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209	150	123	226	236	173	148	82	58	45	146	217	205	180	142	150	164	146	144	161	155	180	231	253	255	250	136	68	164	255	255	255
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Digital Image

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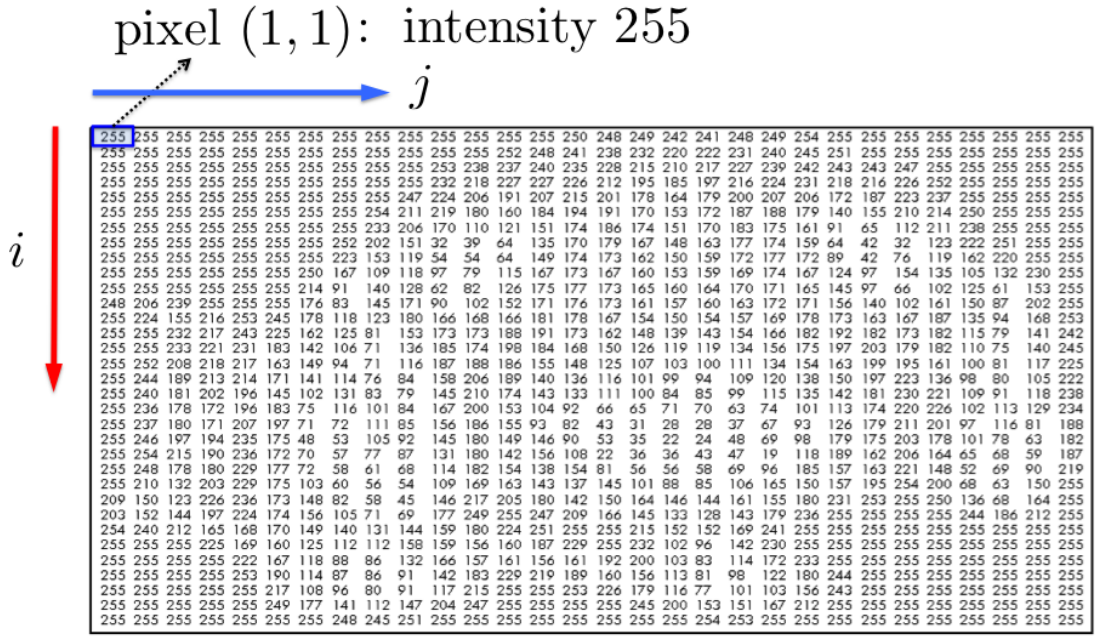

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255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	250	248	249	242	241	248	249	254	255	255	255	255	255	255	255
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255	255	255	255	255	255	255	255	255	255	255	255	255	255	255	250	248	249	242	241	248	249	254	255	255	255	255	255	255	255
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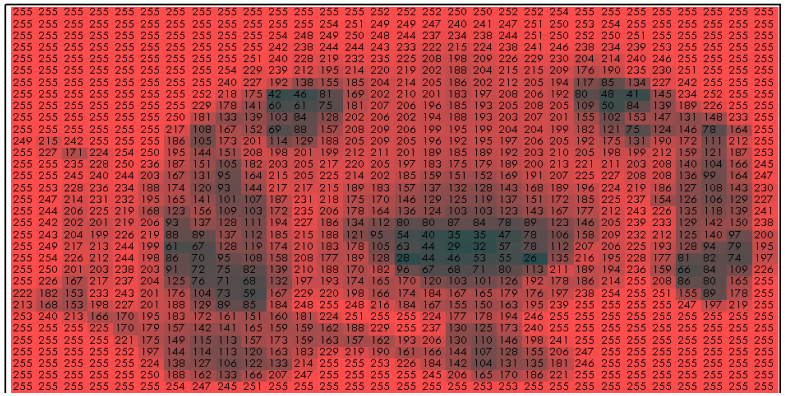
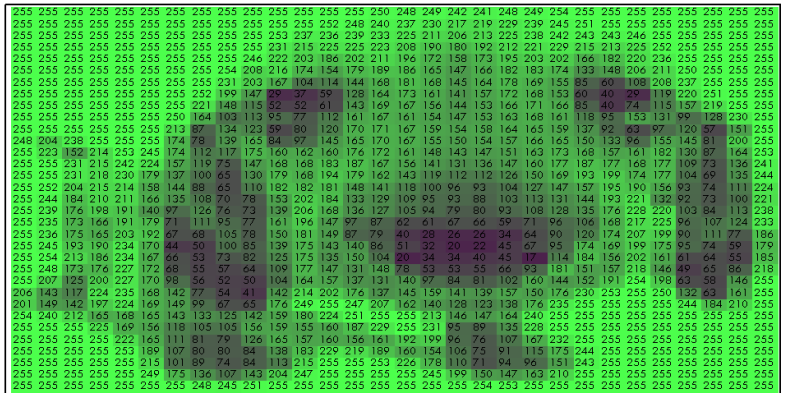
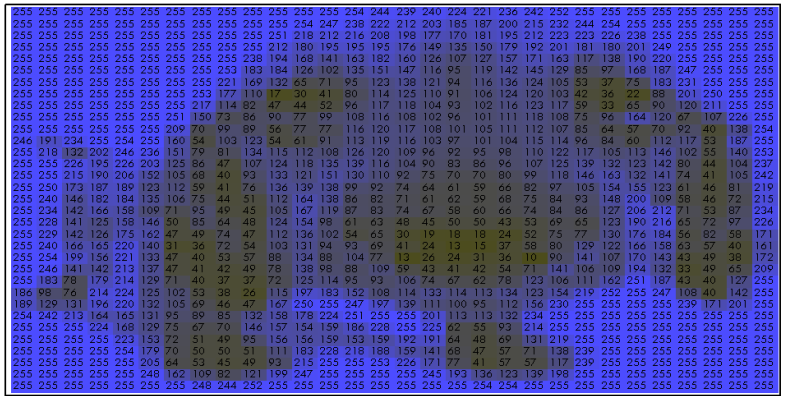
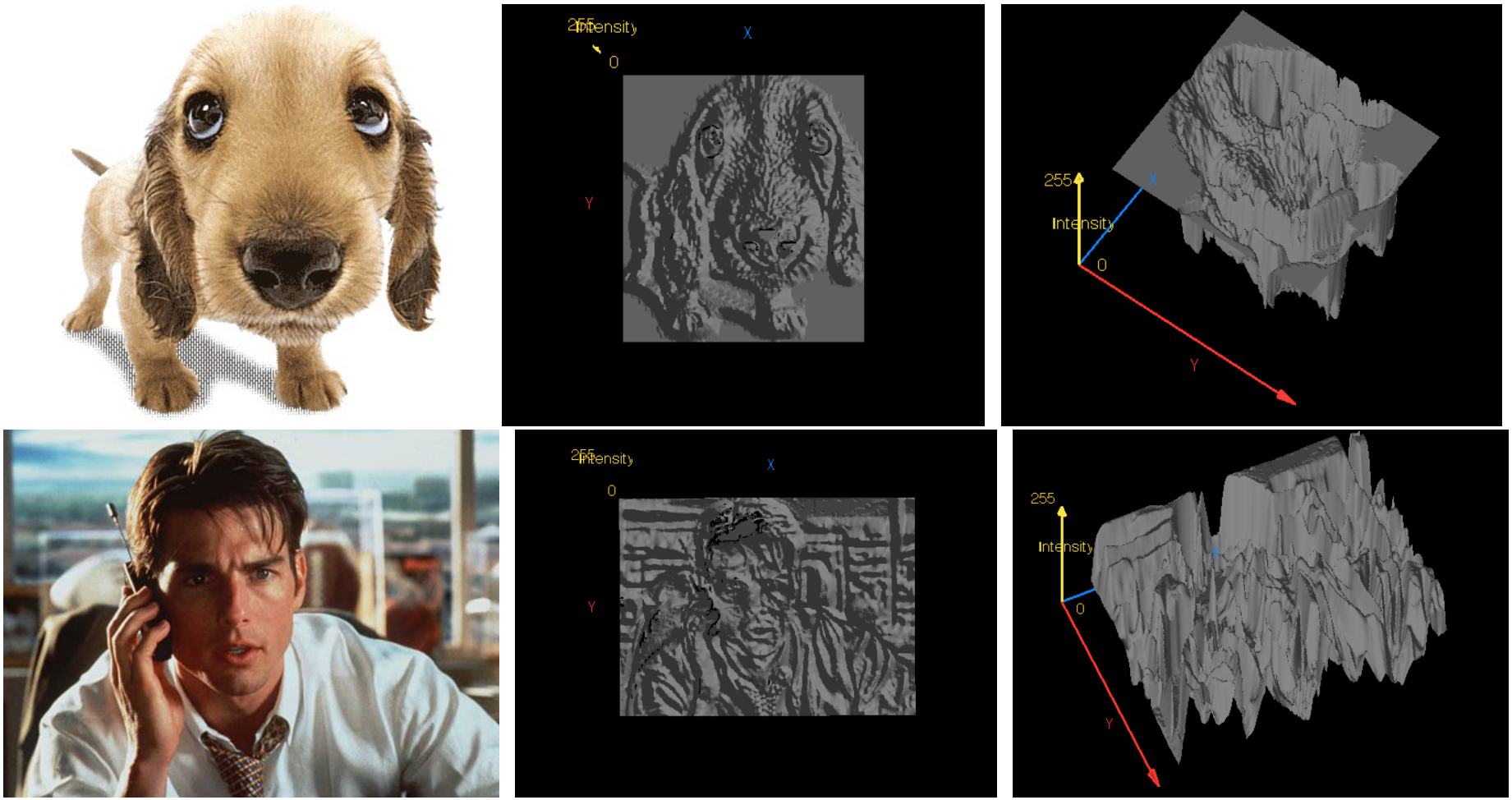






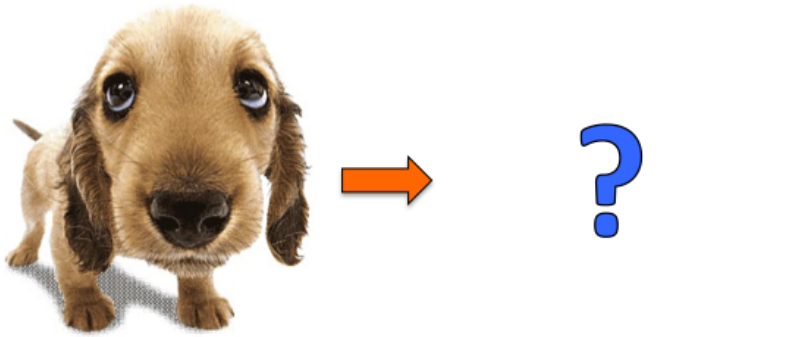
Intensity



- We can think of a (grayscale) image as a function $f : \mathbb{R}^2 \to \mathbb{R}$ giving the intensity at position (i, j)
- Intensity 0 is black and 255 is white

Image Transformations

- As with any function, we can apply operators to an image, e.g.:



$I(i,j)$ $\qquad$ $J(i,j) = I(i,j) + 50$

- We'll talk about special kinds of operators, **correlation** and **convolution** (linear filtering)

[Adapted from: N. Snavely]

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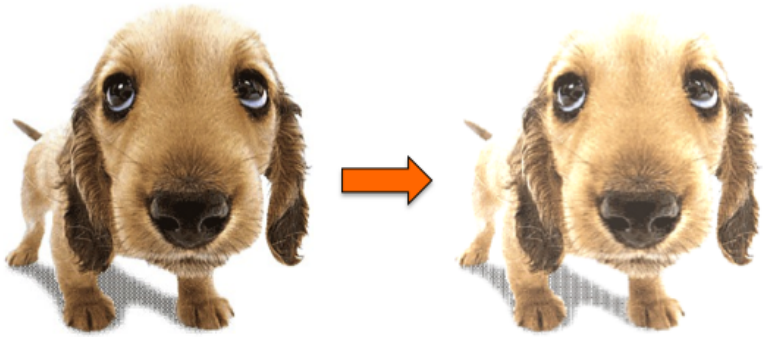

$I(i,j)$ $\quad$ $J(i,j) = I(i,j) + 50$ $\quad$ $J(i,j) = I(i,-j)$

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- As with any function, we can apply operators to an image, e.g.:

$I(i,j)$ $\quad$ $J(i,j) = I(i,j) + 50$ $\quad$ $J(i,j) = I(i,-j)$

- We'll talk about special kinds of operators, **correlation** and **convolution** (linear filtering)

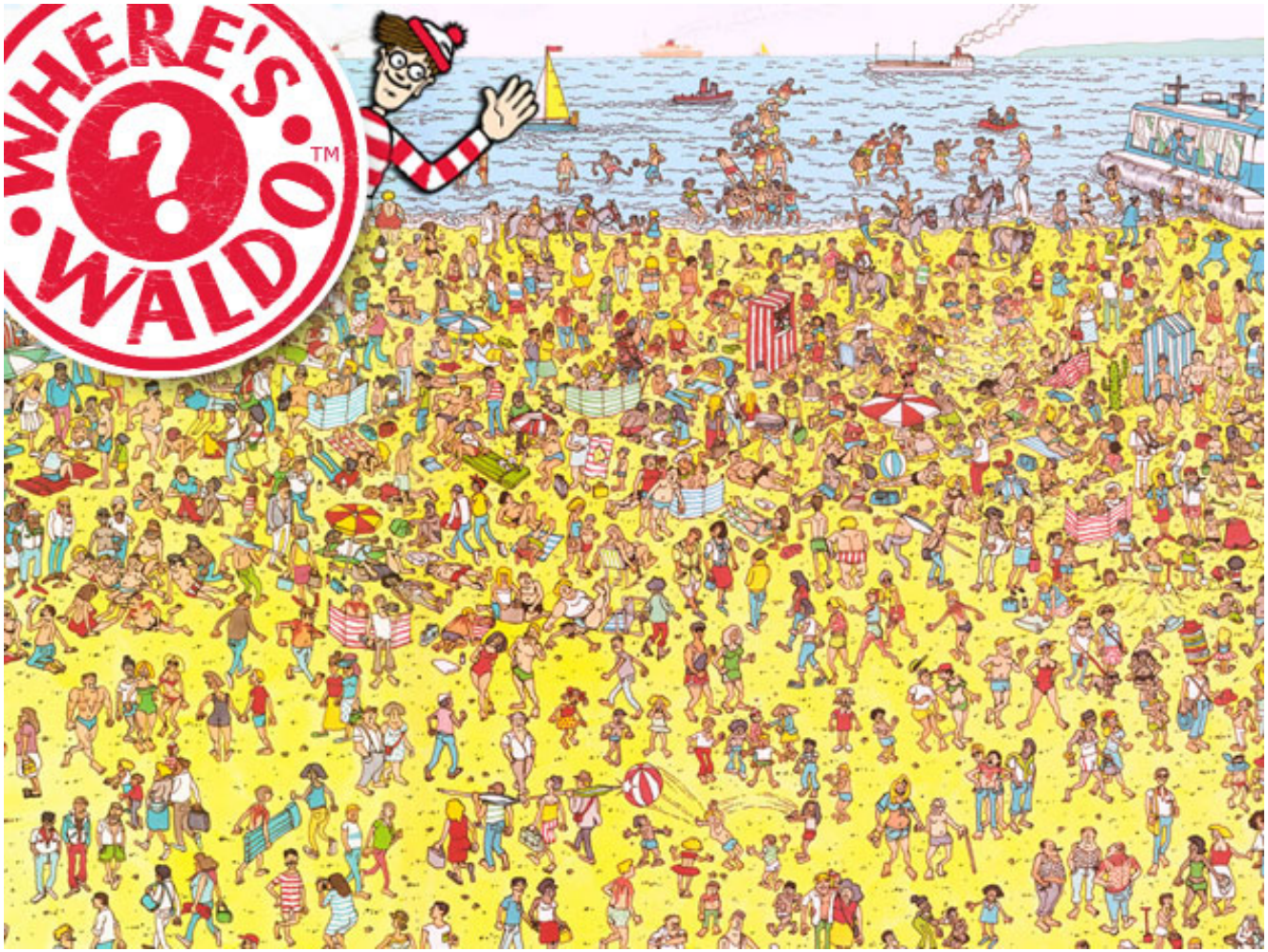
[Adapted from: N. Snavely]

Linear Filters

Reading: Szeliski book, Chapter 3.2

Motivation: Finding Waldo

- How can we find Waldo?



[Source: R. Urtasun]

Answer

- Slide and compare!
- In formal language: **filtering**

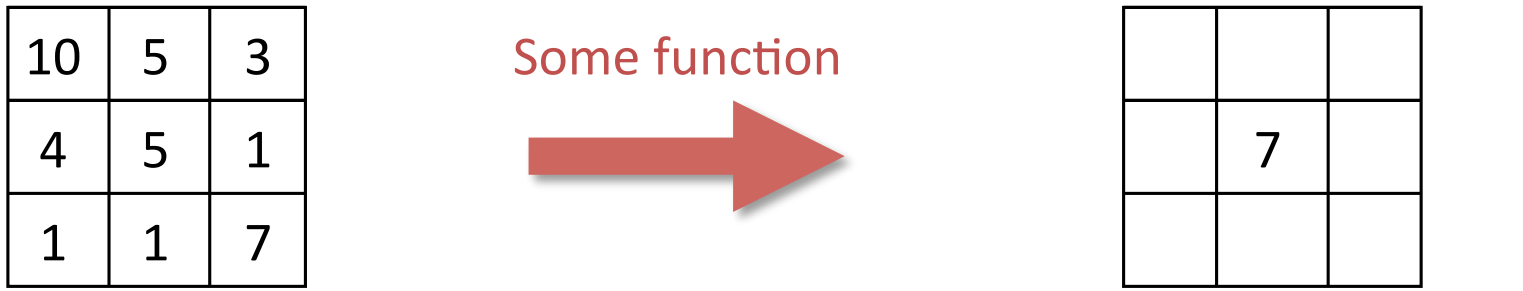
Motivation: Noise reduction

- Given a camera and a still scene, how can you reduce noise?

[Source: S. Seitz]

Image Filtering

- Modify the pixels in an image based on some function of a local neighborhood of each pixel
- In other words... Filtering



[Source: L. Zhang]

Applications of Filtering

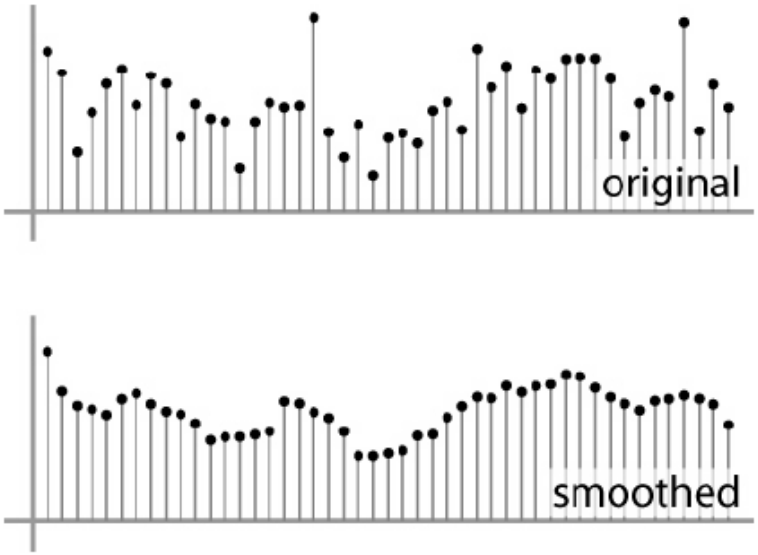
- Enhance an image, e.g., **denoise**.
- Detect patterns, e.g., **template matching**.
- Extract information, e.g., **texture, edges**.

Applications of Filtering

- Enhance an image, e.g., **denoise**. Let's talk about this first
- Detect patterns, e.g., **template matching**.
- Extract information, e.g., **texture, edges**.

Noise reduction

- Simplest thing: replace each pixel by the average of its neighbors.
- This assumes that neighboring pixels are similar, and the noise to be independent from pixel to pixel.



[Source: S. Marschner]

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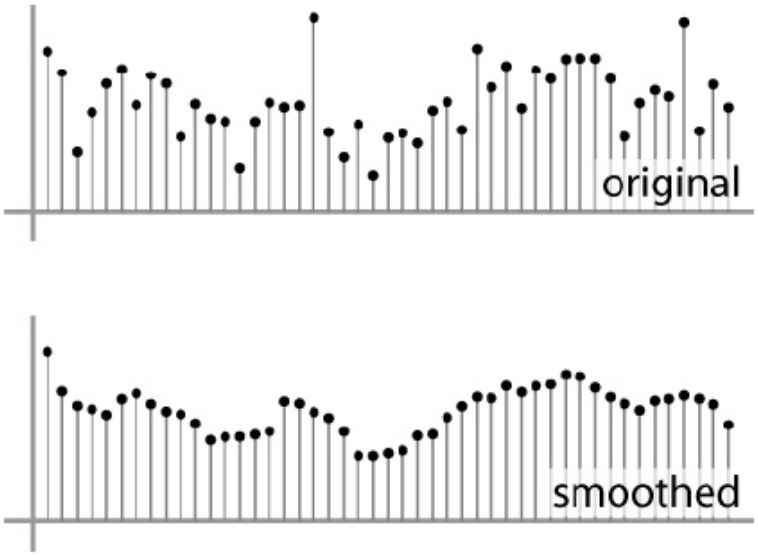
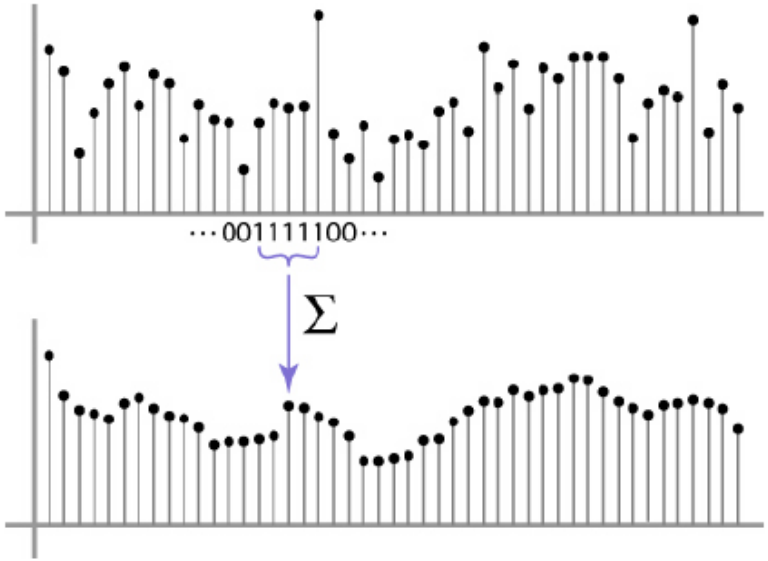


[Source: S. Marschner]

Noise reduction

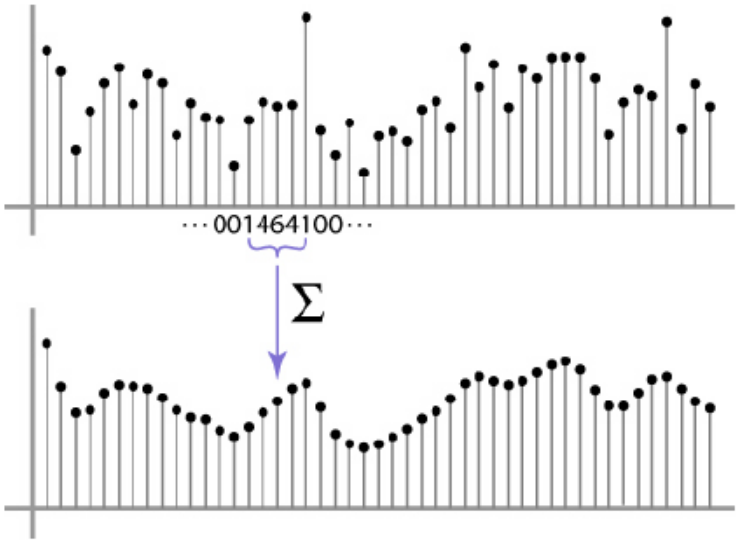
- Simplest thing: replace each pixel by the average of its neighbors
- This assumes that neighboring pixels are similar, and the noise to be independent from pixel to pixel.
- **Moving average** in 1D: [1, 1, 1, 1, 1]/5



[Source: S. Marschner]

Noise reduction

- Simplest thing: replace each pixel by the average of its neighbors
- This assumes that neighboring pixels are similar, and the noise to be independent from pixel to pixel.
- Non-uniform weights [1, 4, 6, 4, 1] / 16



[Source: S. Marschner]

Moving Average in 2D

$I(i,j)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

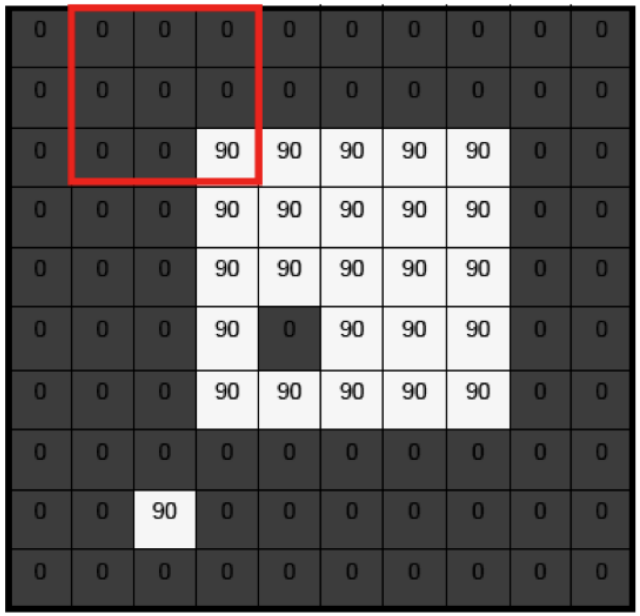
$G(i,j)$

	0								

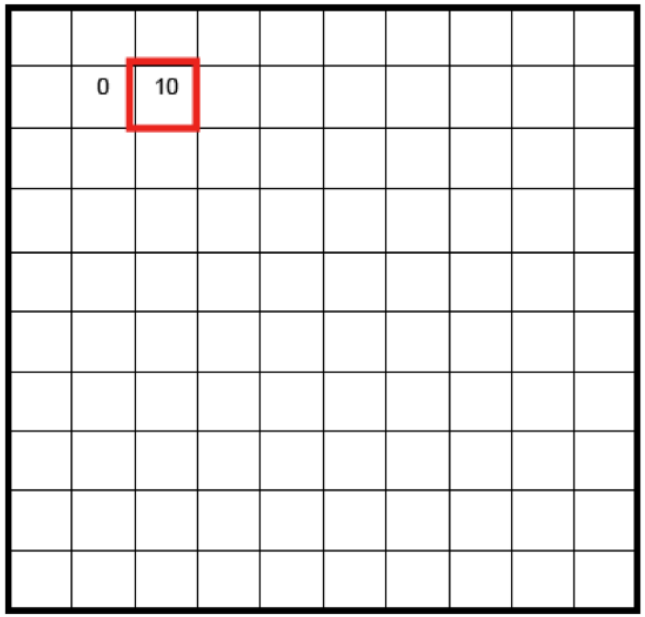
[Source: S. Seitz]

Moving Average in 2D

$I(i,j)$



$G(i,j)$



[Source: S. Seitz]

Moving Average in 2D

$I(i,j)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G(i,j)$

	0	10	20						

[Source: S. Seitz]

Moving Average in 2D

$I(i,j)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G(i,j)$

	0	10	20	30					

[Source: S. Seitz]

Moving Average in 2D

$I(i,j)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G(i,j)$

	0	10	20	30	30				

[Source: S. Seitz]

Moving Average in 2D

$I(i,j)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G(i,j)$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

[Source: S. Seitz]

Linear Filtering: Correlation

- Involves weighted combinations of pixels in small neighborhoods:

$$G(i,j) = \frac{1}{(2k+1)^2} \sum_{u=-k}^{k} \sum_{v=-k}^{k} I(i+u, j+v)$$

- The output pixels value is determined as a weighted sum of input pixel values

$$G(i,j) = \sum_{u=-k}^{k} \sum_{v=-k}^{k} F(u,v) \cdot I(i+u, j+v)$$

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- This operator is the **correlation** operator

$$G = F \otimes I$$

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Linear Filtering: Correlation

- It's really easy!

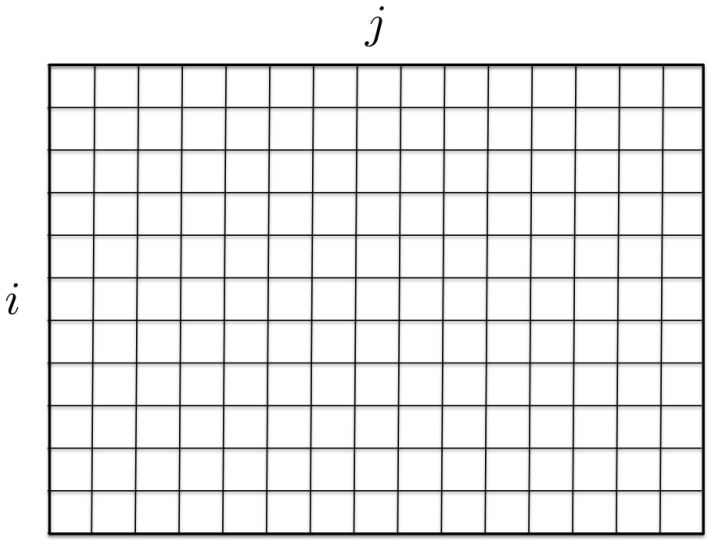
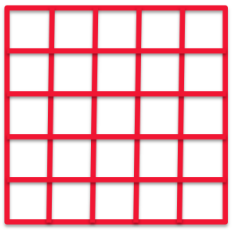


image I



filter F

Linear Filtering: Correlation

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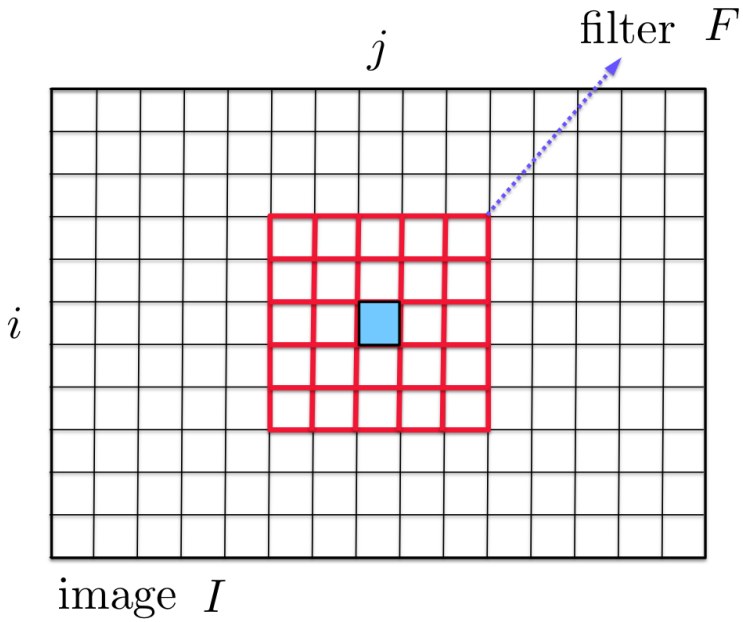

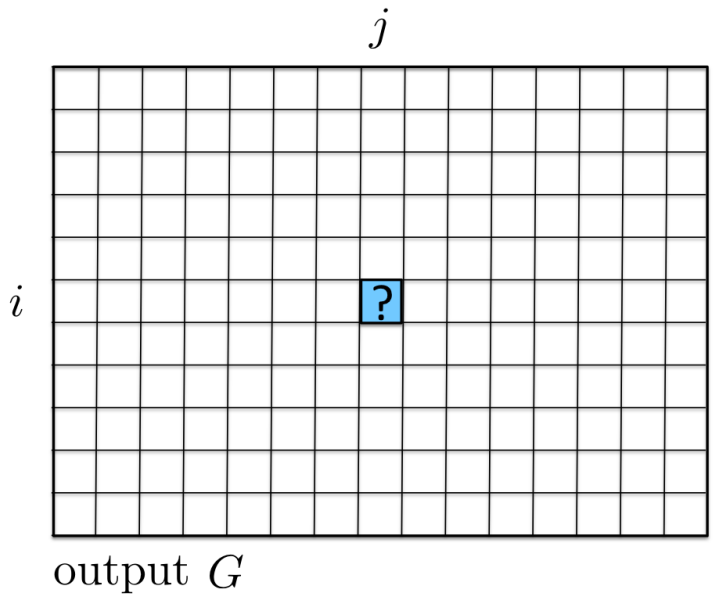

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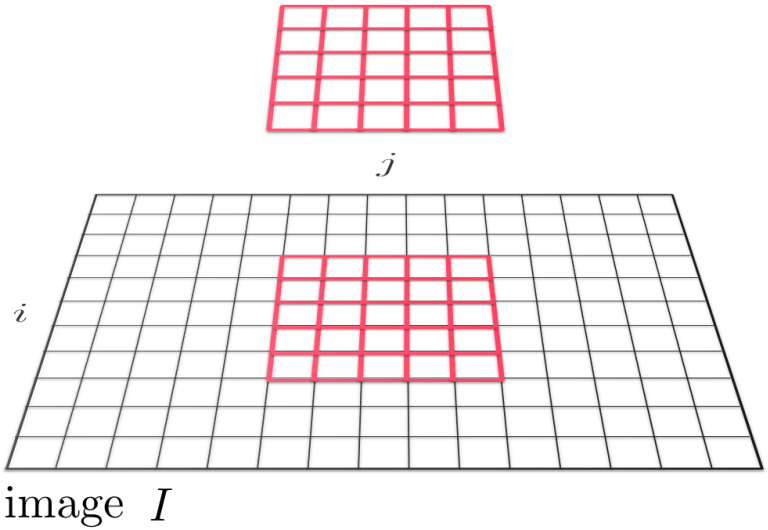

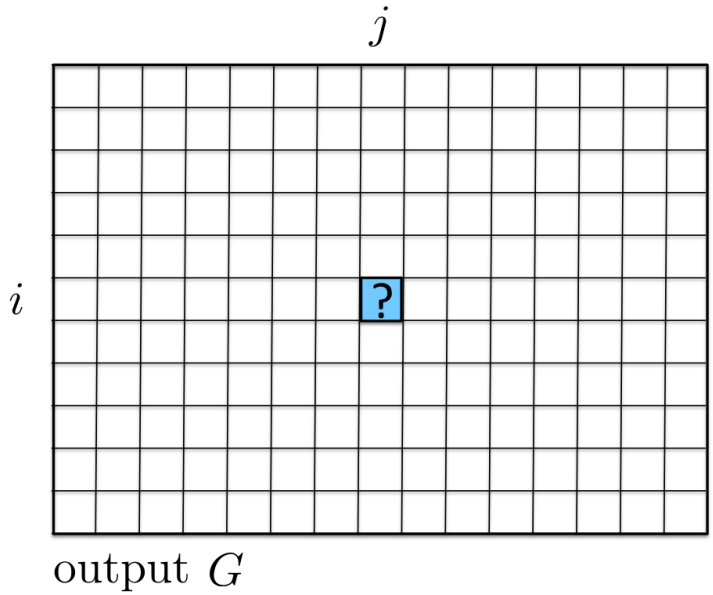

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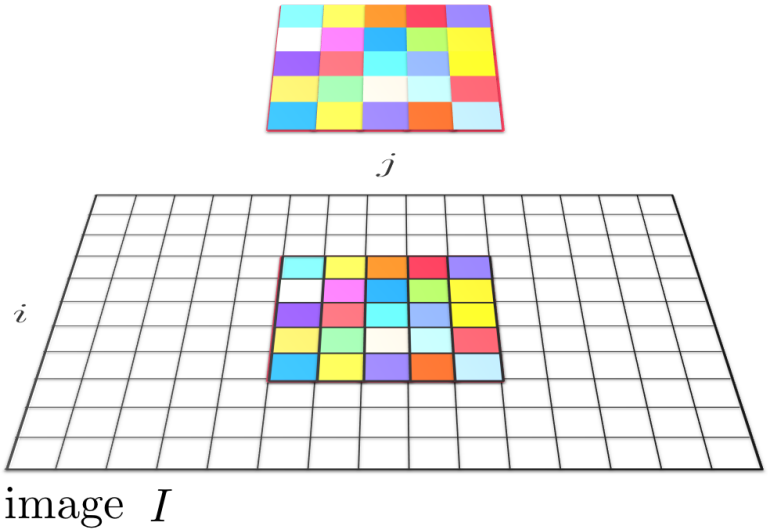
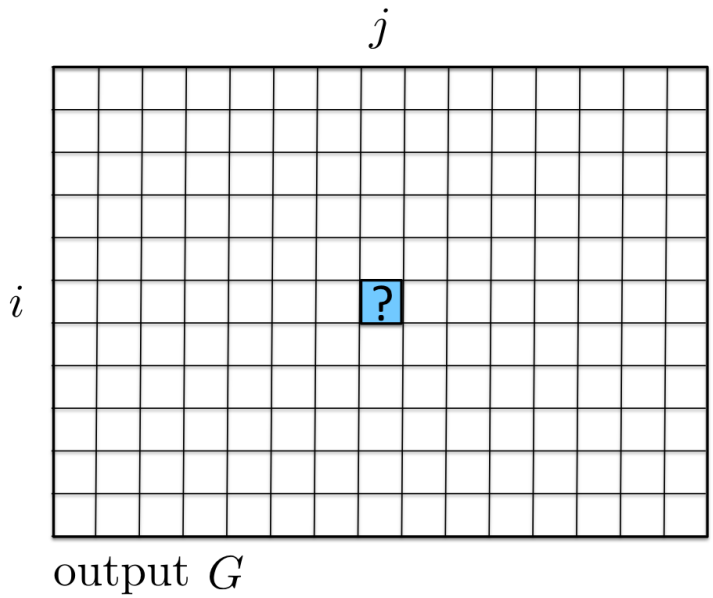


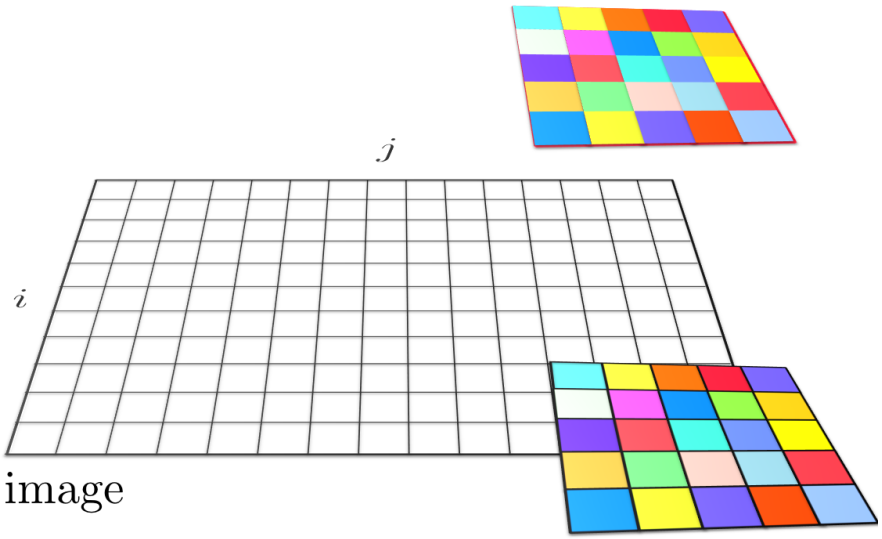


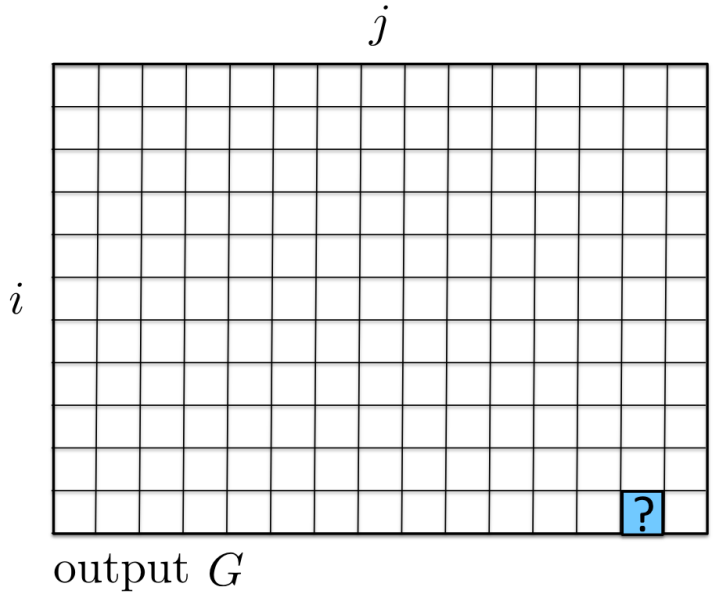
$$G(i,j) = \sum_{u=-k}^{k} \sum_{v=-k}^{k} F(u,v) \cdot I(i+u, j+v)$$

$$G(i,j) = F(\blacksquare) \cdot I(\blacksquare) + F(\blacksquare) \cdot I(\blacksquare) + F(\blacksquare) \cdot I(\blacksquare) + \ldots + F(\blacksquare) \cdot I(\blacksquare)$$

Linear Filtering: Correlation

- What happens along the borders of the image?





$$G(i,j) = \sum_{u=-k}^{k} \sum_{v=-k}^{k} F(u,v) \cdot I(i+u, j+v)$$

$$G(i,j) = F(\square) \cdot I(\square) + F(\square) \cdot I(\square) + F(\square) \cdot I(\square) + \ldots + F(\square) \cdot I(\square)$$

Boundary Effects

- What happens at the border of the image? What's the size of the output matrix?
- OpenCV: cv2.filter2D, MATLAB: filter2(f, I, shape)
- shape = 'full' output size is bigger than the image
- shape = 'same': output size is same as *I*
- shape = 'valid': output size is smaller than the image

[Source: S. Lazebnik]

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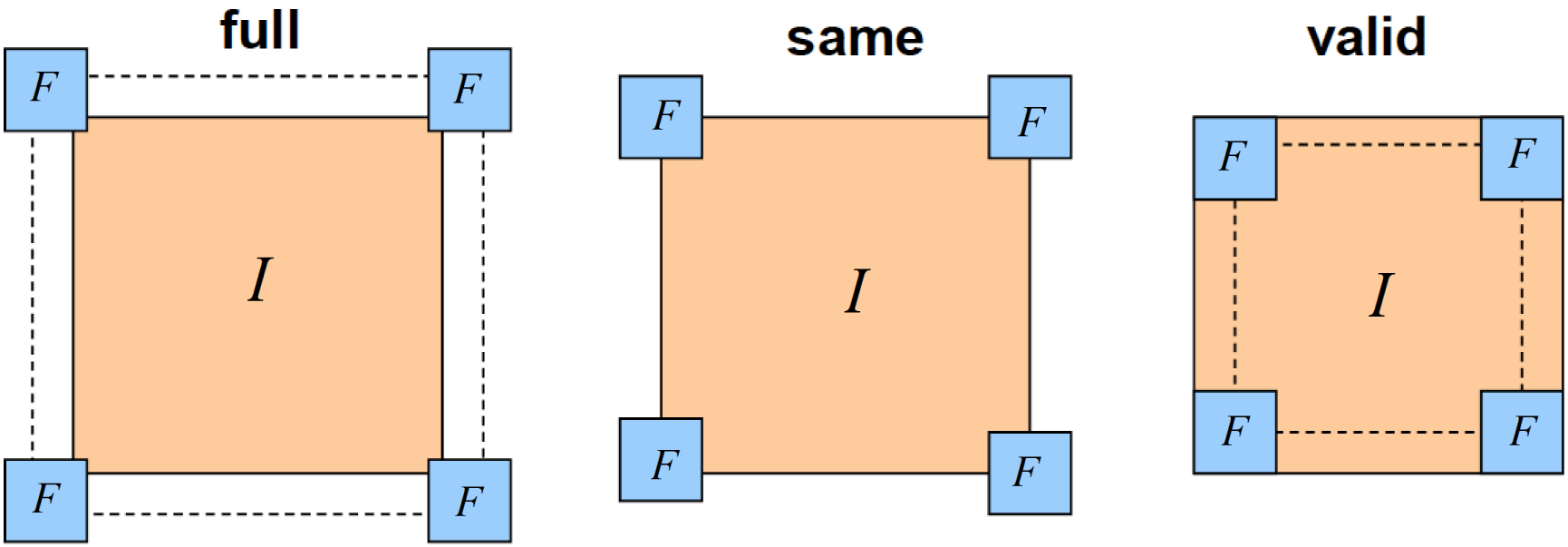
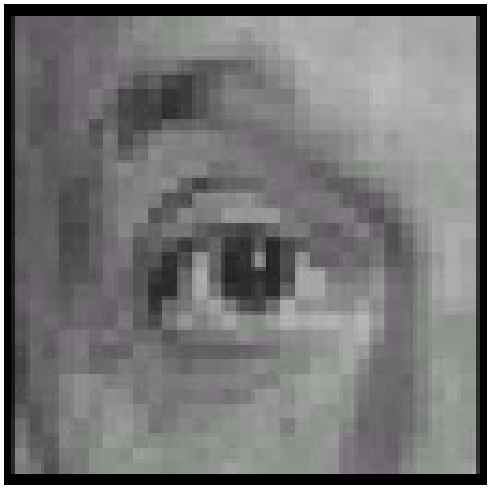


[Source: S. Lazebnik]

Filtering with Correlation: Example

- What's the result?



Original

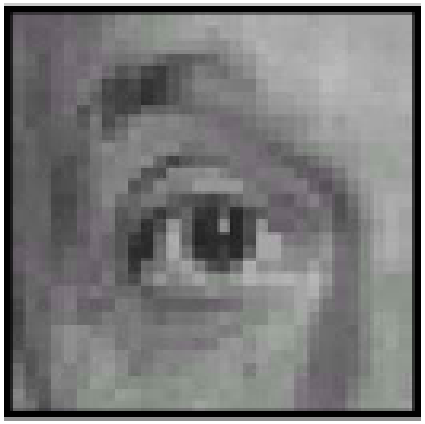
0	0	0
0	1	0
0	0	0

?

[Source: D. Lowe]

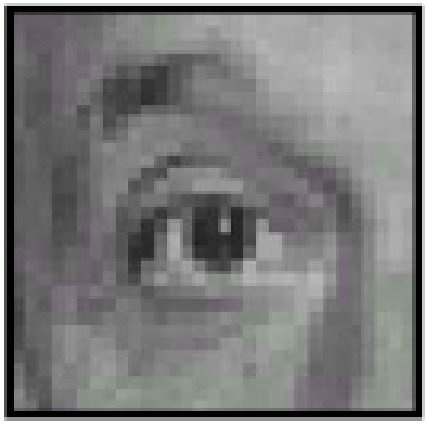
Filtering with Correlation: Example

- What's the result?



Original

0	0	0
0	1	0
0	0	0

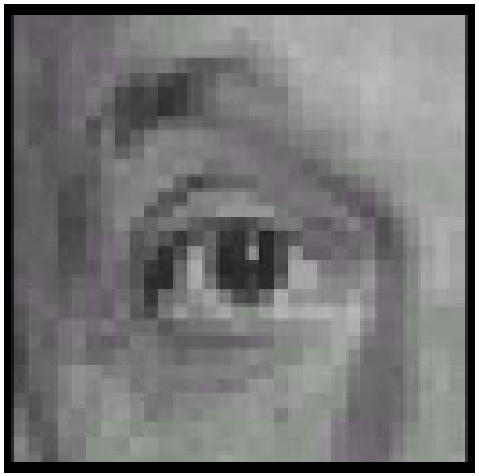


Filtered (no change)

[Source: D. Lowe]

Filtering with Correlation: Example

- What's the result?



Original

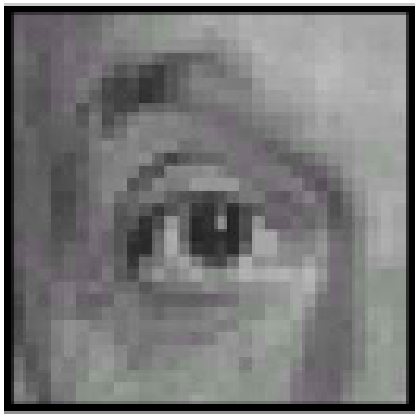
0	0	0
0	0	1
0	0	0

?

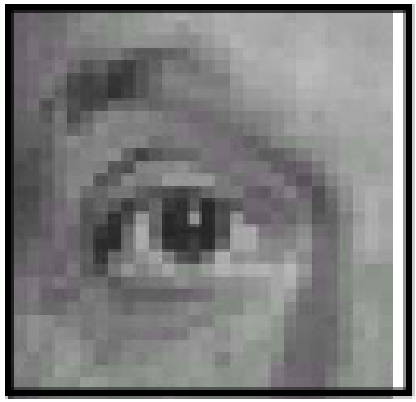
[Source: D. Lowe]

Filtering with Correlation: Example

- What's the result?



0	0	0
0	0	1
0	0	0



[Source: D. Lowe]

Filtering with Correlation: Example

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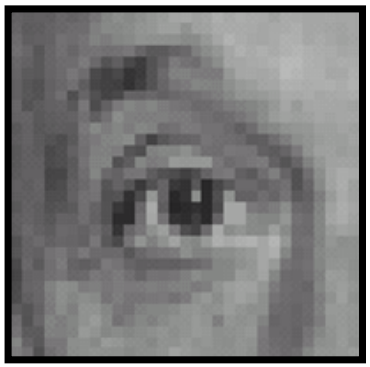


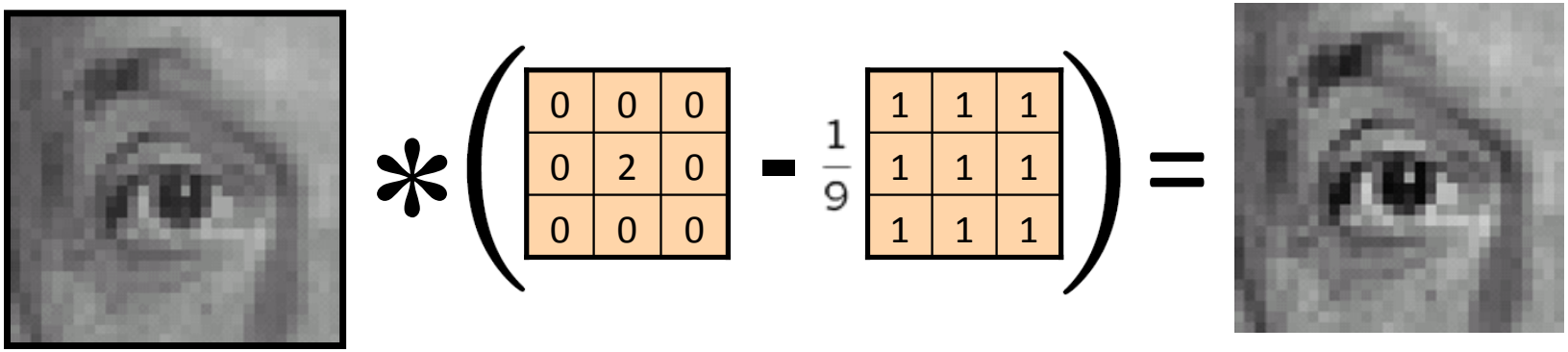
Original

$$* \left(\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \right) =$$

[Source: D. Lowe]

Filtering with Correlation: Example

- What's the result?

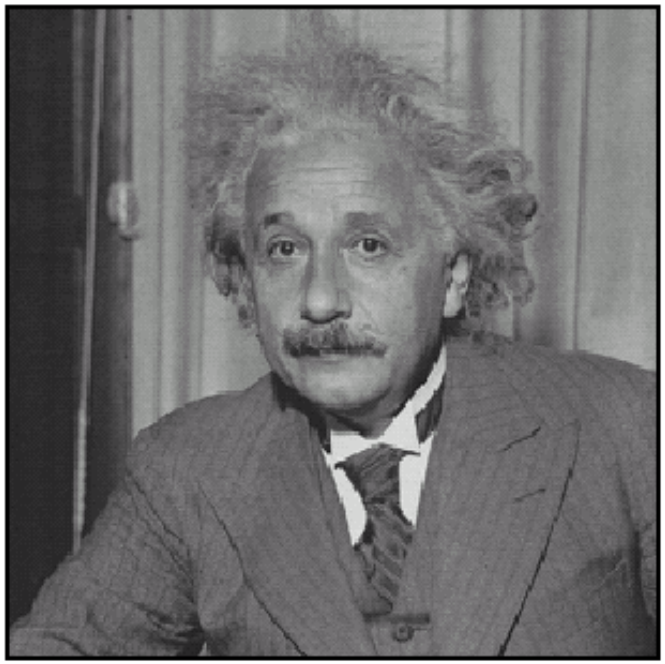


Original

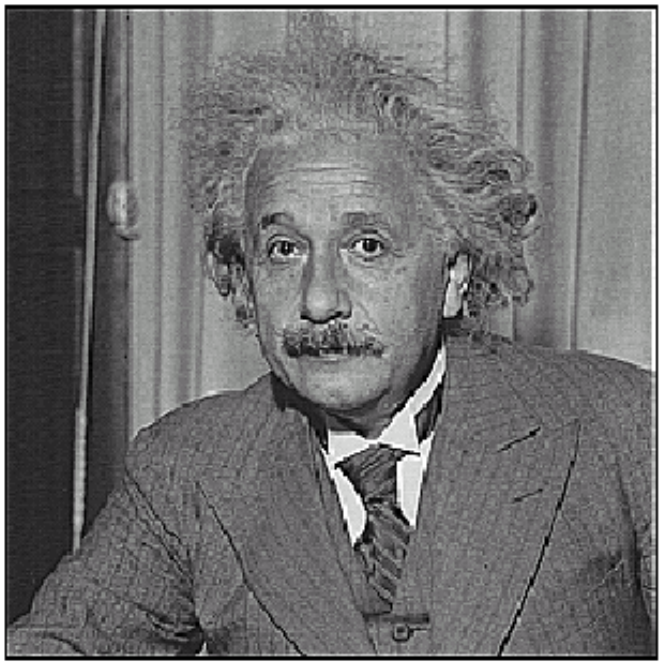
Sharpening filter
(accentuates edges)

[Source: D. Lowe]

Sharpening



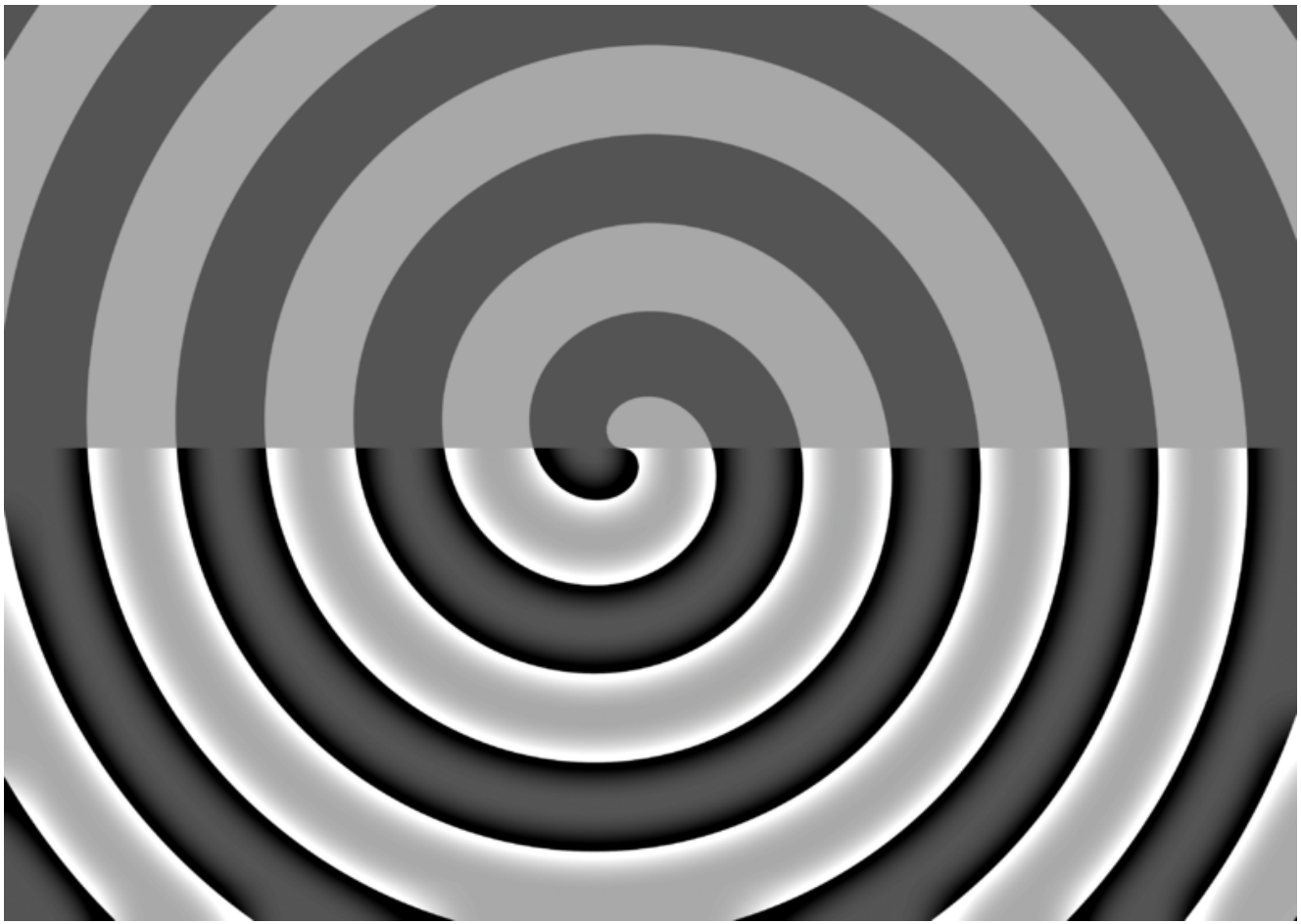
before



after

This is a prelude to edge detection (next time)! [Source: D. Lowe]

Sharpening

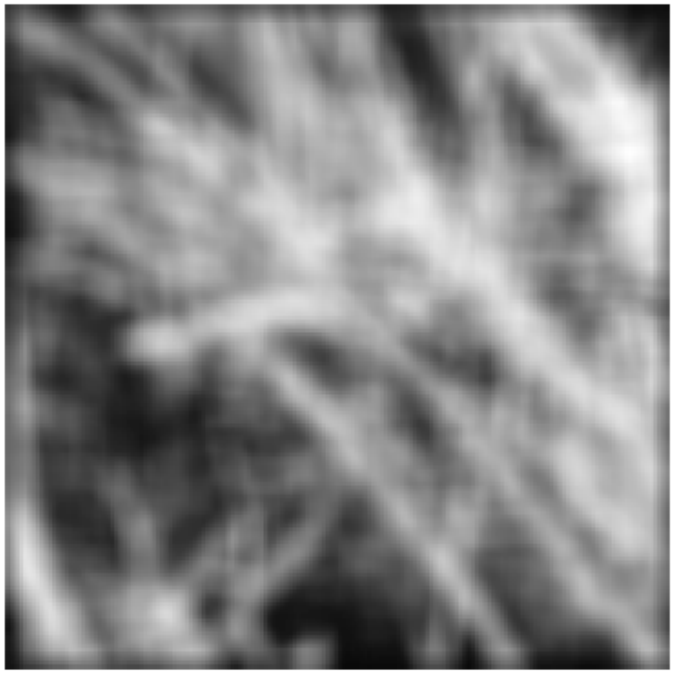


[Source: N. Snavely]

Smoothing by averaging

depicts box filter:
white = high value, black = low value

original



filtered

- What if the filter size was 5 x 5 instead of 3 x 3?

[Source: K. Graumann]

Gaussian filter

- What if we want nearest neighboring pixels to have the most influence on the output?
- Removes high-frequency components from the image (low-pass filter).

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$I(i,j)$

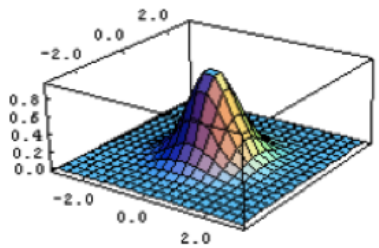
$$\frac{1}{16}$$

1	2	1
2	4	2
1	2	1

$F(i,j)$

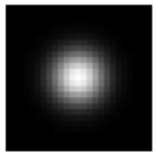
This kernel is an approximation of a 2d Gaussian function:

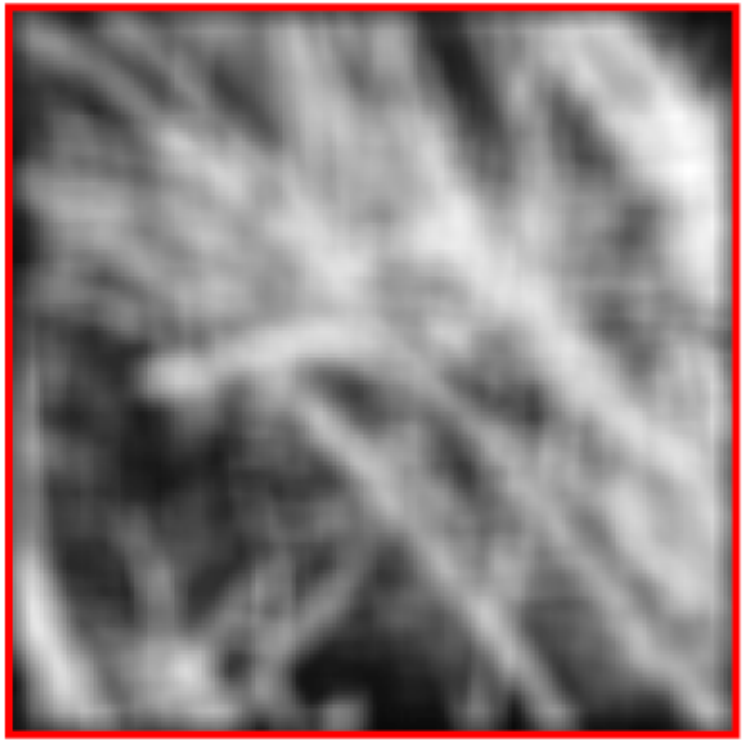
$$h(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}}$$



[Source: S. Seitz]

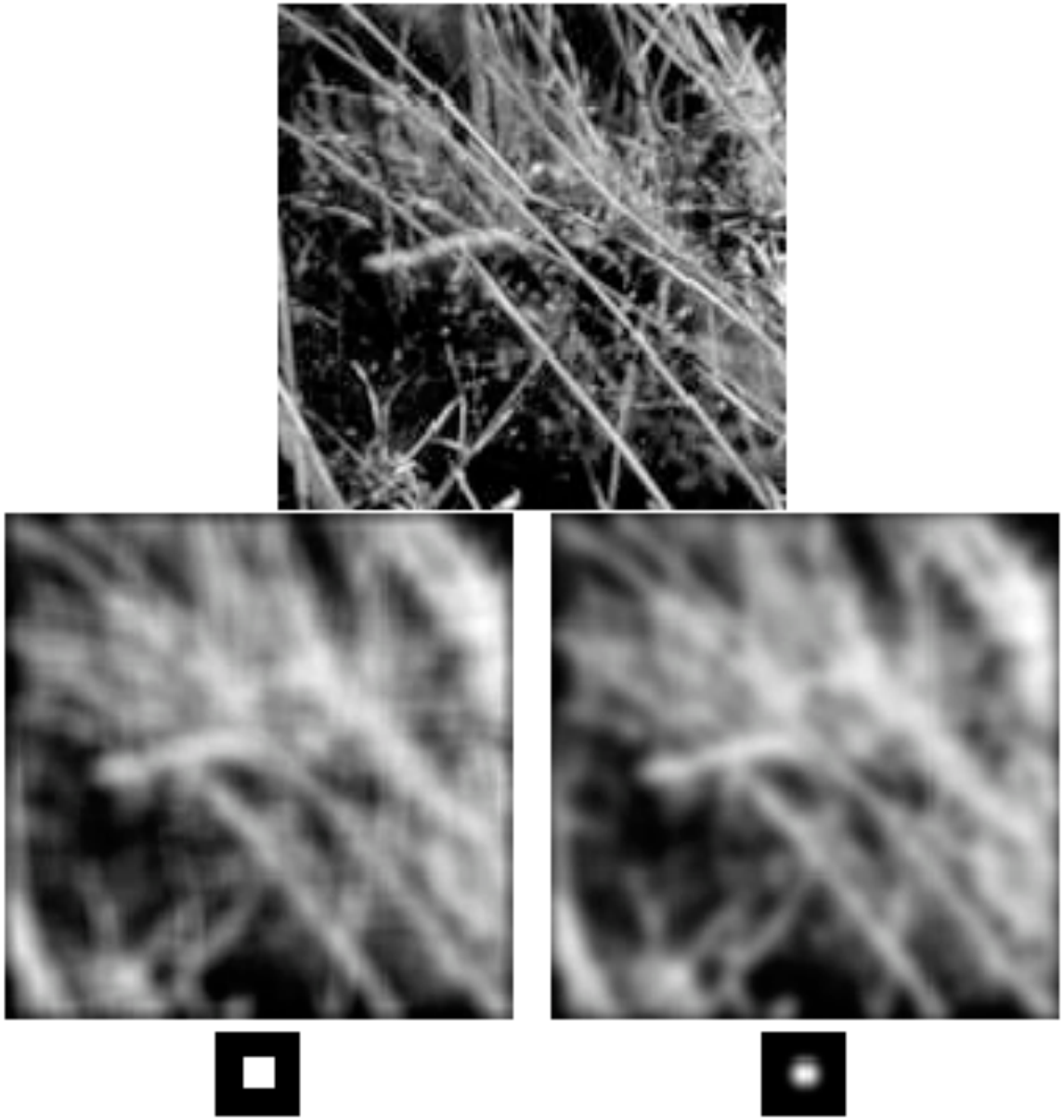
Smoothing with a Gaussian





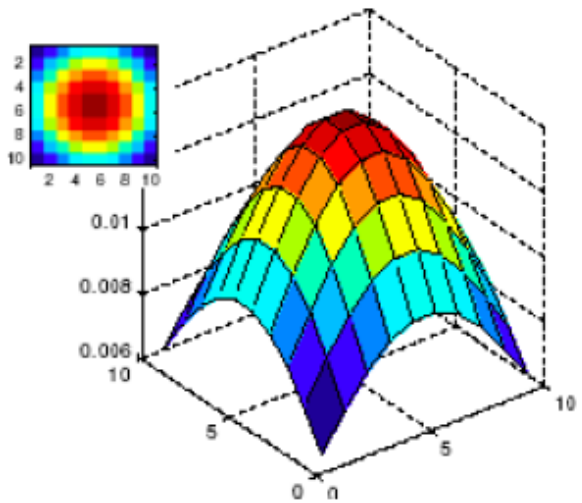
[Source: K. Grauman]

Mean vs Gaussian

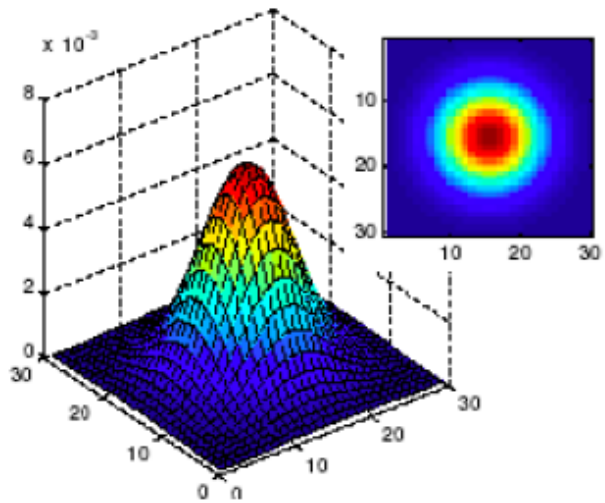


Gaussian filter: Parameters

- **Size of filter or mask**: Gaussian function has infinite support, but discrete filters use finite kernels.



σ = 5 with
10 x 10
kernel

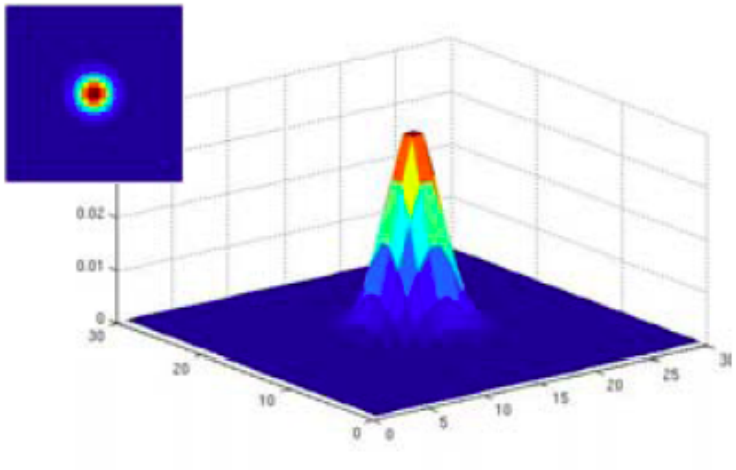


σ = 5 with
30 x 30
kernel

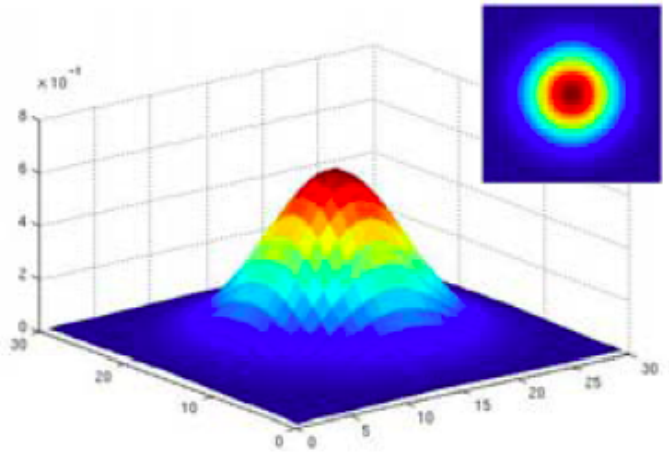
[Source: K. Grauman]

Gaussian filter: Parameters

- **Variance of the Gaussian**: determines extent of smoothing.



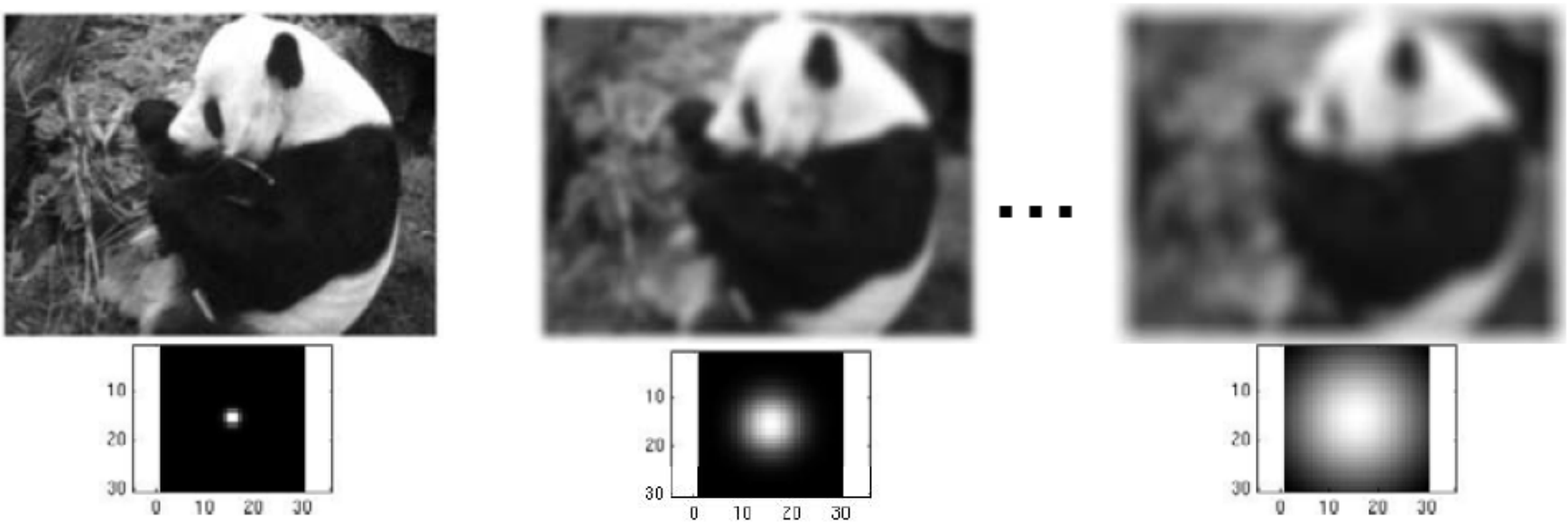
σ = 2 with 30 x 30 kernel



σ = 5 with 30 x 30 kernel

[Source: K. Grauman]

Gaussian filter: Parameters



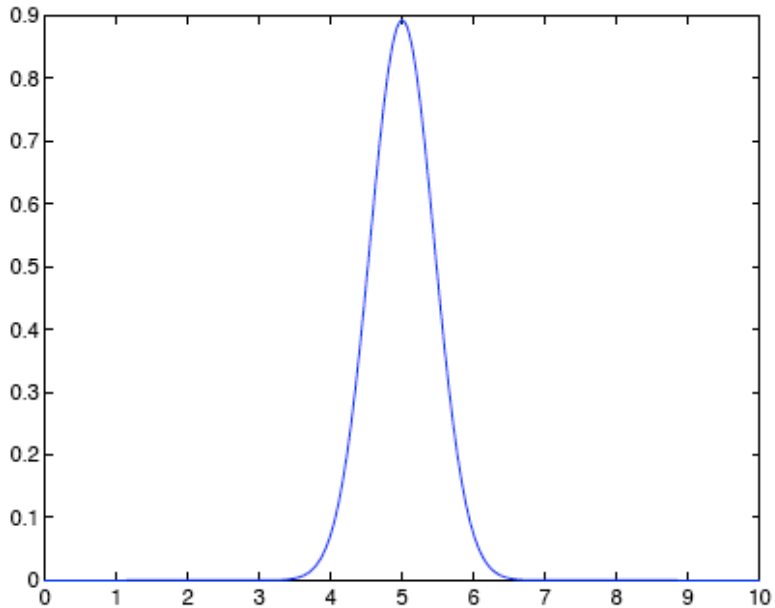

```
for sigma=1:3:10
   h = fspecial('gaussian', fsize, sigma);
   out = imfilter(im, h);
   imshow(out);
   pause;
end
```

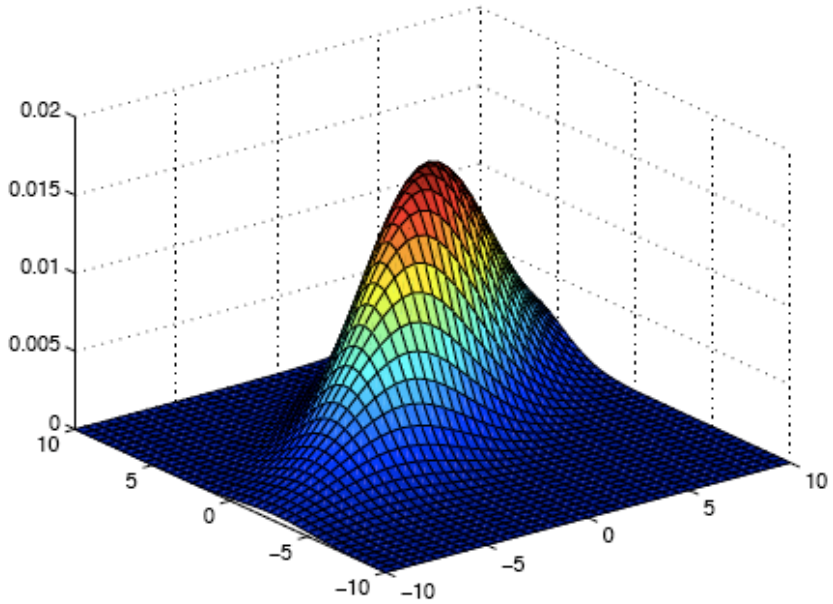
[Source: K. Grauman]

Is this the most general Gaussian?

- No, the most general form is anisotropic (i.e not symmetric) $\mathbf{x} \in \Re^d$

$$\mathcal{N}(\mathbf{x};\, \mu, \Sigma) = \frac{1}{(2\pi)^{d/2}|\Sigma|^{1/2}} \exp\left(-\frac{1}{2}(\mathbf{x}-\mu)^T\Sigma^{-1}(\mathbf{x}-\mu)\right)$$





- But the simplified version is typically used for filtering.

Properties of the Smoothing

- All values are positive.
- They all sum to 1 to prevent re-scaling of the image.

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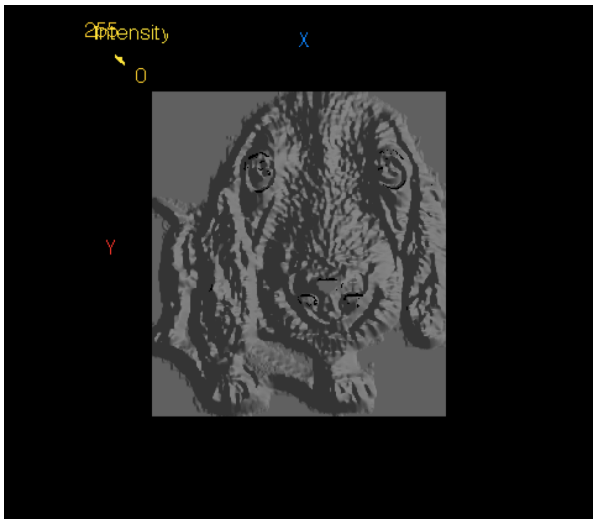

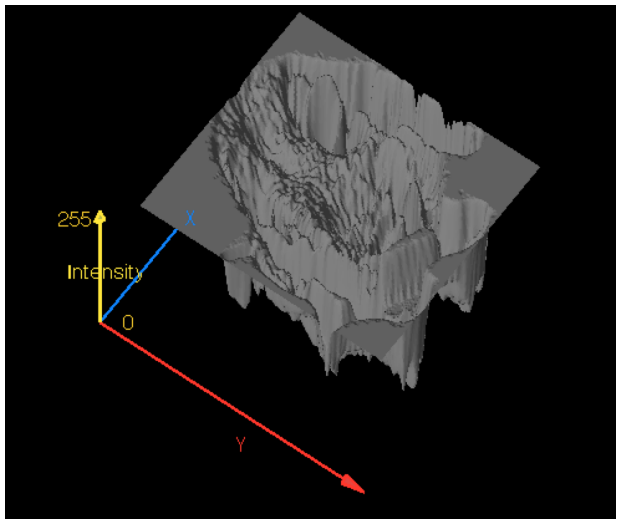

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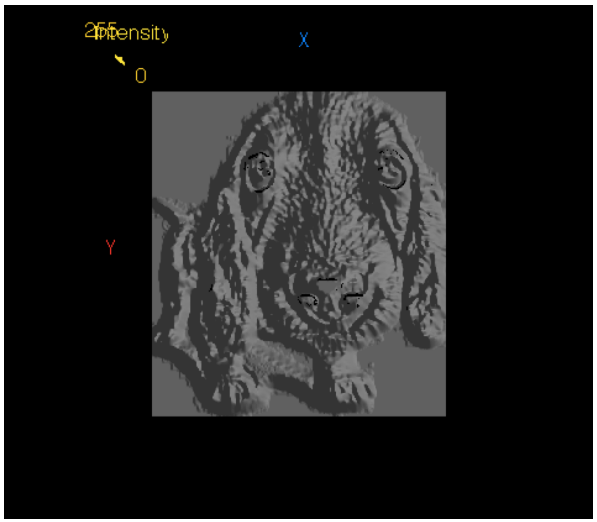

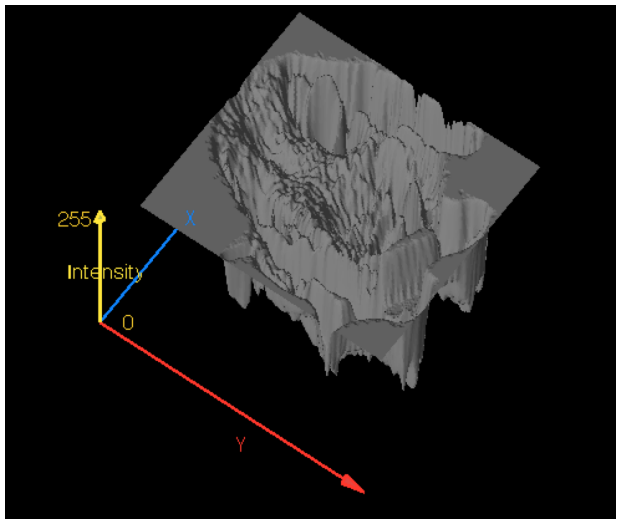

Finding Waldo

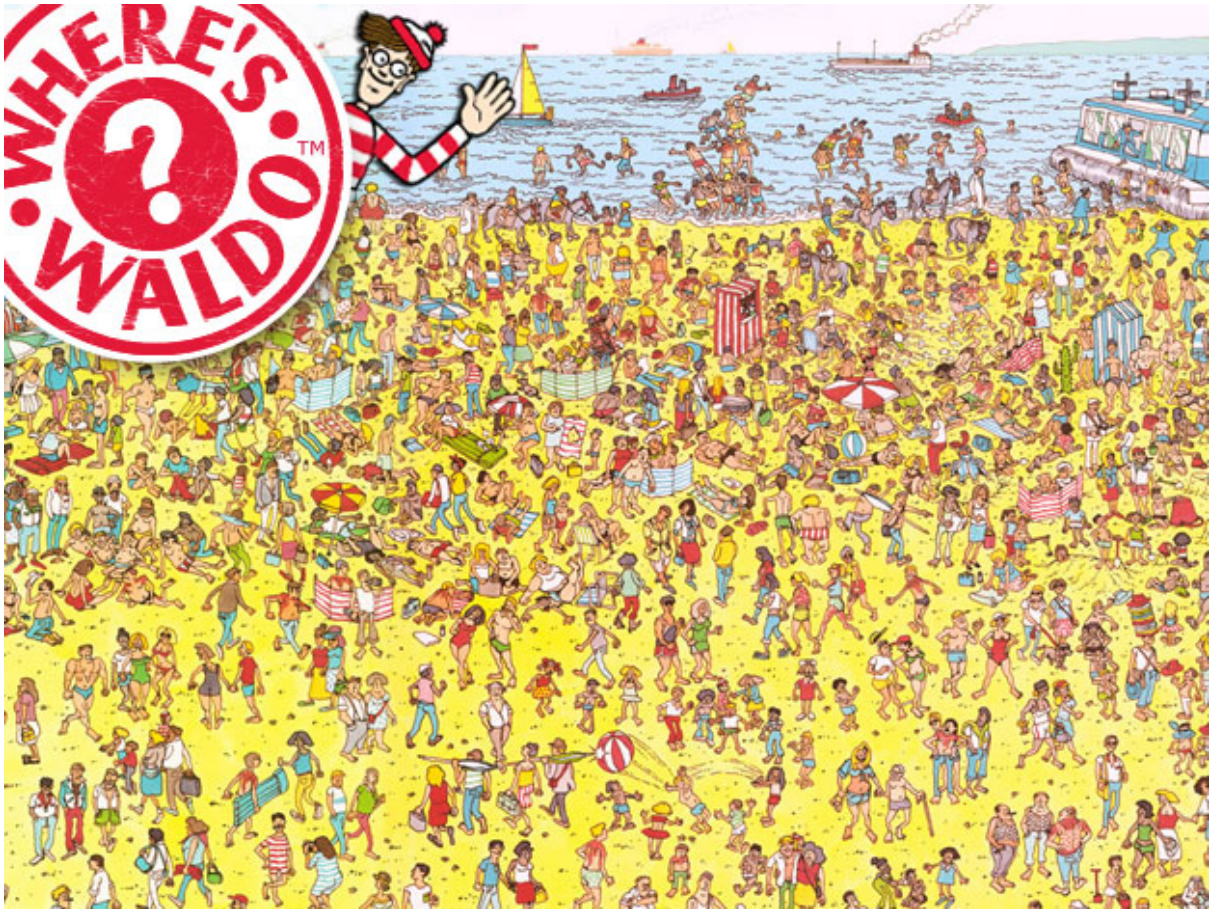

image I

- How can we use what we just learned to find Waldo?

Finding Waldo

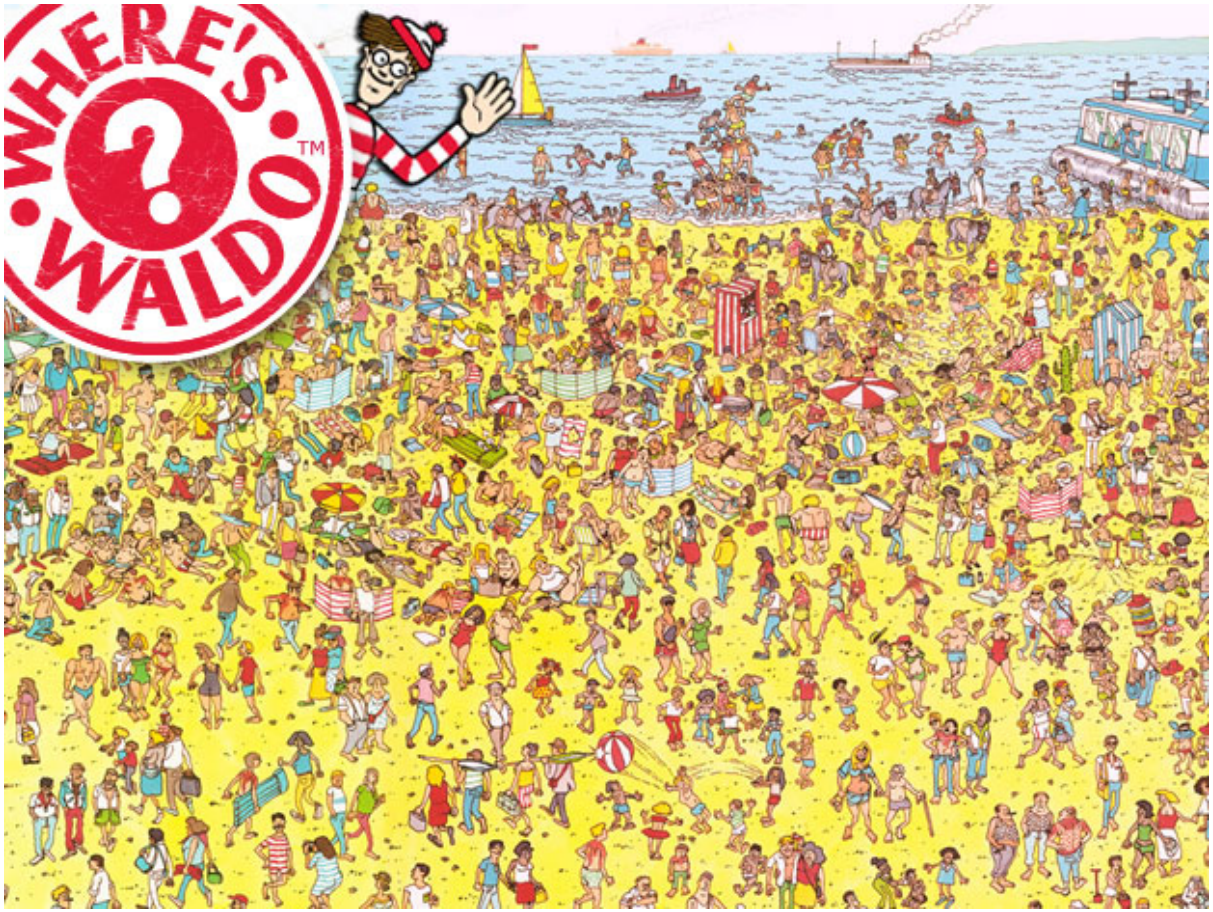

image I

filter F

- Is correlation a good choice?

A Slight Detour: Correlation in Matrix Form

- Remember correlation:

$$G(i,j) = \sum_{u=-k}^{k} \sum_{v=-k}^{k} F(u,v) \cdot I(i+u, j+v)$$

- Can we write that in a more compact form (with vectors)?

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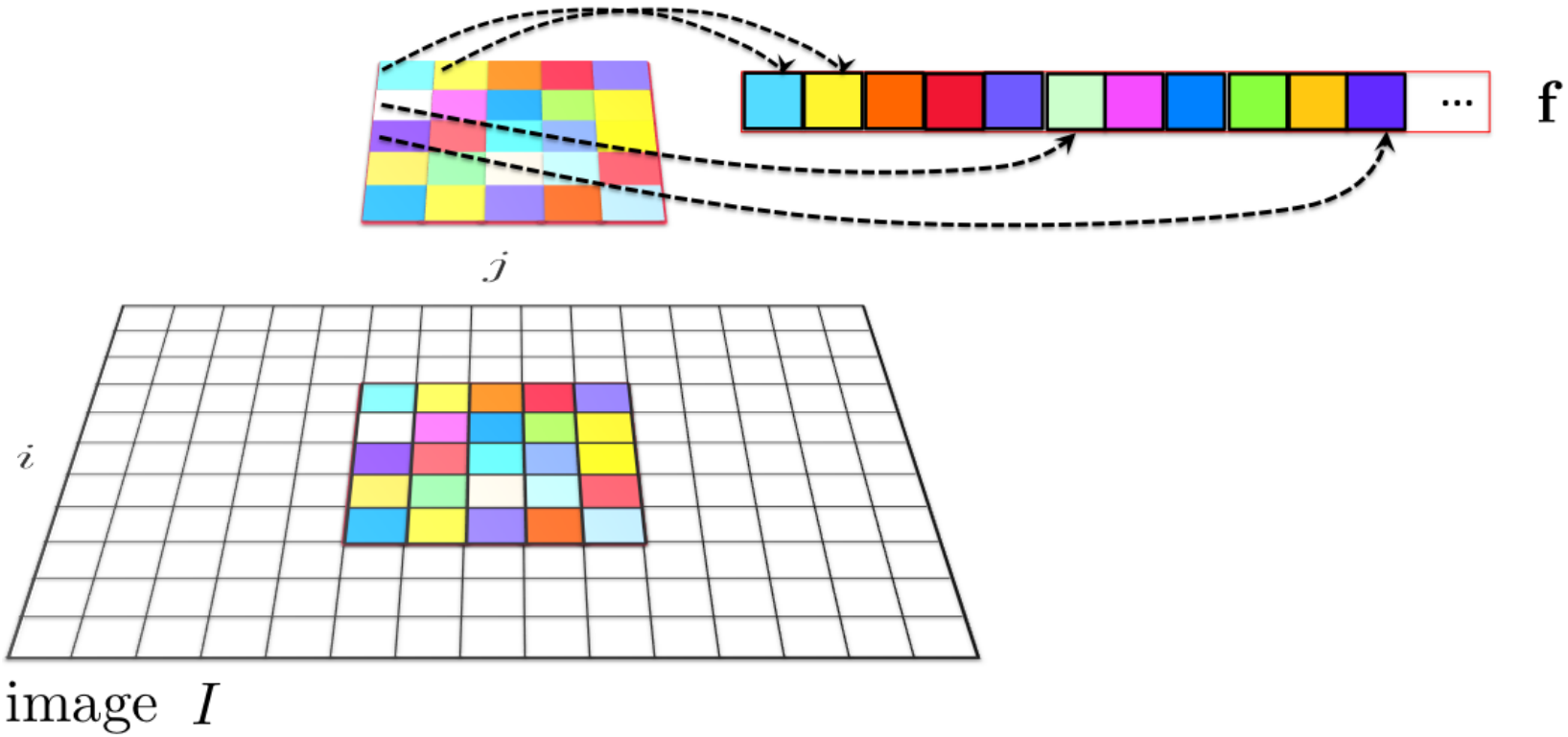

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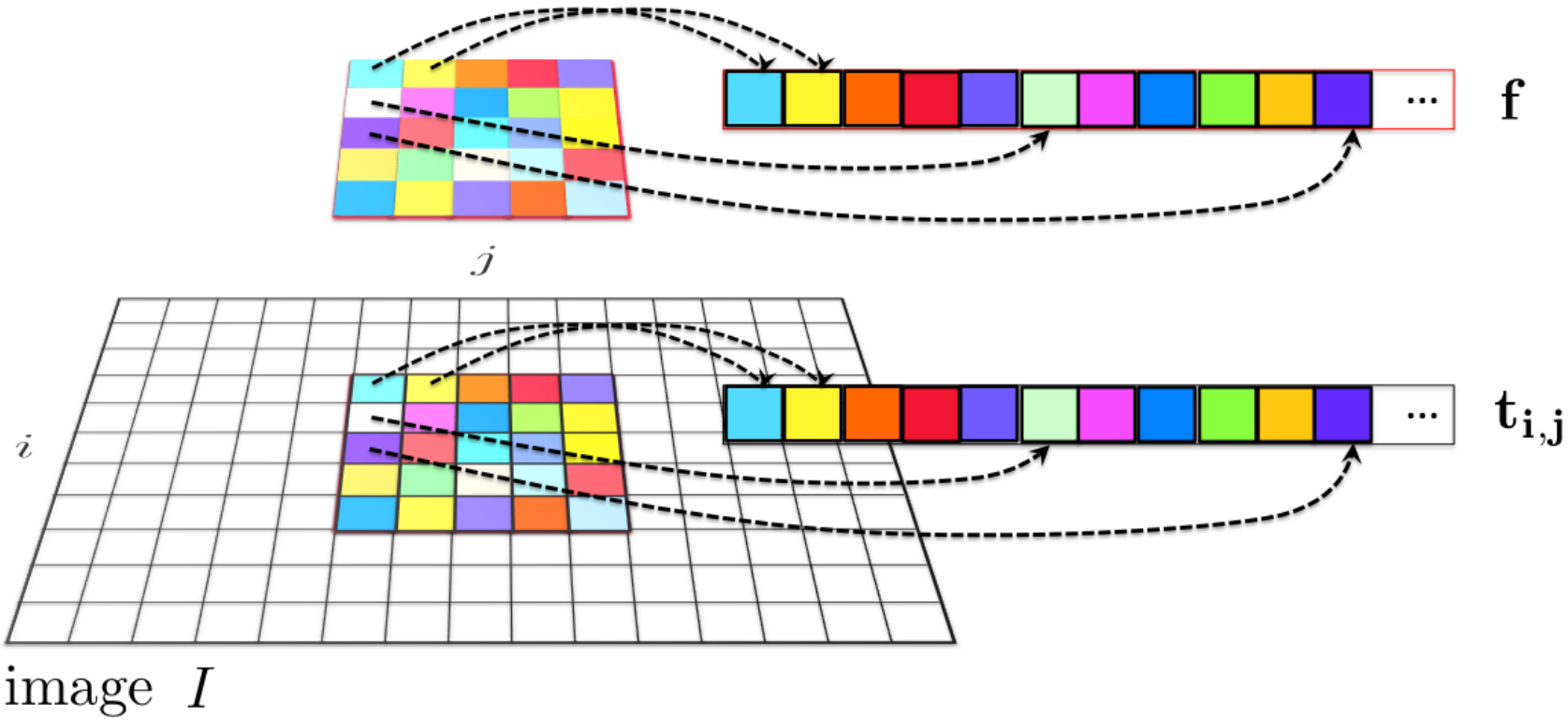

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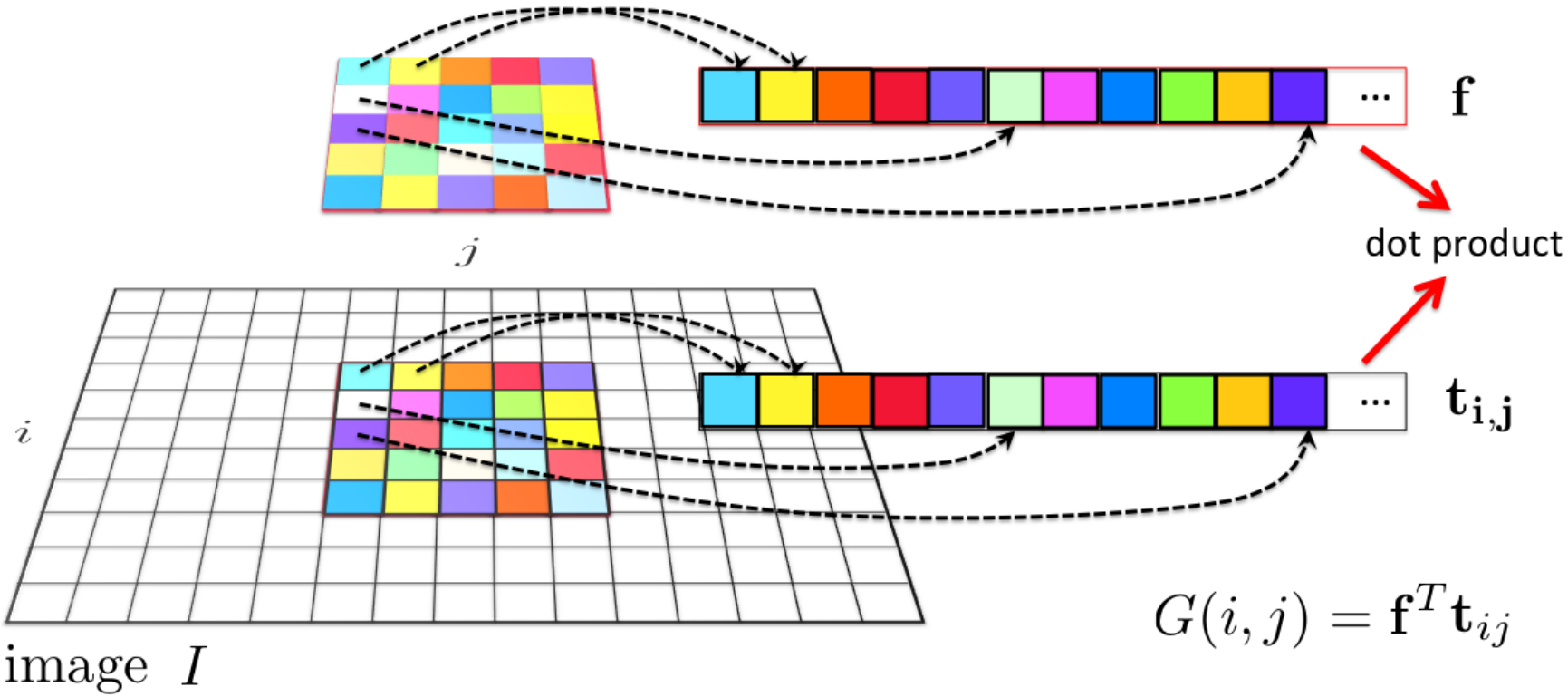

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- Can we write that in a more compact form (with vectors)?
- Define $\mathbf{f} = F(:)$, $T_{ij} = I(i-k : i+k, j-k : j+k)$, and $\mathbf{t}_{ij} = T_{ij}(:)$

$$G(i,j) = \mathbf{f} \cdot \mathbf{t}_{ij}$$

where $\cdot$ is a dot product

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- **Homework:** Can we write full correlation $G = F \otimes I$ in matrix form?

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- **Normalized cross-correlation:**

$$G(i,j) = \frac{\mathbf{f}^T \mathbf{t}_{ij}}{||\mathbf{f}|| ||\mathbf{t_{ij}}||}$$

Back to Waldo

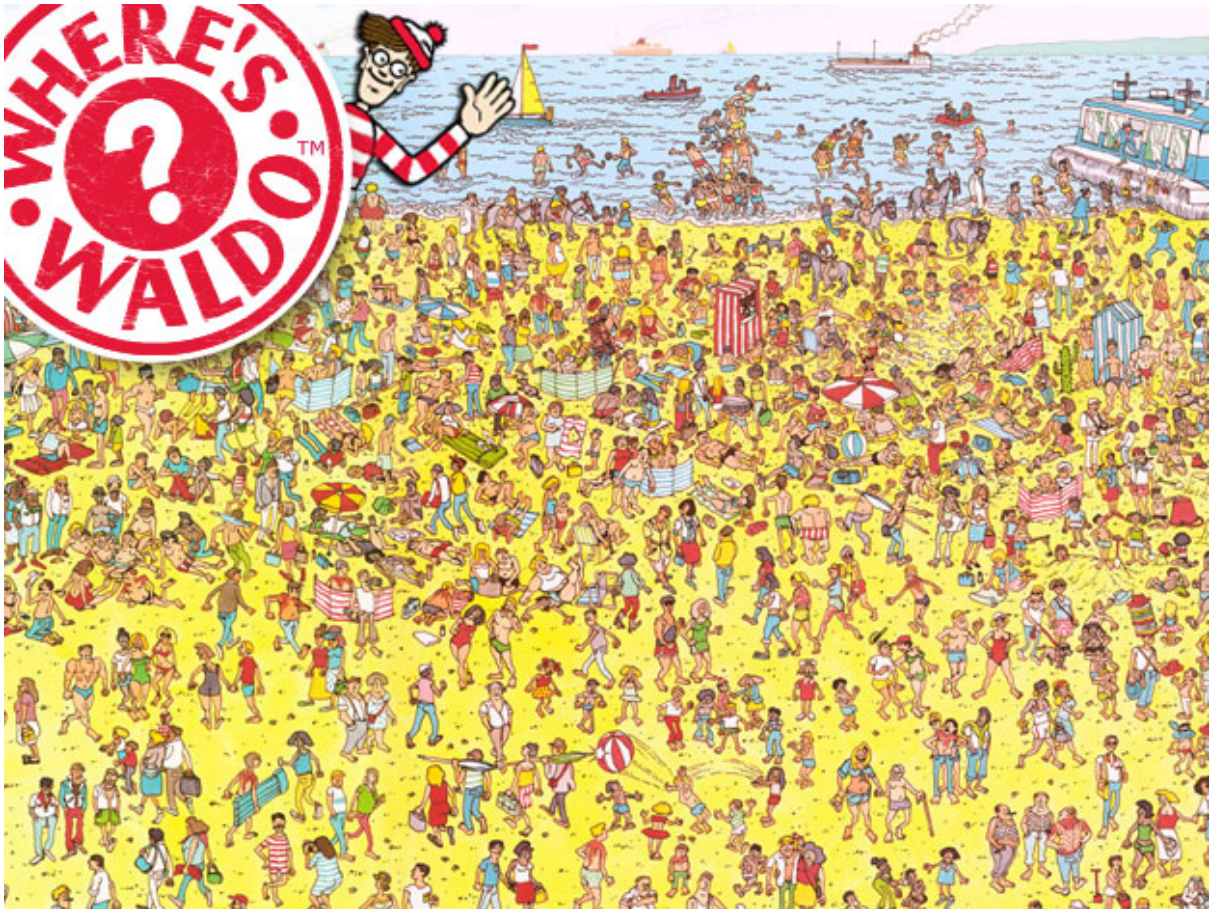
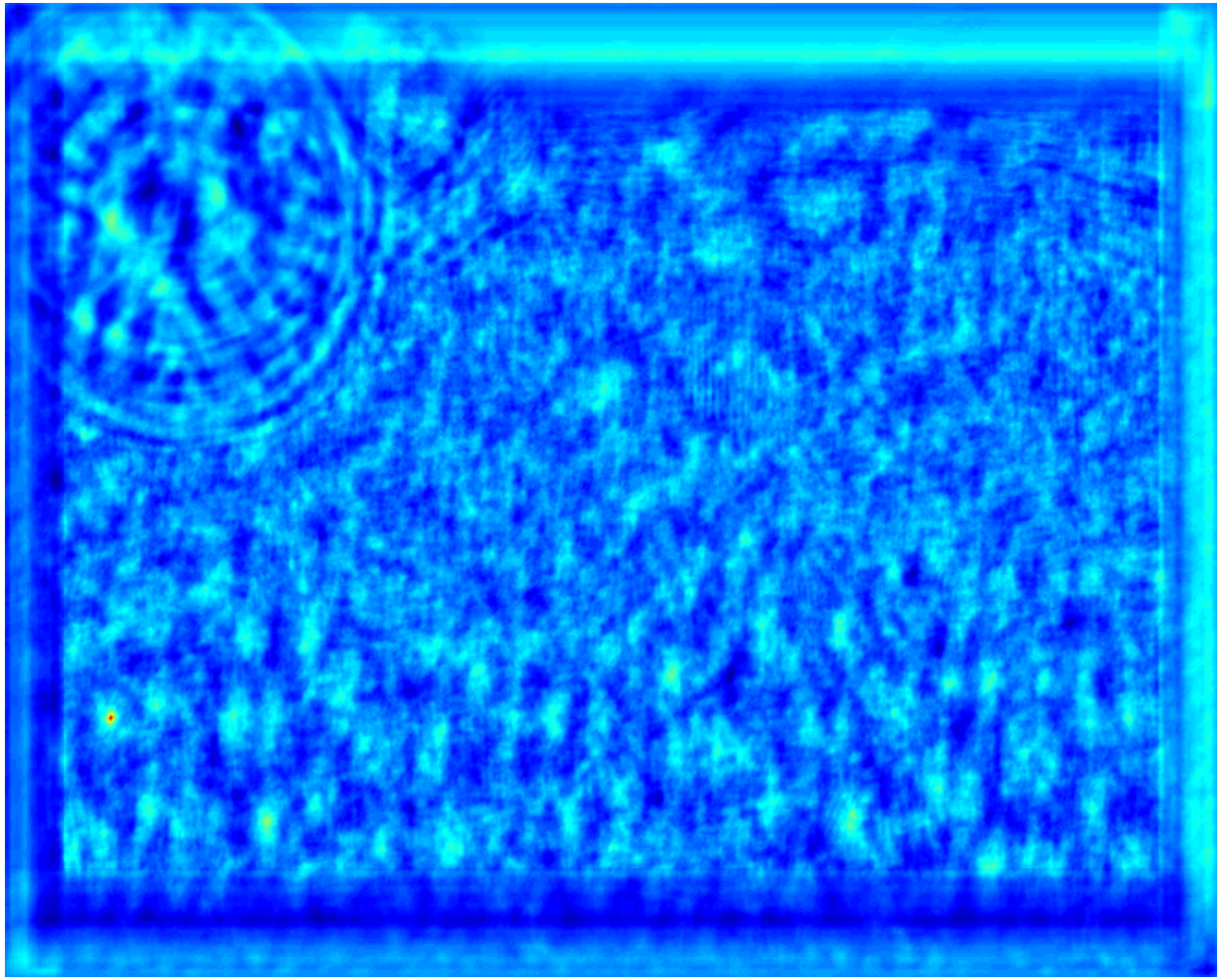


image I

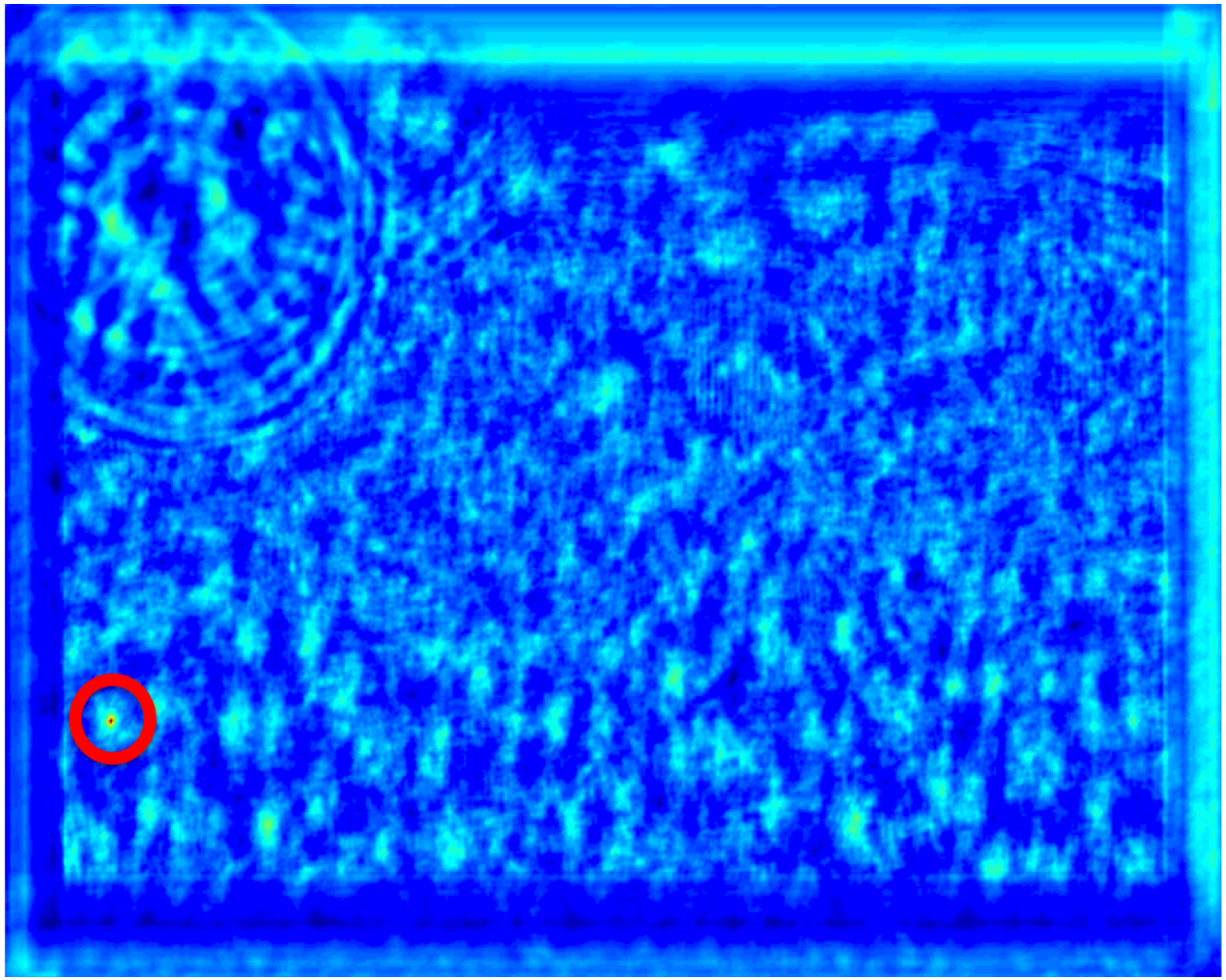
filter F

Back to Waldo



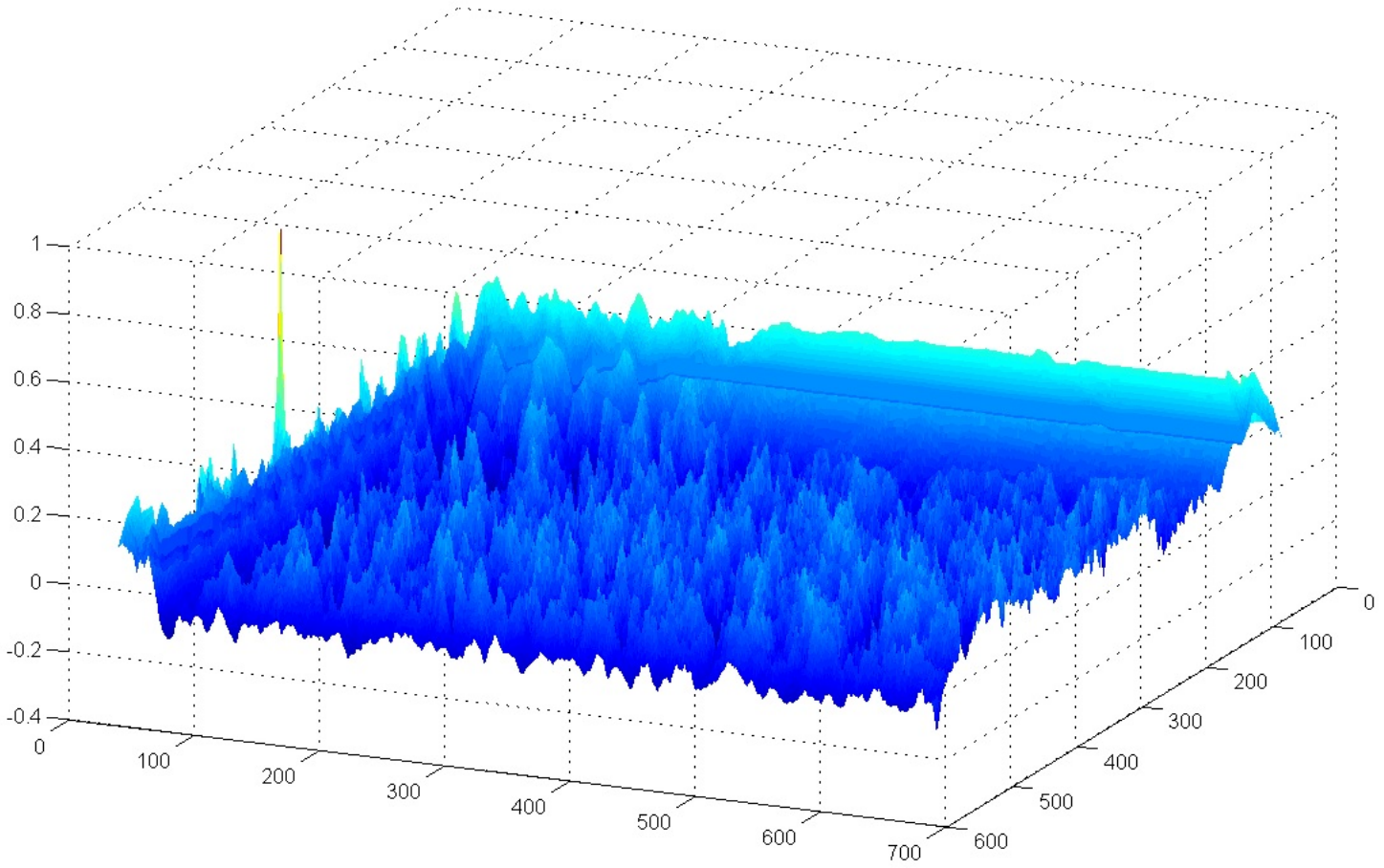
- Result of normalized cross-correlation

Back to Waldo



- Find the highest peak

Back to Waldo



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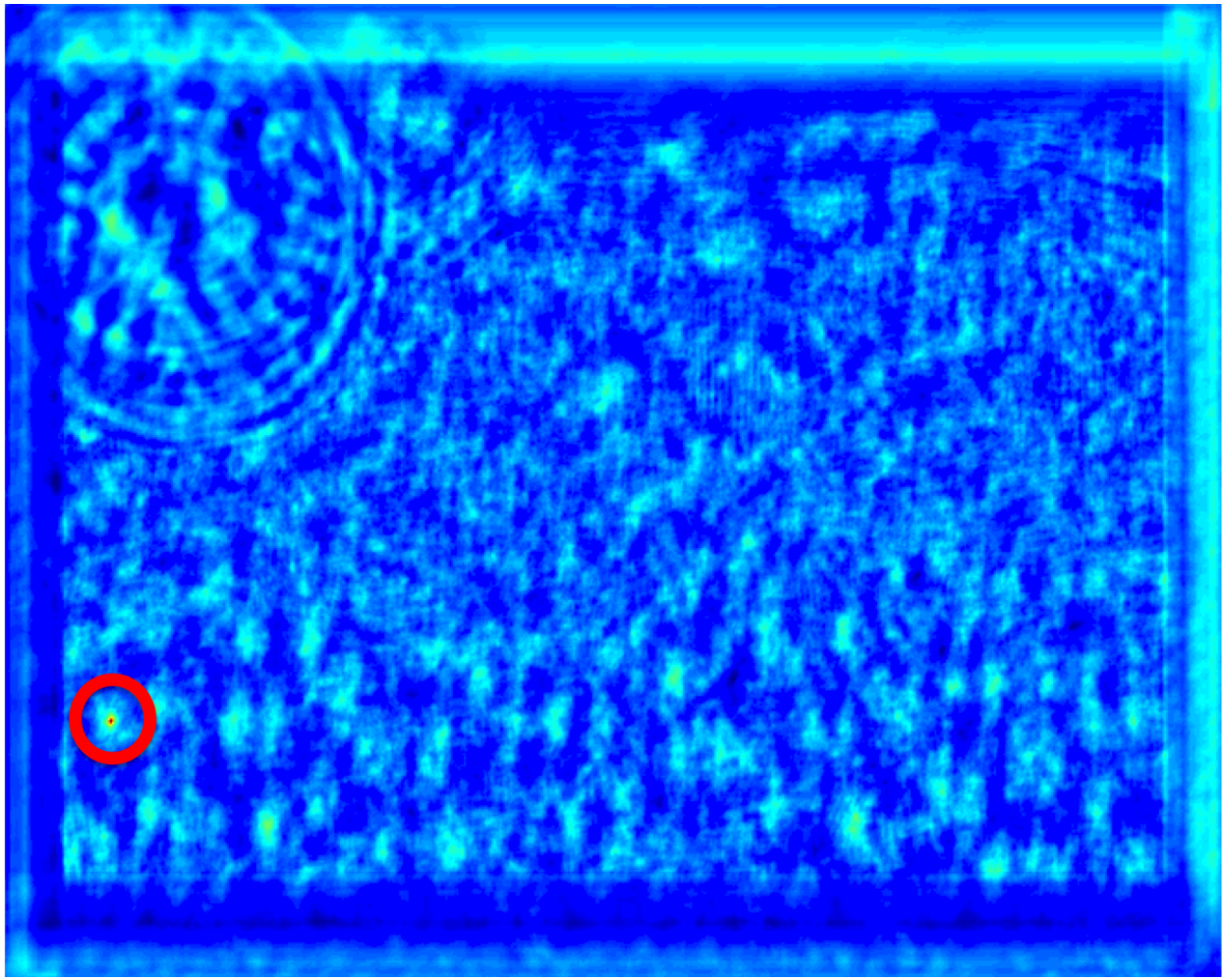

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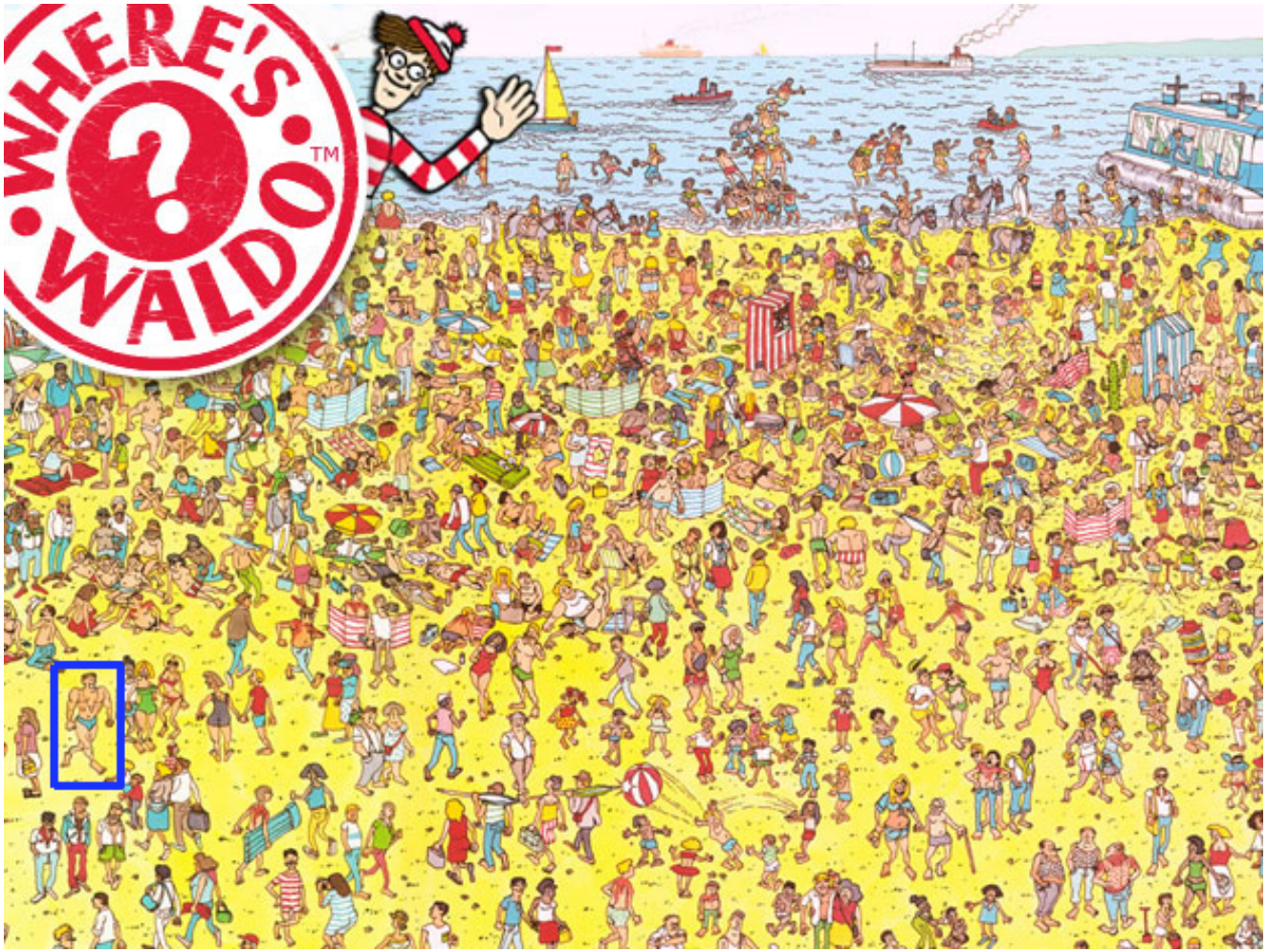
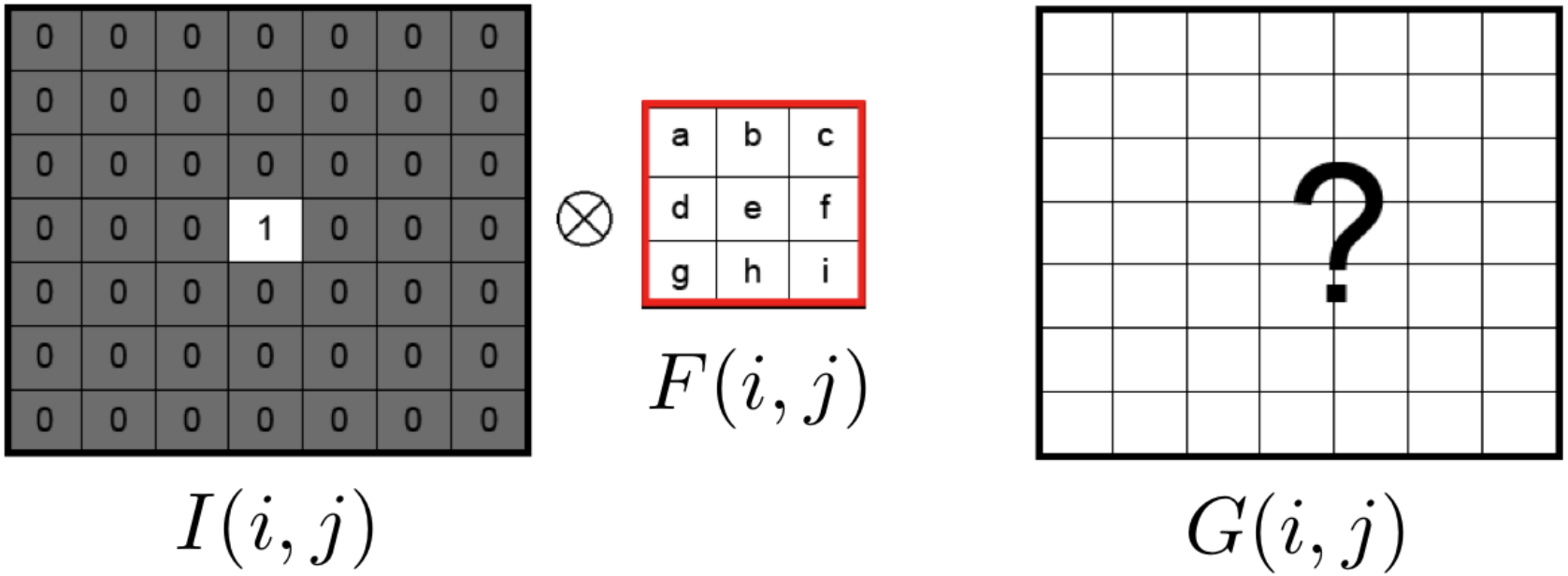


And put a bounding box (rectangle the size of the template) at the point!

Example of Correlation

- What is the result of filtering the impulse signal (image) I with the arbitrary filter F?



[Source: K. Grauman]

Convolution

- **Convolution** operator

$$G(i,j) = \sum_{u=-k}^{k} \sum_{v=-k}^{k} F(u,v) \cdot I(i-u, j-v)$$

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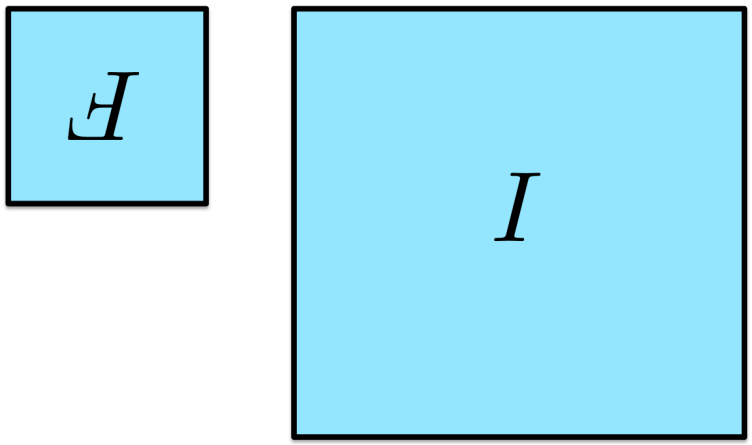
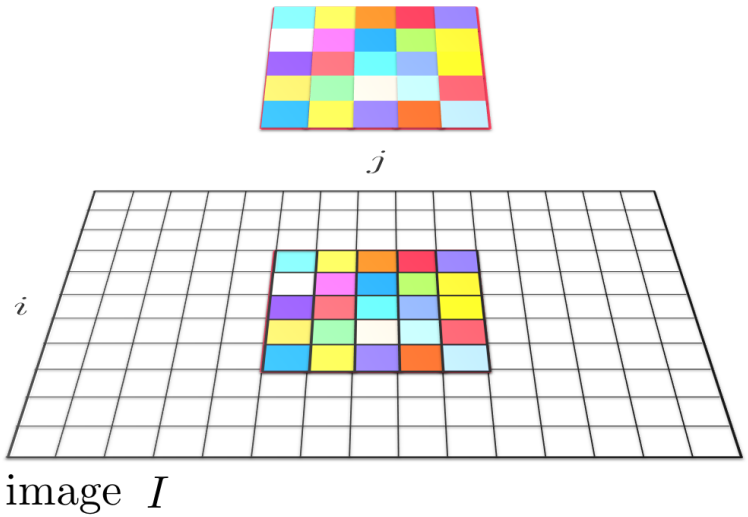
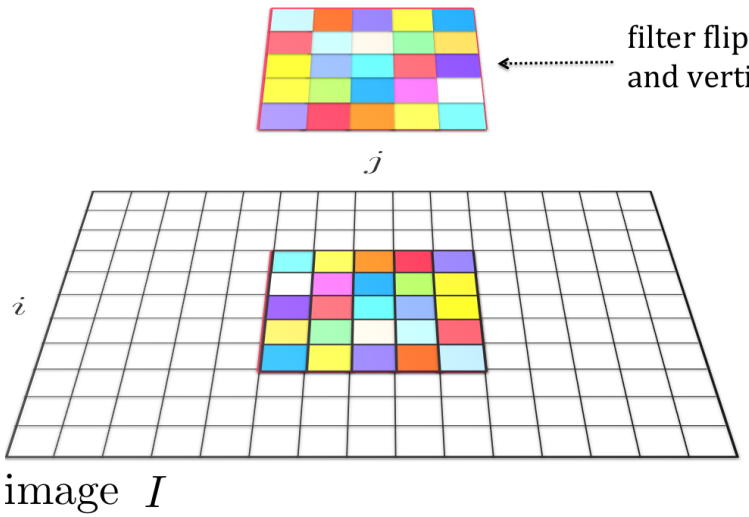
- **Equivalent** to flipping the filter in both dimensions (bottom to top, right to left) and apply correlation.



Correlation vs Convolution



$=$



Correlation

Convolution

Correlation vs Convolution

- For a Gaussian or box filter, how will the outputs $F * I$ and $F \otimes I$ differ?

Correlation vs Convolution

- For a Gaussian or box filter, how will the outputs $F * I$ and $F \otimes I$ differ?
- How will the outputs differ for:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

"Optical" Convolution

- Camera Shake

Figure: Fergus, et al., SIGGRAPH 2006

- Blur in out-of-focus regions of an image.

Figure: Bokeh: `http://lullaby.homepage.dk/diy-camera/bokeh.html` Click for more info

[Source: N. Snavely]

Properties of Convolution

$$\text{Commutative} : f * g = g * f$$

$$\text{Associative} : f * (g * h) = (f * g) * h$$

$$\text{Distributive} : f * (g + h) = f * g + f * h$$

$$\text{Assoc. with scalar multiplier} : \lambda \cdot (f * g) = (\lambda \cdot f) * g$$

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- The Fourier transform of two convolved images is the product of their individual Fourier transforms:

$$\mathcal{F}(f * g) = \mathcal{F}(f) \cdot \mathcal{F}(g)$$

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- **Homework:** Why is this good news?
- **Hint:** Think of complexity of convolution and Fourier Transform
- What if we wanted to undo the result of convolution?

Separable Filters: Speed-up Trick!

- The process of performing a convolution requires K^2 operations per pixel, where K is the size (width or height) of the convolution filter.

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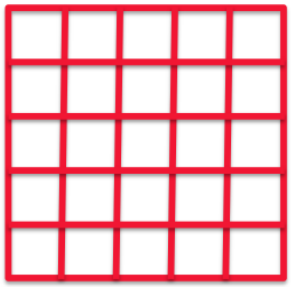
- And it is the outer product of two filters:

$$\mathbf{F} = \mathbf{v}\,\mathbf{h}^T$$

[Source: R. Urtasun]

How it Works



filter

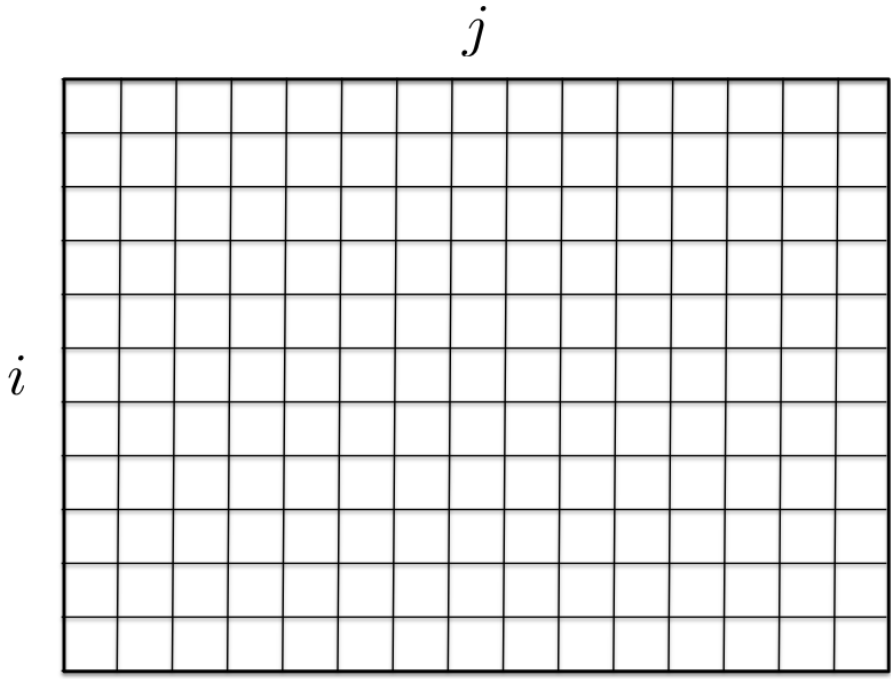


image I

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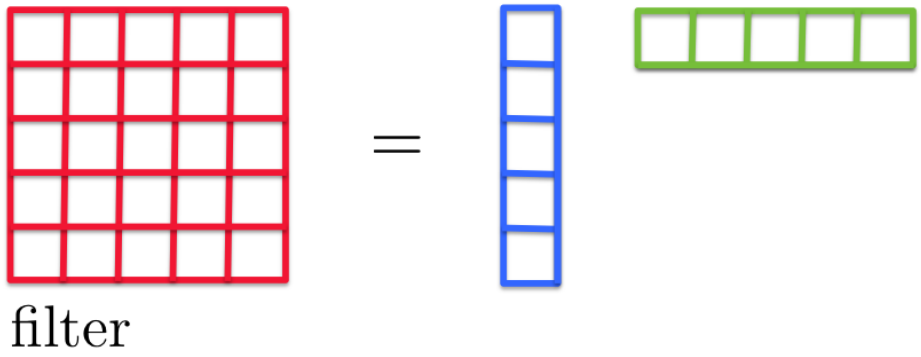

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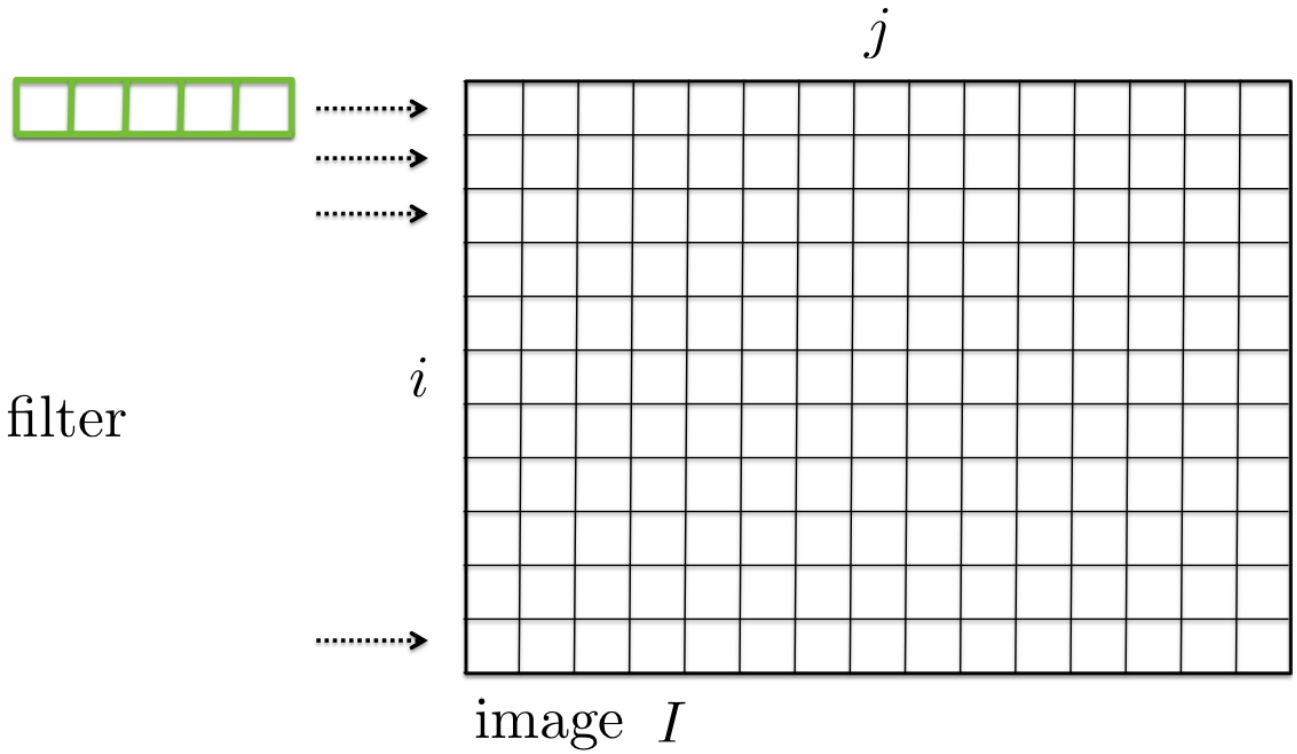

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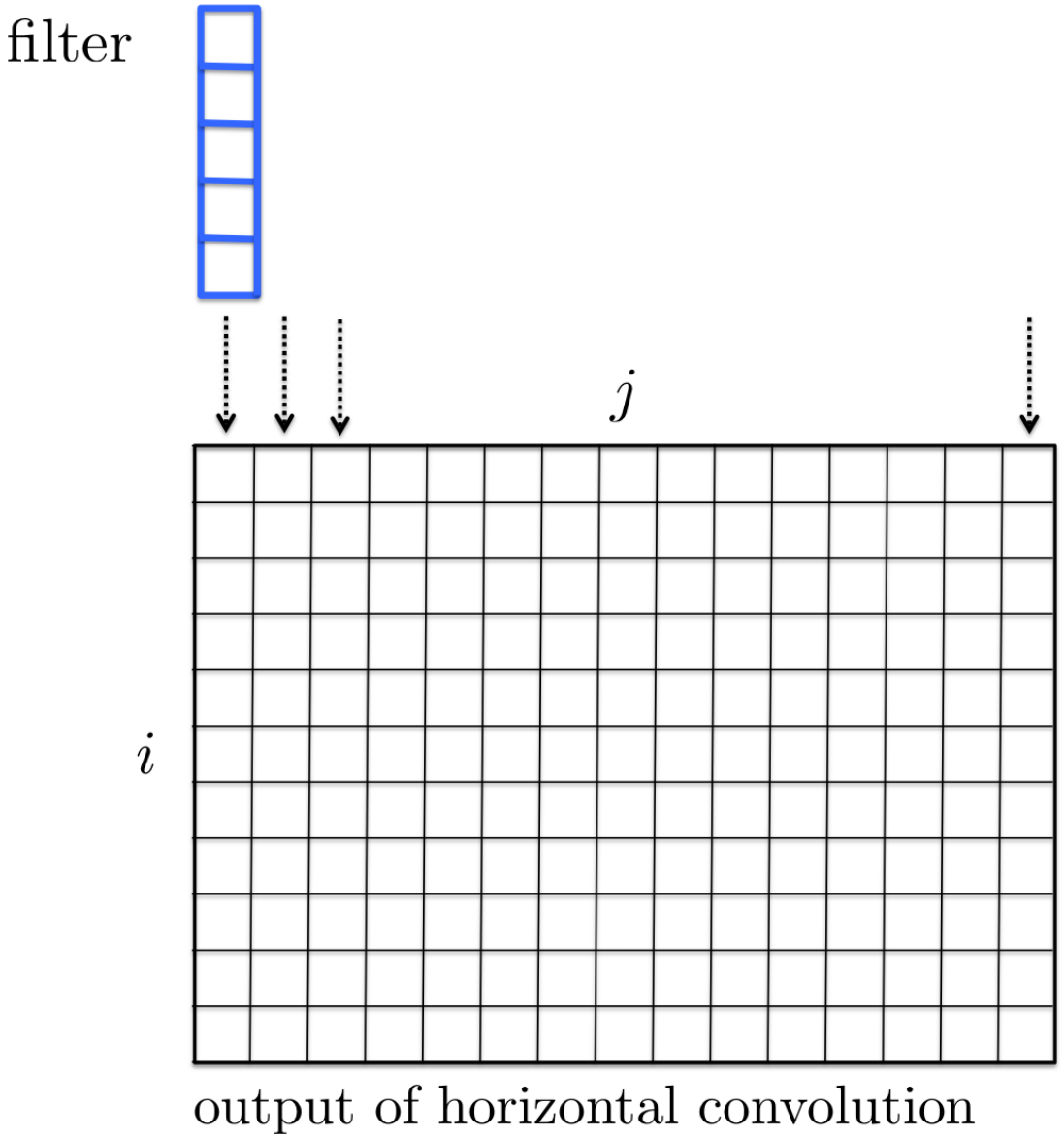
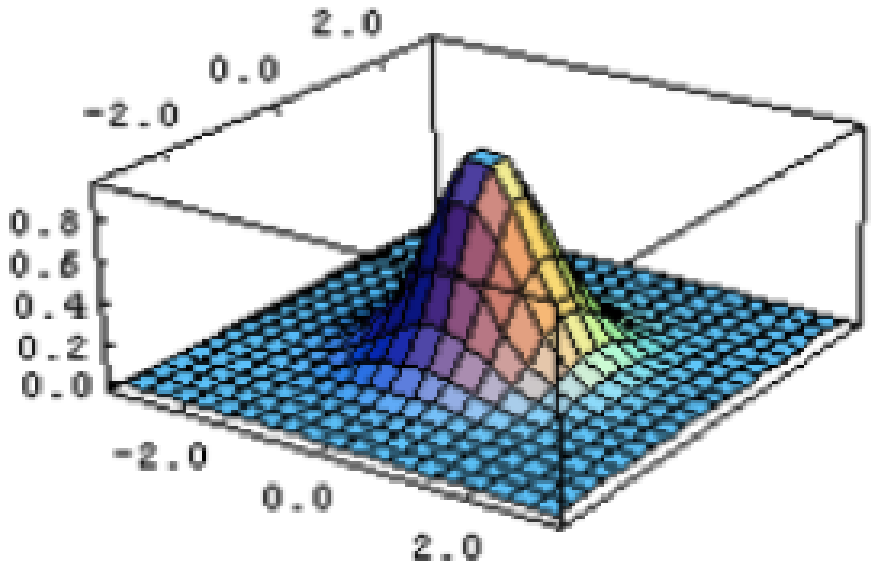


Separable Filters: Gaussian filters

- One famous separable filter we already know:

$$\text{Gaussian} \; : \; f(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{\sigma^2}}$$



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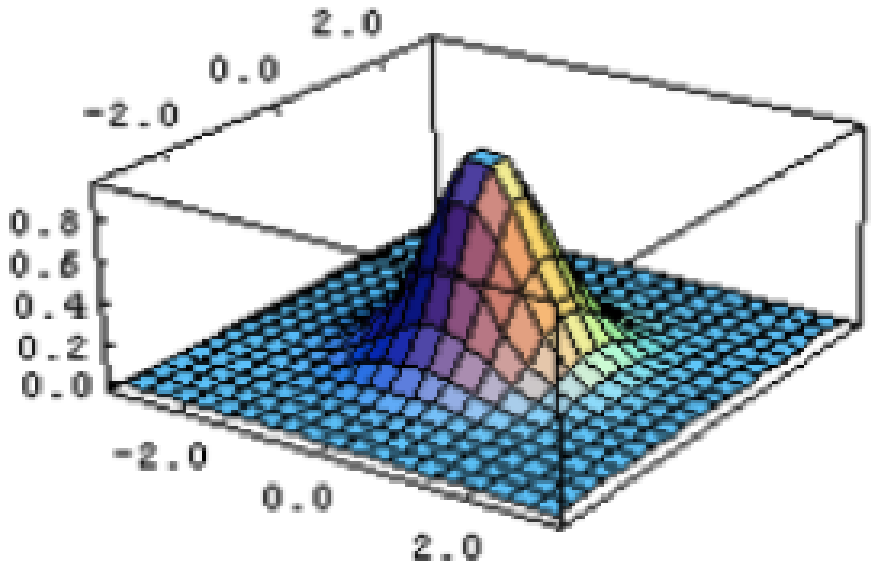
$$= \left(\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{\sigma^2}}\right) \cdot \left(\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{\sigma^2}}\right)$$



Let's play a game...

Is this separable? If yes, what's the separable version?

$$\frac{1}{K^2}\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & 1 & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix}$$

[Source: R. Urtasun]

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What does this filter do?

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[Source: R. Urtasun]

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$$\frac{1}{4}\begin{bmatrix}1 & 2 & 1\end{bmatrix}$$

What does this filter do?

[Source: R. Urtasun]

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Is this separable? If yes, what's the separable version?

$$\frac{1}{8}\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

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[Source: R. Urtasun]

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- Python: NP.LINALG.SVD
- $\sqrt{\sigma_1}\mathbf{u}_1$ and $\sqrt{\sigma_1}\mathbf{v}_1^T$ are the vertical and horizontal filter.

[Source: R. Urtasun]

Summary – Stuff You Should Know

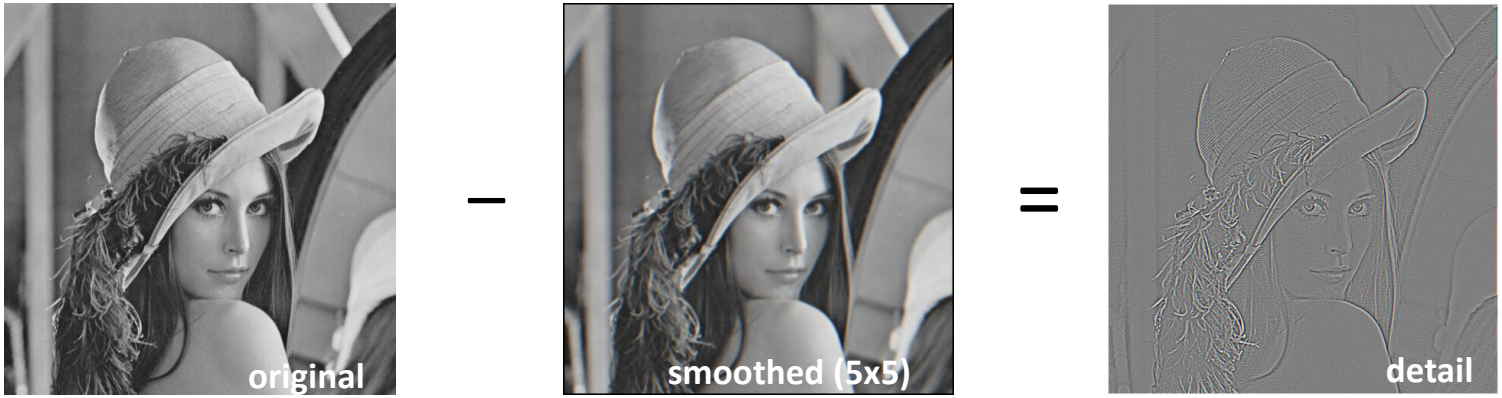
- **Correlation**: Slide a filter across image and compare (via dot product)
- **Convolution**: Flip the filter to the right and down and do correlation
- **Smooth** image with a Gaussian kernel: bigger σ means more blurring
- **Some** filters (like Gaussian) are **separable**: you can filter faster. First apply 1D convolution to each row, followed by another 1D conv. to each column

OpenCV:

- filter2D (or sepFilter2D): can do both correlation and convolution
- GaussianBlur: create a Gaussian kernel
- medianBlur, medianBlur, bilateralFilter

Edges

- What does blurring take away?



[Source: S. Lazebnik]

Next time:

Edge Detection