

1. UpSampling

a. Original: 172 * 170, UpSampled: 685 * 677



No we cannot perform this action in a 2D filter, since we are applying the same vector v twice and the result matrix of $v.T * v$ will just be an identity matrix.

b. For this question I created a filter below and ran convolution with the image,

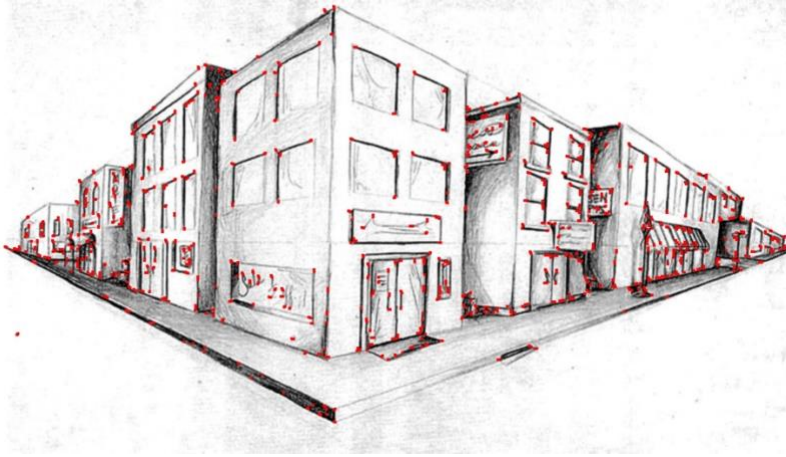
```
[ [0.0625 0.125 0.1875 0.25 0.1875 0.125 0.0625]
  [0.125 0.25 0.375 0.5 0.375 0.25 0.125 ]
  [0.1875 0.375 0.5625 0.75 0.5625 0.375 0.1875]
  [0.25 0.5 0.75 1. 0.75 0.5 0.25 ]
  [0.1875 0.375 0.5625 0.75 0.5625 0.375 0.1875]
  [0.125 0.25 0.375 0.5 0.375 0.25 0.125 ]
  [0.0625 0.125 0.1875 0.25 0.1875 0.125 0.0625]]
```

We obtained the image on the middle, it is a little bit more smoothened than the image from part a, rightmost image is the enhanced difference between these two images

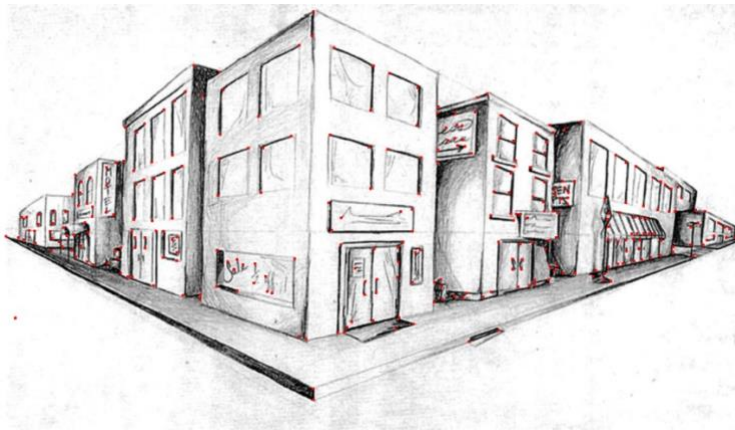


2. Interest Point Detection

a. Result for Harris: threshold = $1e+12$



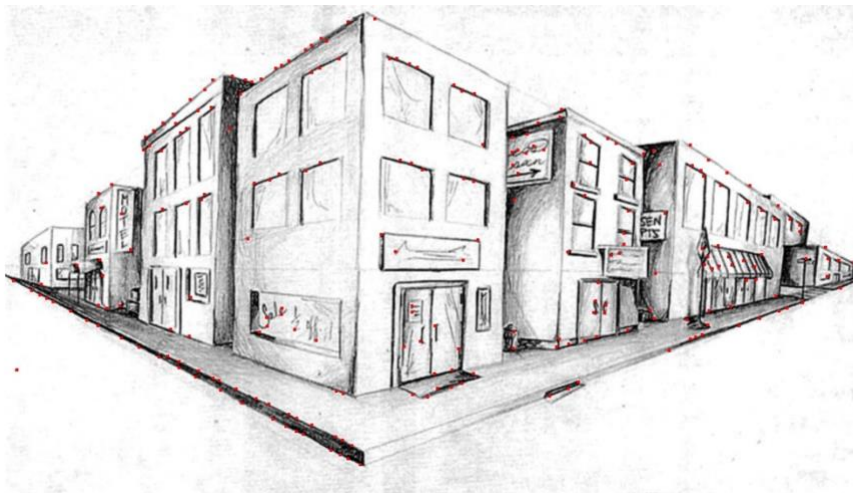
Result for Brown: threshold = 10000



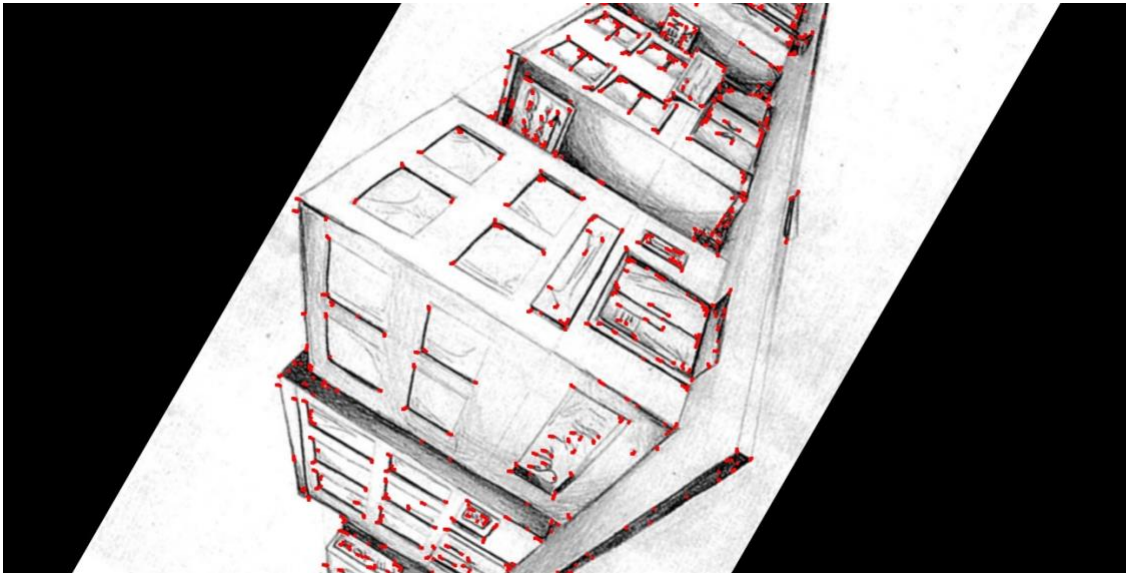
Note: The red dots in both output images are big only for better presentation, each red dot is a 7×7 matrix of red pixels

With reasonable thresholds both metrics provide similar result, Harris's metric has a much higher threshold than Brown.

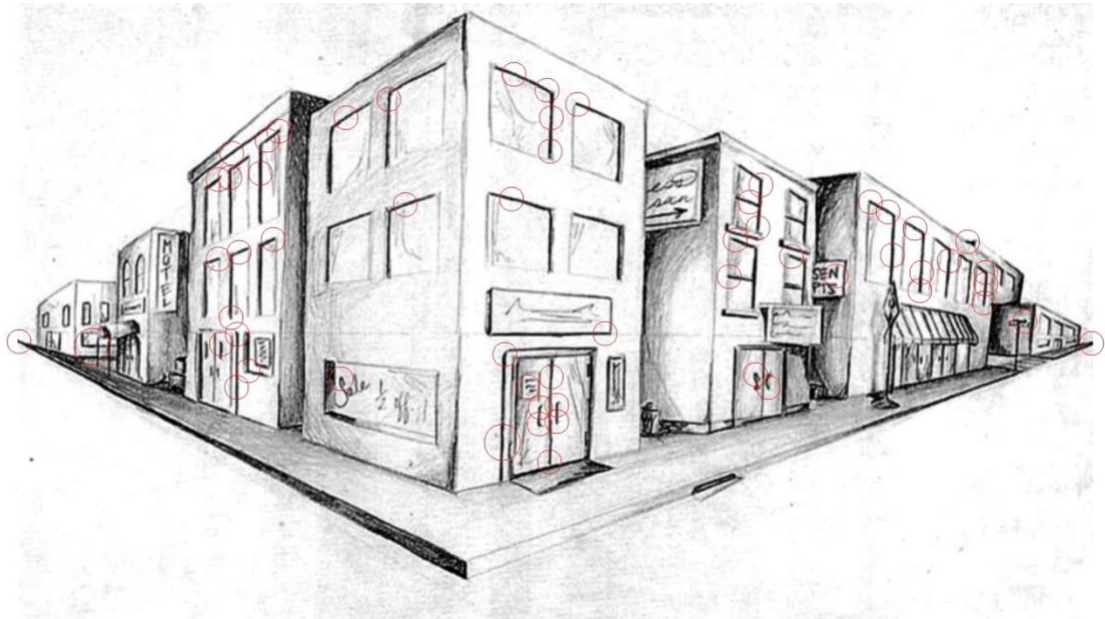
And yes, without using determinant and trace we can still implement Harris detector, we can simply calculate λ_1 and λ_2 and set a threshold, points with λ value greater than threshold are potential corners, threshold for result below is $1e+6$:



- b. Yes the corners do rotate by the exact same degree, since Harris corner detector is rotation invariant.



- c. Result for Laplacian-of-Gaussian blob detection



- d. PCA-SIFT: Principle Component Analysis Scale Invariant Feature Transformation

The main idea for PCA-SIFT: instead of performing a smoothed weight histogram, this algorithm performs a principle component analysis to the normalized gradient patch and thus reduces the dimensionality of the gradient image, ideally in this way we can remove some least important dimension and thus improve the accuracy and speed

Steps:

- i. Pre-compute an eigenspace to express the gradient image of the local patch
- ii. Given a patch, compute local image gradient
- iii. Project the gradient image vector using the eigenspace to derive a compact feature vector

3. Laplacian of Gaussian

$$a. \begin{bmatrix} -\frac{1}{\pi\sigma^2} \left(1 - \frac{(x-1)^2 + (y-1)^2}{2\sigma^2}\right) \exp\left(-\frac{(x-1)^2 + (y-1)^2}{2\sigma^2}\right) & -\frac{1}{\pi\sigma^2} \left(1 - \frac{x^2 + (y-1)^2}{2\sigma^2}\right) \exp\left(-\frac{x^2 + (y-1)^2}{2\sigma^2}\right) & -\frac{1}{\pi\sigma^2} \left(1 - \frac{(x+1)^2 + (y-1)^2}{2\sigma^2}\right) \exp\left(-\frac{(x+1)^2 + (y-1)^2}{2\sigma^2}\right) \\ -\frac{1}{\pi\sigma^2} \left(1 - \frac{(x-1)^2 + y^2}{2\sigma^2}\right) \exp\left(-\frac{(x-1)^2 + y^2}{2\sigma^2}\right) & -\frac{1}{\pi\sigma^2} \left(1 - \frac{x^2 + y^2}{2\sigma^2}\right) \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right) & -\frac{1}{\pi\sigma^2} \left(1 - \frac{(x+1)^2 + y^2}{2\sigma^2}\right) \exp\left(-\frac{(x+1)^2 + y^2}{2\sigma^2}\right) \\ -\frac{1}{\pi\sigma^2} \left(1 - \frac{(x-1)^2 + (y+1)^2}{2\sigma^2}\right) \exp\left(-\frac{(x-1)^2 + (y+1)^2}{2\sigma^2}\right) & -\frac{1}{\pi\sigma^2} \left(1 - \frac{x^2 + (y+1)^2}{2\sigma^2}\right) \exp\left(-\frac{x^2 + (y+1)^2}{2\sigma^2}\right) & -\frac{1}{\pi\sigma^2} \left(1 - \frac{(x+1)^2 + (y+1)^2}{2\sigma^2}\right) \exp\left(-\frac{(x+1)^2 + (y+1)^2}{2\sigma^2}\right) \end{bmatrix}$$

This filter does not appear to be a separable filter, since each row of it are independent and thus it has a rank of 3, therefore it's non-separable

$$b. \sigma^2 \nabla^2 G(x, y, \sigma) = \frac{\partial G}{\partial \sigma} \approx \frac{G(x, y, k\sigma) - G(x, y, \sigma)}{k\sigma - \sigma}$$

Therefore $G(x, y, k\sigma) - G(x, y, \sigma) \approx (k\sigma - \sigma) \sigma^2 \nabla^2 G(x, y, \sigma)$

$$\text{DoG} = G(x, y, k\sigma) - G(x, y, \sigma) \approx (k - 1) \sigma^2 \nabla^2 G(x, y, \sigma)$$

And since $k - 1$ is constant in this case so it will affect the scale but not the position of local extrema

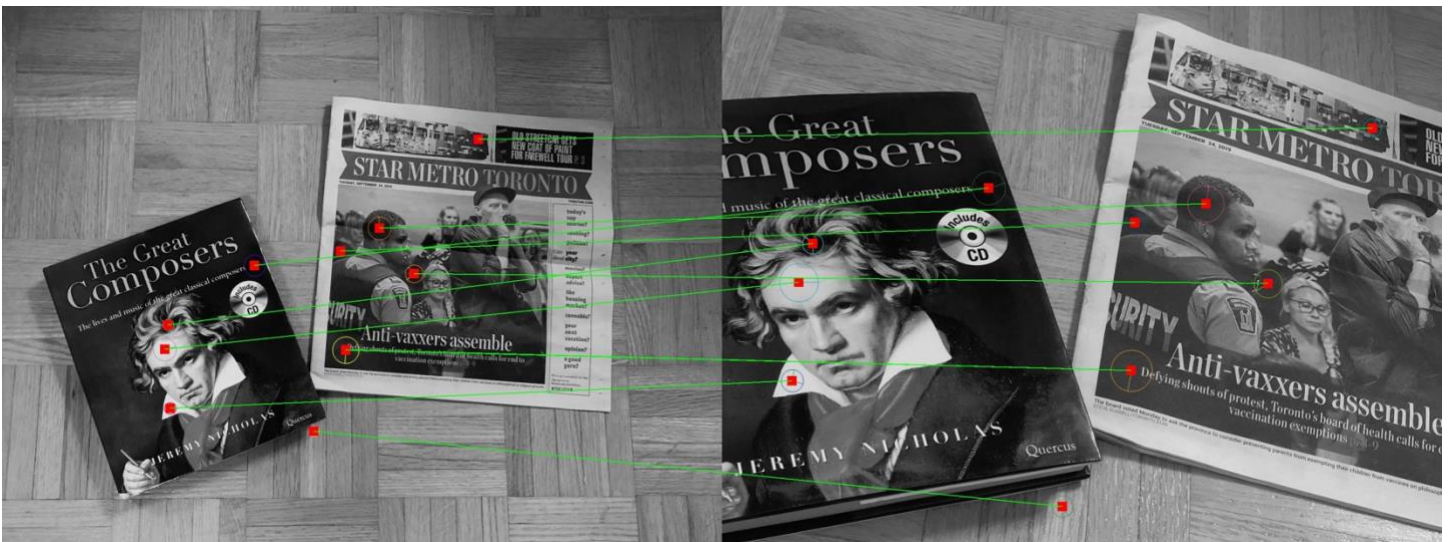
Therefore, the choice of σ_1 and σ_2 will affect the scale.

4. SIFT Matching

- Keypoints generated using opencv2 SIFT, with contrastThreshold = 0.04, edgeThreshold = 10, sigma = 5, setting sigma bigger can reduce some small scale keypoints



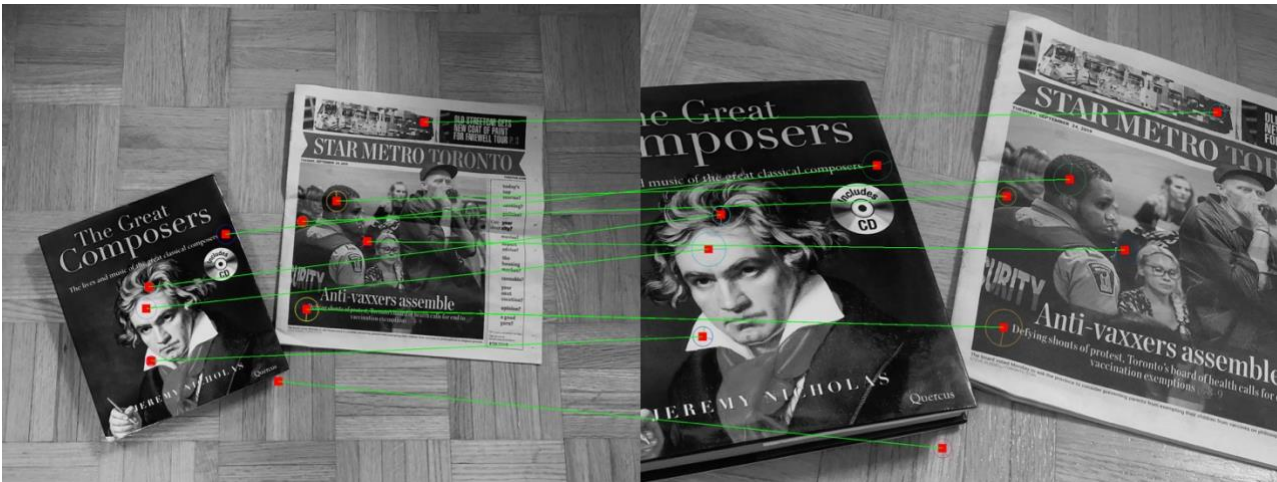
- Using L2norm distance to match the top 10 best matched keypoints in two images



c. L1norm and l3norm



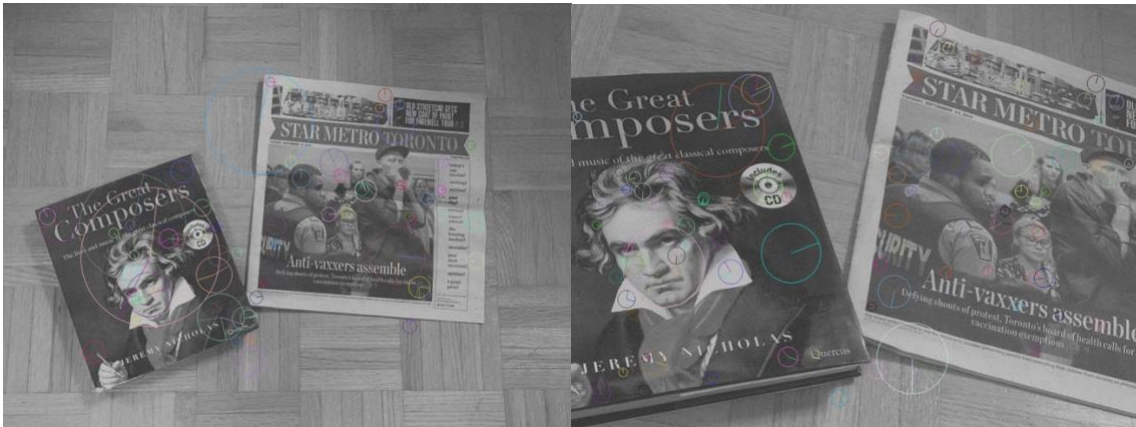
L1norm

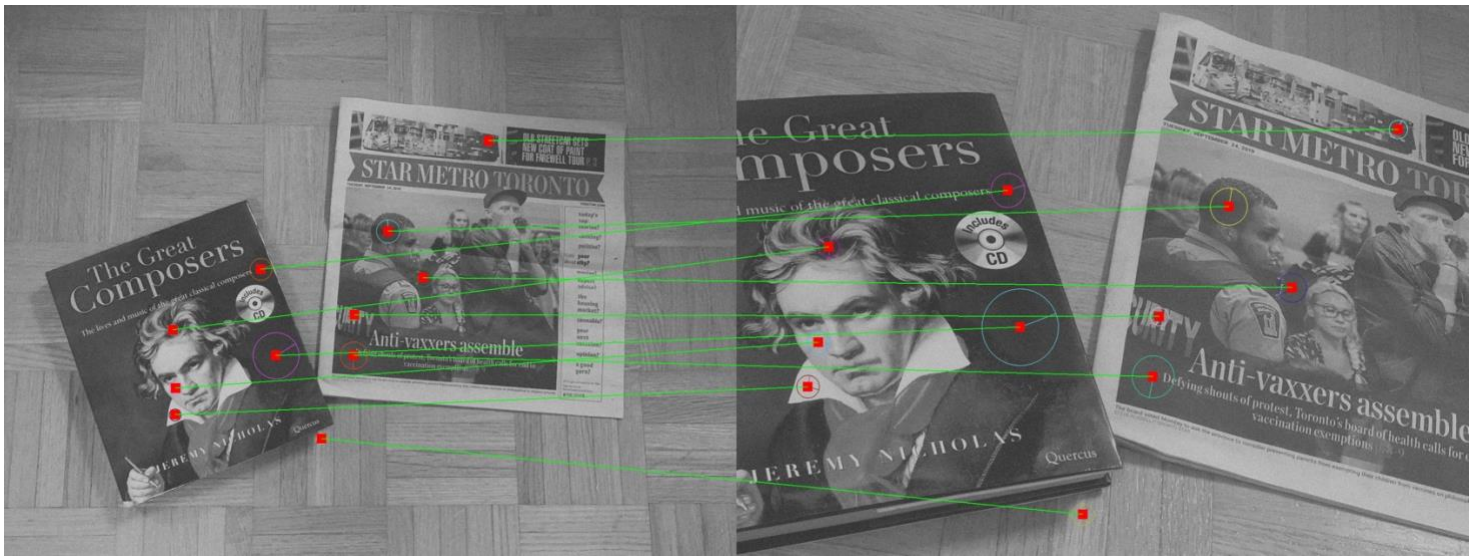


L3norm

Both L2norm and L3norm produced same result, L1norm produced slightly different result which matches some of the flat regions, therefore I think l2norm and l3norm produces better results.

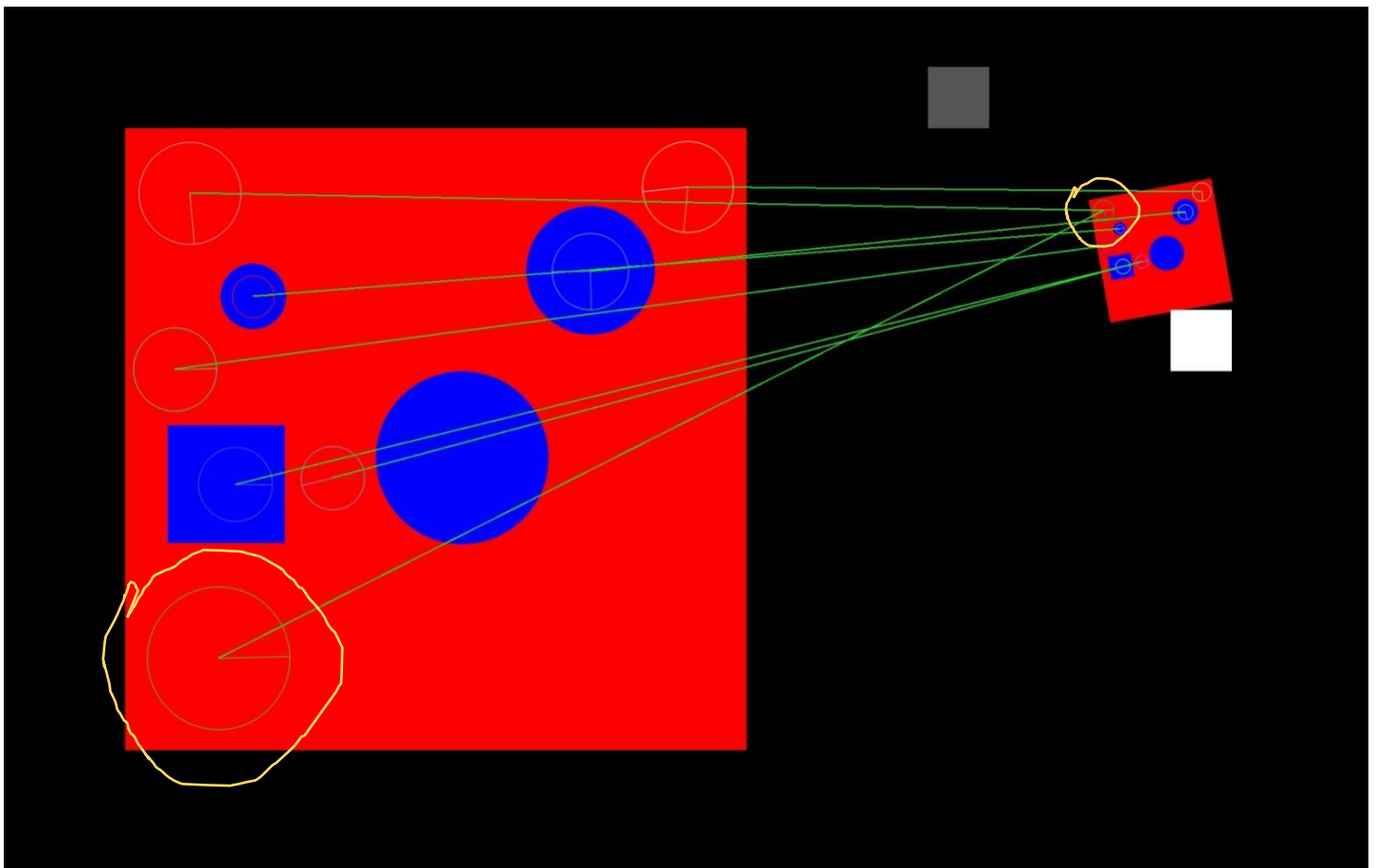
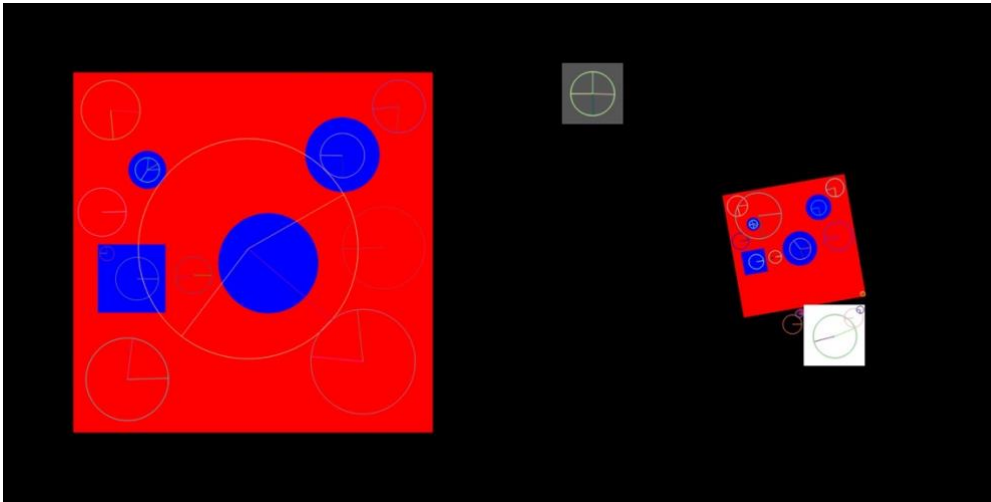
d. Noised images





SIFT produced pretty good result since SIFT is noise resistant

e. To test colored keypoints match



The overall result is okay except one keypoint is miss matched